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This study compares American and Korean curriculum standards, textbooks, and geometry items to help explain the consistent differences that have been found between the achievement levels of U.S. and Korean students in the Program for International Student Assessment (PISA) mathematics. Since PISA measures the mathematical proficiency of 15-year-old students, this study focuses primarily on 8th-grade curriculum standards and textbooks. The findings provide valuable information for curriculum designers and textbook editors to help improve the quality of geometry curricula and textbooks. Teachers can also use these findings to assess the proficiency levels of items in the textbooks they use, so that they can develop lesson plans appropriate to the ability of their students and select relevant problems for their classes.

Keywords: PISA, geometry, mathematics textbook, item difficulty, mathematics curriculum

INTRODUCTION

International comparison studies using international achievement tests, such as Programme for International Student Assessment (PISA) and Trends in International Mathematics and Science Study (TIMSS), provide insightful information on student mathematics achievement and educational practices across countries. PISA is unique to measure "mathematical literacy," which is defined as "an individual's capacity to identify and understand the role that mathematics plays in the world, to make well-founded judgments and to use and engage with mathematics in ways that meet the needs of that individual's life as a constructive, concerned and reflective citizen" (Organization for Economic Co-operation and Development [OECD], 2003, p.24).

Every three years since 2000, administered to a randomly selected group of 15 years old students at the halfway point in their secondary education, PISA results can be used to evaluate the effectiveness of mathematics education for cultivating mathematical literacy up to the first half of secondary education in each and across countries to help mathematics educators reflect, prepare or, if necessary, re-plan the latter half of secondary mathematics education. For the dimension of content, PISA presents space and shape (i.e., geometry), change and relationships (i.e., algebra), quantity (i.e., numbers and operations), and uncertainty (i.e., probability and statistics). Out of the four content domains, American students performed the least in the space and shape (i.e., geometry), which was also found as the weakest area in TIMSS 2003 (Ginsburg et al., 2005). Students from participating Asian OECD countries, such as Hong Kong, Japan, and Republic of Korea (henceforth,

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State of the literature

- Geometry is an important component of mathematics that is used to conceptualize and analyze physical and imagined spatial environments.
- Content and problems in mathematics textbook are important to provide students opportunities to learn and importance of textbook on student mathematical achievement has been recognized.
- Mathematical literacy is recognized as a predictor of success on students' mathematical achievement.

Contribution of this paper to the literature

- This paper provides an extensive review of literature on mathematics textbooks as a primary tool of teaching and learning mathematics.
- Comparative qualitative and quantitative analyses of geometry sections of two countries' 8th grade textbooks and curricula are conducted.
- Results of this study indicate that the U.S. textbook is larger in volume while a larger portion of the Korean textbook problems are of higher levels of proficiency.

Korea) achieved well in PISA 2003, where content focus was on mathematics. Particularly, Korean students obtained the highest scores in the space and shape (i.e., geometry) out of the four content domains, opposite to the trends of American students.

Geometry is an important component of mathematics. It is a complex, interconnected network of concepts, a way of reasoning, and a system of representation that is used to conceptualize and analyze physical and imagined spatial environments (Battista, 2007). It is a most concrete, reality-linked area of school mathematics that should help students learn and link geometrical theory into lived lives, however, in practice, the gap between the two natures of geometry is far and difficult to bridge (Fujita & Jones, 2002). Accordingly, it offers ways to interpret and reflect our physical environment and can serve as a tool for the study of other topics in mathematics and science (National Council of Teachers of Mathematics [NCTM], 2000). PISA 2003 allocated 20 geometry items for geometry recognizing the importance as seen in Table 1. Previous research on geometry education has provided meaningful information including the fact that for teachers as well as students, introduction of formal proofs and theorems is the most difficult topic in geometry (Fujita et al., 2009). In-depth analysis on geometry education of the U.S. and Korea is meaningful to understand the achievement gap between the two countries. It could give possible implications for improving American student achievement in geometry.

PISA (2004) describes students' proficiency in terms of levels, called proficiency level (PL). Students' scores were grouped into six PLs, with level 6 as the highest and level 1 as the lowest with students scored below 358 classified as "below level 1" that representing on average 11% of students across OECD countries. This PL system is used in individual content as well as in overarching four content domains. For individual literacy performance indexing purposes, students identified in the level 6 are very likely to solve a mathematics items in and below the level 6. Technical Report (OECD, 2004) describes how items and students' abilities are aligned using the PL (scale): Using techniques of modern item response theory, PISA designs to "simultaneously estimate the ability of all students taking PISA assessment, and the difficulty of all PISA items, locating these estimates of students and item difficulty on a single continuum." (see Figure 16.1 in OECD, 2005, p.251) shows the relationship between items and students on proficiency scale.

Among many factors that explain students' performance in mathematics, this study focuses on mathematics curriculum standards, textbook structure, and textbook items in relation to PISA mathematics test since it is important to examine both content and problems to understand students' opportunities to learn through textbooks (Li, 2000). The importance of curriculum and textbook on student mathematical achievement has been recognized by researchers (Herbel-Eisenmann, 2007; Reys et al., 2004; Tarr et al., 2008) and examining curriculum can explain teaching and learning expectations and what students are likely to experience in school since curriculum is an outline for teaching and learning that structures students' learning opportunities in schools (Schmidt et al., 1997). It should be noted that curriculum is not the only factor influencing on students achievement and textbook is not the only curriculum material. Nevertheless, growing body of research indicates that understanding one country's curriculum, especially textbook, is significant since curriculum presents mathematics teachers opportunities and challenges for developing classroom instruction (Li, 2008), which informs what students in one country are intended to learn and textbook is the embodiment such intention - mathematics curriculum. In fact, educational reform is usually done in terms of making changes in curriculum because of their close relationship between classroom practice of curriculum

Table 1. The number of items in each dimension of 85 items used in the PISA 2003

Dimension	Number of Items
Quantity	23
Change and relationships	22
Space and shape	20
Uncertainty	20

and student attainment (Senk & Thompson, 2003).

Among many types of curricular materials, "textbooks exert a considerable influence on the teaching and learning of mathematics, so an understanding of how textbooks vary in their content and approach across countries is an important area of investigation." (Howson, 1995, p.5-6) Textbook is a primary tool that teachers rely on and use when making instructional decision (Collopy, 2003) and, serve as a visible and written curriculum, determine many aspects of teaching and learning (Fuson et al., 1988; Herbel-Eisenmann, 2007; Mayer et al., 1995). When implementing curriculum, textbooks help teachers have a clearer picture of what to be taught and learned based on what curriculum developers intend (McKight et al., 1987). Silver (2009) also asserted the importance of textbooks and problems in them that it is likely that weakness found in U.S. students' using high-level cognitive process on test items (Ginsburg et al., 2005) could be a consequence of the limited opportunity that students get to engage in higher level processes during mathematics lessons.

Numerous cross-national studies have been comparing curricula and curricular materials between U.S. and other countries, especially in the field of mathematics education (Cai et al., 2002; Fan & Zhu, 2007; Li, 2000; Mayer et al., 1995; Stigler et al., 1986). Existing textbook comparison studies across countries provide information on content topic coverage in textbook (e.g., Fuson et al., 1988), pedagogical features of textbooks (e.g., Cai et al., 2002), and featured problems in textbooks (e.g., Fan & Zhu, 2007; Li, 2000; Stigler et al., 1986). Since education is greatly influenced by cultural aspects, mathematics textbook comparison studies embed such characters and values as well as values from outside cultural traditions (Haggarty & Pepin, 2002). Cross-national studies in mathematics curriculum and textbooks have been particularly popular to learn from high achieving Asian countries such as Japan, Taiwan, China and Singapore. These are countries of interests in research literature on curriculum and textbook analyses and most of them focus on elementary and middle school mathematics (Stigler et al., 1982). Common purpose of those studies was to investigate origins of students' performance in later grade levels from early learning environment, particularly curricular materials such as textbooks (Cai et al., 2002; Fan & Zhu, 2007; Li, 2000; Stigler et al., 1986). And these cross-national studies in teaching and learning of mathematics provide opportunities to understand students' current learning and areas of possible improvement (Cai, 2001; Stigler & Hiebert, 1999).

Korea as one of high achieving countries in both TIMSS and PISA consistently, has not been a frequent comparison counterpart as previously mentioned Asian countries. For example, in PISA 2009 (OECD, 2010), around 8% of Korean students are at proficiency level 6, while around 5% of students in Japan, Belgium and New Zealand and average of 3.1% students across OECD countries reached to this level. Combining levels 5 and 6, where average of 12.7% of students are proficient to reach, 25.6% of Korean students, which is the highest among OECD countries, are at level 5 or higher (pp. 132). On the other hand, less than 10% of students in Korea and Finland are at or below level 1, which is the lowest among OECD countries, when average 22% students in OECD countries are at the level. Other than PISA 2009, Korean 15 year olds or 8th graders have been consistently recognized with their high performance in mathematics in TIMSS and PISA. A few studies analyzed Korean mathematics curriculum or textbook focused on elementary school mathematics or early secondary grades have been disseminated (i.e., Son & Senk, 2010), it is meaningful to examine geometry textbooks to see if there are any distinctive feature that we can learn from Korean textbooks that we do not find in U.S. textbooks or vice versa.

This study focuses on students' performance expectations based on (1) curriculum standard; (2) textbook structures; and (3) textbook problems' proficiency levels that PISA framework presents. The purpose of this study is to compare mathematics curriculum standards, textbook structure, and textbook items of the U.S. and Korea, focusing on geometry. Three research questions guided this comparative study.

1. *How do the geometry curriculum standards (i.e., topics addressed, structure of topics, and perspectives on computational fluency, algorithm, and technology use) of the two countries compare?*

2. *How do the textbook structures in geometry (i.e., the degree of emphasis they place on geometry and the level of difficulty of 8th-grade geometry topics) of the two countries compare?*

3. *How do geometry textbook items (i.e., mathematical proficiency level required to solve a problem and the relevance of a problem to real-life contexts) of the two countries compare?*

Related Literatures

Review of research suggests that curriculum standards, textbooks, and textbook items could be good points of comparison in student mathematics performance. For example, Jennings and Dunne (1996), after examined and compared French and English mathematics curriculum standards, found that the achievement gap between English and French pupils can be attributed to the fact that the French national curriculum places greater emphasis on more complex methods than on simple methods. Grade placement of topics is offered as another reason some countries

achieve superior results in mathematics because their students have more opportunities to learn a larger number of topics and ultimately more mathematics by comparable grade levels (Fuson et al., 1988).

Curriculum guidelines include more than lists of contents. Learning processes, teacher educations, instructional practices, and instructional materials are some parts of curriculum guidelines. National Mathematics Advisory Panel (2008) reported importance of computational proficiency and fluency with standard algorithms in consideration of instructional practices and students' learning process. Also, they pointed out the extreme length of mathematics textbooks and a lack of coherence. A use of technology is an area of concern, particularly when studies "found limited or no impact of calculators on calculation skills, problem solving or conceptual development" while some instructional software generally showed positive effects on students' achievement in mathematics.

Among other curricular materials and documents, textbook is one of the most influential means in teaching and learning mathematics and reflects cultural and social values (Park & Leung, 2006). Since the emphasis put on textbook varies by cultures and countries with their own strengths and weaknesses, learning strengths from other country's practices can be encouraged to improve one's practice. Textbooks are the primary resources both for teachers and students (Reys et al., 2004) as well as primary expenses for school systems (Education Market Research, 2002). The crucial role of textbooks is reflected in the analysis of textbooks from approximately 50 countries participating in TIMSS and the formation of a discussion group focused on textbooks at the 10th International Congress on Mathematical Education (ICME-10) in 2004 (Fan & Zhu, 2007).

Studies on textbooks also provide valuable information on students' opportunities to learn. Haggarty and Pepin (2002) investigated mathematics textbooks and their use in English, French and German classrooms on the topics of 'angle' and reported that students from three countries have different opportunities to learn mathematics depending on textbooks structures in combination with teachers' uses of textbooks, and development of lessons varied by textbooks.

Volumes and masses of textbooks are another point of consideration in curriculum research. Research on textbook structure shows that American textbooks are distinctive for their length, broad content coverage, and courses with separate content strands (Park & Leung, 2006; Reys et al., 2004). For example, comparing the measurement and geometry parts of American and Korean textbooks, Kim (1993) found that American textbooks are organized in a spiral fashion, with the same topics appearing repeatedly throughout multiple grades while the content and skills of Korean textbooks introduced in each grade are generally independent. The study criticizes the spiral curriculum for lacking teacher time for advanced levels or other topics.

In addition, McKnight and colleagues (1987) reported that the level of the American 8th grade mathematics content is in fact comparable to the level of Japanese 7th grade mathematics. TIMSS video study also showed that some Asian students learn more advanced topics while American students spend more time on review (Hiebert et al., 2005; Leung, 2005). Moreover, McKnight and the colleagues found that American mathematics teachers were found to teach a larger number of topics while their Asian counterparts tend to teach fewer topics but in greater depth.

Regarding textbook items, Fan and Zhu (2007) investigated differences in the problem-solving procedures required to solve textbook items of the American, Singapore, and China based on Polya's model of four general problem-solving strategies: understanding the problem, devising a plan, carrying out the plan, and looking back. While textbooks from all three countries provide problem-solving heuristics through the representation of problem solutions, the distribution of various heuristics was most concentrated in the Chinese textbooks and most widespread in Singapore ones.

Teaching and learning geometry is particularly significant because of its dual nature – practicality and abstract nature. Fujita and Jones (2002) present some textbooks in UK that use a pedagogical approach by introducing practical activities in the early stage and deductive geometry in the later stage that helps students develop geometrical eyes linking intuition with geometrical theory.

The review of literatures supports that curriculum standards, textbook structure, and textbook items are meaningful points of comparison in conducting an international comparison study. It also reveals the lack of a comparative study between the American and Korea and focusing on geometry, which confirms the need of this study.

Theoretical Framework

Once people familiarize themselves with a type of tasks, it is more likely to be successful when they encounter similar types of tasks than dealing with unfamiliar tasks. Studies found it is true in mathematics problem solving situations. Sweller and Cooper (1985) and Schoenfeld (1983) found that students improved their performance on very similar problems after heavy use of worked examples, however failed to improve on different problems using same mathematical rules or to apply them in a new context. Schema theory (Cooper &

Sweller, 1987) explains reasons of such failures. According to Gick and Holoyoak's (1983) study, a schema, the generalized description of two or more problems and their solutions, enhances transfer when students generate more effective schema. A schema is generated through acquainting and solving overlapping elements of related problems and acts as a mediator of transfer on a new context.

Based on the theory of schema, a question if students would perform better in solving mathematics problems that require higher cognitive demanding would be affected by their exposure to such problems will be answered. By examining textbook items using PISA proficiency levels and 15-year-old students' achievement in PISA 2003, we would be able to answer if it would be the case in American and Korean students.

METHODOLOGY

Research Design

This is a comparative case study (Stake, 2000) of the U.S. and Korea. The points of comparison are geometry curriculum standards, textbook structure, and textbook items. Through the comparison, this study intends to produce an "intensive description and interpretation" (Merriam, 1988, p.9) of student performance differences in geometry between the two countries and, thus, to provide implications for geometry education.

Data

Despite of decentralized nature of curriculum in the U.S., the *NCTM Principles and Standards for School Mathematics* (PSSM) (NCTM, 2000) and the *NCTM Curriculum Focal Points* (henceforth, *Focal Points*) (NCTM, 2006) are regarded by most mathematics educators as reliable recommendations for curriculum development and implementation. The *PSSM* for grades 6 to 8 and *Focal Points* for the 8th grade were used for the subjects of comparison of the U.S. For Korea, where the national curriculum standards prevail, the *7th Korea National Mathematics Curriculum Standards for the 8th grade* (KNMCS) (Korean Department of Education [KDE], 1999) was used. This study mainly examines 8th grade materials – textbooks and curriculum documents, but when certain topics in one country cannot be found in the same grade level in the other country and development of mathematics concepts is of consideration, materials for grades 6, 7, and 9 have been consulted.

Similarly, because mathematics curriculum standards that textbook follows vary by state or district in the U.S., it is not possible to make an analysis of "the American mathematics textbook" for 8th grade students. Of many textbooks following the NCTM standards, *Connected Mathematics* (CM) was chosen because it is "*standard-based*" upon scientific research, supported by National Science Foundation and was field-tested in diverse sites across the country (Connected Math Project, 2006; Lappan et al., 2006). To compare textbook structures and items, *Connected Mathematics 2-Grade 8 Student Textbook* (Lappan et al., 2006) for the U.S. and *Middle School Mathematics 8-B* (MSM) (Yang et al., 2001) for Korea are chosen and compared for their alignment with the selected curriculum standards. Among nationally approved Korean textbooks, which present identical topics and difficulty level of items as stipulated by the Ministry of Education (Park & Leung, 2006), *Middle School Mathematics 8-B* was chosen for popularity. Since both are integrated textbooks including geometry and other content stands, two chapters from CM and two chapters from MSM that consider geometry are selected for analysis.

Analyses

Qualitative content analysis, descriptive statistics, and χ-test were used for this study. Each item in relevant chapters of both textbooks was analyzed based on six PISA PLs to examine how complex mathematics items are and to explore if trends in the textbooks reflect students achievement in PISA (Table 2). To enhance the credibility of qualitative content analysis, the two authors with expertise in mathematics curriculum and instruction analyzed the curriculum standards, textbooks structures, and items independently at first and then, discussed findings to arrive at an agreement. Specifically, all items in one chapter of each textbook were analyzed and matched to PLs by two coders and reached agreement over 93%.

Curriculum standards of both countries were compared to investigate the timeline that each topic is taught; appearance of topics through 6-8th grade geometry standards; and perspectives on computational fluency, algorithms, and uses of technology.

Table 2. Mathematical literacy performance band definitions on the PISA scale (OECD, 2005, p.260)

Level	Score points on the PISA scale
6	Above 669
5	607 to 669
4	545 to 607
3	482 to 545
2	420 to 482
1	358 to 420

Table 3. NCTM Curriculum Focal Points for Grade 8 and the Korean National Curriculum Standards

NCTM Curriculum Focal Points for Grade 8-Geometry and Measurement	Applicable Grade Level in Korea
Students use fundamental facts about distance and angles to describe and analyze figures and situations in two- and three- dimensional space and to solve problems, including those with multiple steps.	Grade 7
They prove that particular configurations of lines give rise to similar triangles because of the congruent angles created when a transversal cuts parallel lines.	Grade 8
Students apply this reasoning about similar triangles to solve a variety of problems, including those that ask them to find heights and distances.	Grade 8
They use facts about the angles that are created when a transversal cuts parallel lines to explain why the sum of the measures of the angles in a triangle is 180 degree, and they apply this fact about triangles to find unknown measures of angles.	Grade 7
Students explain why the Pythagorean theorem is valid by using a variety of methods-for example, by decomposing a square in two different ways. They apply the Pythagorean theorem to find distances between points in the Cartesian coordinate plane to measure lengths and analyze polygons and polyhedra.	Grade 9

RESULTS

Curriculum Standards

Geometry topics.—Unlike the *PSSM* released in 2000 by NCTM, which covers all three grades of middle school simultaneously, the NCTM *Focal Points* (Table 3) and the KNMCS (Table 4) present what students should learn grade by grade. Accordingly, to investigate the geometry topics covered in 8th grade, the NCTM *Focal Points* for grade 8 and the KNMCS for 8th grade geometry were compared. The *Focal Points* present geometry standards along with measurement strand while KNMCS separates geometry from other strands. Of the American five 8th grade geometry topics, KNMCS presents that two topics are taught in grade 8, two topics in grade 7, and one topic in grade 9 according to the KNMCS.

Presentations of the topics.—In NCTM geometry standards for grade 6 through 8 and KNMCS for grades 6, 7, and 8, overall, the geometry topics covered during the three years in the middle grades appear to be similar in both countries. Both curriculum standards emphasize understanding the characteristics and properties of geometric shapes, making proofs of geometric relationships, visualizing geometric shapes, specifying locations, and applying transformations and symmetry. However, recognizing and applying geometric ideas and relationships in everyday life, which is one of major emphases of the NCTM curriculum standards as well as the focus of PISA, is suggested as enrichment curriculum rather than the regular curriculum for Korean 6th to 8th graders. In KNMCS, although the 7th grade standards begin to emphasize the introduction of real life into mathematics, a priority is given to the acquisition of mathematical concepts and skills. Applications to real life are left for enrichment curriculum. It is recommended to provide enrichment learning experiences to advanced learners but "it is not specified in the curriculum" (KDE, 1999, p.102).

Another difference was found in the way geometry topics are organized over the three years. While KNMCS presents geometry objectives and activities separately for each grade, NCTM standards present one set of objectives and activities for grades six through eight. *Focal Points*, which are detailed by grade, do not present geometry as a separate content strand but combine it with measurement. This may reflect the NCTM's emphasis on teaching students to understand the interconnectedness of the two areas over three years in the middle grades. The NCTM approach challenges the practice of offering students a one-year course focusing on a few topics, aiming instead to provide students with a background in all mathematics content by the end of the eighth grade (NCTM, 2000). Since the same topics appear and can be expanded upon over the three-year period, they can be repeated or skipped at the teachers' discretion, leading to a spiral curriculum and resulting in slower progression to a higher content level. In KNMCS, geometry is taught for three years in middle school and the curriculum plan for each grade assumes that students have sufficiently acquired concepts and skills arranged to be taught in the previous grades. It avoids repetition of the same level of concepts and skills, introducing advanced concepts and skills at each grade instead. For instance, since the basic characteristics and properties of a triangle are addressed in elementary school, in grade seven, in the course of learning the symbols and terminology of mathematical

Table 4. *Korean National Geometry Curriculum Standard for Grade 6 to 8*

Grade 6	
Prisms and Pyramids, Nets of Pyramids Cylinders and Cones, Nets of Cylinders and Cones Solid of Revolution	
Grade 7	
Basic Figures	Understanding characteristics of a point, a line, a plane, and an angle Recognizing postitional relationship among a point, a line, and a plane. Understanding Characteristics of Parallel lines
Construction and Congruence	Constructing simple figures Characteristics of Congruent figures Postulates of Congruence
Characteristics of Figures	Characteristics of Polygons Understanding the center, the central angle, a sector, an arc, and a chord and the relationship between the central angle and its arc Relationship formed by a Circle and lines
Characteristics of Solids	Characteristics of Polyhedrons Characteristics of Solids of Revolution
Enrichment	Constructing the net of a regular polyhedron Numbers of vertices, edges, and sides of a Polyhedron
Grade 8	
Characteristics of Triangles and Quadrilaterals	Proving simple characteristics of triangles and quadrilaterals using properties of congruence
Similarity of Figures	Understanding the meaning of similarities of figures Characteristics of similar figures Understanding similarity postulates and theorems
Applications of similarity	Proving the ratio of segments between parallel lines and making applications Proving the triangle mid-segment theorem and making application Ratios of Areas and Volumes of similar figures
Enrichment	Finding congruent or similar figure in real-world settings

reasoning, students are expected to be able to reason deductively that the sum of the measures of the angles in a triangle is 180 degrees. In grade eight, while studying methods of mathematical proof and conditions of congruence, students deductively prove theorems regarding characteristics of various kinds of triangles without substantial time spending on reviews of previous lessons. Through this steady progression, students come to understand and prove the conditions of similarity, triangle mid-segment theorem, and the median of a triangle by the end of eighth grade.

Perspectives on computational fluency.—The NCTM standards state that "students need to learn a new set of mathematics basics that enable them to compute fluently and to solve problems creatively and resourcefully" (NCTM, 2000, p.1). Yet, even greater emphasis is placed on the idea that "students must learn mathematics with understanding" (p.10). In KNMCS, the focus is on learning basic computational skills founded on an understanding of numeric concepts, and yet problem solving in geometry is based upon computational fluency.

Perspectives on the algorithm.—NCTM standards recommend that students actively develop new algorithms. The underlying principle is that "students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge" (NCTM, 2000, p.10). In contrast, in KNMCS, understanding and applying standard solutions are required prior to discovering new algorithms. For instance, the standards guide teachers to confirm students' understanding through concrete examples, after proving characteristics of geometric shapes. Since students are given the algorithms immediately with limited time and opportunity to explore further, they are less likely to invent or discover new solutions.

Perspectives on uses of technology.—According to NCTM (2000), "Principles and Standards call for Geometry to be learned using concrete models, drawings, and dynamic software" (p.40). It also affirms that "technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning" (p.24). In Korea, the use of calculators in the classroom was not allowed until

Table 5. Comparison in the number of pages for geometry in the American and Korean textbooks

	U.S.	Korea
Total Number of Pages	704	279
Number of Pages Allotted to Geometry	184	101
Proportion	26.14%	36.20%
Total Number of Chapters	8	7
Number of Chapters for Geometry	2	2

Table 6. Geometry topics of the American 8th Grade textbook

Looking for Pythagoras	*Coordinate Grid*
	Squaring Off
	The Pythagorean Theorem
	Using Pythagorean Theorem
Kaleidoscopes, Hubcpas, and Mirrors (Symmetry and Transformations)	*Three Types of Symmetry*
	Symmetry Transformations
	Exploring Congruence
	Applying Congruence and Symmetry
	Transforming Coordinates

Table 7. Geometry topics of the Korean 8th grade textbook

Characteristics of Triangles and Quadrilaterals	A Proposition and A Theorem
	Characteristics of Triangles
	Characteristics of Quadrilaterals
Similarities of Figures	Similarities of Figures
	Parallel Lines and Ratio of Lengths
	Applications of Similarity

the 7th KNMCS. Now active use of a calculator or a computer is recommended, except when the focus is on developing computational skills. KNMCS encourages Korean educators to consider using technology such as graphing calculators and computer software for calculating complex problems and enhancing mathematical understanding of concepts, principles, and rules and problem-solving ability.

Textbook Structures

Percentage of pages allotted to geometry. The total number of pages in the American textbook is about 2.5 greater than the number in the selected Korean 8th grade mathematics textbook (Table 5). However, the American textbook has proportionately fewer pages of geometry than the Korean textbook: 26.14% of the total pages in the American textbook are allotted to geometry, as opposed to 36.2% of the total pages of the Korean textbook. A proportional z-test value makes it clear that the Korean textbook places more emphasis on geometry than the American textbook.

Numbers and levels of geometry topics.—The American 8th grade mathematics textbook includes two chapters on geometry: one devoted to the Pythagorean Theorem and the other chapter on Symmetry and Transformation (Table 6) as the *Focal Points* recommend the Pythagorean Theorem chapter for grade eight. The Korean curriculum does not introduce this topic until grade nine. The timing of the Symmetry and Transformation chapter differs between the two American curriculum standards. According to the *Focal Points*, Symmetry and Transformation are to be taught in grade four, while *PSSM* includes these topics for grades six to eight.

In comparison, the Korean 8th grade textbook does not include either of the topics covered by the American textbook. As shown in Table 7, the two chapters on geometry in the Korean textbook are Characteristics of Triangles and Quadrilaterals and Similarity of Figures. The first includes advanced topics involving triangles and quadrilaterals, which requires students to write a proof. In the U.S., this skill is expected for students in grades 9 through 12, according to *PSSM*. In the Similarity of Figures chapter, Korean students are expected to deal with two- and three-dimensional figures, proportions of segments related to parallel lines, as well as proofs. In the U.S., both of these topics are more frequently found in high-school level mathematics textbooks.

Textbook Items

Mathematical proficiency required to solve items.—All of the geometry problems in both Korean and American textbooks were analyzed according to the PISA 2003 PLs (OECD, 2004, p.55-56). PLs 1 to 6 were defined by the PISA 2003 framework. Level 0 was defined as a PL below Level 1. PISA specifies the percentage of students across the participating countries who were able and are expected to perform tasks at each level. According to the PISA results, 87% of students should be able perform tasks classified as PL 1; PL 2 should be attainable by 71% of students; PL 3 by 51%; PL 4 by 30%; PL 5 by15%; and only 5% of students in OECD countries perform at PL 6. PLs 4 and above require multi-step problem solving, spatial/geometrical reasoning, or conceptualizing/generalizing complex situations.

Our analysis showed that a larger portion of problems fell into PLs 4, 5, and 6 in the Korean textbook (73.74%) than in the American textbook (58.87%) (see Figure 1). These differences in proficiency levels are significant, especially for PL 6 items and the combination of PL 5 and 6. Students' familiarity with problems at these levels is clearly reflected in the relative PISA scores for each country. As noted, 5% of students in the OECD countries overall achieved the highest proficiency level (PL 6). But, 16% of Korean 15-year-olds demonstrated achievement at PL 6 while only 2.3% of their American counterparts performed at the same level.

Compared to the Korean textbook, the geometry sections of the American textbook contain a larger portion of problems at PL 1 through 3, where students demonstrate basic concepts and simple problem solving strategies or algorithms. Specifically, 0.48% of the items are PL 0 problems and 1.31% represent PL 1. In contrast, the Korean textbook contains no problems at all at PL 1 and 0. The American textbook includes more problems at PL 2 and 3 (39.33%) than the Korean textbook (26.25%) as well. For PL 2 and 3 items, the American textbook includes significantly more by numbers as well as by proportion (Figure 1).

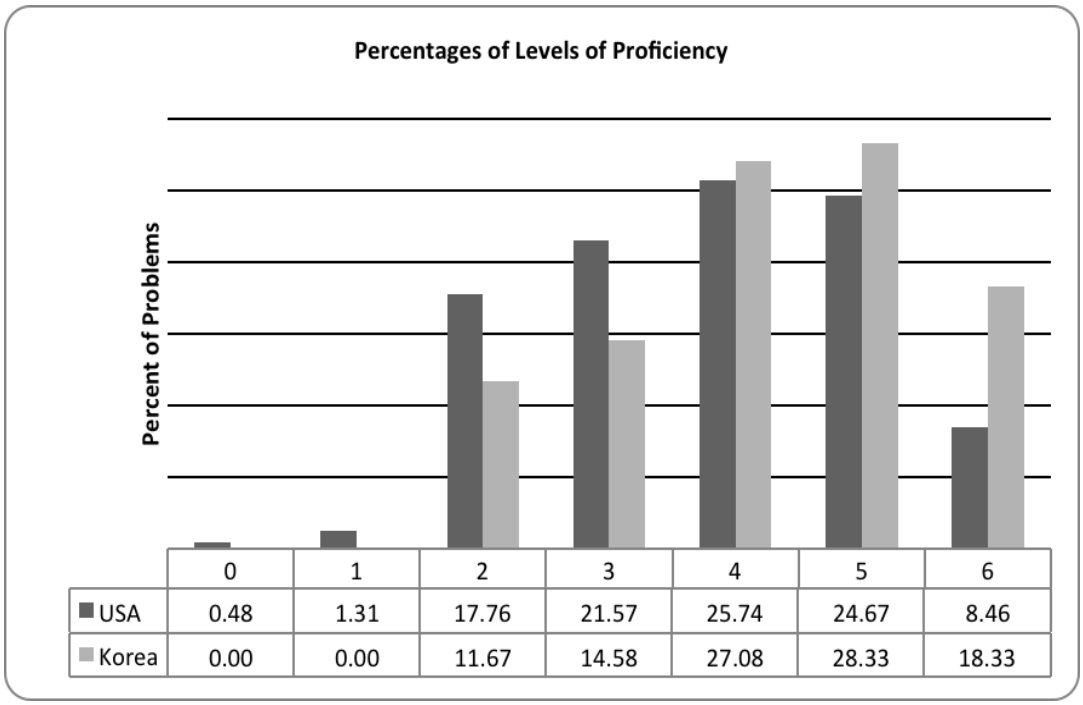
Relevance of problems to a real-life context.—When new mathematical concepts are introduced, the American textbook provides either a real-life context or a hands-on activity. For example, in the chapter "Looking for Pythagoras," before introducing the Pythagorean Theorem, the textbook introduces the coordinate grid system. Instead of offering a traditional two-dimensional coordinate plane, the textbook presents a street system that looks like a coordinate grid, where students look for landmarks using the concept of coordinate points and calculate the shortest distance by car and by helicopter in order to compare them. Another lesson in the same chapter begins with a baseball field. Students are asked to find the distance between the bases and the players on bases as a way of learning how to determine diagonal distances. In "Kaleidoscopes, Hubcaps, and Mirrors," different kinds of design patterns and hubcaps are used to teach symmetry and transformations. Likewise, exercise questions (Applications Connections Extensions [ACE]) in the American textbook include a significant portion of real-life context questions.

The Korean textbook includes a relatively small portion of real-life context problems. In the beginning of each chapter, a few real-life explorations related to the mathematical concepts are provided, but there are no activities or problems that include real-life context applications. While each lesson does include at least one problem involving a real-life context, real-life applications are hardly found from the Korean textbook.

CONCLUSION

This study attempted to answer questions why there is such a big disparity between students' achievement in geometry through analyses of geometry curriculum documents and textbooks of American and Korea.

An analysis on mathematics curricula of the two countries, each country has its unique approaches and perspectives on mathematics education. Korean curriculum standard does not focus on real-life situation and, accordingly, the textbook used in this study includes only few problems of real-world situations. As PISA aims to measure "mathematical literacy" that is individual capacity to apply mathematics to personal, occupational, public, and scientific situations, it is interesting to find that Korean students perform well even though neither Korean textbooks nor curriculum emphasize on students' learning of such types.



	0	1	2	3	4	5	6
USA	0.48	1.31	17.76	21.57	25.74	24.67	8.46
Korea	0.00	0.00	11.67	14.58	27.08	28.33	18.33

Figure 1. Proficiency Levels of Geometry Items

NCTM Focal Points (2006) presents geometry standards in combination with measurement while the strand of measurement is not a part of Korean curriculum standards. Also, NCTM Principles and Standards of School Mathematics (2000) combines three grade bands (6th through 8th) into one when Korean national standards provide individual grade level content objectives. It would be easier for mathematics teachers follow and utilize curriculum standards when objectives are clearly stated for individual grade levels and separated from other strands than all combined together. In that case, teachers can be sure what students learned from the mathematics classes in previous years and do not have to guess what they might have learned. Sometimes, teachers have no other choice but rely on their students claim that they did not learn certain mathematical concepts and end up spending too much time on reviewing materials from previous courses.

Geometry is an area of mathematics that has dual nature with theoretical and practical characters. CMP textbook begins every section with real-life examples and activities that students get familiarized with the abstract idea behind while Korean textbook introduces a real-life story related to the lesson without any activity or example that promote students' engagement in real-world problem solving situations. Only a small number of real-life problems are found at the end of each end-of-lesson example set. The deficiency of efforts connecting abstractness and practicality of geometry can be a possible hindrance of students' recognition of importance of geometry in real world situations. Thus, students are taken their opportunity to learn about geometry in real world away.

In addition, findings of this study imply that the American curriculum needs to pay more attention to student acquisition of adequate levels of mathematical literacy and learning of an accurate application of standard algorithms since these are revealed to be lack in American 8th graders and their geometry textbooks. Although exploring and discovering algorithms is highly important for developing a higher level of mathematical thinking, in practice, all students may not reach that level, and young students do not have the necessary foundational knowledge for this achievement. In this sense, learning underlying basic concepts of standard algorithms and practicing applications will be more educationally effective and meaningful for students. Discovery learning, which encourages a student to discover facts, relationship, or algorithms drawing, should be balanced with acquisition of a substantial amount of knowledge and necessary skills.

Although, in both countries, educators believe that use of technology in mathematics have a great potential for improving students' spatial thinking and data-management ability, heavy reliance on technology uses should be limited if it hinders students' acquisition of basic computational skills.

Analyses of items difficulties based on six PISA PLs signifiy how important it could be that students' exposure to complex items to their success in solving problems with complexity although students' performance on items with complexity does not solely rely on their experiences with complex problems. The findings of this study confirm that American students

proportionally spend more time on solving less complex geometry items that Korean counterparts, which could explain PISA report on students' achievement on geometry regarding six PLs. Teachers, who recognize the importance of students' encounter with geometry problems with various complexity, could allocate instruction time that students familiarity with various types of problems.

The finding of discrepancy between the number of pages and problems allotted to geometry in the American and Korean textbooks give another implication. Even though neither teachers nor students may be expected to complete all of them, the American textbook contains a large number of problems that students may be overwhelmed and discouraged by the task before them. Dealing with the same types of problems repeatedly may also lead students bored and restless before they have the opportunity to develop an interest in mathematics, especially geometry.

Significance of Study

This international comparison study provides valuable information to understand student performance in PISA geometry by comparing multiple factors (curriculum standards, textbook structure, and items). It also gives a series of implications, listed above, useful for curriculum designers, textbook editors, and mathematics teachers to improve geometry education, in which American students experience most difficulty.

Mathematics teachers need to be aware of how their students are experiencing mathematics problems solving situations with various difficulties. Teachers' knowledge in textbooks that they are using in terms of proportions of problems of each PL is particularly important so that students can get exposed to simple mathematics problems as well as very sophisticated problems. When the textbook in use has a lesser proportion of complex problems, teachers should take a note and adopt such problems from additional sources or allocate instruction time so that students get opportunities to learn and can develop schema in solving more sophisticated items and become compatible to their peers in other countries.

Limitation of the Study

This international comparison study provides good information for geometry education of the two countries, however, it does not explain all aspects of educational practices and student achievement in geometry. For example, using a textbook including more items with complexity does not guarantee that students have opportunities to learn such items. Teachers' discretion on selecting appropriate items for instruction is another significant influence on students' exposure to various types of geometry problems. Research on how teachers facilitate students experiences on geometry problems with various difficulties will enhance our understanding in what students actually learn in terms of item complexity and in how students perform in geometry assessment in relation to item complexity. For a comprehensive understanding, complex cultural, social, and educational values embedded in geometry education as well as school, teacher, student characteristics of the two countries should be also considered.

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