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A Comparison of Geometry Problems in Middle-Grade Mathematics Textbooks from Taiwan, Singapore, Finland, and the United States

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ABSTRACT

This study analyzed geometry problems in four middle-grade mathematics textbook series from Taiwan, Singapore, Finland, and the United States, while exploring the expectations for students' learning experiences with these problems. An analytical framework developed for mathematics textbook problem analysis had three dimensions: representation forms, contextual features, and response types. The results showed that the Taiwanese and Singaporean textbooks contained more problems in combined form, whereas the Finnish and American textbooks contained more problems in verbal and visual forms. The problem distribution across various representation forms was more balanced in the Finnish and Singaporean textbooks than in the Taiwanese and American textbooks. Most problems were non-application and close-ended problems compared to other application and open-ended problems. The Taiwanese textbooks contained the lowest proportion of real-world problems, whereas the American textbooks contained the highest proportion of open-ended problems. Implications of this study's findings for textbook developers and future research directions are discussed.

Keywords: middle-grade mathematics textbook, geometry, representation forms, contextual features, response types

INTRODUCTION

Previous studies have shown that mathematics textbooks exert much influence on teaching and learning mathematics (Fan, Zhu, & Miao, 2013; Reys, Reys, & Chavez, 2004; Tarr et al., 2008). School textbooks offer an important resource that mathematics teachers rely on and use when they select teaching activities and approaches (Beaton et al., 1996; Fan & Zhu, 2000; Sun, Kulm, & Capraro, 2009). Of all learning materials, textbooks are a good resource in establishing learning opportunities as well as a good indicator for measuring students'

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State of the literature

- Mathematics textbooks exert much influence on teaching and learning of mathematics.
- In secondary mathematics curriculum, geometry is an important area of mathematics.
- Mathematics textbook is recognized as a good indicator for measuring students' opportunities to learn mathematics.
- It has been recognized that problems in mathematics textbooks can be used as a window through which researchers can view students' opportunities to learn mathematics.

Contribution of this paper to the literature

- This paper provides an extensive review of the literature on "opportunity to learn and textbooks" and "comparing problems in mathematics textbooks."
- The study reported in this paper identified the differences in certain problem types between East Asian and Western mathematics textbooks that may be evidence of cultural impact on the design of textbooks.
- The study also identified the similarities that all selected textbooks may not provide adequate application and open-ended problems which may lead to students' insufficient experience to handle such types of problems.
- The analytical framework identified various problem features of textbooks that may be not only helpful for textbook designers to find out the insufficiency of their textbooks but also for school teachers to enrich their teaching by giving students learning opportunities.
- The implemented methodology of this study could be beneficially applied to analyze other topics in mathematics textbook.
- To complement the previous studies on textbook content comparisons, the findings of this study show the importance of textbook problem analysis.

opportunities to learn mathematics as they reflect the intended curriculum for schools. Learning opportunities are considered an important factor in explaining students' performance differences in mathematics (Törnroos, 2005). It should be noted that textbooks are not the only curriculum materials and not the only factor influencing students' mathematics achievement. Nevertheless, since textbooks are important materials for predicting students' performance in mathematics, some studies emphasized that the analyses of textbooks can provide an important means for explaining the differences in student achievement (Reys, Reys, & Chavez, 2004; Zhu & Fan, 2006). The finding that textbooks are a good resource in establishing students' opportunities to learn mathematics resulted in a series of studies focusing on the analyses of textbooks of different countries after the 1990s (Özer & Sezer, 2014).

Previous textbook studies have been mainly focused on content analysis, including content distribution on textbook pages, content presentation, content-topic coverage and page space devoted to each topic (Delil, 2006; Grishchenko, 2009; Li, 2000; Törnroos, 2005). Less attention has been placed on the analysis of problems presented in textbooks (Delil, 2006; Grishchenko, 2009; Zhu & Fan, 2004). Mathematical problems, however, occupy a central role in teaching and learning of mathematics at all times (Stanic & Kilpatrick, 1988). Problems not only draw students' attention to specific aspects of the content but also to their information processing approaches. Analyses of problems in curriculum materials provide information about one aspect of the kinds of students' opportunity to learn mathematics (Doyle, 1988). The Professional Standards for Teaching Mathematics states that students' learning opportunities are established by the level and the kind of thinking in which students engage with mathematical problems (National Council of Teachers of Mathematics [NCTM], 1991). Problems presented in mathematics textbooks form the bases of students' opportunities to learn mathematics (Doyle, 1983; NCTM, 2000). Besides content analysis of textbooks (Törnroos, 2005), analyzing textbook problems as a window to view students' opportunities to learn mathematics is another useful idea in educational research (Li, 2000).

Textbook problem analysis was conducted in only a few studies. For example, students' failure on some of the problem types may be due to their lack of experience with such situations (Xin, 2007). Textbooks should not present mathematical problems in purely mathematical contexts but they should emphasize real-world problems, such as those dealing with fractions (Alajmi, 2012). The inadequate experience of students with real-world problems may result in their difficulties in solving this type of problems (Wijaya et al., 2015); in contrast, using more real-world problems in classrooms can be beneficial for students' understanding (Gu, Huang, & Marton, 2004). Increasing use of problems with visual representation may also be conducive to students' better conceptual understanding (Cai, 1995; Xin, 2007; Zhu & Fan, 2006). Students' insufficient exposure to open-ended problems may lead to their difficulties in solving complex problems (Cai, 2000). Conversely, providing students with plenty of opportunities to practice open-ended problems can facilitate their divergent thinking skills (Kwon, Park, & Park, 2006). These earlier studies on textbook mathematical problems have shown that a problem's representational forms (e.g., visual representation), contextual features (e.g., real-world problems), and response types (e.g., open-ended problems) are three important dimensions for analyzing mathematical problems. They also showed that these three dimensions possibly affect students' performance in solving mathematical problems.

Previous cross-national studies on textbook problems have examined the similarities and differences in expectations related to the students' mathematical experiences between countries and explained the performance of students by analyzing the specific types of mathematical problems they used (e.g., Stigler, Fuson, Ham, & Kim, 1986; Xin, 2007). These studies comparing textbooks and their problem sets can give new insights into the unique features of what students are learning from the textbook problems. Moreover, such comparisons need to be done on a basis of a framework which captures important features of the textbook design (Kolovou, van den Heuvel-Panhuizen, & Bakker, 2009). Cai and Cirillo (2014) pointed out that researchers may use different ways to analyze mathematical textbooks with various frameworks, and no agreed approach has yet emerged for curriculum analysis. Moreover, they addressed some important methodological considerations in curriculum analysis such as the development of a framework, the selection of textbooks, the formulation of research questions, and so on.

In this study, we considered the aforementioned issues of methodology in conducting textbook analysis. As part of this study, we developed an analytical framework for analyzing the problems in textbooks. Our framework covers three dimensions—representation forms, contextual features, and response types—that were identified in previous studies (e.g., Alajmi, 2012; Kwon et al., 2006; Wijaya et al., 2015; Zhu & Fan, 2006). These dimensions based on the literature (e.g., Fan & Zhu, 2000; Yang & Lin, 2016; Zhu & Fan, 2006) can be defined in a composite way as follows:

1. Representation forms. A purely mathematical form means that a problem includes only mathematical expressions. A verbal form means that a problem is only described in written words. A visual form means that a problem only consists of figures, pictures, graphs, charts, tables, diagrams, maps, and so forth. A combined form means that a problem is presented in a combination of two or three of the above forms.
2. Contextual features. An application problem means that a problem is presented in the context of a real-world situation. A non-application problem means that a problem is presented in a situation without any context.
3. Response types. An open-ended problem means that a problem has several or many correct answers. A close-ended problem means that a problem has only one correct answer.

The mathematics curriculum affects student learning at all educational levels, but the middle school curriculum in particular, is in need of attention (American Association for the Advancement of Science [AAAS], 2000). Geometry is an important component of mathematics that has a dual nature with theoretical and practical characteristics (Choi & Park, 2013). Moreover, geometry is an essential part of international mathematics assessments such as the Program for International Student Assessment (PISA; OEDC, 2013) and the Trends in International Mathematics and Science Study (TIMSS; Mullis, Martin, Foy, & Arora, 2012). Although the topic of geometry is crucial, there has been little research focusing on this topic in middle school mathematics textbooks.

Some studies have identified the differences in textbook features such as topics, topic sequences, and problem presentations between East Asian and Western mathematics textbooks (e.g., Hong & Choi, 2014; Yang & Lin, 2016; Zhu & Fan, 2006). Other studies have suggested that culture may play a role in influencing the design of textbooks (e.g., Fan, 1999; Leung, 2001). Four middle-grade mathematics textbook series from Taiwan, Singapore, Finland, and the United States were chosen for the present study because these four countries—that rank in the top ten in TIMSS 2011 in mathematics achievement at the eighth grade—come from the two largest educational systems in the East and the West.

Therefore, the purpose of this study was to examine how selected middle school mathematics textbooks from Taiwan, Singapore, Finland, and the United States present various types of geometry problems. By conducting this study, we hoped to expand the previous research and add knowledge to the literature by demonstrating the similarities and differences in the mathematical problems of the four textbook series, revealing the expectations for students' learning experiences in using these problems, and exploring the possible ways to improve the presentation of problems in mathematics textbooks, which can, in turn, improve students' learning experiences in mathematics. Based on our chosen topic and countries, we identified two research questions in this study:

1. How different types of problems are presented in the four textbook series?
2. What are the expectations related to students' mathematical experiences in using the four textbook series?

RELATED LITERATURE

Opportunity to Learn and Textbooks

Textbooks are typically the main resource for teachers to make decisions about what to teach and how to teach (Fan, Chen, Zhu, Qiu, & Hu, 2004). Mathematics textbooks mostly determine what teachers teach and what students learn (Stein & Smith, 2010). Several studies have shown a strong correlation between the textbook used and students' mathematics performance (e.g., Törnroos, 2005; Xin, 2007). The concept behind the correlation between what is taught and what is learned is the so-called the opportunity to learn (OTL). According to Husén's (1967) report of the First International Mathematics Study, the OTL was defined as "whether or not … students have had the opportunity to study a particular topic or learn how to solve a particular type of problem" (pp. 162–163). Mathematics textbooks are a significant determinant of students' opportunity to learn. The opportunity for students to experience mathematics in a context is important (NCTM, 2000). Students' mathematics achievement has been found to be positively associated with students' opportunity to learn (Törnroos, 2005). "Opportunity to learn is widely considered the single most important predictor of students' achievement" (National Research Council, 2001, p. 334). Opportunity to learn has been used to account for differences in students' mathematics performance from different countries (Hiebert & Grouws, 2007). Charalambous, Delaney, Hsu, and Mesa (2010) argued that in order to understand such differences in teaching and achievement across various countries, their textbooks should be analyzed. Özer and Sezer (2014) noted that the finding that textbooks are a good teaching and learning resource—for providing learning opportunities as well as a good indicator for measuring learning opportunities—had resulted in a large amount of studies focusing on textbook analysis of different countries since the 1990s. Textbook analysis was applied to many studies examining the OTL offered in textbooks (e.g., Ding & Li, 2010; Kolovou, van den Heuvel-Panhuizen, & Bakker, 2009; Törnroos, 2005; van Zanten & van den Heuvel-Panhuizen, 2014; Wijaya, van den Heuvel-Panhuizen, & Doorman, 2015; Xin, 2007). For example, Wijaya et al. found that a lack of opportunities to learn in Indonesian mathematics textbooks may lead to Indonesian students' difficulties in solving context-based tasks. In Xin's study, it was found that the ability of U.S. students to solve certain problem types better than other types may be related to the design of American textbooks (e.g., unbalanced word problem distribution).

Comparing Problems in Mathematics Textbooks

Comparative studies have placed their attention on the analysis of problems presented in textbooks between countries to understand the similarities and differences in the expectations with regard to students' learning experiences in mathematics.

Stigler, Fuson, Ham, and Kim (1986) compared the presentation of addition and subtraction word problems in American and Soviet elementary mathematics textbooks. The results showed that the Soviet textbooks were more diversified and balanced in distribution of word problems across various problem types compared to the American textbooks. In addition, the Soviet textbooks focused more on complex two-step problems, whereas the American textbooks focused more on simple one-step problems. The findings of Stigler et al.'s study indicated that both mathematical and contextual features of textbooks potentially affect the development of students' mathematical proficiency in solving addition and subtraction word problems. Li (2000) also compared problems involving integer attrition and subtraction in American and Chinese Grade 7 textbooks by using a three-dimensional framework including mathematical features, contextual features, and performance requirements. The results indicated that the vast majority of problems in these two countries' textbooks required single-step solutions and had purely mathematical contexts. Moreover, the American textbook problems were more varied in problem requirements and emphasized more on conceptual understanding compared to the Chinese textbook problems. Li's study offered a glimpse into the potential relationship between the expectations related to students' mathematical experiences and their actual mathematical performance. In another study, Zhu and Fan (2006) examined how different types of problems were presented in Chinese and American textbooks at the lower secondary grade level. The results showed that the large majority of the problems in textbooks from both countries were routine, traditional, and close-ended problems and most of problems were not contextualized in real-world situations. Moreover, the Chinese textbooks focused more on real-life problems or application problems compared to the American textbooks. Additionally, the Chinese textbooks placed greater emphasis on multiple-step problems compared to the American textbooks, whereas the American textbooks placed greater emphasis on problems in a visual form (e.g., with figures or tables). Similarly, Xin (2007) examined word problem distribution across various types in American and Chinese textbooks and its relation to students' problem-solving performance. The results indicated that the Chinese textbooks provided students with relatively balanced opportunities to solve various word problems, whereas the American textbooks showed relatively unbalanced word problem presentation. The relation between textbook word problem distribution and students' performance seems to indicate that the design of American textbooks (e.g., unbalanced word problem distribution) may have a role in shaping U.S.

students' ability to solve certain problem types better than others. More recently, Alajim (2012) made comparisons between American, Japanese, and Kuwaiti elementary school mathematics textbooks regarding the teaching of fractions. The results showed that the American and Kuwaiti textbooks contained more pages than the Japanese textbooks. Moreover, the American textbooks included more real-life problems than the Japanese and Kuwaiti textbooks.

METHOD

Selection of Textbooks

The selected middle-grade textbooks from Finland and Taiwan included those for grades 7 to 9, whereas those selected from the United States included grades 6 to 8, and those from Singapore included grades 7 to 10. To uniformly examine three grades for each textbook series across the four countries, the textbook series for Grade 10 of Singapore was not included in the analysis of this study.

Singapore's New Syllabus Mathematics (NSM) (grades 7-10) was developed by Teh and Loh (2007) in accordance with the national mathematics curriculum guidelines of Singapore's Ministry of Education (2001). NSM has the highest mathematics textbook market share (approximately 80%) in Singapore (Yang, Reys, & Wu, 2010). In this study, only the textbooks for grades 7-9 were selected. These textbooks contain 41 units, 16 of which cover geometry-related topics.

Kang Hsuan (KH) middle-grade mathematics textbooks (grades 7-9) have an approximately 39% market share in Taiwan (Kang Hsuan Educational Publishing Group, 2010). They are the most commonly used middle-grade mathematics textbook series in Taiwan. The KH textbook series contain 62 units in total, 22 of which cover geometry-related topics.

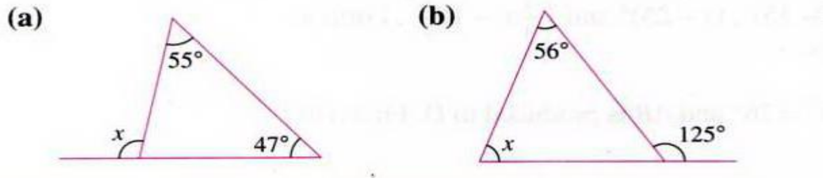
Finland's Laskutaito (grades 7-9) textbook series were published by WSOY (Laurinoli et al., 2008). It was based on the core mathematics curriculum guideline of the Ministry of Education in Finland in 2004, which was in line with new mathematics education trends worldwide. Laskutaito has the highest market share (nearly 70%) in Finland. Laskutaito comprises 41 units, 16 of which cover geometry-related topics.

From the United States, the Connected Mathematics Project (CMP) (grades 6-8), which is sponsored by the National Science Foundation, was selected in this study. It was developed by Lappan, Fey, Fitzgerald, Friel, and Phillips (1996). The CMP textbooks were selected for this study because the CMP, which is used in nearly 2,500 school districts, is the most widely used middle-grade mathematics textbook series in the United States (Cai, Wang, Moyer, & Nie, 2011; Rivette, Grant, Ludema, & Rickard, 2003). The CMP textbooks include 24 units for grades 6–8, 6 of which cover geometry-related topics.

Table 1. An analytical framework for problem analysis in geometry textbooks

Dimension	Category
Representation forms	Purely mathematical
	Verbal
	Visual
	Combined
Contextual features	Application
	Non-application
Response types	Open-ended
	Close-ended

Example 1: Calculate the values of the unknowns in the following diagrams.



Since only one unknown angle (x) in (a) and (b) is required to calculate, they were counted as one single problem.

Example 2: Find the unknown angles marked in each of the following diagrams:

Because there are two unknown angles x and y in (a) and a and b in (b), they were each counted as two problems.

Figure 1. Geometric example problems (Teh & Loh, 2007, p. 361)

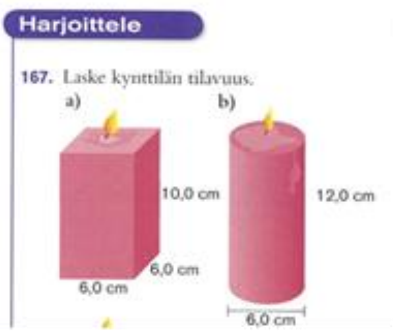
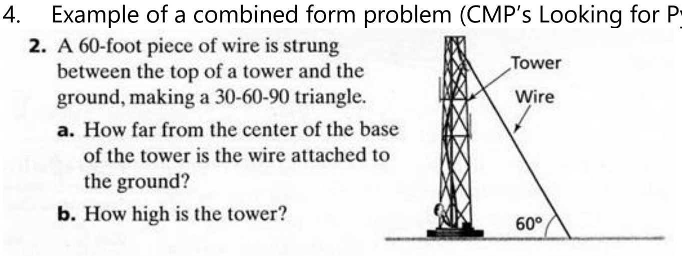
Analytical Framework

The present study defined mathematics problems as the mathematics activities, homework, and exercises related to geometry which students were expected to complete. The analytical framework about the problem types for this study was adapted from Zhu and Fan's (2006) classifications of problems (**Table 1**). The framework for problem analysis in the mathematics textbooks in this study had three dimensions: representation forms, contextual features, and response types.

Analysis of Problems

The problems and exercises in the student textbooks were counted to determine the total number of geometric problems. A problem was considered as the unit of analysis in this study.

Table 2. Sample textbook problems and categories

Example	Category
1. Use the three angles 40°, 60°, 80° to construct a triangle (New Syllabus 3, p. 209).	Purely mathematical
2. "A paper box without a lid is 25 cm long, 16 cm wide, and 5 cm deep. How many square centimeters of paper have been used to make the box?" is a verbal problem (New Syllabus 1, p. 178).	Verbal
3. Example of a visual form problem (Laskutaito 9, p. 49) 	Visual
4. Example of a combined form problem (CMP's Looking for Pythagoras, p. 66) 	Combined
5. Use the three angles 40°, 60°, 80° to construct a triangle (New Syllabus 3, p. 209)	Non-application

2. A 60-foot piece of wire is strung between the top of a tower and the ground, making a 30-60-90 triangle.
a. How far from the center of the base of the tower is the wire attached to the ground?
b. How high is the tower?

In the following figure (**Figure 1**), we present two example problems to explain how we counted an exercise as a problem.

Table 2 shows examples of problems and their categories as listed in **Table 1**.

Interrater Reliability of Coding

All problems in the four textbook series were coded independently by two researchers using the analytical framework and then the two coders compared their codings assigned to the problems. The Cohen's Kappa was found to be 0.92, indicating very high agreement between the two coders (Altman, 1991). The discrepancies between the two coders in coding the problems were discussed and resolved until a consensus was reached.

Table 3. Distribution of problems in representation forms in the four textbook series

	Taiwan		Singapore		Finland		America	
Representation form	*n*	%	*n*	%	*n*	%	*n*	%
Purely mathematical form	36	4.0%	144	8.6%	213	10.3%	4	0.4%
Verbal form	114	12.7%	360	21.4%	577	27.9%	351	37.1%
Visual form	124	13.8%	610	36.3%	854	41.4%	409	43.3%
Combined form	622	69.4%	565	33.7%	421	20.4%	181	19.2%
Total	896	100.0%	1679	100.0%	2065	100.0%	945	100.0%

Table 4. Distribution of problems in contextual features in the four textbook series

	Taiwan		Singapore		Finland		America	
Contextual feature	*n*	%	*n*	%	*n*	%	*n*	%
Application problem	30	3.3%	231	13.8%	425	20.6%	161	17.0%
Non-application problem	866	96.7%	1448	86.2%	1640	79.4%	784	83.0%
Total	896	100.0%	1679	100.0%	2065	100.0%	945	100.0%

RESULTS

Country-Based Variations in Representation Forms

According to **Table 3**, the four textbook series placed greater emphasis on problems in verbal, visual, or combined form compared to other representation forms, whereas they placed the least emphasis on problems in purely mathematical form. The Finnish and American textbooks had higher percentages of problems in verbal form (27.9% and 37.1%, respectively) and visual form (41.4% and 43.3%, respectively); by contrast, the Taiwanese and Singaporean textbooks had higher percentages of problems in combined form (69.4% and 33.7%, respectively). Among the four textbook series, a relatively low proportion (0.4%) of problems in mathematical expressions was found in American textbooks. The problem distribution across various representation forms in the Singaporean and Finnish textbooks was more balanced than that in the Taiwanese and American textbooks.

Country-Based Variations in Contextual Features

Table 4 shows that non-application problems were dominant in the four textbook series. The majority of problems in the four textbook series were not contextualized in real-world situations, especially more obvious in Taiwanese textbooks. Overall, for non-application problems, the Taiwanese textbooks contained the highest percentage compared to the Singaporean, Finnish, and American textbooks (Taiwan: 96.7%; Singapore: 86.2%; Finland: 79.4%; America: 83.0%). On the other hand, for application problems, the Finnish, American, and Singaporean textbooks had higher percentages than that for Taiwanese textbooks (Taiwan: 3.3%; Singapore: 13.8%; Finland: 20.6%; America: 17.0%). Among the four textbook series, the Taiwanese textbooks included the smallest proportion of real-world problems, whereas the Finnish textbooks had the highest proportion of such problems.

Table 5. Distribution of problems in response types in the four textbook series

	Taiwan		Singapore		Finland		America	
Response type	*n*	%	*n*	%	*n*	%	*n*	%
Open-ended problem	39	4.4%	69	4.1%	59	2.9%	165	17.5%
Close-ended problem	857	95.6%	1610	95.9%	2006	97.1%	780	82.5%
Total	896	100.0%	1679	100.0%	2065	100.0%	945	100.0%

Country-Based Variations in Response Types

As shown in **Table 5**, close-ended problems were dominant in the four textbook series (Taiwan: 95.6%; Singapore: 95.9%; Finland: 97.1%; America: 82.5%). The Taiwanese, Singaporean, and Finnish textbooks had higher percentages of close-ended problems compared to the American textbooks. In contrast, the American textbooks had the highest proportion of open-ended problems compared to the textbooks of the other three countries.

DISCUSSION AND CONCLUSION

In this study, when the distribution of problems in the representation forms—in the selected Taiwanese, Singaporean, Finnish, and American textbooks—was evaluated, it was found that all the four textbook series contained higher proportions of problems in verbal, visual, and combined forms compared to the problems in purely mathematical form. Moreover, the distribution of problems in different representation forms was more balanced in the Finnish and Singaporean textbooks compared to that in the Taiwanese and American textbooks. It appears that the Finnish and Singaporean textbooks provided students with relatively balanced opportunities to solve problems with various representations, whereas the Taiwanese and American textbooks showed relatively unbalanced problem presentation. According to Riley, Greeno, and Heller (1983), students' difficulties in solving certain problem types may be related to the unavailability of textbooks to provide sufficient opportunities for them to solve a range of problem types and to facilitate their problem-solving skills. Sweller, Chandler, Tierney, and Cooper (1990) reported that mathematics curriculum (e.g., textbooks) should provide students with opportunities to solve all types of problems and to facilitate their conceptual understanding of problem structures. NCTM (2000) emphasized that students need to be exposed to all types of problems in learning mathematics. Therefore, one implication of this study for student practice in learning mathematics is that Taiwanese and American textbook developers should provide students with sufficient opportunities to practice solving the whole range of problem types comprising problems in mathematical, verbal, visual, and combined representation forms to ensure that students grasp the underlying structures of the problems. Additionally, the results of this study revealed that Taiwanese and Singaporean students from East Asian countries likely have more experiences with problems in combined form, whereas Finnish and U.S. students from Western countries probably have more exposure to verbal and visual problems. Overall, the results indicated a different pattern with respect to problem distribution across various representation forms in East Asian and Western mathematics textbooks. Such problem distribution differences in both educational systems might be due to cultural differences. Thus, the findings of this study appear to support those of previous studies (Fan, 1999; Leung, 2001) that cultural differences may influence the design of textbooks. Another important results of this study showed that the Taiwanese and Singaporean textbooks placed greater emphasis on problems in combined form that required the use of two or three representation forms (i.e., mathematical, verbal, or visual form) compared to other textbooks. Evidently, the Taiwanese and Singaporean students using the textbooks are expected to have more learning and practice opportunities to solve this type of problems with multiple representations than did their Finnish and American counterparts. Brenner et al. (1997) pointed out the importance of using multiple representations in learning mathematics. Dufour-Janvier, Berdnaz, and Belanger (1987) argued that the use of multiple representations is needed in mathematics instruction because such instruction can give multiple concretizations of a concept and can help students overcome certain difficulties when solving problems. Mayer, Sims, and Tajika (1995) described that meaningful teaching methods tend to emphasize the relationships between multiple representations. In this regard, it is possible to state that the Taiwanese and Singaporean textbooks could offer more opportunities for the effective teaching of geometry compared to the Finnish and American textbooks. Such opportunities embedded in both mathematics textbook series in Taiwan and Singapore have the potential to not only allow students to be exposed to more experiences of practicing problems involving more than one representation but also to develop their ability for problem solving in geometry. In Xin's (2007) study, it was found that the ability of U.S. students to solve problems in certain problem types better than in other types may be due to more learning opportunities. In contrast, another study (Wijaya et al., 2015) reported that a lack of opportunities to learn from Indonesian mathematics textbooks may lead to difficulties of students when solving context-based tasks. Thus, the findings of this study further suggest that more opportunities for students to engage in the use of problems containing multiple representations could be one reason for the better mathematics achievement observed in Taiwanese and Singaporean students in international assessments such as PISA and TIMSS in comparison to Finnish and U.S. students. This study also showed that the Finnish and American textbooks offered more opportunities for students to use verbal and visual representations to assist with problem solving than did Taiwanese and Singaporean textbooks. Some studies (e.g., Cai, 1995; Xin, 2007; Zhu & Fan, 2006) reported that more U.S. students than Chinese students preferred to use visual representation in problem solving. Such results revealed that compared to Chinese students, U.S. students seemed to experience much more problem solving with visual information, which is positively related to their performance in this type of problem. Thus, one practical implication of this study is that Taiwanese and Singaporean textbook developers could consider including more problems with visual representation in their textbooks.

It can be noted that the gaps between application problems and non-application problems were evident across the four middle-grade mathematics textbook series. More specifically, the majority of problems in the four textbook series were not set in a real-world context, especially the Taiwanese textbooks with an absolute majority of problems without such context (more than 96%). This means that students using the selected textbooks would have a rare exposure to problems with a relevant real-world context. Considering the application problems or real-world problems, one can see that the Finnish, American, and Singaporean textbooks provided more opportunities with students to work on such problems than did Taiwanese counterparts. Greeno (1991) argued that knowledge of the context plays an important role in mathematics learning. According to Wijaya et al. (2015), the lack of opportunities for students to engage in real-world problems may cause their difficulties in solving context-based tasks. In another study, Gu, Huang, and Marton (2004) found that increasing use of application problems or real-world problems in classrooms can create a learning environment favorable to a higher level understanding. Therefore, another implication of this study is that Taiwanese textbooks should include more application problems or real-world problems that offer students opportunities for using knowledge of the context or selecting relevant information to comprehend and solve the tasks.

It is also interesting to note the great disparity that existed between open-ended problems and close-ended problems in the four middle-grade mathematics textbook series. To be more specific, considerably more close-ended problems were presented in the four textbook series, whereas much less open-ended problems were presented. It appears that students quite likely have more experiences with close-ended problems but relatively less exposure to open-ended problems. In view of open-ended problems, the American textbooks could provide students with more opportunities to experience such type of problems compared to other textbooks. Stein and Smith (1998) mentioned that students' lack of prior experience with open-ended tasks leads to difficulties when solving tasks in which the mathematical procedure is implicit. Similarly, Cai (2000) reported that students who had fewer opportunities to practice open-ended problems would have difficulties in solving complex problems. In another study, Kwon et al. (2006) argued that providing students with more opportunities to practice open-ended problems in mathematics learning can promote their divergent thinking skills such as fluency, flexibility, and originality. Based upon the findings of this study, Taiwanese, Singaporean, and Finnish textbook developers could consider including more open-ended problems in their textbooks.

There existed several limitations in this study. First, this study only examined one mathematics textbook series in each country, although these selected textbook series were representative. Second, this study only investigated students' opportunities to learn mathematics from the perspective of the opportunity to learn (OTL) in textbook problems. For future studies, we suggest that both content and problems should be included in the textbook analysis. This would provide a better overview of the OTL in mathematics textbooks.

CONCLUDING REMARKS

The results of this study showed some similarities and differences in problem distribution across the four textbook series. The problem distribution in textbooks does not refer to the quality of the textbooks; rather it addresses the learning opportunities for students to develop their mathematical understanding. As findings of this study, we have identified differences in certain problem types (i.e., problems in combined, verbal, and visual forms) between East Asian and Western mathematics textbooks. This study has provided evidence that culture may play a role in influencing the design of textbooks. We also have identified similarities in specific problem types (i.e., application vs. non-application and open-ended vs. close-ended problems) across the four textbook series. To be more specific, all the selected textbooks did not adequately represent application and open-ended problems. The insufficient experience of students with real-world and open-ended problems may lead to their difficulties in solving such types of problems. We suggest that more real-world and open-ended problems should be included in middle school mathematics textbooks.

In this study, we grounded the development of our framework in the research literature. The analytical framework helped us identify disparate problem features of textbooks that might contribute to structuring students' learning opportunities. If a textbook provides various geometry problems based on different dimensions (i.e., representation forms, contextual features, and response types) and teachers know the categorization, they can pose these problems to students and strengthen a link connecting their opportunities to learn with their actual learning. The methodology used in this study (textbook problem analysis) can be applied to analyze other topics. In addition, the outline in the development of the methodology can be useful in planning and conducting future research on textbook problem analyses.

This study's focus was on middle school textbook analysis on geometry problems. We believe that our findings have contributed to the body of knowledge about textbook analysis for the mathematics education research community as well as that for textbook developers. To complement previous studies comparing mathematics textbook content, the findings of this study point to the value of comparing problems between mathematics textbooks from different countries.

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