A computer approach to mathematics curriculum developments debugging

Angélica Martínez-Zarzuelo, Eugenio Roanes-Lozano, MªJosé Fernández-Díaz
Universidad Complutense de Madrid, SPAIN

Sequencing contents is of great importance for instructional design within the teaching planning processes. We believe that it is key for a meaningful learning. Therefore, we propose to formally establish a partial order relation among the contents. We have chosen the binary relation "to be a prerequisite" for that purpose. We have applied this approach to the mathematical contents of the compulsory Secondary Education of the Spanish educational system and the information obtained has been modeled as a graph. The amount of contents considered (814) and the number of ordered pairs in the order relation considered (17,782) has produced a big graph. In order to work effectively with that amount of data, we have used software specialized in network analysis. More precisely, we have used the software packages Pajek and Gephi. This software, together with the use of techniques borrowed from graph theory, has allowed providing a tool for debugging curriculum developments (similar to rule based expert systems verification).

Keywords: curriculum development debugging; graph theory; instructional design; network analysis; organization of mathematical contents; sequencing contents.

INTRODUCTION

This work proposes and validates a methodology based on the application of techniques borrowed from graph theory (Kaufmann, 1976; Wilson, 1983; Abellanas & Lodares, 1990; West, 2001; Aldous & Wilson, 2003; Biggs, 2003; Chartrand & Zhang, 2005; Balakrishnan & Ranganathan, 2012) and specialized software in network analysis to debug proposals of organization of educational contents in teaching planning.

By debugging we mean "verifying that the educational contents sequencing proposed is correct" (something similar to what is done in rule based expert systems verification). This way we believe we contribute to the development of models of instructional design. In order to prove the viability of this methodology, it has been applied to the mathematical knowledge (curriculum developments) of the education stage "E.S.O." (compulsory Secondary Education) of the Spanish

Correspondence: Angélica Martínez-Zarzuelo,
Universidad Complutense de Madrid, Rector Royo Villanova, s/n
28040 Madrid, Spain
 E-mail: angelica.martinez@ucm.es

educational system. However, the nature of this methodology makes it possible to easily extend it to other areas of knowledge, as well as to other educational stages. This work summarizes part of the Ph.D. Thesis of the first author, whose advisors were the other two authors (Martínez-Zarzuelo, 2015).

LEGAL FRAME

We shall follow somehow a down to top design: we shall begin by analyzing the example selected (Spanish compulsory Secondary Education) in order to have a panoramic view of the object of study that allows developing the theoretical/computational approach.

The present Spanish K-12 educational system distinguishes 4 educational stages:

- Kindergarten,
- Primary Education,
- compulsory Secondary Education ("E.S.O."), for 12- to 16-year-olds,
- non-compulsory Secondary Education ("Bachillerato").

This study focuses on the third stage. The reasons for choosing this educational stage have been, firstly, to be a compulsory educational stage. This allows performing the study on a stage that involves a huge amount of students of the Spanish educational system. Secondly, it is the last compulsory stage. This fact makes contents more complex and diverse. Consequently, it makes the data of the study richer.

We have analyzed the details of the mathematical subjects of the last three laws regulating compulsory Secondary Education in Spain:

- LOGSE (Ley Orgánica de Ordenación General del Sistema Educativo, 1990; Real Decreto 1007/1991, 1991),
- LOE (Ley Orgánica de Educación, 2006; Real Decreto 1631/2006, 2006), and
- LOMCE (Ley Orgánica para la Mejora de la Calidad Educativa, 2013; Real Decreto 1105/2014, 2014).

These core curricula detail the objectives to be reached, the contents to be taught (not in detail), and the competencies to be developed. It also makes some suggestions on the methodology and assessment (what does not affect this study).

THEORETICAL FRAME AND RELATED WORKS

A correct core curriculum development is generally considered to be the key to a more practical and proactive regulatory framework (Schubring, 1987; Leuders, Barzel & Hußmann, 2005; Goñi, 2011). This characteristic enables an approximation between teaching theory and practice. Although the educational laws usually leave a good degree of freedom to curriculum developers (of textbooks, educative projects, etc.), they usually just translate the suggested ways of organizing suggested by the laws. Moreover, the format of textbooks leads to a contents sequencing mainly

State of the literature

- An appropriate contents sequencing is key for a right instructional design within the teaching planning processes. The literature supports this idea and proposes some specific criteria for contents sequencing. However, these proposals are purely theoretical.
- The foundations of meaningful learning can positively contribute to the development of contents sequencing and to the organization of the knowledge that is intended to be taught.
- Different techniques for a graphical representation of knowledge, like conceptual maps, tridimensional expert maps or pathfinder associative networks, can be used for the organization of knowledge.

Contribution of this paper to the literature

- This work proposes and validates a methodology to debug proposals of organization of educational contents in teaching planning for instructional design.
- We have established a specific epistemological criterion for sequencing knowledge that allows establishing a binary relation between contents, as foundation of this methodology. This criterion has its basis in the meaningful learning.
- The proposed methodology is based on techniques borrowed from graph theory and software specialized in network analysis. Although this kind of software has been applied in a variety of environments, it hasn't been applied to the purpose of this work, as far as we know.

linear, in accordance with their appearance in the textbooks (Ausubel & Barberán, 2002).

Nevertheless, there are many other possible ways to sequence contents. For instance, Zabalza (2009) proposes different kinds of sequencing based on criteria such as the importance of the contents involved in a sequence and the time required for teaching them.

From our point of view, the most interesting criterion is the one based on meaningful learning (Ausubel, 1963; Ausubel, Novak & Hanesian, 1976; Moreira, 2000), where to be an anterior or ulterior content is taken into account.

Regarding this issue, Rodríguez (2008) asserts that "no se puede desarrollar aprendizaje significativo en el alumnado con una organización del contenido lineal y simplista" ("students cannot develop a meaningful learning if the content is organized in a linear and simplistic way") (p. 31). In this sense, two different sequencing for teaching could be considered, for example, the contents *Subtraction*, *Multiplication* and *Division* from the content *Summation* (Figure 1).
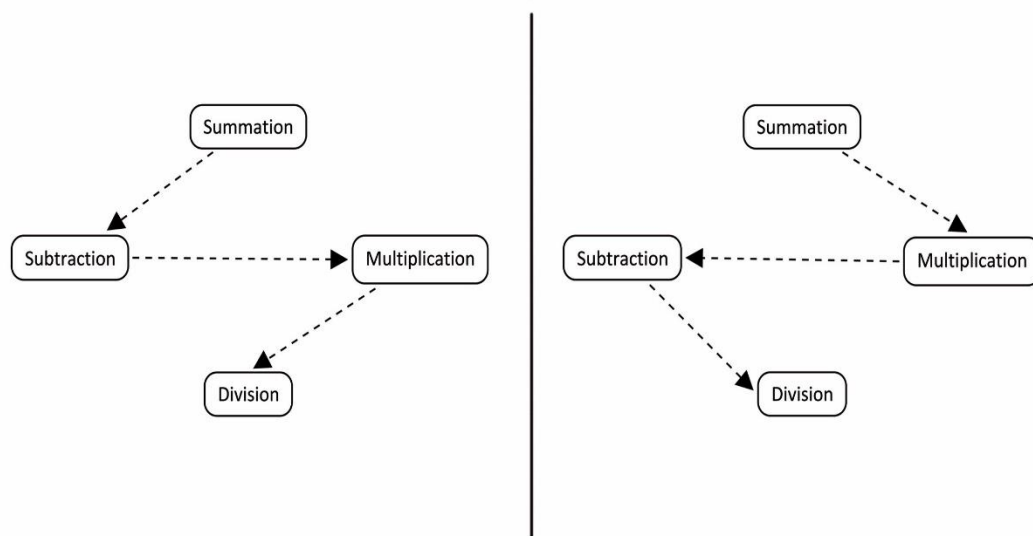

Figure 1. Two alternative contents sequencing (Rodríguez, 1983, p.68).

On the other hand, the internal logic structure of the subject to be taught and a balanced equilibrium between procedural skills and conceptual understanding (Drijvers, Goddijn & Kindt, 2011) are key to plan its contents sequencing. Mathematics has a very special internal logic structure: contents are based one on another. Although this is possibly the reason for the high percentage of students that hate this subject, it is also the key for organizing and sequencing contents, and, consequently, for a meaningful learning. That is why this subject was chosen to exemplify and validate the methodology proposed in this work.

Nevertheless, it is clear that the idea developed here could be applied to other subjects. In this context we can find Ausubel's meaningful learning theory (Ausubel, 1963; Ausubel, Novak & Hanesian, 1976; Moreira, 2000; Ausubel & Barberán, 2002), as this theory holds that new information is meaningful as far as it can be related with information already known.

Regarding the internal logic structure of mathematics as a subject, there is a wide consensus about the importance of understanding it as a network of interrelated concepts and procedures, and not as a collection of rules and isolated facts (Brinkmann, 1999, 2001, 2003, 2005).

The technique underlying conceptual maps (Novak & Gowin, 1988) is well suited for graphically representing knowledge. The reason for this is that it can be used for representing, experimenting, perceiving and learning the network structure of

mathematics (Brinkmann, 1999, 2003; Mwakapenda & Adler, 2003; Brinkmann, 2005). In addition, authors such as Sherborne (2014) discuss how the use of conceptual maps could help curriculum developers and teachers.

Brinkmann (2001) proposes modeling the mathematical connections using a graph too. Nevertheless, how to develop this process is not fully detailed. She underlines the necessity to avoid considering a large number of mathematical objects for the sake of clarity. Both aspects have been addressed in the methodology developed here.

This methodology also inspires tridimensional expert maps (Pérez, 1998; Suero, Montanero & Montanero, 1999; Pérez, Suero, Montanero & Montanero, 2000a, 2000b), that are considered an extension of conceptual maps.

Conceptual maps and tridimensional expert maps are the starting points of the structure used in this work.

Reigeluth's elaboration theory (Reigeluth, 1979; Reigeluth & Stein, 1983; Coll, 1987; Reigeluth, 1987; Reigeluth & Curtis, 1987; Reigeluth & Stein, 1987; Sacristán & Gómez, 1989; Coll & Rochera, 1990; Reigeluth, 1999; Reigeluth, 2013), itself based on conceptual maps and tridimensional expert maps, is also an important theoretical foundation for this study.

One of the purposes of this work is also sequencing and organizing educational contents. The underlying theory includes other theories in the field, such as Ausubel's meaningful learning theory (Ausubel, 1963; Ausubel, Novak & Hanesian, 1976; Moreira, 2000), Gagnè's learning theory (Gagnè, 1970, 1973) and Bruner's spiral theory of learning (Bruner, 1963, 1966, 1991, 1998, 2003, 2004), thereby including its different perspectives about instructional designs (Bruner, 1966; Gagnè, Wager, Golas, Keller & Russell, 2005; Reigeluth, 2013).

Another technique for knowledge graphical representation, the pathfinder associative networks (Schvaneveldt, 1990), is also an inspiration. A recent line of research has applied "pathfinder associative networks" to the study of cognitive structures and has led to the development of the theory of nuclear concepts (Casas, 2002; Casas & Luengo, 2004a, 2004b). The key concepts of the latter theory are the "geographic" organization of knowledge, nuclear concepts, and minimum cost paths.

Finally, regarding the kind of structure used in this work (the graph), we could underline that it has been used by other authors for other didactic goals, such as: determining teaching itineraries (Alarcón, Fernández, González & Martínez, 2007), treating the academic curriculum as a complex system with nodes representing courses and links between nodes representing the course prerequisites (Aldrich, 2015), or suggesting a model for the solution of motion problems with an intelligent tutoring system (Nabiyev, Çakiroğlu, Karal, Erümit & Çebi, 2016).

DESIGN OF THE APPLIED STUDY

The key idea of our work is to firstly manually detect the contents involved in the curriculum proposed. This search is performed on the educational laws and the corresponding curriculum developments (textbooks). Secondly, the binary relation "to be a prerequisite" is manually established in the set of contents (once this set is determined).

The educative contents considered and the binary relation established can then be modeled using a graph structure. Graph theory has been applied in very different environments and diverse goals (Martín & Méndez, 2004; Leydesdorff, 2007; Puchades, Mula & Rodríguez, 2008; Cardozo, Gómez & Parras, 2009; Carlos, Gallardo & Colomer, 2011; Pino, Jiménez, Ruíz & Bailón, 2011; Gómez, Zarrazola, Montero & Yañez, 2012; Sánchez, Moratalla & Sanz, 2012; Nieto, 2013; Wilson, 2013; Clough, Gollings, Loach & Evans, 2014; Romo, Vélez, Solís, Luna & Espinoza, 2015).

Nevertheless, as far as we know, it has not been previously applied to the goal of this work.

As the binary relation is established in a somehow subjective way, we consider the transitive closure of the given relation (the smallest transitive relation that contains the given one). This way the binary relation is completed (although subjected to errata in the data introduction). Finally, if we compute the transitive reduction of the latter graph, a canonical binary relation is obtained.

ABOUT THE SOFTWARE USED

We initially tried to "organize" the contents using pen and paper. The approach looked impractical soon after we began (Figure 2). The information was difficult to manage and it was almost impossible to introduce changes or alterations in it (like new contents, changes in the contents, changes in the binary relation). The first steps of the process were detailed in (Martínez-Zarzuelo, Roanes-Lozano & Fernández-Díaz, 2013).
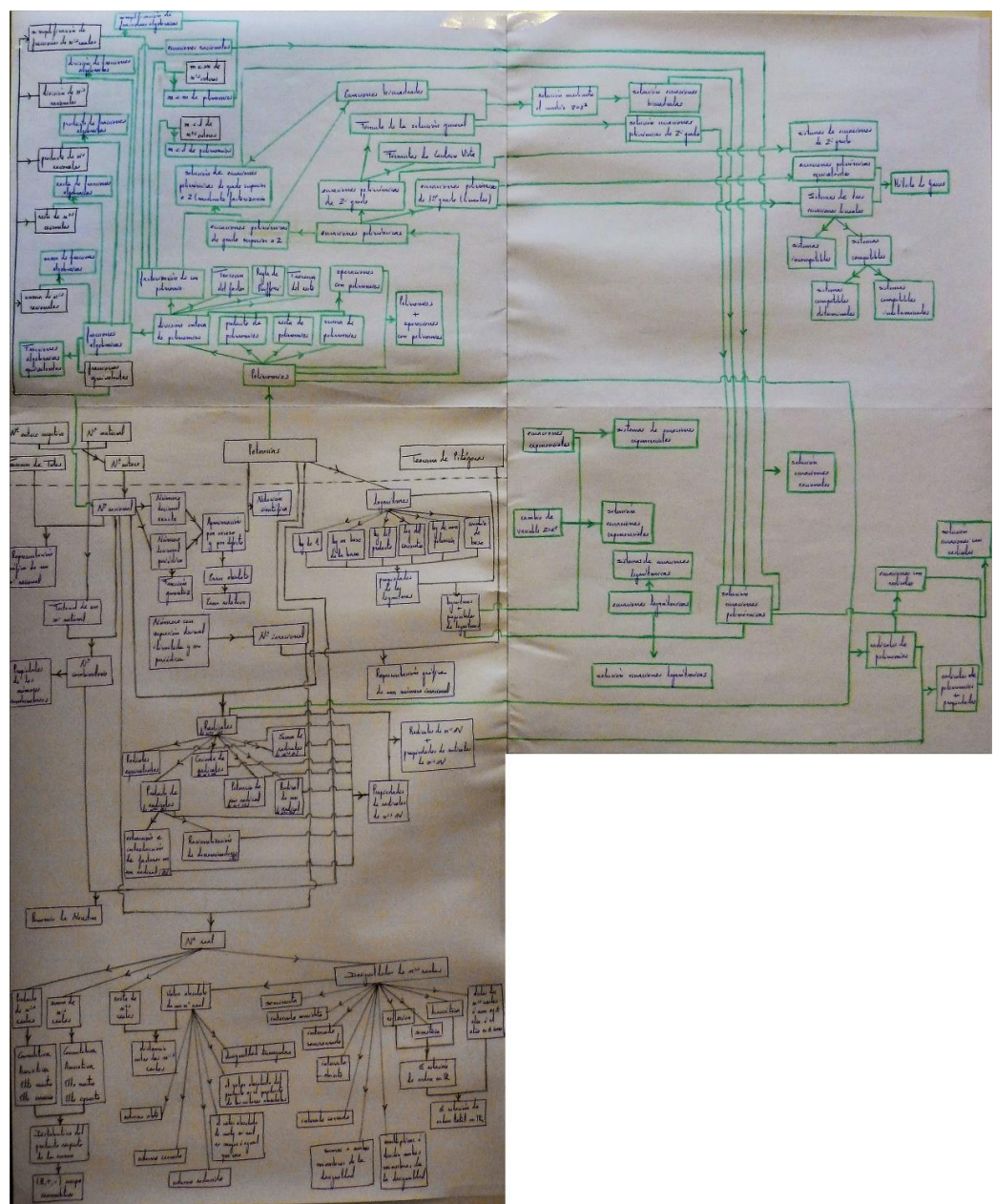

Figure 2. A pen and paper first approach.

It was therefore clear that a network analysis piece of software would be a great help. After evaluating different ones, Pajek[1] (Batagelj & Mrvar, 2014) and Gephi[2] (Cherven, 2013, 2015) were chosen because of their power, their facilities to introduce data from Excel, and the possibility to exchange data between the two of them. Moreover, Pajek is free for noncommercial use and Gephi is open source, they do not require programming skills and can deal with graphs of high order and a big size.

Software specialized in visualization and analysis of information structured as a network in general, and these software packages in particular, have been widely used in a great variety of issues (White, Batagelj & Mrvar, 1999; Peñaranda, Quiñones & Osca, 2009; Camino, 2012; Conway & White, 2012; Raper, 2012; Álvarez, Kuz & Falco, 2013; Heymann & Le Grand, 2013). Nevertheless, they have not been used for the goal of this work.

DEVELOPMENT OF THE APPLIED STUDY

The binary relations "to be a prerequisite" and "to be an immediate prerequisite" have been defined in a theoretical way. Specifically, we have used the following definitions:

Definition 1: Let C be the set of mathematical contents of Spanish compulsory Secondary Education.

Definition 2: Let $c_1, c_2 \in C$. c_1 is a prerequisite of c_2, denoted $c_1 \blacktriangleright c_2$ if and only if understanding content c_1 is required to understand content c_2.

Definition 3: Let $c_1, c_2 \in C$. c_1 is an immediate prerequisite of c_2, denoted $c_1 \rhd c_2$ if and only if understanding content c_1 is required to understand content c_2 and $\nexists$ $c_3 \in C$ such that $c_1 \rhd c_3 \rhd c_2$.

Once the two binary relations have been defined, the ordered pairs of mathematical contents (within the educational stage of Spanish compulsory Secondary Education) that define them are determined. In order to obtain these ordered pairs, we have used the guidelines given by the educational laws mentioned above and we have explored in detail 40 textbooks of the main Spanish publishers. More precisely, we have used ten textbooks for each of the four courses of the educational stage considered.

The definitions of each of the mathematical contents have been carefully analyzed. For example, we can find in one textbook that: "The set of reals numbers is formed by all the rational numbers and all the irrational numbers." Therefore: real number, rational number, irrational number $\in C$ and:

rational number $\blacktriangleright$ real number

and: irrational number $\blacktriangleright$ real number.

The pairs of contents established this way are modeled using a digraph. When applied to the four courses of the compulsory Secondary Education in Spain, the digraph associated to the binary relation $\blacktriangleright$, denoted $G = (C, A)$, turns out to have 814 nodes and 3,925 arcs.

This digraph, as is initially introduced, is not really a graph, but a multigraph. It has multiple arcs because a kind of Bruner's spiral curriculum (Bruner, 1963, 1966, 1991, 1998, 2003, 2004) is adopted by the Spanish law and some arcs appear in different academic years. Therefore, a first step in the preprocessing is the elimination of multiple arcs. Let us denote the digraph obtained this way: $G_1 = (C, A_1)$.

The second step of the preprocess is to construct the transitive closure of G_1, denoted: $G_1^c = (C, A_1^c)$. In our case, G_1^c consists of 814 nodes and 17,782 arcs! Nevertheless, it can be handled without problems by both Pajek and Gephi.

DEBUGGING THE DIGRAPH (LOOKING FOR LOOPS AND CYCLES): VERIFICATION OF CURRICULUM DEVELOPMENTS

Once the digraph G_1^c is obtained, it is necessary to debug it. The objective is to detect possible inconsistencies in the contents sequencing. These inconsistencies arise when contents that are prerequisites of themselves are found, that is, when there are loops in the digraph G_1^c associated to the prerequisite relation.

Thanks to the computational approach to the digraph G_1^c developed (using Pajek and Gephi), six loops have been detected. Figure 3 shows a graphic representation of the digraph. There are so many arcs that it is almost opaque. Nevertheless, the six loops can be seen in the upper left and right corners. They are more clearly seen in Figure 4.
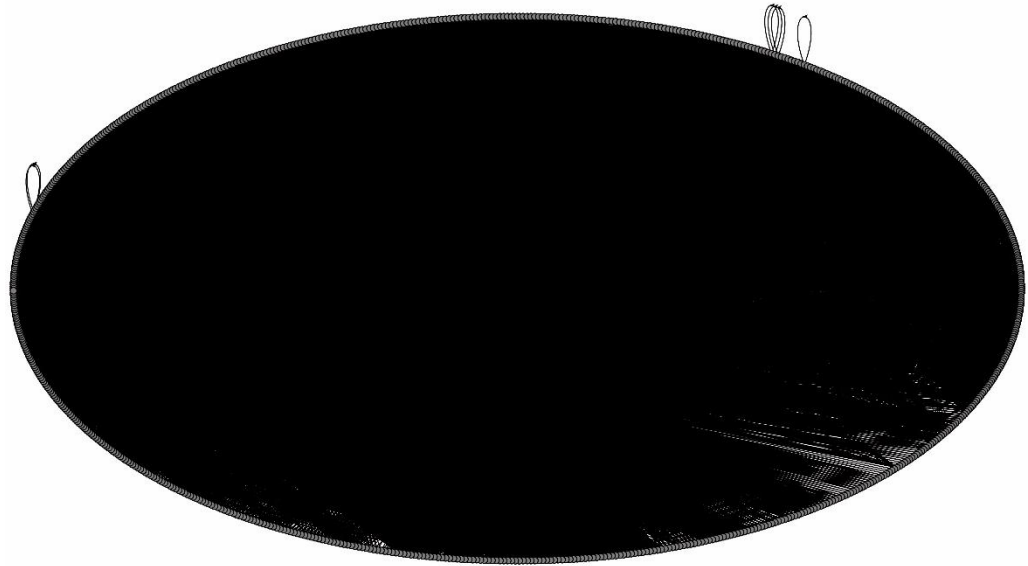

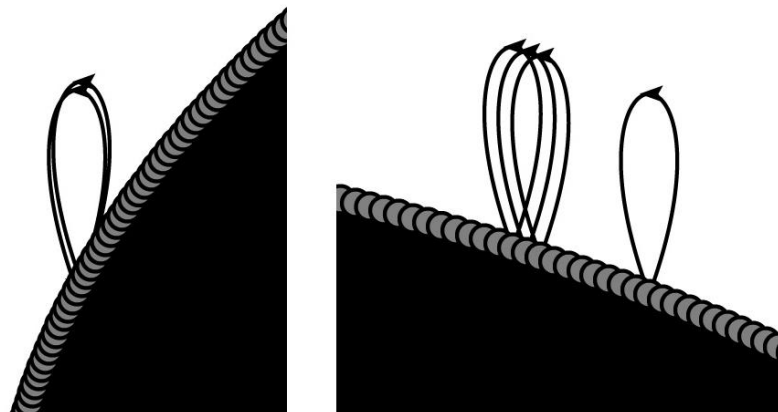**Figure 3**. A graphic representation of the arcs in digraph G_1^c.

Figure 4. Loops in digraph G_1^c (magnified).

It is necessary to determine the reasons why those loops do appear in the digraph G_1^c. First, the origin and destination nodes of those loops are identified using Pajek. This way, 6 nodes are identified. More precisely, those nodes correspond to the contents:

- *Factor de un número entero (Factor of an integer number)*,
- *Multiplicación de números enteros (Product of integers)*,
- *Representación gráfica de una función (Graphic representation of a function)*,
- *Función convexa (Convex function)*,

- • *Función cóncava (Concave function),* and
- • *Puntos de inflexión de una función (Inflection points of a function).*

If we focus on the contents corresponding to *Factor de un número entero (Factor of an integer number)* and *Multiplicación de números enteros (Product of integers)*, and the subgraph of G_1^c containing just these two nodes, the subgraph has 4 arcs, two of which form a directed cycle and the two others are loops (Figure 5).

Figure 5. Two loops in digraph G_1^c (factors of an integer number & product of integers). Remark: In this and the following figures, the contents appear in Spanish, as they are software output captures.

If we check the digraph G_1, these two arcs forming a cycle already appear there, so that is the reason for finding these loops in G_1^c.

But digraph G_1 should be antisymmetric, so no loops should arise. In this case the software has detected an erratum in the concept selection. The concept "factors of an integer number" should really be divided into "factorization of an integer number" and "factors of a product of integers". Then "factors of a product of integers" would be a prerequisite for "product of integers" and "product of integers" would be a prerequisite for "factorization of an integer number". This was exactly the kind of errata we expected to be able to debug.

Following the same procedure, the subgraph of G_1^c containing the contents *Representación gráfica de una función (Graphic representation of a function)*, *Función convexa (Convex function)*, *Función cóncava (Concave function)* and *Puntos de inflexión de una función (Inflection points of a function)*, is represented in Figure 6.
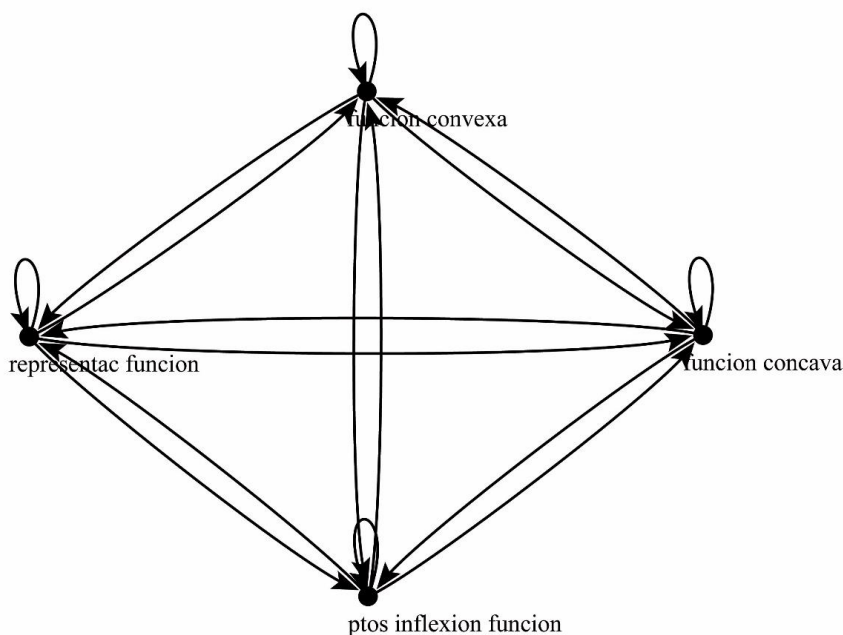

Figure 6. The other loops in digraph G_1^c.

In Figure 6 it is clear that there are two arcs between each pair of nodes, so the subgraph is not antisymmetric. Again, these cycles already appear in digraph G_1. The corresponding subgraph of G_1 can be found in Figure 7.
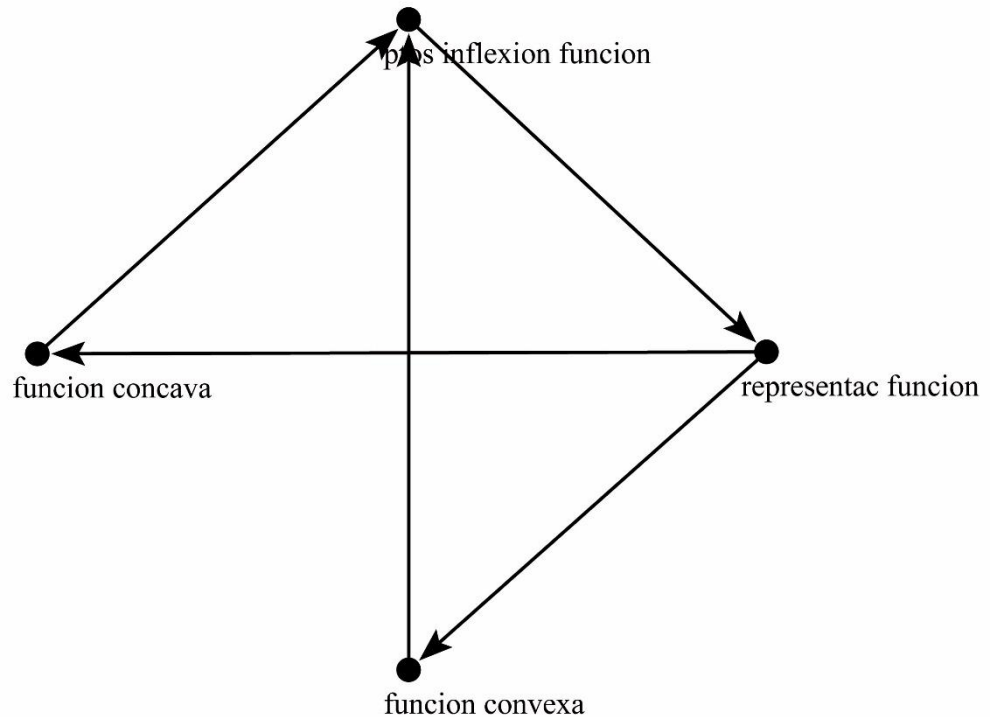

Figure 7. Subgraph of G_1.

Two directed cycles are detected in digraph G_1, one involving: *Representación gráfica de una función, Función convexa* y *Puntos de inflexión de una función (Graphic representation of a function, Convex function, Inflection points of a function);* and another one involving: *Representación gráfica de una función, Función cóncava* y *Puntos de inflexión de una función (Graphic representation of a function, Concave function, Inflection points of a function).*

This case is really interesting. It reveals a finding of the software regarding concepts: what comes first? The concept "graphic representation of a function" or the concept "concavity and convexity"? Probably the concept "graphic representation of a function" needs to be split into two different concepts:

- "intuitive ideas regarding graphic representation of a function and plotting (in the sense of computers, by representing many points)"

and

- "formal graphic representation of a function (based on the determination of local extrema and inflexion points)".

Therefore the system has discovered a detail that we, that considered that mastered the subject, had not taken into account!

The situation is similar to rule based expert systems verification (Laita & Ledesma, 1997; Laita & Roanes-Lozano, 1999): an error in the knowledge of the expert cannot be detected at the verification step, but logical contradictions can be detected.

CONCLUSIONS

Techniques borrowed from graph theory and the use of networks analysis software have allowed us to propose a methodology for debugging curriculum developments. This methodology has been developed and validated in the field of mathematics. More precisely, it has been applied to the mathematical knowledge of the education stage "E.S.O." (compulsory Secondary Education) of the Spanish educational system.

The peculiar internal logic structure of this subject has allowed determining an epistemological criterion of knowledge structuring based on meaningful learning. A binary relation has been formally defined on this basis. Defining this binary relation has allowed us establishing a concise sequencing of educative contents. Moreover, starting from graphic representations techniques of knowledge such as conceptual maps and pathfinder associative networks, the graph structure has turned out to be ideal for modeling contents and the binary relation between them.

Using software has made possible to deal with a big amount of information. The software used, specialized in network analysis, has been key for analyzing the digraph created. It has been necessary to apply some preprocesses to make it practical (such as eliminating multiple arcs or computing the transitive closure).

Finally, thanks to the use of graph theory techniques and this specialized software, loops have been found in the graph and have been analyzed. These loops, interpreted as the existence of contents that are prerequisite of themselves, lead to interesting findings.

Two loops derive from an erratum: certain contents should be split into two different ones.

The other loops lead to an unexpected and far more interesting finding: the convenience of refining a standard concept ("graphic representation of a function") and distinguishing a concept similar to "manual plotting of a function" from "formal graphic representation of a function".

This kind of findings and reflections are contributions to the processes of sequencing contents and, therefore, to the processes of planning teaching and instructional design.

NOTES

[1] http://vlado.fmf.uni-lj.si/pub/networks/pajek/
[2] http://gephi.github.io/

ACKNOWLEDGEMENTS

REFERENCES

Abellanas, M. & Lodares, D. (1990). *Análisis de algoritmos y teoría de grafos*. Madrid: RA-MA.

Alarcón, J. D., Fernández, J. R., González, L. A. & Martínez, J. J. (2007). Precedencia Difusa y Generación de Itinerarios Docentes en Sistemas LCMS. In M. Benito, J. Romo, J. Portillo (Eds.), *Post-Proceedings del IV Simposio Pluridisciplinar sobre Diseño, Evaluación y Desarrollo de Contenidos Educativos Reutilizables, SPDECE*. Volume 318 of CEUR Workshop Proceedings, CEUR-WS.org.

Aldous, J. M. & Wilson, R. J. (2003). *Graphs and applications: an introductory approach* (Vol. 1). Springer Science & Business Media.

Aldrich, P. R. (2015). The curriculum prerequisite network: Modeling the curriculum as a complex system. *Biochemistry and Molecular Biology Education, 43*(3), 168-180.

Álvarez, A., Kuz, A. & Falco, M. (2013). Gephi: Análisis de Interacciones en un Foro, a través de ARS en el aula. *TE & ET, 11,* 66-75.

Ausubel, D. P. (1963). *The psychology of meaningful verbal learning*.

Ausubel, D. P. & Barberán, G. S. (2002). *Adquisición y retención del conocimiento: una perspectiva cognitiva*. Barcelona: Paidós.

Ausubel, D. P., Novak, J.D. & Hanesian, H. (1976). *Psicología educativa: un punto de vista cognoscitivo*. México: Trillas.

Balakrishnan, R. & Ranganathan, K. (2012). *A Textbook of Graph Theory.* New York: Springer Science & Business Media.

Batagelj, V. & Mrvar, A. (2014). Pajek. In R. Alhajj & J. Rokne (Eds.), *Encyclopedia of Social Network Analysis and Mining.* (p. 1245-1256). New York: Springer.

Biggs, N. L. (2003). *Discrete Mathematics.* New York: Oxford University Press.

Brinkmann, A. (1999). Investigating the Use of Concept Mapping as Tools in Mathematics Education. *Network, 48,* 1-13.

Brinkmann, A. (2001). Mathematical Networks–Conceptual Foundation and Graphical Representation. In R. Soro (Ed.), *Current State of Research on Mathematical Beliefs X, Proceedings of the MAVI-10 European Workshop.* (p. 7-16). Sweden: University of Turku, Department of Teacher Education.

Brinkmann, A. (2003). Graphical knowledge display–mind mapping and concept mapping as efficient tools in mathematics education. *Mathematics Education Review, 16,* 35-48.

Brinkmann, A. (2005). Knowledge maps–tools for building structure in mathematics. *International Journal for Mathematics Teaching and Learning*.

Bruner, J. S. (1963). *El proceso de la educación.* México: Unión Tipográfica Editorial Hispano Americana.

Bruner, J. S. (1966). *Toward a theory of instruction* (Vol. 59). Cambridge: Harvard University Press.

Bruner, J. S. (1991). *Actos de significado: más allá de la revolución cognitiva.* Madrid: Alianza editorial.

Bruner, J. S. (1998). *Desarrollo cognitivo y educación.* Madrid: Morata.

Bruner, J. S. (2003). La estructura básica de una disciplina. *C. June Maker, Teaching models in education of the gifted*.

Bruner, J. S. (2004). *Curriculum en espiral.* Madrid: Morata.

Camino, L. (2012). Herramienta de visualización para un simulador de propagación de enfermedades contagiosas (Trabajo fin de grado). Universidad Carlos III, Madrid.

Cardozo, O. D., Gómez, E. L. & Parras, M. A. (2009). Teoría de grafos y sistemas de información geográfica aplicados al transporte público de pasajeros en Resistencia (Argentina). *Revista Transporte y Territorio, 1,* 89-111.

Carlos, M., Gallardo, A. & Colomer, F. (2011). Comparación de métodos de optimización de rutas en la recogida de residuos sólidos urbanos aplicado a Castellón de la Plana. In Redisa (Eds.), *Proceedings del Simposio Iberoamericano de Ingeniería de residuos (SIIR): Hacia la sustentabilidad: Los residuos sólidos como fuente de energía y materia prima.* (p. 145-150). México.

Casas, L. M. (2002). El estudio de la estructura cognitiva de alumnos a través de Redes Asociativas Pathfinder. Aplicaciones y posibilidades en Geometría (Tesis doctoral). Universidad de Extremadura, Badajoz.

Casas, L. M. & Luengo, R. (2004a). Representación del conocimiento y aprendizaje. Teoría de los Conceptos Nucleares. *Revista Española de Pedagogía, 227*, 59-84.

Casas, L. M. & Luengo, R. (2004b). Teoría de los Conceptos Nucleares: Aplicación en Didáctica de las Matemáticas, un ejemplo en Geometría. In R. Luengo (Ed.), *Líneas de Investigación en Educación Matemática.* (p. 127-164). Badajoz: Sociedad Extremeña de Educación Matemática "Ventura Reyes Prósper".

Chartrand, G. & Zhang, P. (2005). *Introduction to Graph Theory.* New York: McGraw-Hill.

Cherven, K. (2013). *Network graph analysis and visualization with Gephi.* Birmingham: Packt Publishing Ltd.

Cherven, K. (2015). *Mastering Gephi Network Visualization*. Birmingham: Packt Publishing Ltd.

Clough, J., Gollings, J., Loach, T. & Evans, T. (2014). Transitive reduction of citation networks. *Journal of Complex Networks, 3,* 189-203.

Conway, D. & White, J. M. (2012). Analyzing Social Graphs, Visualizing the Clustered Twitter Network with Gephi. In D. Conway & J. M. White (Eds.), *Machine Learning for Hackers.* (p. 261-267). O'Reilly.

Coll, C. (1987). *Una aproximación psicopedagógica a la elaboración del currículo escolar*. Barcelona: Laia.

Coll, C. & Rochera, M. J. (1990). Estructuración y organización de la enseñanza: las secuencias de aprendizaje. In J. Palacios, A. Marchesi & C. Coll (Eds.), *Desarrollo psicológico y educación.* (p. 373-394). Madrid: Alianza Editorial.

Drijvers, P., Goddijn, A. & Kindt, M. (2011). Algebra education: Exploring topics and themes. In P. Drijvers (Ed.), *Secondary algebra education. Revisiting Topics and Themes and Exploring the Unknown*. (pp. 5-26). Rotterdam: SensePublishers.

Gagnè, R. M. (1970). *Las condiciones del aprendizaje.* Madrid: Aguilar.

Gagnè, R. M. (1973). Learning and instructional sequence. *Review of research in education, 1*, 3-33.

Gagnè, R. M., Wager, W. W., Golas, K. C., Keller, J. M. & Russell, J. D. (2005). *Principles of instructional design*. Belmont, CA: Thomson Learning Inc.

Gómez, D., Zarrazola, E., Montero, J. & Yañez, J. (2012). Clasificación no supervisada y jerárquica basada en coloración en gras: una aplicación a imágenes astronómicas. In G. I. Sainz, J. Alcalá, R. García, B. Llamazares & M. J. de la Fuente (Eds.), *XVI Congreso español sobre tecnología y lógica Fuzzy (ESTYLF).* (p. 492-497). Valladolid: Universidad de Valladolid.

Goñi, J. M. (2011). El currículo de Matemáticas en la educación secundaria obligatoria y el bachillerato. In J. M. Goñi (Ed.), *Matemáticas. Complementos de formación disciplinar*. (p. 9-28). Barcelona: Graó.

Heymann, S. & Le Grand, B. (2013). Visual Analysis of Complex Networks for Business Intelligence with Gephi. *1st International symposium on Visualization and Business Intelligence, in conjunction with the 17th International Conference on Information Visualization.* (p. 307-312). ISC-PIF Publications.

Kaufmann, A. (1976). *Puntos y flechas: teoría de los grafos.* Barcelona: Marcombo.

Laita, L. M. & de Ledesma, L. (1997). Knowledge-based systems verification. In A. Kent and J. G. Williams (Eds.), *Encyclopedia of Computer Science and Technology.* (p. 253-280). New York: Marcel Dekker.

Laita, L. M., Roanes-Lozano, E., de Ledesma, L. & Alonso, J. A. (1999). A computer algebra approach to verification and deduction in many-valued knowledge systems. *Soft Computing*, *3*(1), 7-19.

Leuders, T., Barzel, B. & Hußmann, S. (2005). Outcome standards and core curricula: a new orientation for mathematics teachers in Germany. *ZDM, 37*(4), 275-286.

Ley orgánica de ordenación general del sistema educativo (LOGSE) (Ley Orgánica 1/1990, de 3 de octubre). *Boletín Oficial del Estado*, *nº 238*, 1990, 4 de octubre.

Ley orgánica de educación (LOE) (Ley Orgánica 2/2006, de 3 de mayo). *Boletín Oficial del Estado, nº 106*, 2006, 4 de mayo.

Ley orgánica para la mejora de la calidad educativa (LOMCE) (Ley Orgánica 8/2013, 9 de diciembre). *Boletín Oficial del Estado, nº 295*, 2013, 10 diciembre.

Leydesdorff, L. (2007). Visualization of the citation impact environments of scientific journals: An online mapping exercise. *Journal of the American society for Information science and technology, 58*(1), 25-38.

Martín, E. & Méndez, A. (2004). Aplicaciones de la teoría de grafos a algunos juegos de estrategia. *Suma, 46*, 31-35.

Martínez-Zarzuelo, A. (2015). Selección, organización y secuenciación del conocimiento matemático mediante teoría de grafos (Tesis doctoral). Universidad Complutense de Madrid, Madrid.

Martínez-Zarzuelo, A., Roanes-Lozano, E. & Fernández-Díaz, M.J. (2013). About Organizing and Structuring the Contents of Mathematical Subjects using Graph Theory, in Cavalcante, A. (eds.) *Graph Theory: New Research*, New York: Nova Science Publishers, Inc.

Moreira, M. A. (2000). *Aprendizaje significativo: teoría y práctica*. Madrid: Visor.

Mwakapenda, W. & Adler, J. (2003). Using concept mapping to explore student understanding and experiences of school mathematics. *African Journal of Research in Mathematics, Science and Technology Education, 7*(1), 51-62.

Nabiyev, V. V., Çakiroğlu, Ü., Karal, H., Erümit, A. K. & Çebi, A. (2016). Application of Graph Theory in an Intelligent Tutoring System for Solving Mathematical Word Problems. *Eurasia Journal of Mathematics, Science & Technology Education, 12*(4), 687-701.

Nieto, Á. (2013). Aplicación de nuevas herramientas basadas en estructuras de grafos para la caracterización de redes de áreas cortafuegos en la planificación forestal a escala del paisaje (Tesis doctoral)*.* Universitat de Lleida, Lleida.

Novak, J. D. & Gowin, D. B. (1988). *Aprendiendo a aprender*. Barcelona: Martínez Roca.

Peñaranda, M., Quiñones, E. & Osca, J. (2009). La revista" Anales de Psicología" desde una perspectiva de redes sociales. *Anales de psicología, 25*(2), 199-208.

Pérez, Á. L. (1998). *Mapa de experto tridimensional de dinámica: materiales para elaborar una macrosecuencia instruccional basada en la teoría de la elaboración de Reigeluth y Stein*.

Pérez, Á. L., Suero, M. I., Montanero, M. M. & Montanero, F. M. (2000a). *Mapas de experto tridimensionales.* Extremadura: Consejería de Educación, Ciencia y Cultura de la Junta de Extremadura.

Pérez, Á. L., Suero, M. I., Montanero, M. M. & Montanero, F. M. (2000b). *Mapas de experto tridimensionales: aplicaciones al diseño de secuencias instruccionales de física, basadas en la teoría de la elaboración*. Badajoz: Junta de Extremadura.

Pino, J., Jiménez, E., Ruíz, R. & Bailón, R. (2011). Evaluación de redes tecnocientíficas: la red española sobre áreas protegidas, según la Web of Science*. Revista española de documentación científica, 34*(3), 301-333.

Puchades, V., Mula, J. & Rodríguez, A. (2008). Aplicación de la Teoría de Grafos para mejorar la planificación de rutas de trabajo de una empresa del sector de la distribución automática. *Revista de Métodos Cuantitativos para la Economía y la Empresa, 6*, 7-22.

Raper, S. (2012). Graphing the history of philosophy. *Drunks and Lampposts blog, 13*.

Real decreto por el que se establecen las enseñanzas mínimas correspondientes a la Educación Secundaria Obligatoria (Real Decreto 1007/1991, de 14 de junio). *Boletín Oficial del Estado, nº 152,* 1991, 26 de junio.

Real decreto por el que se establecen las enseñanzas mínimas correspondientes a la Educación Secundaria Obligatoria (Real Decreto 1631/2006, de 29 de diciembre). *Boletín Oficial del Estado, nº 5*, 2007, 5 de enero.

Real decreto por el que se establece el currículo básico de la Educación Secundaria Obligatoria y del Bachillerato (Real Decreto 1105/2014, de 26 de diciembre). *Boletín Oficial del Esta*do, nº 3, 2015, 3 de enero.

Reigeluth, C. M. (1979). In search of a better way to organize instruction: The elaboration theory. *Journal of Instructional Development, 2*(3), 8-15.

Reigeluth, C. M. (1987). *Instructional theories in action. Lessons illustrating selected theories and models*. New Jersey: Lawrence Elrbaum Associates.

Reigeluth, C. M. (1999). The elaboration theory: Guidance for scope and sequence decisions. In C. M. Reigeluth (Ed.), *Instructional design theories and models: A new paradigm of instructional theor*y, 2, 425-453. New Jersey: Lawrence Erlbaum.

Reigeluth, C. M. (2013). *Instructional-design theories and models: A new paradigm of instructional theory*. New York: Routledge.

Reigeluth, C. M. & Curtis, R. V. (1987). Learning situations and instructional models. In R. Gagnè (Ed.), *Instructional technology: foundations*. (p.175-206). New Jersey: Lawrence Erlbaum.

Reigeluth, C. M. & Stein, F. S. (1983). The elaboration Theory of Instruction. In C. M. Reigeluth (Ed.), *Instructional design theories and models: An overview of their current status*, (p. 335-381). New Jersey: Lawrence Earlbaum.

Reigeluth, C. M. & Stein, F. S. (1987). Lesson blueprints based on the elaboration theory of instruction. In C. M. Reigeluth (Ed.), *Instructional theories in action: Lessons illustrating selected theories and models*, (p. 245-288). Hillsdale: LEA.

Rodríguez, J. L. (1983). La estructura del mensaje en el acto didáctico: revisión del problema y propuesta metodológica. *Enseñanza & Teaching: Revista interuniversitaria de didáctica, 1*, 57-76.

Rodríguez, M. L. (2008). *La teoría del aprendizaje significativo en la perspectiva de la psicología cognitiva*. Barcelona: Octaedro.

Romo, R., Vélez, H., Solís, G., Luna, B. C. & Espinoza, A. (2015). Estudio de la probabilidad de conexión en epilepsia: grafos. In *XII Congreso participación de la mujer en la ciencia.* León.

Sacristán, J. G. & Gómez, A. P. (1989). *La enseñanza: su teoría y su práctica* (Vol. 57). Madrid: Akal.

Sánchez, J. M., Moratalla, A. & Sanz, A. (2012). Grafos y diseño arquitectónico. In *II Jornada Internacional Matemáticas Everywhere (CIEM)*, (p. 203-216). Castro Urdiales.

Schubring, G. (1987). On the methodology of analysing historical textbooks: Lacroix as textbook author. *For the learning of mathematics, 7*(3), 41-51.

Schvaneveldt, R. W. (1990). *Pathfinder associative networks: Studies in knowledge organization*. New Jersey: Ablex Publishing.

Sherborne, T. (2014). Mapping the curriculum: How concept maps can improve the effectiveness of course development. In A. Okada, S.J. Buckingham Shum & T. Sherborne (Eds.), *Knowledge Cartography.* (p. 193-208). London: Springer.

Suero, M. I., Montanero, M. M. & Montanero, F. M. (1999). Los fenómenos físicos como contenido organizador. Los mapas de experto tridimensionales. *Cátedra Nova, 10*, 361-372.

West, D. B. (2001). *Introduction to graph theory* (Vol. 2). New Jersey: Prentice Hall.

White, D. R., Batagelj, V. & Mrvar, A. (1999). Anthropology Analyzing Large Kinship and Marriage Networks With Pgraph and Pajek. *Social Science Computer Review, 17*(3), 245-274.

Wilson, R. J. (1983). *Introducción a la teoría de grafos.* Madrid: Alianza Editorial.

Wilson, R. (2013). *Four colors suffice: how the map problem was solved.* New Jersey: Princeton University Press.

Zabalza, M. A. (2009). *Diseño y desarrollo curricular* (Vol. 45). Madrid: Narcea Ediciones.

❖ ❖ ❖