A Qualitative Study on the Development of Pre-service Teachers' Mathematical Knowledge for Teaching in a History-based Course

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This study explores the effect of a history-based course on pre-service teachers' mathematical knowledge for teaching (MKT). Data were collected from 10 pre-service teachers at a normal university in China. At the beginning of the course, participants were asked to submit their teaching plans followed by a teaching simulation on selected topics. They then received a history-based course particularly stressing the value of the history of mathematics in education. After being educated on the historical background of these topics, the participants were asked to think back while watching their simulation videos and then redesign their instructional plans on the same topics. We investigated these pre-service teachers' MKT through the simulated teaching videos, conducting semi-structured interviews, and analyzing their teaching plans and learning journals. Results suggest that these pre-service teachers' MKT had improved. Their changes in PCK were more significant than those in SMK. Nevertheless, the history-based course seemed to have had less influence on the participants' CCK and KCC. Possible reasons for the positive and negative effects are addressed.

Keywords: History of mathematics, mathematical knowledge for teaching, pedagogical content knowledge, pre-service teacher, subject matter knowledge

INTRODUCTION

With its significant role in influencing teachers' instructional practices, teacher knowledge has received much attention from educational researchers in recent decades (Ball, Hill, & Bass, 2005; Fennema & Franke, 1992; Hill, 2010; Kleickmann et al., 2013; Leninhardt, 1988). Ball and Bass (2003) have studied that the goal of improving students' learning mostly depends on improving teacher knowledge. The

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National Council of Teachers of Mathematics (NCTM, 2007）pointed out that

knowledge of teaching, of mathematics, and of students are essential aspects of what a teacher needs to know to be successful; and effective teaching in mathematics depends on a sophisticated knowledge about the interconnection of the three conceptual constructs. Teachers need to understand the big ideas of mathematics and be able to represent mathematics as a coherent and connected enterprise (Ball, Lubienski, & Mewborn, 2001; Hill, Rowan & Ball, 2005; Ma, 1999). In addition, teachers should develop a grounded comprehension of students as learners and a solid pedagogical knowledge to create a dynamic learning environment so as to enable students to build up a conceptual understanding of content knowledge and a procedural proficiency in mathematics.

Given the importance of teacher knowledge for students' progress, the teacher education program can therefore be regarded as a key target and lever of educational reform (Kleickmann et al., 2013). However, knowledge about how the teacher education program may affect pre-service teachers' development of their professional preparation remains limited (Cochran-Smith & Zeichner, 2005; Yasemin, 2012).This study aims at investigating how Chinese pre-service mathematics teachers developed their teaching knowledge in a history of mathematics course. Additionally, we are also concerned about how different teaching approaches may have an impact on these pre-service teachers' knowledge for teaching.

State of the literature

- Previous researches have suggested that improving students' learning mostly depends on improving teacher knowledge, but the understanding of how teacher education program may affect pre-service teachers' knowledge remains limited.
- For its reliability and validity for interpreting teachers' practices, MKT theory has been adopted by many relevant studies.
- Though the relationship between the history of mathematics and teachers' MKT has been discussed theoretically, it has rarely been studied empirically, which is the focus of the present study.

Contribution of this paper to the literature

- This study aims at investigating how Chinese pre-service mathematics teachers developed their teaching knowledge in a history of mathematics course.
- The result reveals that the history of mathematics did have a positive effect on pre-service teachers' MKT in almost all dimensions, especially in KCT.
- Findings also reveal that different types of content exerted various effects on MKT in which evolutionary type of knowledge affected widely.

THEORETICAL FRAMEWORK

As early as the 1930s, Dewey (1938) had indicated that the teacher is a mature and knowledgeable individual responsible for effective teaching and successful learning in the classroom. Among all variables contributing to effective teaching, teachers' training had been viewed as the most significant one. However, in the 1960s, studies investigating the relationship between teachers' mathematics knowledge and students' achievement suggested otherwise (Begle, 1979). Begle (1979) conducted a meta-analysis surveying the correlation between these two aspects and indicated that there were only 10% of studied cases suggesting that the variable of "course taking" may produce positive effects on student achievement. Even worse, 8% of them yielded negative effects. In the 1970s, influenced by the study of cognitive psychology, scholars turned toward investigating the relationship between teachers' knowledge and their cognitive processes; including teachers' decision making and thinking, and teacher's knowledge individuality and subjectivity. Some others (Carter, et al., 1987; Leninhardt, 1988) conducted comparison studies to contrast novice teachers' and expert teachers' instructional practices to identify their differences in implementing course content. It was found

that, compared to their novice counterparts; expert teachers were more likely to implement materials at their disposal. However, studies along this line paid much more attention to pedagogical issues and neglected a wide range of teachers' knowledge.

By taking both variables of knowledge and pedagogy into account, Shulman (1986) distinguished three categories of teacher knowledge: (1) content knowledge, (2) pedagogical content knowledge, and (3) curricular knowledge. Shulman (1987) expanded the number of categories to seven by adding (4) general pedagogical knowledge, (5) knowledge of learners, (6) knowledge of educational context, and (7) knowledge of educational purposes, values, and their philosophical and historical grounds. Shulman's work had a significant influence on teacher education because it highlights the complexity of teaching and sets an agenda for future studies. Since then, these issues have received worldwide relevant researchers' attention to investigate in what way and to what extent teacher knowledge may have an impact on teaching and learning (An, Kulm, & Wu, 2004; Ball, Thames & Phelps, 2008; Fennema & Franke, 1992; Grossman, 1995; Hsien, 2013; Ma, 1999). These categories can be generally summarized into subject matter knowledge (SMK) and pedagogical content knowledge (PCK) (Ball et al., 2005, 2008; Holmes, 2012; Jones, 2000; Kleichmann et al., 2013; Olanoff, 2011), which will be explored in more detail below.

SMK and PCK

Subject Matter Knowledge (SMK), or also known as Content Knowledge (CK), refers to the basic knowledge of subject content. In mathematics, SMK refers to conceptual knowledge and procedural knowledge, including mathematical concepts, theorems, properties and formulas. SMK is usually considered one of the most important domains of teacher knowledge. Many studies have posited the importance of profound mathematical content knowledge in mathematics teachers' knowledge. (An & Wu, 2012). Before the 1970s, it was believed that the more a teacher knew about his subject matter (SMK), the more effective he would be as a teacher (Begle, 1979). However, as new evidence appeared, this belief faced more challenges. Although, SMK still remains essential in a teacher's knowledge, it is not as critical (Fennema & Franke, 1992；Porter & Brophy, 1988); at least, its relationship to the quality of teaching is not linear. In the 1980s, a relevant but different dimension, pedagogical content knowledge (PCK), was detached from SMK and received a lot more attention in the education community.

Pedagogical content knowledge (PCK), also known as pedagogical knowledge (PK), refers to a teacher's knowledge about how to teach, including how to select curriculum content, how to organize lesson plans, and deciding what is the best way to convey the concept etc. (Shulman, 1986). Many studies have suggested that both teachers' PCK and SMK affect student learning, but some found that despite the high correlation between SMK and PCK, SMK had a lower predictive power for student progress than did PCK (Baumert et al., 2010; Monk, 1994). This may be due to the idea that teachers' understanding of knowledge to be taught, curriculum design, and students' cognitive ability can be transmitted via PCK for effective practice in the classroom (An, Kulm, & Wu, 2004; Fennema & Franke, 1992; Seung, 2012).

Furthermore, PCK has had decisive impact on key aspects of instructional quality (Kleichmann et al., 2013). There is some consensus and preliminary evidence for the notion that SMK might be a prerequisite for PCK development (Kahan, Cooper, & Bethea, 2003; Kleichmann et al., 2013). Despite the clear theoretical distinction between SMK and PCK, findings on their empirical separateness are not that clear. Hill, Schilling, and Ball (2004) found that elementary teachers' SMK and PCK in

mathematics are merged in a single body of knowledge. They then combined the two interrelated dimensions and named it mathematical knowledge for teaching.

Mathematical knowledge for teaching

Shulman's PCK not only provides a conceptual orientation of the nature and types of knowledge needed for teachers, but also highlights the content-intensive nature of teaching. Particularly, Shulman specified the ways in which content knowledge for teaching is distinct from disciplinary content knowledge. Nonetheless, PCK alone may not be able to prepare schoolteachers' capabilities of carrying out all practices in the real and complicated classroom setting. After analyzing several classroom lessons at the elementary level, Ball (1990) distinguished teachers' knowledge of mathematics (concepts and ideas), knowledge about mathematics (procedures and strategies), and knowledge about completing mathematical problems (validity of claims and appropriateness of representations). Though there was a lot of research following Shulman's paradigm, "the actual mathematical content that teachers must know how to teach has yet to be precisely mapped" (Hill, Schilling, & Ball, 2004). Through a series of educational investigations, including classroom observations, interviews, analyzing exam sheets, and coding of students' achievement (Ball & Bass,
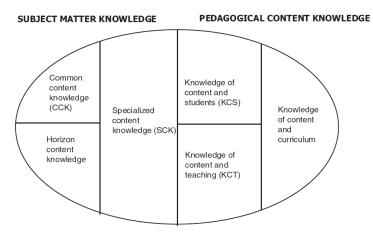

Figure 1. Domains of Mathematical Knowledge for Teaching (Ball et al., 2008)

2003; Hill, Rowan, & Ball, 2005; Hill, Schilling, & Ball, 2004; Schilling & Hill, 2007), Ball and her colleagues constructed a Mathematical Knowledge for Teaching (MKT) framework that expanded Shulman's theories and applied them to mathematical teaching at the elementary level (Figure 1, Ball, Thames, & Phelps, 2008).

In figure I, SMK contains three sub-domains in which Common Content Knowledge (CCK) refers to the mathematical knowledge known to be in common with others, Specialized Content Knowledge (SCK) is the kind of detailed knowledge corresponding to a particular situation for teaching, and Horizon Content Knowledge (HCK) is teachers' awareness of how mathematical topics are related over the span of the curriculum. On the other hand, PCK consists of Knowledge of Content and Students (KCS), Knowledge of Content and Teaching (KCT), and Knowledge of Content and Curriculum (KCC). KCS stresses how teachers know about students' understanding of curriculum; KCT denotes the knowledge of arranging lesson plans; KCC is about knowledge of the relationship between content being taught at a particular moment and the whole curricular development. For its reliability and validity for interpreting teachers' practices, MKT theory has been adopted by many relevant studies. For instance, in determining the work for teaching prospective teachers fractions, multiplication, and division, Olanoff (2011)

conducted a qualitative study on three experienced educators on the basis of MKT framework. Yasemin (2012) explored in what way and to what extent the changes in teachers' MKT would be related to the changes in their instructional behavior and student achievement. Though, as Ball, Thames, and Phelps (2008) indicated, the MKT framework is not a comprehensive and complete conceptual model that always calls for ongoing revision, it does provide a scientific basis for surveying and analyzing complicated interrelationships between teachers' mathematical knowledge and practices.

Teacher knowledge and the history of mathematics

The value of the history of mathematics in education has been addressed by many studies, and several of them have discussed the influence of the history of mathematics on teachers (e.g., Clack, 2012; Liu, 2003, 2009; Tzanakis & Arcavi, 2000). By proposing several inspiring questions, Freudenthal (1981) had early indicated that the history of mathematics may help teachers to appreciate the process of mathematical creativity, rather than its product. Schubring (2000) claimed that schoolteachers may have a better understanding of curricular development through realizing past development of mathematics. Liu (2003) and Tzanakis and Arcavi (2000) considered history as a means for revealing the nature of mathematical activities, which may provide teachers a guide for teaching and enrich their instructional strategies.

The aforementioned arguments were supported by several studies. For instance, Furinghetti (2007) found that teachers demonstrated different beliefs of mathematics after participating in a program involving reading and discussing historical materials; Arcavi and Isoda (2007) reported how a 3-stage workshop about ancient Egyptian mathematics may promote pre-service teachers' professional development. Charalambos, Panaoura, and Philippou (2009) tried to induce pre-service teachers' changes in their beliefs and attitudes toward mathematics through incorporating the history of mathematics in a teacher education program. They did find some promising outcomes, such as their strong Platonist beliefs about the nature of mathematics thinking that mathematics is a fixed body of eternal precise truths, became weaker and their development of efficacy beliefs was in the expected direction. Nonetheless, they also reported that not every participant saw the merit of the history of mathematics for their teaching. In Clark's study (2012), pre-service teachers participated in a history of mathematics course in which they were asked to develop history-based course plans after reading, discussing, and analyzing relevant historical topics and events in mathematics. She found that these pre-service teachers not only gained a better appreciation of mathematical knowledge, but also developed an emotional attitude toward mathematics.

Although many studies have explored the role of the history of mathematics, investigating the effect of the history of mathematics on teacher knowledge has just begun to receive attention. Jankvist, Mosvold, Fauskanger, and Jakobsen (2012) took negative numbers and number systems as instances to make a theoretical argument about why MKT should become one of the significant strands of the area of history and pedagogy of mathematics (HPM). Instead of focusing on why and how of using history in mathematics education, they stressed that future research should elaborate more on "whats" (i.e., what kinds of historical knowledge are required for the teaching of mathematics). Following this appeal, Mosvold, Jakobsen, and Jankvist (2014) discussed how each domain of MKT may profit from the history of negative numbers, equations, functions, logarithmic and exponential functions, and Lagrange's and Cayley's algebra. Though the relationship between the history of

mathematics and teachers' MKT has been discussed theoretically, it has rarely been studied empirically, which is the focus of the present study.

PURPOSE OF THE STUDY

Studies consistently indicated that what mathematics teachers need to know, even at the elementary level, is much more complex than it was originally thought (e.g., Ball, Lubienski, & Mewborn, 2001; Ma, 1999). Furthermore, researchers have shown that elementary teachers, either pre-service or in-service, may not be well equipped with this deeper knowledge (e.g., Ball, 1990; Ma, 1999; Yasemin, 2012). The growing knowledge of teaching mathematics demands teacher educators to help teachers acquire this knowledge (Olanoff, 2011). While a large amount of researchers have drawn attention to mathematical knowledge necessary for teachers, the understanding of how a history-based course in the teacher education programs might affect the development of teachers' professional knowledge remains limited (Cochran-Smith & Zeichner, 2005). The main purposes of the present study are:

(1) Determining that after receiving the history-based course, in what way and to what extent do the pre-service teachers' mathematical knowledge for teaching change?

(2) If these pre-service teachers demonstrated any change in their mathematical knowledge for teaching, how were their changes related to, if at all, the course content?

METHDOLOGY

Participants

One hundred mathematics majors from a medium sized university in China attended the course. Most of these students chose teaching as their future career and enrolled in the teacher education program at a university in China. They were in their junior year and had received fundamental courses of college mathematics, including calculus, advanced algebra, and analytic geometry. Ten of them were randomly selected and invited to participate in the study.

The instructor of this course

The researcher of this study is also the instructor of the course. The researcher studied the history of mathematics when he was a graduate student and soon recognized that the historical development of mathematical knowledge not only demonstrates the zigzag path of the progress of mathematics but also shows how mathematicians exerted creative approaches, either naïve or sophisticated, to resolve problems. Particularly, history made him realize that logic and deduction are not the whole story of mathematics. Rather, survey and induction are also play significant role. He therefore gradually comprehended that the history of mathematics may help teachers appreciate the true nature of mathematics as well as assist students to gain a real understanding of mathematics. In this manner, he was convinced that the history of mathematics could benefit pre- and in-service teachers' MKT.

Instruments and Processes

By considering their historical backgrounds, ten mathematics topics ranged from seventh grade to ninth grade were selected and then four in-service mathematics teachers were asked to evaluate the levels of difficulties for secondary students. Five

topics, including irrational number, negative multiplication, the Pythagorean Theorem, solution of quadratic equation, and the application of similar triangles, were taken into account as a result.

A course of the history of mathematics

A 2-credit history of mathematics course was offered for these pre-service teachers. In addition to the regular content of a traditional history of mathematics course, such as mathematics of ancient civilizations (Babylonians, Egyptians, Greeks, and China), mathematics in the Middle Ages (the Europeans, the Arabs, and the Indians), and modern mathematics, this course particularly emphasized the value of the history of mathematics in education by reminding these pre-service teachers in what way the historical episodes can be related with the mathematics curriculum. The reason for doing so was to develop their historical sensation of mathematics and encourage them to include historical background of mathematical concepts in their future teaching. In every lecture, they were shown some cases about the way of integrating the history into the mathematics curriculum.

Simulation teaching

To identify ten participating pre-service teachers' initial mathematics knowledge on the five topics, they were asked to do simulation teaching during the course. Prior to introducing the historical background of the five topics, participating pre-service teachers were asked to hand in their teaching plans and demonstrate how they will convey the topics in the future. Each simulation teaching lasted 20 minutes and was video-taped. Therefore, 50 instructional episodes in total were collected and transcribed for the subsequent analysis.

Semi-structured interviews and reflection

After the historical background of each topic was introduced in class, all participating pre-service teachers were invited to participate in follow-up interviews to reflect upon their simulation teaching. While watching teaching videos during the interviews, they did think-back activities on their teaching and explained why and how they were doing so. They then submitted a new teaching plan on the same topic highlighting which parts they would change and how they would make the change, and why they would make that change in future teaching.

Researcher's and pre-service teachers' journals

The teacher of the history-based course was the researcher himself. In order to reduce researcher's potential bias to a minimum level and keep the researcher on the right track, the researcher noted student teachers' feedback and his own reflections every week. The ten participating pre-service teachers also handed in learning journals that described their responses and feedback about the course at the beginning, the middle phase, and the end of the semester. The researcher's and pre-service teachers' journals serve as complimentary evidence for subsequent data analysis.

Data analysis

Data collection and analyses were conducted in five rounds under the framework of MKT. For each round, there were five kinds of data (original teaching plan, video of simulation teaching, transcription of interview, updated teaching plan, and pre-service teachers' learning journals) available for subsequent analysis. A particular focus was made to compare and contrast, if any, the differences of participants' SMK and PCK between the original and updated teaching plans. If any changes in the teaching plans were found, the researcher would look into their transcriptions of

interviews to identify factors causing the differences and determine if these potential changes could be attributed to the course of the history of mathematics. An external reviewer was invited to evaluate the appropriateness of the researcher's judgment. Any different interpretations about respondents' claims were negotiated to reach agreement. If agreement could not be made on certain aspects, the data were excluded from this study.

RESULTS

Research findings are demonstrated in two main domains. First, participating pre-service teachers' changes in their SMK and PCK will be reported. Second, in what way and to what extent the course of the history of mathematics caused these changes will be discussed.
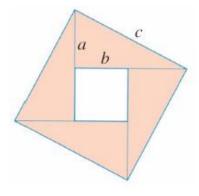
Pre-service teachers' changes in SMK

Research findings indicate that these pre-service teachers did show some changes in their SMK, but the connection with the history of mathematics was not linear since evidence suggests that the length of time exposed to the history of mathematics was significantly related to the degree of changes in SMK. Furthermore, the degree of change in their CCK, SCK, and HCK were varied for each round of observations.

In the first round, some participants (PT1, PT3 and PT10) learned that, by means of proving by contradiction, $\sqrt{2}$ can be shown to be an irrational number (CCK aspect). These pre-service teachers realized the connection between non-cyclic characteristics of decimals and irrationality through the history of mathematics (SCK aspect). Prior to the history-based course, their knowledge regarding irrational numbers was discrete, yet they turned to understand that the birth of irrational numbers is a product of social needs. Furthermore, history also helped them to be aware that the cardinality of irrationals is greater than that of rational numbers and the definition of irrational numbers in the textbook is more suitable for learning than other definitions of history (HCK aspect).

In the second round of investigation, PT3, PT4, and PT6 concurrently claimed that the history of mathematics helped them realize that the rule of "two negatives make a positive" can be proved in the ring of integer. Because they did not learn how to prove the rule before but only knew that it was provable, it can be categorized that their increased understanding was a minor SCK change. Some participants' HCK had also been changed, but the degree was insignificant. For instance, PT7 indicated that she did not realize that the definition of negative numbers is based upon the theory of integers until she learned how Weierstrass and Peano defined negative numbers. PT2 mentioned history helped her understand how the demand of extending computational domain in daily life gave birth to negative numbers, which became well-defined later.

Pre-service teachers' CCK seemingly showed minor change, but SCK

Figure 2. Diagram of Hypotenuse

demonstrated significant change in the third round. They began to appreciate that there were several ways of proving the Pythagorean Theorem and recognize inappropriate methods introduced in textbooks. This allowed them to have multiple ways to explain the rationality of the Pythagorean Theorem (SCK aspect). For instance, before attending the history-based course, PT3 only taught the Pythagorean Theorem using the Diagram of Hypotenuse (Figure 2) provided in the textbook. While knowing about its history, she demonstrated multiple ways of proving this proposition and stressed their historical backgrounds in the lesson plan. It appears that her SCK had been improved.

Furthermore, some of them had a better understanding about the link between the Pythagorean Theorem and other mathematical knowledge (HCK aspect), which can be manifested by PT2 and PT10's claim that history reveals the relationship between the Pythagorean Theorem and other areas, such as trigonometry, vector, Fermat's Theorem, and distance between two points.

The course introduced the development of linear equations with two variables in the fourth round and demonstrated various historical approaches for solving the equation. Findings indicate that history helped some pre-service teachers to be aware of multiple ways for solving linear equations with two variables, particularly geometrical fashion in ancient Arabic mathematics, and realized the reason why negative roots were not admitted. PT7 used to implement the algebraic style of "completing the square" to solve the quadratic equation. However, she shifted the algebraic approach to the geometric one by adopting the Arabic "diagram-cutting method", transforming $x^2 + 10x = 39$ into $(x+5)^2 - 25 = 39$ (Figure 3). Such historical insight may not only help students appreciate the problem of solving equations in an intuitional way but also create a dynamic image of concept in their minds. This is another sign of improvement of pre-service teachers' SCK.

Nonetheless, they showed minor changes in CCK and HCK. Furthermore, their SMK almost remained the same even though they learned of the discussions about similar triangles in history in the last round of the investigation, indicating that the influence of the history of mathematics may not be continuous and gradually accumulated. Aforementioned results can be summarized in Table 1.

The influence of the history of mathematics on these pre-service teachers' SMK can be concluded as below:

The influence is non-linear

It is generally assumed that pre-service teachers' literacy could have been improved as long as they learned more historical knowledge about mathematics. However, our findings reveal that it is not always the case. Pre-service teachers' SMK only demonstrated significant differences in the first, third, and fourth round. There was not any change identified in the fifth round.
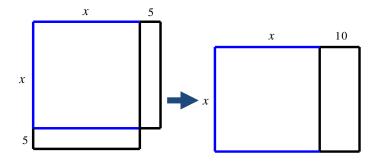

Figure 3. The Arabic diagram-cutting method

Table 1. Pre-service teachers' changes in SMK

No. of Rounds	Highlights of Changes	Categories	Remark
1st round	Realizing that by means of proving by contradiction, $\sqrt{2}$ can be shown to be an irrational number.	CCK	PT1, PT3, PT10
	Knowing the origins of irrationals.	SCK	All participants
	Understanding the cardinality of irrationals is greater than that of rational numbers.	HCK	All participants
2nd round	Realizing that "two negatives make a positive" can be proved in the ring of integer.	SCK	Minor change
	Understanding that definition of negative numbers is based on the theory of integers; other definitions of negative numbers and their relations.	HCK	Minor change
3rd round	Appreciating more proof about the Pythagorean Theorem and realizing that it is inappropriate to prove Pythagorean Theorem by means of $\sin^2 x + \cos^2 x = 1$.	SCK	All participants
	Understanding the relationships among Pythagorean Theorem, vector, and Fermat Last Theorem.	HCK	PT2、PT10
4th round	Being aware of ancient geometrical approaches for solving linear equations with two variables and why only positive roots are admitted.	SCK	All participants
5th round	None	None	None

The degree of change in CCK, SCK, and HCK were varied

As indicated in Table 1, pre-service teachers' changes in SCK were noted in four out of the five rounds of investigation. Their improvements in HCK were found in three of the five rounds and the change in CCK was only exhibited in the first round. It suggests that the history of mathematics had a minor impact on pre-service teachers' ontological understanding of mathematics, but was benefitted by their appreciation of connections among all mathematical topics, which can be manifested by PT10's reflection in his journal:

> I had a comprehensive understanding about the developmental processes of irrationals and negative numbers in the first and second round. In the third and fourth round, I learned more ways of proving the Pythagorean Theorem and solving linear equations with two variables. In the fifth round, I realized the application of similar triangles in history. All of these can be modified to become instructional sources.

Pre-service teachers' changes in PCK

In this study, we found the evolution of their PCK was continuous and related to knowledge of the history of mathematics, but degrees of influence for KCT, KCS, and KCC were varied.

In the first round, prior to knowing the historical development of irrationals, they were more likely to show $\sqrt{2}$ as an irrational number by using hand-set calculators to demonstrate that it cannot be represented as terminating or repeating decimals. After that, they will give the definition of an irrational number, and then let the students do the exercises in the class. Similarly, in the second round, many of them showed the rule of two negatives makes a positive by induction. After introducing the rule, they let the students do the exercises in the class.

After they know the history of irrational numbers and negatives, the participants realized that the irrational numbers and negatives have a winding development process; the students cannot easily accept and therefore the teaching design should not be too simple. Then, these pre-service teachers adjusted their teaching design, increased explanation time, demonstrated and discussed, and reduced the number

of exercises. For instance, some of them began to prove the irrationality of $\sqrt{2}$ by contradiction. Similarly, in the second round, many of them explained the rule of two negatives make a positive by telling stories instead of merely showing rules of multiplication in the beginning. The reason for doing so can be understood by PT6's claim that

> Negative numbers could not be accepted until thousands of years later. It may not be easy for students to realize the rule of two negatives make a positive. Besides, the stories of how mathematicians explained this rule could make them feel they are not alone. (PT6, 2nd round interview)

It can be seen that the history of mathematics helped these pre-service teachers have a better understanding about students' thinking processes and thus they can catch the point of designing teaching plans (KCT and KCS aspects). However, we also found that some of them (e.g., PT7) proved $\sqrt{2}$ is an irrational number by using the Pythagorean Theorem, neglecting the fact that the Pythagorean Theorem is introduced at 8th grade while $\sqrt{2}$ appears at 7th grade. This suggests history of mathematics may fail to benefit their KCC aspect.

Note that, in the third round, participating pre-service teachers tried to learn more historical knowledge from the internet or books on their own accord and exhibited more willingness to integrate history into micro-teachings. As PT3 indicated,

> I used to teach Pythagorean Theorem according to the method shown in the textbook. I was reluctant to allow students to share their methods because I was afraid that they showed me something that I was not aware of. But now, I have become more confident and like to encourage students to develop their own ways for proving the Pythagorean Theorem. If they are able to make it, I'll let them know their approaches happen to be the same as one of the mathematicians' ideas. (PT3, 3rd round interview)

History provided them with multiple methods for proving the Pythagorean Theorem and enriched their teaching methods (KCT aspect). In addition, history helped these pre-service teachers realize that the Pythagorean Theorem is gradually developed from daily experiences and thus it would be hard for students to discover the rule. Instead, encouraging students to test the reasonableness and make an argument for it should be the focus of teaching (KCS aspect).

In the 4th round of investigation, the pre-service teachers not only showed more diverse instructional strategies for solving quadratic equations, but also used or modified several historical problems in the classroom. For instance, some pre-service teachers use the methods of ancient Arabic geometrical approaches to solve quadratic equations in simulated teaching. The reason for doing so can be understood by PT5's claim that, through demonstrating old methods students could be impressed and enlightened by this naïve but creative thinking, which is also a cultural lesson. For most cases, mathematical methods have matured and become more rigorous with time. Teachers' instructional designs may be benefited from the history of mathematics because historical approaches remind them of the potential flaws in students' thinking. PT10 indicated that, "we always have trouble figuring out students' difficulties, but historical episodes gave us help for designing curriculum." This finding supports the effect of the history of mathematics on teachers' KCS dimension. Furthermore, any changes in KCT and KCC were seen in the fourth round. The characteristics of the methods of solving quadratic equations in different periods helped the pre-service teachers to understand students' thinking (KCT). The historical development of the quadratic equations also increased their

understanding of the relationship between teaching knowledge and the course (KCC).

The pre-service teachers' use of history became more evident in the 5th round of investigation. They implemented and adjusted a lot of historical problems about similar triangles in their instructions, which could be a sign of their development of KCT. In the interview, PT6 professed that there were few appropriate examples in the textbooks and those regular problems could hardly draw students' attention. However, historical mathematicians' creative manipulations of similar triangles in solving practical problems may humanize mathematics as well as broaden their vision for teaching, which is a development of KCS.

The effect of the history of mathematics on these pre-service teachers' PCK in each round can be summarized in Table 2.

Aforementioned analysis indicates that history of mathematics had two particular effects on pre-service teachers' PCK:

The influence of history increased along with the time

Data indicates that, throughout the five rounds of investigation, not only the history of mathematics had an impact on these pre-service teachers' PCK, but its effect increased along with time. In the first two rounds, they understood the concepts of irrationals and negative numbers developed in a zigzag path, and this understanding urged them to restructure teaching plans for responding to students'

Table 2. Pre-service teachers' PCK development in each round

No. of Round	Highlight of Development	Categories	Individual Development
1st round	The zigzag developmental path of irrationals help them to realize that its conception may not be easily understood by students. Introducing more historical background of irrationals is necessary.	KCT KCS	All participants
	History motivated some of them to reorganize curriculum.	KCC	PT7 induced irrationals from Pythagorean Theorem, which is not in accordance with the sequence in the textbook.
2nd round	The history of negative numbers helps them realize that the rule of two negatives makes a positive not being accepted by students intuitively. More detail is necessary in curriculum design.	KCT KCS	All participants
3rd Round	They learned various ways of proving the Pythagorean Theorem in history.	KCT	All participants
	History reminded them that discovering Pythagorean Theorem is difficult. It had better involve students to experience its validity.	KCS	All participants
4th Round	They learned more solutions of quadratic equations from history.	KCT	All participants
	While learning different solutions of quadratic equations in history, they were more likely to comprehend students' thinking processes and willing to redesign their teaching plans.	KCS	All participants
	Some of them realized the solution of quadratic equations is based on linear equations and students' understanding of quadratic equations is subject to their conceptions of quadratic functions.	KCC	PT2, PT3, PT6, PT7, PT10
5th Round	They learned more applications of similar triangles from history and used these historical examples in their teaching.	KCT	All participants
	While learning different applications of similar triangles in history, they were more likely to comprehend students' thinking processes and willing to redesign their teaching plans.	KCS	All pre-service teachers

difficulties. While entering the second half of the study, they showed more eagerness to learn historical knowledge and used them in teaching when appropriate. It appears that, in a broad sense, the effect of history of mathematics on PCK had been accumulated along as time went by. For instance, PT2 was skeptical about the effect of the history of mathematics in the beginning of the course, yet he turned to realize that history not only provides him a lot of ideas for designing curriculum, but also helps him comprehend students' difficulties in learning mathematics.

Degrees of influence varied across different sub-domains

Despite the influence of history on PCK being positive, the finding also suggests it may not benefit all sub-domains of PCK. As shown in the Table 2, the pre-service teachers demonstrated a greater degree of changes in their KCT and KCS, whereas KCC was not so significant. The history of mathematics provides available materials for teaching and enriches their teaching strategies, which is a sign of KCT improvement. In addition, historical knowledge developed a sympathetic attitude toward students' difficulties (KCS aspect). Nevertheless, their KCC were changed in the first and fourth round only. There even was a negative effect in the first round and only half of the participants had minor change in the fourth round.

Pre-service teachers' changes in MKT

The study also aimed to determine in what way and to what degree history of mathematics may influence pre-service teachers' MKT. We'll report the outcome in terms of content and method.

Historical content and MKT

In the 1st and 2nd rounds, the pre-service teachers learned the evolution of concepts of irrationals and negative numbers. Such "evolutionary type of knowledge" provided them a holistic vision with which each topic is related and increased their understanding of students' potential struggles, promoting their HCK and KCS. In the 3rd round, various forms of proofs about the Pythagorean Theorem were introduced to them and this "synthetic type of knowledge" strengthened their appreciation of mathematical creativity, benefiting their SCK and KCT. In the last two rounds, the researcher demonstrated how different ancient civilizations solved quadratic equations in diverse ways and how concepts of similar triangles were used to resolve practical problems. It was found that these pre-service teachers were impressed by the "ethnographic type of knowledge", which has a significant impact on their KCT. A typical response can be read in PT2's learning journals:

> I was impressed by the history of irrationals and negative numbers. This information gave me a whole understanding regarding the development of irrationals and negative numbers. It deepens my recognition about the two topics, enriches my teaching, and helps me understand students' learning processes much more. Learning the history of Pythagorean Theorem is quite a special experience. I not only learned a lot of ways for proving the theorem, but was also aware that, without a careful teaching plan, encouraging students to do measurement or cut-and-rearrange triangles may not be so meaningful. The history of quadratic equations and similar triangles provide us a lot of examples that can be used directly or adapted when necessary. Students can comprehend the merit of modern methods when they realize how a quadratic equation had been solved in different places and times.

Note that the degrees of pre-service teachers' change in their MKT were varied. PT2's MKT in general, PCK in particular, demonstrated more significant change than that of PT1. It can be attributed to PT2's effort devoted to this course. A summary of the effect of historical content on MKT is shown in Table 3.

Table 3. Effects of historical content on MKT

Domains of MKT	SMK			PCK		
Types of Content	CCK	SCK	HCK	KCT	KCS	CCK
Evolutionary	○	○	●	○	●	--
Synthetic	--	●	○	●	○	--
Ethnographic	--	○	--	●	○	○

● *strong influence*   ○ *week influence*   --*insignificant influence*

Instructional method and MKT

Data suggests that the complimentary teaching approach employed by the researcher also caused a certain degree of effect on these pre-service teachers' MKT. For encouraging these pre-service teachers to reflect on what they had learned about the history of mathematics and how historical knowledge may be involved in their teaching, the researcher organized discussion groups to share conceptions of (1) what mathematical insight they gained from history, (2) when is the best time for implementing historical instances, and (3) whether the historical approaches is appropriate strategies or not. They generally claimed that group discussion urged them to contemplate how historical instances can be implemented as well as promotes their mathematical vision. In PT5's final reflection journal, she said:

> Group discussion in the classroom is good for me. I learned a lot of others' ideas through communications and debates. For instance, I have a better judgment about how and when to use history in my teaching.

Classroom observations showed that, in group discussions, everyone was free to express their points of view, especially the doubt in their minds, helping them to realize that it is not always a better strategy to put more history in teaching. It can be seen that organizing group discussion is critical for developing their MKT.

It was also found that micro-teaching may play a role in improving their MKT. The pre-service teachers were required to hand in their instructional plans and micro-teach. They complained about this assignment at the beginning, but soon recognized that this activity urged them to clarify the value of history as well as facilitated their comprehension of integrating history into teaching, as indicated by PT9 in his reflection journal:

> Frankly speaking, I regretted to attending this course when I was asked to do these assignments. But I later found that my understanding of history of mathematics was upgraded whenever I finished micro-teaching. It is more helpful than just sitting and listening in the classroom. Spending more time preparing for micro-teaching is worthwhile.

The aforementioned evidence suggests that encouraging pre-service teachers to write teaching plans and then micro-teaching are essential for them to connect isolated historical knowledge. Their MKT could be advanced in this manner.

CONCLUSION AND DISCUSSION

Through investigating ten pre-service teachers lasting a whole semester in five rounds, our findings suggest that (a) history of mathematics promoted MKT and (b) content and instructional methods generated different effects that are summarized and discussed below:

History of mathematics promoted MKT

In each round of investigation, the history of mathematics did have a positive effect on pre-service teachers' MKT in almost all dimensions. Results indicate the history of mathematics has more significant effects on the dimensions of HCK, SCK, KCT, and KCS, yet it was less likely for CCK and KCC. The development of HCK can be attributed to their understanding of the connection among various mathematical concepts that were improved by the history of mathematics, which is particularly so in the 1st and 2nd rounds. Being aware of multiple solutions and proofs for certain problems in history (such as the proof of the Pythagorean Theorem and solutions of quadratic equations) not only helped them gain creative insight into mathematical problem solving, but also empowered them to better interpret mathematical concepts, benefiting their SCK.

In addition, history provides them several examples, which can be directly used or adapted, to stimulate students' mathematical thinking. Compared to regular exercises practiced in the class, these historical instances are more realisitc and humanisitc, hence better for motovating students' interest. This appreciation could advance their KCT. Data suggests their KCT was improved in all of five rounds.

On the other hand, the pre-service teachers were short on practical teaching experiences and less likely to have an appropriate understanding of students' modes of thinking and learning obstacles. The zigzag developmetal path of mathematical concepts (e.g., irrationals and negative numbers) reminded them of future students' potential difficultuies. This is the merit of the history of mathematics for their KCS. In general, we observed that the history of mathematics has a more significant effect on PCK than on SMK, which could be due to PCK being more observable than SMK.

Content and instructional methods generated different effects

Findings also reveal that different types of content exerted various effects on MKT in which evolutionary type of knowledge was widely affected, particularly on HCK and KCS. Because the evolutionary type of knowledge shows the development of mathematical knowledge, it not only could help the pre-service teachers to have a deeper understanding about the nature of mathematical knowledge, including contextual connections among different concepts, but can also develop their sympathetic attitudes toward students' mistakes and difficulties in the process of learning. Synthesized type of knowledge, demonstrating diverse mathematical approaches in history affected their SCK, HCK, KCT, and KCS in general, and SCK and KCT in particular. The study found the synthesized type of knowledge was favored by these pre-service teachers and used, either directly or indirectly, in their micro-teaching.

Ethnographic type of knowledge focused on diverse mathematical ideas among different regions in the world, which may cultivate their understandings of cultural creativities and humanistic appreciations. Such understandings and appreciations could have a positive effect on the development of SCK, HCK, KCT, and KCS, in which KCT was more significant. These results may have implications for future relevant studies aiming to develop specific dimensions of teachers' MKT. Our observations also indicate that micro-teaching is crucial in developing MKT because it promoted these pre-service teachers' abilities of integrating theoretical and practical knowledge. In group discussions, the pre-service teachers received several critical opinions regarding integrating history in mathematical teaching, preventing them from accepting the HPM paradigm without a second thought, which is beneficial for their MKT.

In an empirical setting, Clark (2012) found that the study and application of the history of solving quadratic equations enlightened prospective mathematics teachers' historical and cultural knowledge for teaching secondary school algebraic

topics. On the other hand, Mosvold, Jakobsen, and Jankvist (2014) theoretically discussed how and why history of mathematics may have its place in improving teachers' MKT. Our findings generally endorse their claims but still indicate some other issues. Despite the aforementioned positive effect on these pre-service teachers' MKT in several sub-domains, the present study nevertheless showed its weakness in promoting their CCK and KCC. The CCK (Common Content Knowledge) refers to the mathematical knowledge that is shared with others and KCC (Knowledge of Content and Curriculum) is about knowledge of the relationship between the content being taught at a particular moment and the whole curricular development. The former is knowledge that can be shared by other colleagues, which is so general that history of mathematics might only play a minor role in improving it. Whereas the latter demands teachers' sophisticated understanding about what content that student should learn and the place of content in learning, which is so critical that history of mathematics is supposed to have its influence, yet seemingly failed to achieve in this study. Reasons for the failure could be attributed to the curriculum design and teaching strategies, or the duration of the course. Further study of this line may help to reveal the doubt. It should also be noted that because part of our findings were based on pre-service teachers' simulation teachings and instructional plans, rather than observing their teaching in the real setting, any over-implication or over-generalization is inappropriate.

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