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# A study of mathematical understanding levels in set theory based on the APOS framework by using python programming language for secondary school students

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**Abstract**
This study used the APOS theoretical framework to assess secondary school pupils' set math knowledge. Princess Chulabhorn Science College Lopburi randomly sampled 20 8th graders in the second semester of 2022. There are a total of six plans, with each plan comprising a 180-minute. The objectives are the instructional goals. Content is the topic. Learning materials include resources and tools. Last, activities include learning assignments and exercises. Assessment includes three subtests to assess set understanding and a test to assess mathematical comprehension. These activities increased secondary school students' set comprehension. The result of the study indicated that they had demonstrated a strong conceptual understanding of set theory to pass the criteria over 60% of the total students with a statistical significance level of .01. Furthermore, delving into all four APOS theoretical levels we found similar impressive results, with over 60% of students exceeding expectations at each level ($p<0.01$).

**Keywords:** mathematical understanding level, APOS theory, set, secondary school students, constructivist theory

## INTRODUCTION

A fundamental aspect of mathematical proficiency is the ability to comprehend mathematical concepts and principles. As it is abstract in nature, it is not possible to measure it directly (Kilpatrick et al., 2001, p. 118). The process necessitates careful observation of the learning behaviors exhibited by individuals engaged in the learning process (Curriculum Development and Supplement Materials Commission, 2006, p. 1-14). In the pre-21st century era, numerous educators endeavored to develop a comprehensive framework for measuring and assessing the extent of learning. Mathematical comprehension plays a crucial role in numerous educational endeavors, including the implementation of pedagogical frameworks like the SOLO taxonomy or the exploration of mathematical understanding growth as proposed by Pirie and Kieren in their work, "A theory of growth of mathematical understanding." This alternative methodology was deemed suitable within the educational setting of that era, characterized by the prevalent utilization of lecture-based instructional techniques. In the context of the 21st century, the education system has experienced a paradigm shift, wherein the student has emerged as the focal point of the learning process. The integration of technology with traditional teaching methods and curriculum is observed (Partnership for 21st-Century Skills, 2007, p. 1-9). Hence, assessing the comprehension of the theory may not align with the contemporary learning management approach in the 21st century, which emphasizes the integration of technology. In 2018, Vidakovic made significant contributions to the field of education by further developing Dubinsky's APOS theory. This theory encompasses teaching methods and content and has gained recognition as a valuable framework within the academic community. The classification of mathematical understanding falls within the framework of constructivism, a theoretical perspective rooted in Piaget's concept of reflective abstraction (Arnon et al., 2014, p. 6-26). This development resulted in a novel conceptualization of


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**Contribution to the literature**

- The researcher reviewed documents and research related to APOS theory, which serves as a conceptual framework for assessing mathematical comprehension.
- This study presents findings on the variables influencing mathematics instruction and learning to develop instructional exercises that are appropriate for the modern-day setting of the 21st century, employing technology as a means of teaching and learning.
- This study provides findings from the examination of the Python programming language that can be applied to enhance the management of mathematics education.

mathematics that encompasses four distinct levels (Cetin, 2015, p. 156-157). The action level, denoted as actions, represents the initial stage of the first level of comprehension, wherein the learner is exposed to external stimuli that serve as catalysts for the learning process. The learners will adhere to instructions, conditions, and procedures in a sequential manner. The sequential progression of steps must not be omitted. Conversely, the process level represents the phase of comprehension that follows the action level, wherein understanding is acquired through repeated actions or actions that do not necessitate external process instructions. The object-level component refers to the phase in which knowledge is collected from the process and action levels. The process by which learners integrate their existing knowledge with newly acquired knowledge results in the formation of novel knowledge. Cognitive linkage, also known as schema, refers to the cognitive ability to enhance knowledge acquisition by establishing connections between levels A, P, and O, which can then be utilized in the construction of advanced mathematical concepts. Nevertheless, Vidakovic et al. (2018, p. 451-456) emphasizes the utilization of mathematical programming languages as a means to enhance learners' mathematical comprehension within the framework of APOS theory.

Python has emerged as the most widely adopted programming language in the contemporary era due to its inherent simplicity and suitability for mathematical programming purposes. Numerous nations across the globe are implementing measures to equip students with fundamental knowledge of Python programming concepts. According to Khummanee (2019, p. 6), one approach to enhancing mathematical comprehension in Thailand involves utilizing Python, a programming language that bears a resemblance to both mathematical symbols and the widely employed English language for human communication (Barry, 2017, p. 5; Dierbach, 2013, p. 9). Additionally, the Deputy Minister of Education, has placed emphasis on programming in this language, which has led to the inclusion of Python in the Ministry of Education of Thailand's core curriculum (Wichanon, 2019). The Basic Education Curriculum, initially implemented in 2008 and revised in 2017, serves as the foundation for educational instruction. The field of technology, specifically computing science, encompasses fundamental scientific subjects. Its objective is to combine the knowledge acquired from Python programming with other scientific disciplines in order to facilitate product development (Institute for the Promotion of Teaching Science and Technology, 2019, p. 20-28). It is evident that complex mathematical problems can be effectively resolved by utilizing pre-existing math modules, which offer efficient computational capabilities (Perkovic, 2015, p. 320-333; Richard, 2018, p. 4-5) and the functionality to invoke external packages (Jason, 2013, p. xx-xxiii). Program developers have endeavored to devise strategies aimed at enhancing the acceptance of diverse data types, including list, string, tuple, dictionary, and set. When examining the method of set, it becomes apparent that it possesses a distinctiveness that sets it apart from other methods. This distinctiveness arises from the inherent connection between the method of set and the broader set of methods. Mathematics, for instance, encompasses union set operations. The concepts of intersection, difference, and complement can be utilized to determine various properties of sets, including empty sets, members of sets, equal sets, and subsets (Udemy, 2020). According to Vidakovic et al. (2018, p. 451), enabling students to engage with programming serves as a significant instrument that not only fosters their comprehension of mathematics but also enhances their capacity for mathematical reasoning. Upon considering the utility of sets, it becomes evident that a fundamental understanding of sets is necessary for the study of modern mathematics, also known as modern math. The inclusion of sets in the mathematics curriculum at esteemed national institutions like Princess Chulabhorn Science College Lopburi is of significant importance, as it mandates junior high school students engage with this mathematical concept (Regional School of Science Curriculum, 2014, p. 16). This observation indicates that the educational institution places significant emphasis on the study of mathematics pertaining to sets. According to Vejjajiva (2018, p. 1), it has been asserted that the entirety of mathematics can be conceptualized and defined within the framework of sets. Based on the findings of the researcher's survey, which examined the current factors influencing the teaching and learning of mathematics among both mathematics teachers and secondary school students, this observation highlights the importance of considering not only technological awareness, but also technological knowledge, mathematical content, and teaching and learning activities as influential factors in the current landscape of mathematics education. Furthermore, this tool aids in the enhancement of mathematical comprehension (Tiengyoo et al., 2023, p. 831), providing researchers with a means to enhance the efficacy of teaching and learning activities.

Furthermore, it is consistent with the APOS theoretical framework, as previously mentioned, that the measurement of mathematical comprehension should incorporate teaching and learning activities aligned with the principles of constructivism. The constructivist theory is considered a fundamental framework for guiding teaching and learning activities (Sophakham, 2013, p. 210). Given that APOS theory is grounded in constructivist principles, it is imperative for educators to provide opportunities for learners to engage in practical experimentation, frequent survey administration and regular practice sessions. The utilization of technology as a significant medium for enhancing learners' knowledge power during activities is facilitated by a four-step process: the persuasion step, the exploration step, the new knowledge step, and the knowledge application step (Calik et al., 2007, p. 257-270). This process enables learners to absorb new knowledge through observation, thereby highlighting the importance of technology as a tool for knowledge acquisition (Gilakjani et al., 2013, p. 49-60). Consequently, the researcher expressed a keen interest in investigating the extent of mathematical comprehension pertaining to sets within the APOS theoretical framework subsequent to implementation of instructional and educational interventions centered around sets. The utilization of the Python programming language for secondary school students aligns with the principles of constructivism.

The researcher conducted an analysis of mathematical understanding at each level, utilizing the APOS theory framework, which is grounded in constructivist concepts. This approach aligns with the teaching and learning activities discussed in the introduction as well as the content and learning standards outlined in the mathematics curriculum. In order to ascertain the learning behaviors that align with each level of mathematical comprehension of sets as per the APOS theoretical framework, the framework is divided into four distinct levels: the action level (A), the process level (P), the object level (O) and the scheme level (S). The user's text refers to an intellectual association, which is represented by a schema as depicted in **Table 1**.

**Table 1.** APOS theoretical framework suggests that set-related behavior might indicate mathematical comprehension

| Content | Activities | | Instructional & learning objectives |
|---|---|---|---|
| Introduction to sets | 1 | | Activity 1: What is in the set? (Lesson plan 1: Acquisition of knowledge regarding set members & set representation in writing) |
| | Level | A | Students can use Python to verify membership. Alternatively, "or does not belong to a specified set." |
| | | P | Students can perform two tasks. Firstly, they can accurately record the symbols representing both members and non-members of a given set and provide a comprehensive explanation for their choices. Secondly, they can construct sets that allocate members and establish the conditions for membership within those sets. |
| | | O | Students can formulate conclusions regarding the characteristics of sets and the methods employed to represent them. |
| | | S | The proposition that students possess the ability to develop a program aimed at resolving scenarios pertaining to the identification of prime numbers within the range of 2021 to 2565 is accurate. |
| | 2 | | Activity 2: Seek set (Lesson plan 2: Acquire knowledge pertaining to the various classifications of sets and sets that possess an equivalence relation). |
| | Level | A | Python can be utilized by students to verify the equality of sets and inequivalences. |
| | | P | Students possess the ability to elucidate the significance of empty sets, finite sets, infinite sets, equal sets, and unequal sets. A subset and its complement. |
| | | O | Students possess the capability to differentiate between sets and non-sets, as well as identify empty sets, finite sets, and infinite sets within a given set. Additionally, it is important to elucidate the characteristics and attributes of sets and subsets. |
| | | S | Students can develop a computer program that can effectively address scenarios pertaining to the "Division of the Apostle Clan's Inheritance." |
| | 3 | | Activity 3: The influence of the moon subset and power set (Lesson plan 3: Acquire an understanding of Venn-Euler diagrams, subsets, and power sets). |
| | Level | A | Students can use Python to verify the subset of a given set. |
| | | P | Students can engage in logical thinking and analysis when considering sets that are either equal or unequal. The concept of being a subset and not being a subset of a given set. |
| | | O | Students possess the capability to: (1) articulate and establish the powerset of a finite set; and (2) construct a conjecture that elucidates the correlation between the cardinality of set A and the cardinality of its powerset, if set A contains n elements. |
| | | S | Students can develop a program that addresses the problem of determining the number of members in power set A. Additionally, they can establish the correlation between the number of power sets and the number of ways to select all n distinct objects, irrespective of their order. |

**Table 1 (Continued).** APOS theoretical framework suggests that set-related behavior might indicate mathematical comprehension

| Content | Activities | | Instructional & learning objectives |
|---|---|---|---|
| | 4 | | Activity 4: The concept and application of Intersection and union sets (Lesson plan 4: Acquire an understanding of the operations involved in intersection and union sets) |
| Set operations | Level | A | Students can use Python to verify outcomes of operations involving intersection & union of a particular set. |
| | | P | Students possess the capacity to: (1) elucidate the significance of intersection and union; and (2) ascertain the relationship between the intersection and union of two specified sets. |
| | | O | Students possess the capability to: (1) ascertain the outcomes of operations involving three or more sets using intersections and unions; and (2) articulate the essential nature of the imperative property. The uniqueness of the treasure can be interchanged with the properties of the group. |
| | | S | The act of designing a program by students to address situations pertaining to the identification of the complete set of ordered pairs ($x$, $y$) that adhere to the specified conditions is accurate. |

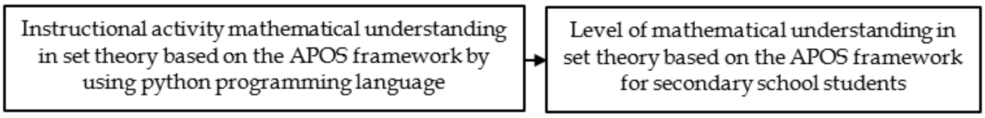


**Figure 1.** Scope of research (Tiengyoo, 2023)

## MATERIALS & METHODS

### Research Objectives

The objective of this study is to examine the extent of secondary school students' comprehension of sets based on the theoretical framework of APOS theory.

### Research Hypothesis

1. Secondary school students who have participated in the researcher's teaching and learning activities have a mathematical comprehension of sets exceeding 50% of the total number of students and 60% of the overall score criteria.
2. Secondary school students have a mathematical comprehension of sets after participating in the researcher's teaching and learning activities. According to the APOS theoretical framework, each level exceeds the threshold of 60% of the total score and accounts for more than half of the total number of students.

### Scope of Research

The scope of this research refers to the extent and boundaries within which the study will be conducted, including the specific objectives, variables, and population that will be examined. The researcher established specific criteria for sample selection, which included students possessing fundamental programming skills in either language. Based on the raw scores obtained by students in subjects related to programming, and as an individual with limited prior exposure to the study of sets, Researcher find myself in the position of a novice student. The research sample consisted of 20 8th grade students enrolled at Chulabhorn Science College Lopburi, during the second semester of the academic year 2022. The participants were selected using a simple random sampling technique, resulting in a total of 20 individuals. The intended audience for an extensive examination of the mathematical comprehension of sets, utilizing the APOS theoretical framework, is secondary school students. The data regarding assertiveness was derived from interviews conducted with the class teachers of the sampled students, which facilitated the exchange and delivery of individual students' ideas. The selection of participants for this study was conducted using purposive sampling. Four individuals were chosen from a larger sample of 20. The participants were divided into three groups based on their raw scores in basic mathematics subjects: one group consisted of high-scoring students, another group consisted of medium-scoring students, and the final group consisted of low-scoring students. Video surveillance devices were utilized, as well as written tasks, evaluative assessments, and mathematical aptitude examinations pertaining to sets. **Figure 1** shows scope of research.

### Research Conceptual Framework

By utilizing the APOS theory framework, this study presents a conceptual framework that the researcher developed to look into the level of mathematical comprehension of sets among secondary school students, by utilizing the findings obtained from the examination of the fundamental principles pertaining to sets. As per the basic core curriculum, 2008 (revised 2017) (Ministry of Education, 2008, p. 6), the researcher has the ability to condense three primary subjects into a concise overview of the fundamental understanding of sets. This academic text explores the concept of set operations and the application of set theory to problem-solving in various scenarios.

Regarding the arrangement of instructional and educational endeavors in accordance with the constructivist framework, the researcher conducted a study on the concept proposed by Calik et al. (2007, p. 257-270), which led to the development of a guideline for teaching and learning activities comprising four distinct steps. In the initial stage, referred to as persuasion, the instructor introduces a scenario pertaining to the set, which may be connected to real-life contexts or other scientific disciplines. This approach is employed to stimulate students' interest and encourage their engagement in the subject matter.

The situation selected by the teacher should be suitable and personally relevant. Furthermore, in accordance with the students' capabilities, it incites curiosity among them. Alternatively, this stage entails the teacher's review of prior knowledge for the students in anticipation of the subsequent phase.

The second step, known as the survey step, entails the execution of predetermined activities by students to enhance their practical skills by utilizing Python commands as a means of investigation and inquiry, and adhering to the guidance provided by the instructor. Alternatively, one can acquire knowledge of Python programming language commands by independently studying the activity sheet. To optimize the level of engagement between students and teachers, students can gain access to technology prior to the discovery of the outcomes of the dataset based on the execution of the program. The third step involves the acquisition of new information or understanding. This is the location, where students can provide an explanation regarding the origin of the output generated by executing the program. In this inquiry, we aim to generate predictions or provide a concise summary of new knowledge pertaining to sets. In the absence of utilizing the Python programming language and excluding the fourth step, the application of acquired knowledge is being considered. This step involves the application of knowledge pertaining to sets acquired in the second step and the third step to address problems within a specified context, utilizing the Python programming language for program design. The APOS theory proposed by Cetin (2015, p. 156-157) and Vidakovic et al. (2018, p. 450) was examined by the researcher, who discovered that mathematical comprehension can be categorized into four distinct levels denoted as A, P, O, and S, as presented in **Table 1**.

### Instrumentation

The research instruments included six different lesson plans, each with a single class session of 180 minutes. Objectives, instructional materials, and activities for learning knowledge are discussed. Each lesson will be evaluated using a rubric based on the APOS theory framework, and students will take a total of three examinations to assess their mathematical comprehension of sets. The reliability of the three sections of the mathematical comprehension exam was 0.75, 0.71, and 0.80, respectively, when calculated using Cronbach's alpha coefficient ($\alpha$-coefficient), a context-based narrative. The test's reliability was 0.88 when used to examine knowledge of sets in mathematics in light of the APOS theoretical framework. The qualitative data analysis techniques include

(1) semi-structured interviews,

(2) computers, and

(3) written work from activity sheets or files from the Jupyter notebook programmed built in Python.

### Methodology of Research

The researcher used a single-sample research design, specifically a one-group posttest-only design. Audience members received the independent variable, as well as post-experiment testing. We analyzed the data from the experiments using both quantitative and qualitative methods. As a result of this lesson planning and administration software, the researcher has access to data from eleven separate 180-minute classes. In addition to emphasizing that "Throughout the experiment, students had to utilize laptops with Jupyter notebook apps to execute tasks," it took three classes to get the language and comprehension of the students on the same page. The institution fully backs the use of this hardware. The researcher also served as a teacher over the course of the trial. In addition to regular class time, the researcher worked with 8$^{th}$ grade students in the second semester of 2022 to develop teaching and learning activities. Between November 2022 and February 2023, after finishing all assignments for unit 1: Introduction to sets studying lessons 2: Set operations and lesson 3: Putting your set knowledge to work. In order to investigate the differences in students' mathematical set knowledge across grade levels, the researcher gave students a series of subtests numbered 1, 2, and 3. Students take a mathematical comprehension exam on sets after completing each sub-test to evaluate their knowledge of sets in light of the APOS theoretical framework, and all three sections take a quantitative and qualitative exam of set-theoretical comprehension.

## RESEARCH RESULT

The research encompassed a total of seven distinct exercise sheets, three individual examinations, and an assessment evaluating comprehension of mathematical sets. The computation of the mean and standard deviation involves straightforward mathematical procedures. The results of the data analysis are presented in **Table 2**.

Based on the data presented in **Table 2**, the overall average score on the activity sheets was found to be 24.13 (equivalent to 80.43%), with a standard deviation of 3.02. Additionally, the average scores on the first, second, third subtests, and sets mathematics comprehension test were determined to be 7.24 (equivalent to 72.40%), 7.74 (equivalent to 77.40%), 8.18 (equivalent to 81.80%), and 32.20 (equivalent to 80.50%), respectively. The overall

**Table 2.** Statistics on mean & standard deviation (SD) of students' performance on sets of three subtests & a mathematical comprehension test

| Score source | Full score | Mean | Arithmetic mean as a percentage of full score | SD |
|---|---|---|---|---|
| Activities 1-7 (individual) | 30 | 24.13 | 80.43 | 3.02 |
| Subtest no. 1 | 10 | 7.24 | 72.40 | 0.99 |
| Subtest no. 2 | 10 | 7.74 | 77.40 | 0.83 |
| Subtest no. 3 | 10 | 8.18 | 81.80 | 0.85 |
| Sets mathematics comprehension test | 40 | 32.20 | 80.50 | 3.37 |
| **Sum** | **100** | **79.49** | **79.49** | **6.63** |

**Table 3.** Hypothesis test results of research no. 1

| Target students | Number of students | Number of students who have a mathematical understanding of sets according to the APOS theoretical framework | p-value |
|---|---|---|---|
| | 20 | 16 students (equivalent to 80%) | .006** |

Note. **At the level of statistical significance .01

**Table 4.** Hypothesis test results of research no. 2

| Number of students | Level of understanding | The number of students who have a mathematical understanding of sets according to the APOS theoretical framework. | p-value |
|---|---|---|---|
| 20 | A | 20 students (representing 100%) | .000** |
| | P | 20 students (representing 100%) | .000** |
| | O | 20 students (representing 100%) | .000** |
| | S | 16 students (equivalent to 80%) | .006** |

Note. **At the level of statistical significance .01

average scores on the instructional activity mathematical understanding in set theory based on the APOS framework by using python programming language was found to be 79.49 (equivalent to 79.49%), with a standard deviation of 6.63. The distribution of scores for each level of mathematical comprehension regarding sets was analyzed across four levels. Specifically, the A level exhibited an average score of 93.40, which accounted for 93.40% of the total score for that level. The standard deviation for this level was calculated to be 5.95. These findings were derived from three distinct sources that classified the level of mathematical understanding of sets based on the APOS theoretical framework. A total of twenty students achieved scores above the 60% threshold. The P level, which represents the performance level, had an average score of 80.85, indicating that it accounted for 80.85% of the total score. Additionally, the standard deviation for the P level was 6.71. The remaining 60% is comprised of a group of twenty individuals. Students who received a score of 60% or higher passed the O-level mathematics test about sets. The arithmetic mean of these passing scores was 77.65, which corresponds to 77.65% of the overall O-level score. The standard deviation for these passing scores was 7.19. The test had a total of 20 participants. On average, the S-level score for these participants was 71.30, equivalent to 71.30% of the overall S-level score. The standard deviation for the S-level scores was 10.99. It is worth noting that 16 students achieved a passing score of 60 or higher on the test. The evidence presented indicates that a group of 16 high school students possesses a theoretical understanding of mathematical sets within the APOS framework.

Based on the research conducted, two hypotheses have been formulated. The acquisition of a conceptual understanding of sets among middle school students is facilitated through teaching and learning activities devised by the researcher. Additionally, over half of the students and more than 60% of the total available points are achieved in this process. The number of students at the advanced level of mathematical comprehension of APOS theoretical sets exceeds the number of students at the novice level by more than twofold. The analysis results are presented in **Table 3** and **Table 4**.

Based on the findings obtained through the utilization of the instructional materials developed by the researcher (as depicted in **Table 3**), High school students possess a comprehensive understanding of the mathematical concept of sets. Over 50% of the student population and over 60% of the total achievable points have significance at the 1% level in statistical analysis by Binomial test.

Based on **Table 4** and the researcher's teaching activities, secondary school students demonstrated a strong understanding of sets**.** All four APOS theoretical levels (A, P, O, and S) achieved average scores above 60% of the full score, with over 50% of students in each category. These results were statistically significant at the .01 level by binomial test.

## CONCLUSIONS & DISCUSSION

The findings of the study examining the level of mathematical understanding of sets among secondary school students, as per the APOS theoretical framework, indicate that students who participated in teaching and learning activities designed by the researcher demonstrated a proficient comprehension of sets. Over 60% of the maximum score criteria and over 50% of the overall student population, with a level of statistical significance set at the 0.01 threshold. Upon examination of the various levels of mathematical comprehension pertaining to sets, as outlined by the APOS theoretical framework, it was determined that secondary school students exhibited a level of understanding surpassing the 60% threshold in the A, P, O, and S levels. The category labeled "full and accounted for" constituted a majority, surpassing 50% of the overall student population. A statistically significant finding was observed at the significance level of.01. The potential cause can be attributed to the teaching and learning activities devised by the researcher. The problem situations were arranged in ascending order of difficulty. By employing pedagogical strategies and instructional methods, one can cultivate a comprehensive understanding of sets within the realm of mathematics. Based on the theoretical framework of constructivism, the researcher initiates the persuasion stage by presenting a hypothetical scenario pertaining to sets. To enhance student motivation towards academic pursuits, it is imperative to establish connections between the subject matter and its relevance to daily life or other scientific disciplines. The researcher must carefully select a suitable situation that is relevant to the individual and aligns with the students' capabilities and fosters student curiosity.

This stage entails the researcher's examination of existing knowledge for the benefit of students. To adequately prepare for the subsequent stage of academic pursuit. The exploration stage denotes the point at which students possess a comprehension of level A, which represents the initial stage of understanding. The pedagogical focus is on students utilizing the Python programming language to investigate various sets, with the objective of succinctly summarizing and elucidating set-related concepts grounded in mathematical principles. The process of enhancing students' comprehension to a higher level, referred to as P level, occurs when they acquire proficiency in additional subject matter and can generate novel concepts independently, indicating comprehension at O level. Researchers can facilitate this progression by presenting challenging problem situations that require the application of acquired knowledge. To effectively utilize the Python programming language, students at different educational levels, namely A, P, and O, must possess a solid foundation in mathematical comprehension of sets. This understanding represents the highest level of proficiency, denoted as S level. This aligns with the research outcomes presented by Chamwan (2018, p. 981-986). The study revealed that the mathematical multimedia effectiveness index, as measured by the axis of symmetry within the APOS theoretical framework, yielded a value of 0.8148. This value suggests that students achieved an academic progress of 81.48%, indicating their ability to learn. In addition to the systematic progression from A level to P level, P level to O level, and from O level to S level, there is a concurrent development of self-understanding facilitated by group learning processes. Furthermore, when examining the group of students who met the requirements for levels A, P, O, and S, it was observed that there were 20 individuals in each of the first three levels, while the S level had 16 students. This data indicates that there were four students who lacked a comprehensive understanding of mathematical sets at the S level, potentially attributable to the inability of students to effectively establish connections between pre-existing concepts, methods, or mathematical principles to address programming challenges, including errors in programming structure. The Python grammar and the concept of sets, among other topics, are discussed in the works of Schwalbach and Dosemagen (2000, p. 90-98) and Wiggins and McTighe (2005, p. 35-55). These authors argue that students who possess the ability to establish intellectual frameworks that connect pre-existing mathematical concepts, methods, principles, or facts are better equipped to tackle complex problems and assimilate new knowledge structures. According to Pinter (2014, p. 10-24) and Sfard (2013, p. 14), the integration of concepts across content or disciplines is contingent upon their ability to be interconnected. Furthermore, it was discovered that students exhibited deficiencies in English reading and translation after the execution of the program. This discrepancy aligns with a declaration made by Tim Cook (DailyGizmo, 2019), asserting that individuals proficient in programming ought to possess a comprehensive understanding of mathematics, the English language, and creative thinking. In addition to the systematic progression from A level to P level, P level to O level, and from O level to S level, individuals also enhance their self-understanding through a collective learning experience. This study aims to analyze the characteristics of secondary school students' level of mathematical understanding of sets based on the APOS theoretical framework.

The researcher will consider a specific target group comprising four individuals, namely B1, B2, B3, and B4, due to B1's involvement. Aspire to become a student who attains exemplary academic performance. By participating in academic camps focused on programming with the C language, I have developed a strong ability to concentrate, think analytically, and approach problems systematically. Additionally, I possess a natural inclination to inquire and seek clarification, as well as a propensity for effectively communicating complex concepts to my peers. Simultaneously, it is imperative to attentively consider the viewpoints expressed by others. In addition, being


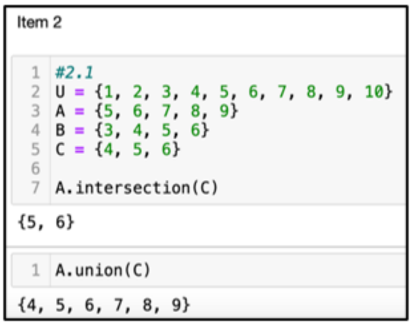


**Figure 2.** B3's A.intersection(B) & A.union(B) clauses (Tiengyoo, 2023)

an attentive listener is an essential attribute. B2 and B3 are students who demonstrate average academic performance. The individual exhibits a keen enthusiasm for programming, demonstrating a strong inclination towards inquisitiveness and a capacity for sustained focus and mindfulness during demanding tasks. Furthermore, they actively engage in the process of acquiring knowledge and resolving complex programming-related challenges. Ultimately, B4 assumes the role of an individual exhibiting subpar academic performance. Through the application of diligence, patience, and concerted effort, individuals can acquire the necessary skills to effectively address and resolve programming-related challenges. In addition, they possess strong listening skills and demonstrate assertiveness. To address the research inquiry, "What is the level of mathematical comprehension of sets among high school students based on the APOS theoretical framework at each stage?" The researcher presents a taxonomy of levels of mathematical comprehension for sets, as follows:

For the comprehension of sets from a mathematical perspective at the operational level (A), students employ Python to investigate the outcomes of program execution. Despite the inability of students to provide a comprehensive explanation for the outcomes derived from executing the program, in this context, students can perform three operations. Firstly, they can assess the membership of a set by utilizing the "in" command. Secondly, they can determine the cardinality of a set by employing the "len( )" command. Lastly, they can evaluate the equivalence and inequality of sets by utilizing the "A=" statement. Additionally, the study aims to examine the outcomes of performing intersection, union, and difference operations on sets, as well as the use of the A.union(B), A.intersection(B), and A.difference(B) commands (as illustrated in **Figure** 2) When students independently engage in activities to study fundamental commands, it demonstrates their ability to comprehend and apply instructions when utilizing the Jupyter Notebook program with the Python language. In addition to documenting the outcomes derived from executing the program accurately. Mathematically comprehending sets at the procedural level (P), students may


**Figure 3.** Programming results for B1 A-B (Tiengyoo, 2023)

(1) write the membership and non-membership symbols of a set,

(2) write the membered and member-conditional sets,

(3) identify the set members,

(4) define their meaning, refers to sets (e.g., empty sets, finite sets, infinite sets, equal and unequal sets, being a subset and not being a subset, and operations between sets), and

(5) find the relation between the number of members of set A or set B and the number of elements of set A, B, or C without Python.

The researcher also found that students who can construct flow charts and explain their own programming process accurately will discuss or demonstrate mathematical problem-solving concepts in the same context. It indicates that students can execute the same A level many times, can illustrate how to get those results without Python, and can explain topics connected to level A activities. This matches the target group interviews in activity 5: Complement and difference. The researcher emphasizes methodical thinking by promoting and discussing with pupils the link between mathematics and Python's operations. Students can identify operations between set A-setB by knowing the A level, as indicated in **Figure 3**.

When pupils obtain outcomes from running the program. The following is what the researcher and the target group discussed:

> Researcher: Why do you suppose Python returns {'c'}?
>
> B3: The difference between A and B.
>
> B4: What exactly is it?

B3: Distinction English signifies distinction. Lol. And what about you, researcher? Haha.

B2: Oh, a and b belong to both sets A and B, but c belongs to A but not B. How about e? Do not you consider it? So, let's alter the faces of sets A and B for other members.

B1: Is this how it is? Python, for example, only accepts the first element but not the second. Because I changed the members from set A to set B and then back to set A, it turned out to be {'e'}.

During that time, the researcher and the target group of students collaborated to describe the concepts derived from the above-mentioned program's outcomes. The researcher then posed the following interesting questions to the students:

Researcher: What do you think about Python, kids? Explain the notion from start to finish.

B1: Python must first understand what sets A and B look like. Show whether or not we have to take sets A and B first.

B1: Sure, blah blah.

Researcher: Eh ! What symbols must the flowchart have in order to get sets A and B? Everyone should draw a parallelogram (with a flowchart).

Researcher: What should I do now that I know the faces of sets A and B? If we run the command A. difference(B), we will get.

B1: Represents the members of set A who are not members of set B.

Researcher: Naturally. What do youngsters do when they want to know which member is in set A but not in set B?

B2: Examine them one by one.

Researcher: Python has the same thoughts that we do. Python will check set A to determine if the next member is in set A or not, because this is the correct condition (meanwhile, all four target students are in set A). The creator of the accompanying flowchart: What if it is there? What will occur?

B1: It will keep checking to see if it is in set B. If it is, it will go to the next element of set A, and so on; otherwise, it will remain in the result.

Students will be able to create a flowchart based on the Q&A findings, as shown in **Figure 4**.

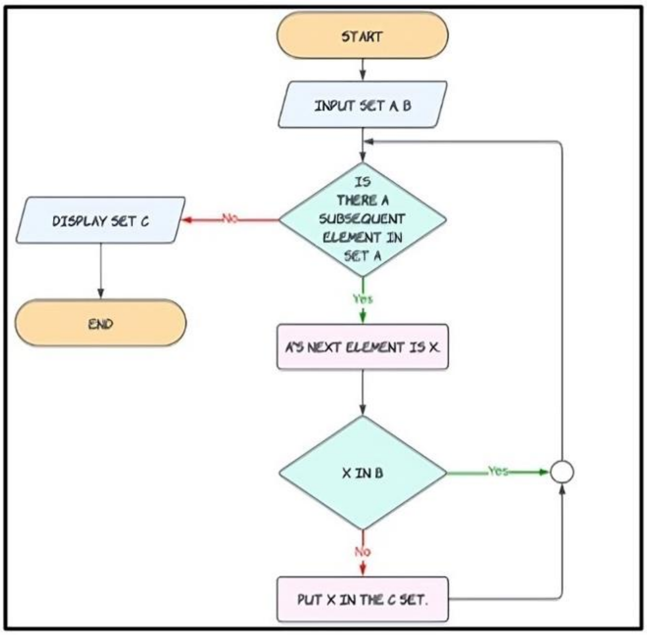


**Figure 4.** A-B flowchart (Tiengyoo, 2023)

Subsequently, the researcher proceeds to elucidate for the students that the outcomes derived from employing the command A.difference(B) shall be denoted. In the field of mathematics, it is commonly acknowledged at a foundational level that the distinction between sets A and B is symbolized as A-B. Subsequently, the students are encouraged to independently derive conclusions regarding the significance of the disparity observed between sets A and B. The interpretation of the distinction between sets A and B was accurate, as were the remaining three students of interest. At the level of objects (O), students will be able to understand sets in math by being able to:

(1) draw conclusions about the properties of sets and represent them symbolically;

(2) classify sets and things that are not sets; and

(3) classify the different kinds of sets.

Section 4 of the document discusses properties pertaining to sets, including properties of subsets and operations performed on sets. Section 5 explores the correlation between the cardinality of set A and the cardinality of its powerset. Indeed, when set A consists of n members, one can leverage knowledge pertaining to the cardinality of a finite set-in order to address problem scenarios. In the absence of employing the Python programming language, this study aims to illustrate the capacity of students to integrate the knowledge acquired from both the A and P levels to articulate descriptions of properties or principles as well as formulate predictions that reveal relationships pertaining to a specific concept. The researcher presents a representative sample. Activity 6 aims to assess the level of mathematical comprehension pertaining to sets. The understanding demonstrated in **Figure 5** and **Figure 6** seems to align with an O-level level of comprehension. This level

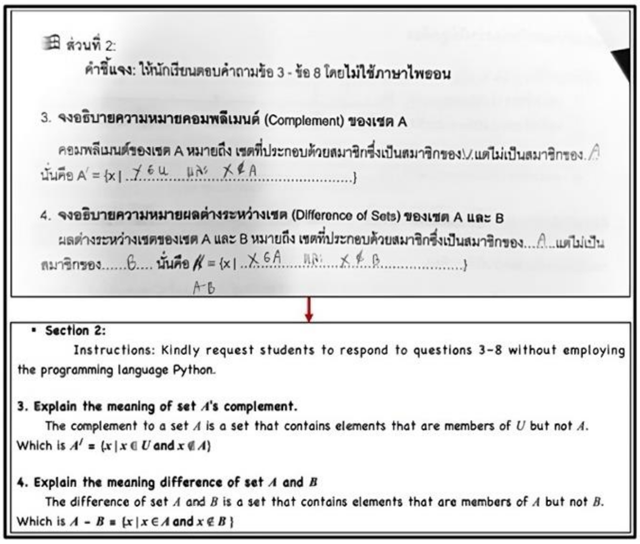


**Figure 5.** Explaining A-B (Tiengyoo, 2023)

emphasizes the application of knowledge pertaining to the elements within a limited set, with the aim of problem-solving. To determine the cardinality of a set S, one can use the len(S) function to manually count the number of elements in the set.

Furthermore, the students acquire fresh knowledge in this course. It remains imperative to draw upon the knowledge acquired during activities four-five conducted in the preceding class. The proposed approach can be employed to determine the outcomes of operations performed on sets that are combined using the union operation. This discussion will focus on the concepts of intersection, complement, and difference in the context of sets.

By engaging in the tasks outlined in this section, the students at the target level utilize their A-level comprehension skills to generate speculative statements and derive conclusions regarding the union of sets A and B, denoted as n(A∩B). The formula for the cardinality of the union of two sets A and B, denoted as n(A∪B), is given by the sum of the cardinalities of A and B minus the cardinality of their intersection, n(A∩B). Similarly, the formula for the cardinality of the union of three sets A, B, and C, denoted as n(A∪B∪C), is given by the sum of the cardinalities of A, B, and C minus the cardinalities of their pairwise intersections, n(A∩B), n(A∩C), and n(B∩C), plus the cardinality of their intersection, n(A∩B∩C). To access the animation media, we consider the cardinality of the union of sets A and B, denoted as n(A∪B), as well as the cardinality of the union of sets A, B, and C, denoted as n(A∪B∪C). The researcher developed a comprehensive understanding of the students' abilities at the P level. Upon examination of the target student population, it was determined that only students at the B1, B2, and B3 levels could make inferences regarding the cardinality of the union of sets A and B, denoted as n(A∩B). The solution to the given problem can be accurately obtained without utilizing the Python programming language by employing the set notation of the union of sets A, B, and C, denoted as n(A∩B∩C).

With respect to the comprehension of sets in mathematics at the level of cognitive structure (S), students in secondary school demonstrate the ability to establish connections between their existing knowledge and the content being learned. This study aims to explore the utilization of mathematical concepts such as matters, sets, and other related mathematical entities in conjunction with the Python programming language for the purpose of solving a given problem scenario. This demonstrates that students can utilize their knowledge of A, P, and O levels to effectively employ program design principles using the Python language to address attainable problem scenarios. Subsequently, the researcher proceeds to provide an illustrative instance. Activity 7 involves the use of cryptography to assess the level of mathematical understanding of sets, specifically at an S-level comprehension (as depicted in **Figure 7**). This activity requires students to draw upon their prior knowledge gained from previous activities. Activity 6: Stock market analysis: students may possess prior

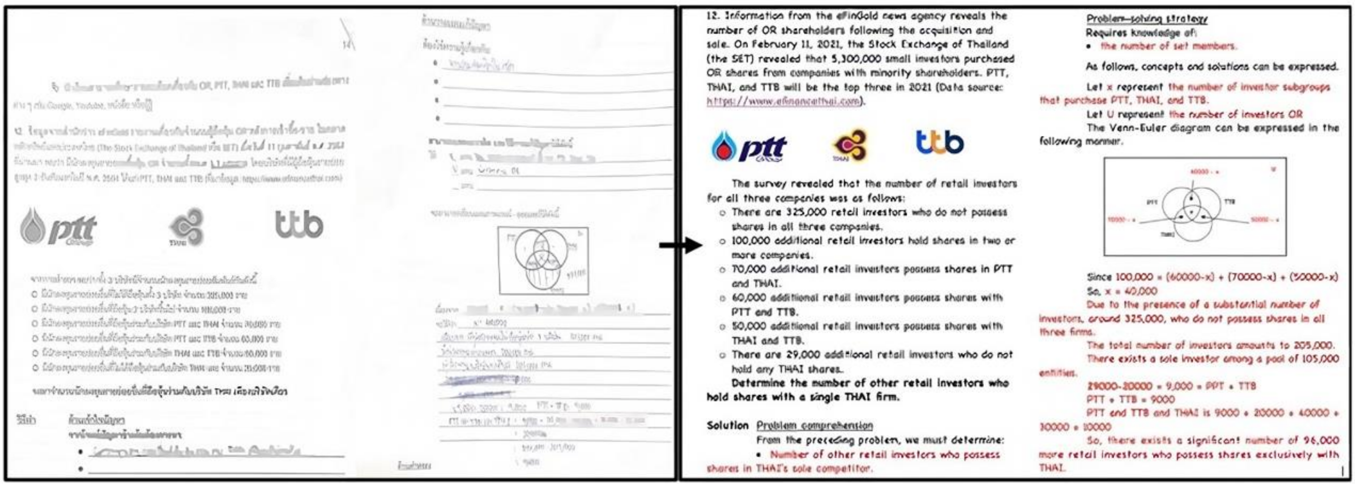


**Figure 6.** B1 activity 6 writing task 12 (Tiengyoo, 2023)

```
1 A = int(input('n(BTC) = '))
2 B = int(input('n(ETH) = '))
3 C = int(input('n(BNB) = '))
4 D = int(input('n(DOT) = '))
5 E = int(input('n(BTC.intersection(ETH)) = '))
6 F = int(input('n(BTC.intersection(BNB)) = '))
7 G = int(input('n(BTC.intersection(DOT)) = '))
8 H = int(input('n(ETH.intersection(BNB)) = '))
9 I = int(input('n(ETH.intersection(DOT)) = '))
10 J = int(input('n(BNB.intersection(DOT)) = '))
11 K = int(input('n(BTC.intersection(ETH,BNB)) = '))
12 L = int(input('n(BTC.intersection(ETH,DOT)) = '))
13 M = int(input('n(BTC.intersection(BNB,DOT)) = '))
14 N = int(input('n(ETH.intersection(BNB,DOT)) = '))
15 O = int(input('n(BTC.intersection(ETH,BNB,DOT)) = '))
16 sum = A+B+C+D-E-F-G-H-I-J+K+L+M+N+O
17 print(sum)
```

```
n(BTC) = 5
n(ETH) = 4
n(BNB) = 3
n(DOT) = 3
n(BTC.intersection(ETH)) = 3
n(BTC.intersection(BNB)) = 2
n(BTC.intersection(DOT)) = 1
n(ETH.intersection(BNB)) = 3
n(ETH.intersection(DOT)) = 2
n(BNB.intersection(DOT)) = 2
n(BTC.intersection(ETH,BNB)) = 4
n(BTC.intersection(ETH,DOT)) = 3
n(BTC.intersection(BNB,DOT)) = 2
n(ETH.intersection(BNB,DOT)) = 1
n(BTC.intersection(ETH,BNB,DOT)) = 2
14
```

**Figure 7.** B3 instruction file "SET7" programing result (Tiengyoo, 2023)

knowledge of calculating the cardinality of the sets A∪B and A∪B∪C. Activity 7: Crypto requires them to further extend their understanding to determine the cardinality of the set A∪B∪C∪D. Please develop a program that calculates the total number of individuals who have invested in all four cryptocurrencies simultaneously. The researchers viewed a video presentation on the fundamental concepts of cryptography and engaged in a comprehensive discussion to enhance their understanding prior to engaging in programming activities. When examining the programming contributions of B3 as demonstrated in the command file "SET7," it is evident that B3 successfully computed the union of sets A, B, C, and D, denoted as n(A∪B∪C∪D). This can be observed by referring to line 16 of the command file. The B3 programming language adheres to the structure and grammar conventions of the Python language. While the programming involved in this activity may be straightforward, it may still pose a challenge for students attempting to write programs. There is a necessity to augment the existing body of knowledge by generating novel insights and information. During regular instructional activities, teaching in accordance with the IPST guidelines requires that students' focus on the computation of n(A∪B) and n(A∪B∪C), while excluding the integration of mathematics and programming in the mathematics curriculum.

Based on the provided illustration, depicted as Figure 7, the study revealed that B3, like B1 and B2, possesses the capacity to facilitate the acquisition of novel information, particularly in the realm of programming. The data indicates that the three student target groups possess a level of comprehension classified as "S." It is crucial to develop a comprehensive understanding of the activities examined by the researcher at the A, P, and O levels. Based on what we've learned so far, it's clear that the APOS theory is a new theoretical framework that was made to fit with the modern educational landscape, which puts more emphasis on using technology to teach and get students involved. In this context, it is imperative for learners to possess the ability to effectively apply problem-solving techniques in intricate scenarios by integrating mathematical expertise with technological proficiency via programming. Simultaneously, numerous educators who possess an interest in incorporating mathematical programming languages into the realm of education have provided their support for this endeavor. To cultivate a proficient level of mathematical comprehension within the framework of the APOS theory, which posits a similar viewpoint, it is imperative to engage in appropriate learning activities and instructional strategies. The utilization of programming languages such as C or Java is a contributing factor that fosters expedited comprehension among learners in the field. In addition to facilitating the integration of mathematical concepts with computational systems, this interdisciplinary approach enables students to resolve intricate problems, as well as its role in the advancement of programmers, innovation, and artificial intelligence (Ginat, 2004, p. 165-181).

The findings of Vidakovic et al. (2018, p. 451-456) align with the research results, indicating that learners demonstrated a consistent level of mathematical comprehension of functions when utilizing the ISETL language within the framework of the APOS theory. Specifically, learners effectively employed the ISETL language to derive outcomes and establish connections among elements within the transformed domain of members. This activity aims to foster students' ability to critically analyze the program's process within the given range. Consequently, the learners acquired a deeper understanding of the concept of function by engaging in cognitive exercises. In addition to students' ability to articulate the overarching scenario that the operational functioning of a computer when the function "*f* " is activated for utilization through activities at the action level is a subject of inquiry. The instructor provides the code or instructs the students on the commands to be used, enabling them to execute the given instructions. Subsequently, the students are required to engage in an analysis of the function's purpose. It is imperative to enhance students' comprehension at a more advanced cognitive level. The instructor initiates a problem scenario and facilitates the students' resolution of the problem by employing the ISETL programming language to construct their own functions. With *func* the learner extrapolates the action performed based on the given problem scenario of the function. In the field of computer programming, individuals progress from employing specific representations to executing generic actions. Based on the analysis, it is evident that the application of the action level is extended to the process level. During this process, learners will be motivated to accumulate knowledge at the object level, with the goal of being able to compose functions that can accept one or more function arguments. As an example, A function, denoted as D, can be implemented to generate a function, *f*, and subsequently return it as a computational function capable of evaluating at various values of x within the domain of the function. Subsequently, a function shall be devised with the purpose of constructing a program capable of determining the slope of any given function. Other learners will possess the ability to compose functional code. Let us consider the function f and return it as the resulting function. Moreover, students perceive functions as entities that facilitate the transfer of information between two sets. For instance, in the activities, the participants utilized the ISETL programming language. The utilization of the ISETL language was discovered to be a significant facilitator in expediting the comprehension of mathematical concepts.

By fostering an environment that promotes the identification of connections and relationships within computer processes, code, and mathematics, students are encouraged to develop a deeper understanding of these subjects. Furthermore, students can provide an explanation of the concept of a function. Could you please provide an explanation of the operational process? (Weller et al., 2003, p. 97-100) Teaching in accordance with the IPST guidelines necessitates students' focus on the computation of n(A∪B) and n(A∪B∪C), while excluding the integration of mathematics and programming in the mathematics curriculum.

### Recommendations

Developing online instructional modules to assess the proficiency level of individuals in the domain of set theory. Alternatively, the instructor may opt to have the students engage in the activity collaboratively, either in pairs or small groups. I am interested in conducting research on the topic of creativity within the field of mathematics. When developing instructional materials for mathematics education, Python can be employed as a tool for event organization.

**Author contributions: SS:** develop study's subject of inquiry and design & use methods from statistical analysis to analyze the data being analyzed; **KT:** analyze the previous work done on APOS theory by doing a literature review, develop a guiding theoretical framework for the research, collect data from the students, & apply APOS theory to the analysis of the data on the learning outcomes of the students; & **ST:** review the existing literature on methods used to teach set theory using the Python programming language. All authors have agreed with the results and conclusions.

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**Declaration of interest:** No conflict of interest is declared by authors.

**Data sharing statement:** Data supporting the findings and conclusions are available upon request from the corresponding author.

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