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A Study of Pre-service Primary Teachers' Discourse when Solving Didactic-Mathematical Tasks

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ABSTRACT

From a commognitive approach, this article focuses on the discourse generated by pre-service primary teachers who are solving didactic-mathematical tasks. Our aims are to study the characteristics of the aforementioned discourse and, through these characteristics, identify whether a discourse close to the one of primary teachers is beginning to emerge. The sources of data were audio-recordings of group discussions and group reports. Two different discourses were identified in our results. One is the discourse generated by pre-service teachers when adopt the role of students of any level who have to solve a task proposed in the classroom. The other discourse is linked to the adoption of a role close to their future professional work. If we consider that the acquisition of a specific discourse enables future teachers to integrate into the community of practice of primary teachers, the role of the different discourses becomes a relevant element in teacher education.

Keywords: commognitive approach, didactic-mathematical task, discourse, pre-service primary teachers

INTRODUCTION

Over the past few decades, the importance of mathematics teacher education has significantly increased its visibility and reinforced its relevance in diverse areas of activity. Specific conferences such as the XV ICMI, dedicated to Professional Education and Development of Teachers of Mathematics (2005), specialized journals, many books and articles on this particular subject have substantially contributed to building public awareness worldwide about teacher training. This awareness has led to a modification of curricular orientations of different countries (UK Department for Education, 2013; US Common Core State Standards Initiative for Mathematics, 2010), and new mathematics teacher education programs and courses have been implemented at some universities and colleges of education (Lee, 2005; Lin & Hsu, 2018; Ponte, 2012). Nevertheless, although it may be assumed that the impact of these programs on teacher education has been significant, more information about what occurs some years after the initial process of implementation is needed. In this context, sociocultural approaches have provided new and important insights for analyzing the learning processes of pre-service teachers. In particular, the importance assigned to discourse in these perspectives has been emphasized by different authors from different approaches.

On the basis of sociolinguistic theories and other fields such as cognitive science and philosophy, authors such as Gee (1996) have highlighted the importance of discourse and provide tools to investigate the discourse and social practice affecting the interrelationships between language, social identity and social context (MacKay, 2003). Gee (1996; 2001) admits the existence of a great amount of discourses (for example, the discourse maintained by different collectives such as doctors, lawyers, athletes, etc.) that can be considered a signal of identity or form of life. He maintains that discourses' meanings are linked to the experiences and perceptions in which the language has been used.

Focusing on mathematics education in general, many researchers have considered the discourse with very different aims and contexts. For instance, among many others, authors such as Lampert, Rittenhouse and Crumbaugh (1998) have shown how discourse can encourage mathematical learning, Hunter and Anthony (2011) have provided information on the emergence of specific mathematical discourses in school classrooms and Caspi

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Contribution of this paper to the literature

- Our work adapts the four properties provided by the commognitive framework to study the discourse generated by pre-service primary teachers when they solve didactic-mathematical tasks.
- This study has shown the coexistence of two different discourses: a predominant discourse as students and a minority discourse as future teachers.
- The development of a discourse as primary teachers is important for the integration into the community of practice of primary teachers, aim of the professional development.

and Sfard (2012) have followed the progress of elementary students from their informal algebraic talk to the formal algebraic discourse, as taught in school. More recently, Erath, Prediger, Quasthoff and Heller (2018) have shown the interrelationship among discourse competence, participation in classroom discourse and mathematical learning opportunities in secondary classrooms.

Special relevance has in our research the studies that have dealt with the discourse in the context of pre-service teacher education. Authors such as Tatsis and Koleza (2008) have studied the discourse of pre-service teachers focusing on social and socio-mathematical norms that are established during the interactions of pre-service teachers when they are solving mathematical problems, Sánchez and García (2014) have also studied the discourse of pre-service teachers identifying the norms that regulate it when they are solving a task related to the mathematical definition. Other authors as Tabach and Nachlieli (2015) have explored how pre-service teachers use a definition of function (to identify functions) by examining changes in their discourse and Martín-Molina, Toscano, González-Regaña, Fernández-León and Gavilán-Izquierdo (2018) have studied the discourse of pre-service teachers in order to identify the routines that they use when they are describing and defining solids.

To sum up, we can say that the notion of discourse has extensively exceeded the field of linguistics and currently forms part of very different scientific fields related to social sciences (sociology, education, mathematics education, etc.). Perhaps one of the reasons for this extensiveness could lie in the expansion of this notion of a type of practice that belongs to collectives rather than to individuals. This has led us to dig deeper into the discourse generated by pre-service primary teachers who are in the process of solving a didactic-mathematical task in a mathematics teacher education course. From some years, this course has been included in a primary teacher education program. Inquiring about the characteristics of the abovementioned discourse can provide information about what really happens in that course. In particular, our aims are to study the characteristics of the aforementioned discourse and, through these characteristics, identifying whether a discourse that is close to the one of primary teachers is beginning to emerge.

THEORETICAL FRAMEWORK

The sociocultural approaches share the idea of "communication, thinking and learning as related processes which are shaped by culture" (Mercer, 2004, p. 138). According to Mercer (2004), communicative events are modeled by historical and cultural factors, and thinking and learning cannot be understood without taking the social and communicative nature of human life into account. Authors such as Tabach and Nachlieli (2016) have pointed out the existence of different ways of considering communication in different perspectives, highlighting that "in some traditions, communication is seen as a window to thinking processes whereas, in others, communication is tantamount to thinking" (p. 300).

In this study, we adopt the commognitive approach of Sfard (2008). In line with Sfard's point of view (2006; 2008), we consider thinking to be an individualized form of communication, with interpersonal communication being "the collective activity that morphs into thinking through the process of individualization" (Sfard, 2006, p. 158). To highlight the unity of communicating and thinking (cognition), Sfard suggested the word commognition. She considers that "in the proposed discourse on thinking, cognitive processes and interpersonal communication processes are thus but different manifestations of basically the same phenomenon" (Sfard, 2008, pp. 82-83). This author defines discourse as a "special type of communication" (Sfard, 2008, p. 297), clarifying that this communication need not necessarily involve other people, because individuals frequently communicate with themselves. In this theory of commognition, learning is seen as a discursive change, which is stimulated through the commognitive conflict, in other words, through the situation that arises when different interlocutors are acting according to different discursive rules (Sfard, 2007).

We are in agreement with different researchers (Gee, 1996; Moschkovich, 2007) that state that discourse depends on the social community in which it occurs, has several meanings and implies language, representations and behaviors embedded in the practice. The communication could be verbal or may involve gestures, drawings, or other symbolic systems.

In this paper, the discourse generated among pre-service primary teachers when solving didactic-mathematical tasks becomes the main object of study. We assume that this discourse can be considered a colloquial discourse, which incorporates characteristics of spontaneity and informality of everyday conversations. In that colloquial discourse, we consider that characteristics of a discourse related to the students/teacher/content triad can exist, and expressions related to mathematics teaching and learning are included therein. In contrast to mathematical discourse, in which: "unlike many other discourses, the objects of mathematics, those things that are being talked about, do not pre-exist the talk; rather, they arise as byproducts of the ongoing mathematical conversation" (Nachlieli & Tabach, 2012, p. 10), in the abovementioned colloquial discourse two types of objects can coexist. One of them originate from mathematics itself (triangles, functions, etc.), while other objects have linguistic meanings associated with teaching and learning processes (understanding, problem, etc.).

Sfard (2008) considers mathematics as a special type of discourse characterized by four properties: word use, narrative, visual mediators and routines. The first property, word use, refers to the use of mathematical words (such as triangle, angle, etc.) and ordinary words used with mathematical meaning (for example pointy referring to acute angle). The second property, narrative, is considered as "a series of utterances, spoken or written, that is framed as a description of objects, of relations between objects, or processes with or by objects, and is subject to endorsement or rejection, that is, to being labeled as "true" or "false"" (Sfard, 2008, p. 300). Example of narratives are mathematical definitions or theorems, they are labeled as endorsed narratives because they are assumed by the mathematical community. The visual mediators are understood as the visual objects that participants of the mathematical discourse use to coordinate the communication (for example, written symbols, drawings, graphs, etc.). Lastly, routines are repetitive patterns in the actions of the participants of the given discourse (for example, how calculations are made or conjectures are proved).

In our case, we adapt Sfard's proposal for a didactic-mathematical discourse related to students/teacher/content triad, in which we focus on words that provide important information about processes of teaching/learning as an object of study. These words may appear in individual form or combined with others, forming a sole meaning. For example, "explaining", "children's understanding", "levels of reasoning", "modes of representation", etc.

With respect to narratives, on the basis of aforementioned meaning, we consider as objects any kind of entities related to students/teacher/content triad. For example, "In the first level of van Hiele the pupils perceive the figures as a whole" is a narrative.

Regarding visual mediators, they are specific objects, pictures and symbols that are utilized as a part of the process of communication among pre-service teachers. For example, the picture of the school activity included in a task could be used as a mediator. Finally, routines are repetitive patterns characteristic of the considered discourse, for example, repetitive ways of analyzing pupil's school works.

We want to point out that this research deals with the discourse that is not established between the pre-service teachers and the teacher educator in the classroom, but with the one that arises in the interaction between the pre-service teachers themselves. This choice was made in order to minimize the teacher's influence on the discourse generated in the resolution process of the task (Tatsis, 2007).

METHODOLOGY

Context

This research has been developed with pre-service primary teachers in a mathematics teacher education course. This course is part of the second year of a primary teacher education program of four years of a big public university of Spain. Some years ago, 'situated knowledge' and 'cognitive apprenticeship' formed the initial framework that allowed teacher educators to address the knowledge and the learning process of these students (Collins, Brown, & Newman, 1989; García & Sánchez, 2002; García, Sánchez, & Escudero, 2007). Through the process of solving 'authentic activities' (Brown, Collins, & Duguid, 1989), they were expected to develop their approach to the professional activities of a primary teacher, and begin to develop a discourse close to that community of practice.

At present, the mathematics teacher education course comprises four different sections (Numerical sense, Magnitude and Measure, Geometrical sense and, finally, Probability and Statistic) and is 90 hours long. There are two parts to each one of the four sections: a theoretical part (60 hours) and a practical part (30 hours). The theoretical part is dedicated to providing pre-service teachers with conceptual tools. These tools are understood to be the concepts and theoretical constructs that have been generated from research in mathematics teacher education, which can lead to the understanding and handling of situations in which mathematics is taught and learned. The practical part is dedicated to putting these tools into practice by asking the pre-service teachers to solve didactic-mathematical tasks (understood to be authentic activities) proposed by teacher educators. These tasks are focused on bringing future teachers closer to the reality of the professional activities of a primary teacher. For instance, a

didactic-mathematical task can be related to a professional activity like interpreting pupils' mathematical productions or analyzing mathematical school problems taken from school textbooks. All the pre-service teachers have to solve the tasks, analyzing and informing about aspects they have learned.

In our study, the data was collected when pre-service elementary teachers were solving didactic-mathematical tasks posed in the last two sections of the practical part of the mathematics teacher education course in the first semester of the year 2014. In this article, we focus on the development of two didactic-mathematical tasks, one in each section. There was no intervention on the part of the researchers in the design of the tasks of the course.

The choice of these didactic-mathematical tasks was conditioned by two reasons. On the one hand, they encompassed professional activities like analyzing, interpreting and posing questions, which are very present in the work of teachers. On the other hand, they were situated at the end of the course, thus the working groups are more cohesive and more familiar with their way of working, which leads us to suppose that the colloquial discourse could be more fluent.

The first task, corresponding to the practical part of the Geometry section, asked the pre-service teachers to analyze a school activity and interpret the primary school pupils' productions (see Appendix). On the basis of a proposal of Burger and Shaughnessy (1986), the school activity was related to quadrilaterals, part of the geometry section of the primary school. In the school activity, primary school pupils had to answer questions related to the identification and classification of said quadrilaterals. In the didactic-mathematical task posed to pre-service teachers, first, they had to analyze the school activity in relation to the processes of geometric reasoning and the skills that could be developed and then interpret the pupils' answers (students aged 6-12) taking into account several interviews that the pre-service teachers had previously conducted. For solving this task, the conceptual tools came from information that had been provided in the theoretical part and from some articles (Gutiérrez & Jaime, 1998; Hoffer, 1981; Jaime & Gutiérrez, 1990). The university teacher of the course also gave the pre-service teachers some orientation on the subject in the Spanish curriculum at the grade-level in question.

In the second didactic-mathematical task (corresponding to the practical part of Probability and Statistic section), the pre-service teachers had to analyze school problems related to probability and formulate questions to ask their future pupils (see Appendix). For this purpose, the task included a collection of school problems. These problems were taken from primary school textbooks or websites. In this didactic-mathematical task, the pre-service teachers had to analyze said problems looking for the underlying mathematical concepts and procedure in them, and consider what possible questions they might put to their pupils in the classroom for each problem. In this case, the conceptual tools came from the theoretical part of the course and articles from journals (Figueiras, 2013).

The processes of solving these didactic-mathematical tasks are the context in which the data were generated.

Participants

This study is part of an ongoing research carried out with pre-service primary teachers enrolled in the abovementioned mathematics teacher education course. All the pre-service teachers were older than 19 and coming from diverse socio-economic backgrounds. They were informed of our research (characteristics, aim, confidentiality issues, etc.), and they decided for themselves whether or not they wished to participate in the study. 88 pre-service primary teachers (66 females and 22 males) voluntarily agreed to participate.

This work focuses on 45 of the 88 pre-service teachers, who had the same university teacher. In the practical part, they were split into 11 subgroups (named SG1, …, SG11 on our research). The subgroups were formed according to the preferences of the pre-service teachers themselves, and the university teacher only recommended that there should be three to five people on each subgroup. There was no intervention on the part of the researchers in the forming on the subgroups.

Data Collection Procedures

The sources of data were:

- Audio recordings of subgroup discussions when the pre-service teachers were solving the two didactic-mathematical tasks corresponding to the practical part of the Geometry and Statistics/Probability sections. These recordings were collected during ten sessions (five sessions for each abovementioned section) of one hour per week for each one of eleven subgroups, approximately, 110 hours of recording. Every session, one of the researchers distributed the corresponding recorders to the participating subgroups (with the non-participating subgroups remaining in the classroom). The researcher gave them specific instructions on the dynamics of the recording process, such as ensuring a clear recording of what was said, to state their name in order to facilitate their identification and so on. Considering that some of the pre-service teachers could have had some doubts about the recording process, the researcher stayed in the classroom without intervening or interfering at all in the work of

the university teacher. The transcriptions of these audio recordings allowed us to access pre-service teachers' spoken discourse.

- Subgroup reports, containing the decisions that were taken by the pre-service teachers in the process of solving the didactic-mathematical tasks. At the last session of each task, reports were provided by each pre-service teachers' subgroup. These reports were used by the university teacher as a means of evaluating pre-service teachers' work. For us (the researchers), these final reports allowed us to access the pre-service teachers written discourse.

The data were not collected in English. In order to present part of the data in this article, some excerpts were translated into this language by the authors, trying to preserve the original meaning.

Data Analysis

The data analysis adopts a qualitative/interpretative approach. The process of analysis was developed in three steps.

In a first step, once each of the subgroups' audio recordings were transcribed, taking at all times the context in which the discourse had been generated into account (conversation between pre-service teachers when solving a didactic-mathematical task) two of the researchers identified the parts of the transcript that were related to the students/teacher/content triad. The common identifications were selected, and the problematic identifications were discussed to be selected or rejected.

Sfard (2007; 2008) considers that the four properties are both part of the discourse and a mean to analyze it. The selected parts of the transcripts were analyzed according to our adaptation of four properties proposed by Sfard (2007; 2008) developed in the theoretical framework section. The words were highlighted in bold, the narratives were underlined, and the visual mediators were indicated in italics. Repetitive patterns of actions in the narratives allowed us to infer different routines. These guidelines are shown in all the excerpts presented in the section of Results.

In our analysis, we consider narratives that were explicitly or implicitly endorsed across discourse, and with a large presence in one subgroup or a small presence in several subgroups. In addition, we identified narratives around a single idea. They could appear in individual form or as a series of narratives. In the case of the series of narratives, they were grouped as one narrative in our analysis. In this study, we use the term *narrative* for expressing that duality. For example, in our analysis, the following excerpt extracted from a subgroup's transcript "because for example [when the child] says [this] he observes the opposite sides and realizes a property of the parallelograms, this is one of the things of the level…", which was accepted by all the members of a subgroup, has been identified as a narrative.

For the process of analysis, we relied on the Atlas.ti software, which has allowed us to perform both this first step and the second step, which we will develop below.

In a second step, following a constant comparative method based on the work of Glaser and Strauss (1967), we began to establish categories based on similarities and differences about that which was identified in each of the four properties. In doing so, we tested the viability of our categories in the analysis process. This back-and-forth was a means of establishing the reliability of our analysis. This allowed us to document specific features of each property. For instance, in the case of the narratives, there were narratives which expressed a common idea related to the way of considering the tasks proposed that were categorized as "when analyzing school activities, you have to consider pupils' possible responses" or "when interpreting pupils' responses, you have to consider a specific conceptual tool to address those responses". These categories were included in a broader category related to the elements of the triad, in this case, mainly related to two elements: students (pre-service teachers) and content (professional task).

In a third step, on the basis of previous analysis, and taking into account that words, narratives, mediators and routines are properties that characterize jointly a specific discourse, we perform a transversal analysis of these four properties to establish relationship between those characteristics that could configure a particular discourse.

Finally, with respect to the analysis of the subgroup reports, we relied on the categories inferred from the narratives identified in the abovementioned analysis of the transcripts to see if the narratives included in the pre-service teachers' reports could been included or not in them. This allowed us to collect information about the differences between spoken narratives and the narratives that pre-service teachers wrote in their final subgroup reports. For instance, in our analysis, the following excerpt extracted from a subgroup's report "Level of recognition because the description of the figures is limited to its physical appearance and, in addition, we should not expect answers that refer to parallelism, angles, etc., but that will be observe the shape of the figures, its color, or size", has been included in the same category of the narrative that we show as example previously (excerpt from subgroup's transcript).

Table 1. Narratives associated with the relationship between student and teacher

Narratives related to:	Subgroups of pre-service teachers and number of times in which narratives have been found
the relationship between pre-service teachers as students and their university teacher	SG1(8), SG3(26), SG4(17), SG5(25), SG6(29), SG7(4), SG9(16), SG10(3), SG11(9)
the relationship between pre-service teachers as teachers and their future students (pupils)	SG3(3), SG6(5), SG7(1)

RESULTS

The analysis we have presented allowed us to approach pre-service teachers' discourse when solving two didactic-mathematical tasks, documenting the characteristics that underlay it in order to highlight the different discourses that could coexist in that colloquial discourse. We present our results in two stages. Firstly, we focus on the characteristics identified in the pre-service teachers' colloquial discourse on the basis of the four properties. Secondly, we deal with the discourses identified through a transversal analysis of the previous findings, and we present information about the differences between the spoken colloquial discourse collected through the recordings of the subgroup discussions and the written discourse included in the subgroup reports.

Delving into the Words

Our results allowed us to establish differences related to the origin, meaning and use of words.

The origin of the words in the discourse showed some differences. Some words had a general origin coming from previous mathematical courses (**quadrilateral**, **polygon**) or other school contexts (**teacher**, **schoolwork**, etc.), and maintained the same meaning throughout the entire conversation. One might say these words already formed part of previous discourse as students in other courses. Other words were linked with the school activity that is part of the task (words that appear in the description of the school situation). Representative examples can be "**scale**" or "**identifying**". Finally, some words were related to professional activities such as "**pupils could say**", "**involved concepts** [in the task]", etc., or conceptual tools involved in the tasks such as "**van Hiele's levels**" or "**modes of representation**".

Regarding the meaning and use of words, some maintained their meaning throughout the discourse. Nevertheless, we identified some words whose meaning changed depending on the context in which they were used. Sometimes, the use of the words was also different with respect to the person or persons performing the action of the verb (whether pre-service teachers as subject or future pupils as subject). Representative of this situation is the word "answer" whose meaning changed depending on the people involved: "we have to answer" versus "pupils have to answer". Other words, through discussion, expanded their meaning or changed it, showing learning. On some occasions, their use caused conflicts during task resolution. A representative example taken from SG5 could be the use of "pairwise parallel" in relation to the sides of a rectangle/square. The meaning of these combined words enlarges its meaning from exclusive to inclusive definition across the discourse.

Delving into the Narratives

The results of the analysis conducted with the Atlas.ti software allowed us to recognize among the identified narratives two different sets of narratives. The first set grouped all the narratives that had to do with the relationship between student and teacher, and in the other set those that had their focus on the relationship between the pre-service teachers and the professional activity. We will now detail each of these two sets.

Set 1. Narratives associated with the relationship between student-teacher

One of the sets of narratives identified was associated with the relationship between student and teacher. In this set were identified two broader categories according to the role that pre-service teachers adopt when they are solving the tasks proposed: on the one hand, when pre-service teachers adopt the role as students, we have identified narratives associated with the relationship between pre-service teachers as university students and their university teacher. On the other hand, when pre-service teachers adopt the role as teachers, we identified the relationship between pre-service teachers as future teachers and their future students (pupils). In **Table 1**, based on an ATLAS.ti report, we show these two broader categories of narratives and the subgroups of pre-service teachers in which these narratives have been found. These broader categories arose from categories of narratives that were previously identified.

As can be seen in **Table 1**, we have identified a first broader category with 137 narratives in which pre-service teachers are situated as students of any level solving a task proposed by a teacher, in this case, a university teacher.

Table 2. Narratives associated with the relationship between pre-service teachers and the task

Narratives related to:		Subgroups of pre-service teachers and number of times in which narratives have been found
Pre-service teachers considering the task as a "traditional" schoolwork		SG1(3), SG2(20), SG3(27), SG4(7), SG5(17), SG6(30), SG7(16), SG8(3), SG9(10), SG10(17), SG11(15)
Pre-service teachers considering the task as a means of developing a future professional activity	In all three professional activities	SG1(2), SG3(13), SG5(5), SG6(4), SG7(6), SG9(1), SG10(17), SG11(3)
	Only when interpreting	SG3(1), SG4(2), SG8(2), SG9(5)
	Only when analyzing	SG2(6), SG3(11), SG5(10)
	Only when posing questions	SG3(9), SG5(1), SG9(15), SG10(12), SG11(2)

In this regard, categories as "they have to call the university teacher to clarify doubts", "they have to call the university teacher to validate their findings" and so on, are included in this broader category. On occasion, they try to guess what the university teacher wants them to do. Any comment by the university teacher is automatically accepted by all members of the subgroup without them questioning or even discussing it. In this way, the pre-service teachers show features related to what the teacher's role is for them. These narratives have a strong presence in the discourse of these pre-service teachers, since they appear in practically all the subgroups. We may take this to be a normal occurrence in teaching, and that we have not identified anything new. However, two aspects emerge from these results. Firstly, how a particular way of contemplating the teacher/student relationship in the classroom is perpetuated. Secondly, how it could affect the professional development of these pre-service teachers with respect to the expected results of the 'authentic activities' posed in the course.

We have also identified a second broader category. This included others narratives, less numerous and shared by fewer subgroups, in which pre-service teachers assume the role of primary teachers (see **Table 1**). In this role, aspects providing information on their view of the professional activity of a primary teacher begin to emerge. For instance, the following series of underlined narratives were grouped as one narrative in our analysis.

Excerpt 1

SG3. Context: Pre-service teachers try to interpret the pupils' answers.

70:   A5: Yeah! yeah! I know there have to be mistakes but what I'm saying is <u>we need</u>

71:   A1: <u>A template</u>

72:   A5: Of course, for instance, <u>for activities children do in didactic units the answers are given on the back [in the textbooks] in some cases</u>

73:   A6: But it's not the same thing

74:   A1: <u>Solved exercises</u>

75:   A6: But I don't know what you want the answers for

76:   A5: <u>To correct the children if…</u>

77:   A6: But that's not what it's about

78:   A5: Yeah! OK! I know I don't have to <u>correct</u> it, it's for when it comes to <u>analyzing</u> them <u>to see where they went wrong</u>. <u>Imagine if in one case a child has put an R</u> [referring to a rectangle] <u>and should have put an S</u> [referring to a square], well <u>I'll tell him he was wrong here</u>…

This narrative shows the importance these pre-service teachers as future teachers give to having a template to assess the children's responses. This fact could be associated with the typical form of assessment they have experienced in previous educational levels, which disagrees to some extent with what is asked of them in the task (to interpret the children' answers). For them, the meaning of the words "analyze" and "correct" is not well differenced. Although they place themselves as future teachers, in some way they have moved their own school experiences to their future professional activity, indicating the need for a template where the pupils' answers can be situated.

Set 2. Narratives associated with the relationship between pre-service teachers and the professional task

In another set of narratives, the relationships are established between pre-service teachers and the professional activity included in the tasks. As can be seen in **Table 2**, two broader categories arose from categories of narratives that were previously identified. On the one hand, the pre-service teachers consider professional activities as a "traditional" schoolwork that they have to solve. And, on the other hand, the pre-service teachers consider those tasks as a means to develop a future professional activity.

With respect to the first broader category of this set (see **Table 2**), there are 165 narratives that showed that the pre-service teachers considered the proposed task as a "traditional" schoolwork, regardless of the professional

activity that the task poses. These narratives were identified, to a greater or lesser extent, in all the subgroups. One of the categories included narratives related to the consideration of the task as an application of already given content. The following narrative, in which we identify the underlined sentences as a series of endorsed narratives that are considered in our study as a single narrative, has been included in this category:

Excerpt 2

SG1. Context: Pre-service teachers attempt to situate children's answers with respect to van Hiele' Levels.

106: A3: <u>I have here a copy of the different levels, and level 1 is recognition. I'm going to read it out</u> [A3 starts to read], the child perceives the figure as a whole, the figures are individual objects, they simply describe the physique, sentences such as it looks like …

107: A1: <u>It's not in 1</u>

108: A3: OK! <u>In 2 [continues reading]</u> the children perceive the figures formed by parts, and they can deduce other properties from experimentation. However, they are not able to relate some figures to others …

109: A2: <u>Yes</u>

110: A3: <u>Let's go on, Ok, [continues reading] level 3, related to classification</u> commences with the capacity of formal reasoning, where they are capable of recognizing the fact that some properties are deduced from others … [<u>continues reading</u>]

This excerpt shows how a previous consideration of mathematical tasks as an application of already given content, associated with the consideration of school problems as exercises, moves to a context in which a didactic-mathematical task is posed. Pre-service teachers tend to apply this idea. In different narratives, they seem to consider the van Hiele' levels as a kind of "list", where children's answers might fit.

Furthermore, other categories as "it is necessary to write a lot to give a good answer", or "it is not right to finish without having filled in all the sections of the task" are situated in this broader category. We would like to point out that six of the eleven subgroups showed narratives of both categories, which could indicate a certain complementarity between them. In-depth exploration of all these issues can shed light on whether it is the school culture that lingers over time or whether it is due to the proposed tasks, which have not achieved the purported learning.

Regarding the second broader category of this second set of narratives (see **Table 2**), the pre-service teachers' focus was on the particularities of the task that establish differences between a specific task for future teachers and any other task. They consider the professional activities proposed in the tasks (interpreting, analyzing, posing questions) in different ways: shared or specific to each one.

With respect to the 51 narratives which were shared in all three professional activities, one of the categories is inferred from some narratives where the pre-service teachers solve the school problems included in the task, as a first step of the proposed professional activity. The following excerpt is representative of this category:

Excerpt 3

SG1: Context: Analyzing school problems related to probability.

7: A2: [Reading the school problem] There are 6 red, 4 yellow and 2 blue balls in the bag. Indicate with a cross in the following table the type of event in the experiment of picking a ball from the bag and writing down its color. Pick out a yellow ball …

8: A3: <u>But we don't do that or do we? Do we do this?</u>

9: A2: <u>Us, No? To see which concept moves…</u>

In the narrative represented by this excerpt, pre-service teachers agree on the importance of their solving the school problem as a means of seeing the concepts that appear in it.

With respect to the specific narratives, we were able to identify some particularities linked to each one of the proposed professional activity. For instance, one of the categories included in the case of analyzing came from the narratives where the pre-service teachers are situated as future teachers, and they try to consider pupils' possible responses to the school problem included in the task. Somehow, the pre-service teachers start to consider this ability to predict pupils' answers as something necessary to carry out their work as future teachers, as we can see in the following excerpt. This could mark a transition between their role as students who merely solve a task and students that try to approach to their future professional activity.

Excerpt 4

SG8. Context: Pre-service teachers try to think about the interpretation of possible answers of their future pupils.

4: A1: <u>I think a child would say that</u> what to look for... what <u>course</u> are they in? Second cycle

5: A4: But <u>they know that</u> …

Table 3. Identified routines

Routines linked to:	Subgroups in which iterative patterns have been found
Ways of working	SG1(1), SG2(1), SG3(3), SG4(8), SG5(16), SG6(2), SG11(2)
Presentation of answers to the questions	SG2(1), SG3(7), SG4(1), SG5(11), SG6(5), SG7(1), SG11(1)
Search for sources of information	SG1(1), SG3(20), SG4(6), SG5(11), SG6(19), SG7(1), SG9(3), SG10(4), SG11(7)

6:    A1: That <u>they have two equal sides, two to two and thinner than the square</u>

7:    A3: <u>I think that if they knew what a right angle is, they would say it is a polygon or a figure that has four right angles and parallel sides two to two, that is second cycle of primary school</u> and that's it, isn't it?

Finally, if we consider the results included in **Table 1** and **Table 2** together, we can see some aspects we would like to highlight. The first broad category included in each of the tables (the relationship between pre-service teachers as students and the university teacher and pre-service teacher considering the task as a "traditional" schoolwork) informs us about how these pre-service teachers, according to the number of narratives identified, were mostly situated as students of any level who are facing a proposed task. On the other hand, a smaller number of narratives included in the second broad category of each of the tables (the relationship between pre-service teachers as teachers and their future students (pupils) and pre-service teacher considering the task as a means of developing a future professional activity) informs us about an incipient presence of a role as future teachers, and an approach to their future professional activity.

This leads us to think about the posed tasks. If a characteristic of these tasks, considered as "authentic activities", must be that they replicate situations faced in the real world, perhaps these situations are not coherent with the context in which they had to be developed (a university classroom).

Delving into the Visual Mediators

The visual mediators identified in the pre-service teachers' colloquial discourse basically come from the school problem included in the task. What is more outstanding is pre-service teachers' use of the pictures or objects, which appear or are mentioned in the school problem, as a direct support when they solved the school problem included in the task. A representative example is given below.

Excerpt 5

SG3. Context: Pre-service teachers are analyzing a probability problem.

309:    A1: [reading a probability school problem included in the task] Now it says, Anne and Mary are playing at throwing a pushpin on a table. If the pushpin falls on its side, Anne wins, and if it falls on its head, Mary wins. The questions would be, would that game be fair to Anne? Why? What could be done to try to find a way to assign probabilities to such events?

[….]

315:    A3: Wait, *I am going to see if I have a pushpin somewhere*

These visual mediators (drawings of figures, objects) have much more presence in the discourse of pre-service teachers when used as support while solving the school problem than when used to reinforce the refutation. For example, in the SG9, a pre-service teacher uses two coins to convince others pre-service teachers of the possibilities that can result when throwing two coins.

We would like to highlight pre-service teachers use visual mediators to help themselves with the resolution of school problem, at no time they are thinking about how they could use visual mediators with their future pupils to communicate in a classroom, therefore they are adopting the role as students and not as future teachers.

Delving into Routines

Regarding routines, taken to be patterns in the activity that appear in the discourse repeatedly, we were able to infer the ones that we show in **Table 3**.

As can be seen in **Table 3**, routines linked to the ways of working appear in seven subgroups. We were able to identify patterns of action in which pre-service teachers made individual contributions, which are coupled without previous discussion to lead to a collective response to the task. This individual work can be both something done by a member of the subgroup (for instance, an interview of a pupil and its analysis) and information extracted from the theoretical part of the course, articles provided or lectures given by the university teacher. The presence of these routines points out possible difficulties that these pre-service teachers could have to promote a real collaborative work between their future pupils. Other kinds of identified routines are linked to how pre-service teachers organize their responses to the tasks (in a table, in text format, etc.), trying to follow a single format throughout all their work. Sometimes, this leads to repeated expressions, starting with the same words, etc. As a consequence of the

individual contributions, the pre-service teachers also show repetitive patterns aimed at unifying the format before giving their work to the university teacher. Finally, in a large number of subgroups we have identified routines linked to the way of searching for information using different sources. Some of these routines showed a reiterated support of articles provided in the task, information provided in the theoretical part or originating from the Internet. Despite having these sources of information, other subgroups considered the university teacher as the sole source of information. This last aspect of the routine is closely related to the traditional view of teacher. As can be seen in the following representative example, sometimes pre-service teachers reject the possibility of consulting the articles provided and they prefer to ask the university teacher.

Excerpt 6

SG3. Context: Pre-service teachers are solving the first question of the first didactic-mathematical task (see Appendix).

47: A3: But this is what the teacher said of…

48: A5: Let's see, the teacher gave the names yesterday

49: A3: Ah are those, right?

50: A4: And the explanation is here [referring to the articles provided]

51: A2: But the teacher says the next day [referring to the next class] [the teacher] will start with the skills

52: A5: To explain them

53: A3: Of course, the teacher is going to explain the skills, right?

54: A2: Of course

55: A3: And cannot we wait for the teacher explain it? I understand better when the teacher explains it on the slide

A Transversal Analysis

From the results coming from the four properties, we were able to identify different characteristics in the colloquial discourse of these pre-service teachers in their solving of the proposed didactic-mathematical tasks.

On the one hand, we have identified characteristics such as:

- inclusion of words with a general origin linked mainly to previous experiences as students (in previous mathematical classrooms or in other school subjects), which maintain their meaning; and words that appear in the task itself. They mostly use the words to respond to the task as an activity that is part of the university course and, in this use, the pre-service teachers are subjects of the actions;

- narratives in which a relationship between pre-service teachers as students and the university teacher predominates (see **Table 1**), and a vision of the tasks posed in the course as schoolwork that has to be done by the pre-service teachers, and valued and directed by the university teacher (see **Table 2**);

- use of mediators for the purpose of clarifying themselves as university students doing a task proposed in the classroom and not thinking about their future students (pupils);

- routines that show a standard procedure to present the reports related to the tasks, and routines related to searching information which they need in order to give response to those tasks, considered as an exercise in which a previous content has to be applied.

All these characteristics lead us to infer that, sometimes, some subgroups of pre-service teachers maintain a discourse that can be considered widely accepted as an usual form of language in a group of students when they solve a task proposed in a classroom by the teacher, which we have called a "discourse as students".

On the other hand, we have identified characteristics such as:

- inclusion of words related to the professional activities demanded in the task. These words were used by the pre-service teachers as if they were future teachers who take into account their pupils. In this case, the use of words is conditioned by their future pupils, who are the subjects who perform the action of the verb;

- narratives associated with the relationship between pre-service teachers as teachers and their future students (pupils) (see **Table 1**), and the tasks as a means of approaching the different professional activities of primary teachers, either in a general or specific form of each of them, establishing differences depending on the professional activity considered (see **Table 2**);

- none of visual mediators that have been found refer to the ones that pupils could use or that the pre-service teachers as future teachers could use in relation to their future pupils. We want to highlight the difficulties presented by approaching the visual mediators, since these pre-service teachers do not have had direct contact (past or present) in this role with pupils;

- through established routines, the pre-service teachers sought out information using several sources in order to have knowledge about the contents implicit in the tasks and thus be able to carry out what the professional activity demanded of them.

All these characteristics lead us to infer that, on other occasions, the pre-service teachers are situated as future teachers. In this case, we have identified what we have called a "discourse as future teachers", in which what must become a discourse for teaching begins to be manifested (Gresalfi & Cobb, 2011).

To sum up, in our results we have been able to identify two discourses in the colloquial discourse of several subgroups of pre-service primary teachers when solving two didactic-mathematical tasks. One is the discourse generated by the pre-service teachers when they adopted the role of students who have to solve a task proposed in the classroom by a university teacher, with features originating from previous experiences as students of mathematics or other subject matters. The other discourse is linked with the adoption of a role close to their future professional work, related to aspects such as the way they consider mathematics as a school subject that has to be taught and learned in a school context.

Pre-service teachers generate a mixed discourse, in which they combine, to a greater or lesser extent, both discourses. The way of combining these discourses throughout the process of resolution of the two tasks has originated what we have called "discursive itineraries". With respect to the reasons and characteristics of this mixed discourse, we have been able to observe that the alternation or permanence in one or the other of the discourses depended on the interactions between the pre-service teachers, which role the pre-service teachers adopted and which discourse came to the fore at each moment. Sometimes, the change of discourse depended on the conviction force of a pre-service teacher or several pre-service teachers within the subgroup (Sfard, 2001). This leads to two possibilities: all pre-service teachers adopt the change of discourse or some pre-service teachers do not adopt the change in that moment, provoking a conflict or a coexistence of the discourses. A future focus of research could be to study the reasons and characteristics of that alternation.

Finally, the endorsed narratives that the pre-service teachers eventually wrote as the final decision in their subgroup reports revealed some characteristics of the learning achieved through the proposed tasks. From an academic point of view, in most cases these endorsed narratives included in the final report showed that these subgroups of students were able to pass the requirements established for the practical part of the course. From our research, we can say that all the narratives of **Table 1** that inform us about the student-teacher relationship are missing in the pre-service teachers' written responses, since we have no explicit evidence of them in the subgroup reports. Taking this into account, the spoken discourse that arises in the interaction between pre-service teachers becomes a key element to obtain information on how they consider the student-teacher relationship.

Regarding the narratives included in **Table 2**, which inform us about the pre-service teachers-task relationship, they have been identified in the subgroups reports, but they have provided less information that the subgroup oral discussions. In the reports of the pre-service teachers, we appreciate narratives related to the application of already given content, repetition of arguments in the response, or inclusion of content little related to the task, all them included in the broader category "pre-service teachers considering the task as a "traditional" schoolwork". However, there are narratives that have been not fully identified within of the other broader category "pre-service teachers considering the task as a means of developing a future professional activity". For example, in the reports appear narratives related to children's possible responses, but they have provided less information than the narratives of the spoken discourse.

In this sense, the subgroup discussions (spoken discourse) have allowed us to obtain more information than the subgroup reports (written discourse). This could be an aspect that the teacher educators would have to take into account when they use written responses to proposed tasks as the only instrument to assess the learning achieved by their students.

CONCLUSIONS

The work presented here adopts a commognitive approach to identify the characteristics of pre-service primary teachers' discourse when solving didactic-mathematical tasks on a mathematics teacher education course. In this study, we have adapted the four properties provided by Sfard (2007; 2008) to analyze the abovementioned discourse. Our results extend the work of authors such as Gee (1996), Caspi and Sfard (2012), among many others, identifying other discourses and expanding their works to other contexts. The initial aim of characterizing pre-service primary teachers' discourse concluded with the identification of two discourses which coexist in pre-service primary teachers' colloquial discourse. This has enabled us to provide information to researchers and teacher educators about what really happens when the future teachers of our study solve a didactic-mathematical task.

Regarding the two identified discourses, we wish to point out that they are not opposing options. There are clearly two discourses, with different identifying aspects that endow them with specific characteristics. These

discourses alternate over time, forming "discursive itineraries". These itineraries could allow us to identify social forms of temporality, approaching us to the idea of trajectories in the sense of Wenger (1998).

Our results lead us to pose different questions related to the different aspects that intervene in the generation and development of these discourses. We wonder whether working with more subgroups would allow us to identify different discourses, and whether these discourses depend on the tasks. Perhaps the tasks posed in the mathematics teacher education course have failed in an important aspect pointed out by Brown et al. (1989): they have not encouraged the pre-service teachers to abstract from a classroom situation (in which they are students) to a professional context (in which they are teachers). Both the proposed task and the actual context need to be experienced by pre-service teachers in order to minimize the "discourse as students" and foster the "discourse as future teachers". Our study has also show differences between the spoken discourse and the written discourse, being the pre-services' written discourse closer to the one of primary teachers. In this sense, contributions of researchers as Ivars, Fernández, Llinares and Choy (2018) have showed how the use of hypothetical learning trajectories as guide can help to develop the professional written discourse of pre-service teachers. Enlarge this proposal to spoken discourse could give information about whether this type of task fosters also the spoken discourse. It is important that teacher educators know the differences and similarities among the two discourses to assess what really pre-service teachers learn.

In relation to the discourse as students, our results have brought to light some of the problems that exist with regard to the traditional view of teachers' work that emerges from the pre-service teachers' role as students. Given the characteristics we have identified in that discourse, there remains the question of the extent to which future teachers translate traditional ways of viewing the role of the teacher, shown in said discourse as students, to their future work. In the case of the proposed professional activities (interpreting, analyzing and posing questions), we think that the broader adoption of the abovementioned discourse could, in some way, affect both pre-service teachers' considerations about said professional activities and their way of implementing them in the classroom.

With respect to the emergence of a discourse close to primary teachers' discourse, the results have shown that this kind of discourse is beginning to appear but its presence is minor in most of the subgroups. Of note are the few narratives associated with the relationship between pre-service teachers as future teachers and their future students (pupils), unlike what was happening in the discourse as students. This could be due to the difficulty that the pre-service teachers have in assuming the role of a teacher, despite this being what the tasks demanded. This leads us to consider whether the task itself has been suitable enough to foster that role. However, with respect to the relationships established between pre-service teachers and the tasks, in our study we have identified an emerging development of the professional activities related to their future work.

In relation to the limitations of this study, it focuses on what happens in a very specific context, with some tasks and some pre-service teachers. We need enlarge the study with different pre-service teachers and tasks to examine whether other discourse could arise or whether the discourse is closely related to the tasks or the pre-service teachers. Specifically, this study need be developed with other pre-service teachers of the same context and with different didactic-mathematical tasks that include different mathematics content and/or different professional activity.

Finally, if we are not conscious of the possible coexistence of discourses, teacher educators may not take into account their presence and mathematical education research may ignore their existence. In short, if we consider that the acquisition of a specific discourse enables future teachers to integrate into the community of practice of primary teachers, the role of the different discourses becomes a relevant element in teacher education. As researchers in the mathematics teacher education field, in particular in the processes of learning to teach mathematics, we have tried to ascertain whether theoretical ideas coming from Sfard's commognitive framework extend our understanding of pre-service primary teachers' mathematical discourse. On the basis of this idea, we consider that the theory of commognition has been useful for both our research and for our practice as mathematics teacher educators.

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APPENDIX

First didactic-mathematical task: Analyses of a school activity and interpretation of primary school students' productions.

The following activity is proposed to students in the second cycle of Primary Education:

Given a sheet of quadrilaterals, the student was asked to put an S on each square, an R on each rectangle, and if he or she was familiar with the terms, a P on each parallelogram and a B on each rhombus. The student was asked to justify his or her markings and, if necessary, why some of the figures had been omitted.

The student was asked, "What would you tell someone to look for in order to pick out all the rectangles on a sheet of figures? Could you make a shorter list? Is No. 2 a rectangle? Is No. 9 a parallelogram?"

Figure 1. School activity (extracted from Burger & Shaughnessy, 1986, pp. 34-35)

We ask you as pre-service teachers:

a) Analyze the type of skills and geometric reasoning processes that could be developed when solving this school activity.

b) What van Hiele levels of geometric reasoning could be manifested when the students answer the different questions of the school activity? Justify the answer.

c) Raise this school activity to primary students. Collect the answers and transcribe them. Analyze them, identifying the type of skills/geometric reasoning processes and level in which the primary students are.

Notes:

- Each member of the subgroup must provide the transcript of at least one interview with a primary student to be analyzed by the subgroup. The transcripts of the interviews will be submitted as an annex to the report.

Second didactic-mathematical task: Analyses of school problems and formulation of questions to ask primary students.

You are thinking about working on some ideas related to probability in your primary classroom, and you have selected some problem of textbooks or some links of website on the topic that could be useful. Before you decide what problems use, you remember what was said in mathematics teacher education course about the importance of conducting a previous analysis and you consider:

a) identify the mathematical concepts and procedures that your students could put into play in the resolution of each one of school problems.

b) think "good questions" that could be considered for each problem in a teaching situation. At least 4 questions must be included per problem and indicate what is intended with each one of them.

Examples of school problems proposed taken from primary school textbooks:

Problem 1

In the rotating drum (see figure) there are 6 red, 4 yellow and 2 blue balls. Indicate with a cross in the following table the type of event in the experiment of picking a ball from a rotating drum and writing down its color.
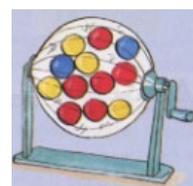
	Sure event	Possible event	Impossible event
Pick out a yellow ball			
Pick out a green ball			
Pick out a ball that is not blue			
Pick out a ball that is not green			
Pick out a red ball			

Problem 2

This is to complete the previous problem, trying to assign (in addition to one word) a number to each of the events in the table, using the scale shown below:

Impossible event (0)   Sure event (1)

Problem 4

Abel and Rose play by throwing a dice. Abel wins if the result is a 5, and Rosa wins if the result is less than 3. How many times will each one has won, approximately, after throwing the dice 60 times?

Problem 8

Anne and Mary are playing at throwing a pushpin on a table. If the pushpin falls on its side, Anne wins, and if it falls on its head, Mary wins. Would that game be fair to Anne? Why? What could be done to try to find a way to assign probabilities to such events?

http://www.ejmste.com