Eurasia Journal of Mathematics, Science & Technology Education, 2015, **11**(5), 1181-1196

A Study of Van Hiele of Geometric Thinking among 1st through 6th Graders

Hsiu-Lan Ma
Ling Tung University, TAIWAN
De-Chih Lee & Szu-Hsing Lin
Da Yeh University, TAIWAN
Der-Bang Wu
National Taichung University of Education, TAIWAN

•*Received 10 February 2015*•*Revised 15 April 2015* •*Accepted 6 June 2015*

This study presents partial results from the project "A Study of perceptual apprehensive, operative apprehensive, sequential apprehensive, and discursive apprehensive for elementary school students (POSD)", which was undertaken to explore gender differences and passing rate of van Hiele's geometric thinking level. The participants were 5,581 elementary school students randomly selected from 23 counties/cities in Taiwan. The conclusions drawn from this study were: (a) evidence supporting the hierarchy of the van Hiele levels, (b) students at different levels have different concepts of basic figures, and (c) for elementary school students, the passing rates of boys and girls have no significant differences in van Hiele's geometric thinking level.

Keywords: discursive, operative, perceptual, sequential, van Hiele

INTRODUCTION

Geometry is one of the most important topics in mathematics (Ministry of Education of Taiwan, (MET), 1993, 2000, 2003, 2008; National Council of Teachers of Mathematics (NCTM), 1989, 1991, 1995, 2000). The geometry curriculum is developed and designed according to the van Hiele model of geometric thought in Taiwan. (MET, 1993, 2000, 2003, 2008). Most 1 and 2 grade students achieve Level 1; most 3 and 4 grade students achieve Level 2; most 5 and 6 grade students achieve Level 3 (National Academy for Educational Research, 2001, Škrbec and Čadež, 2015). Therefore, this study focuses on the first three van Hiele levels.

In 1957, the van Hiele model was developed by two Dutch mathematics educators, P. M. van Hiele and his wife (van Hiele, 1957). Several studies have been conducted to discover the implications of the theory for the current K-12 geometry curricula, and to validate aspects of the van Hiele model (Burger & Shaughnessy, 1986; Chen, Wu, Ma, & Sheu, 2011; Eberle, 1989; Fuys, Geddes, & Tischler, 1988; Gutierrez, Jaime, & Fortuny, 1991; Ma & Wu, 2000; Ma, Wu, & Wu, 2012; Mayberry, 1983; Molina, 1990; Senk, 1983, 1989; Pegg, 1985; Pegg & Davey, 1989, 1991;

Correspondence: Szu-Hsing Lin,
Program in Management, Da Yeh University, No.168, University Rd., Dacun, Changhua 51591, Taiwan (R.O.C.).
E-mail: szuhsing@gmail.com
doi: 10.12973/eurasia.2015.1412a

ISSN: 1305-8223

Usiskin, 1982; Wu, 1994, 1995, 2003; Wu & Ma, 2005a, 2005b; Wu & Ma, 2006; Wu, Ma, & Lan, 2005; Wu, Ma, & Chen, 2006; Wu, Ma, Lan, & Yao, 2006). Besides the researches of Wu and Ma (2005a, 2005b; 2006), most researchers focus on the geometrical curricula of secondary school; however, discovering the implications of the van Hiele theory for elementary school students is also very important, thus, the focus of this study is at the elementary level. The main objectives of this study were to determine the passing rates of van Hiele levels of geometric thinking among 1st through 6th graders.

Hyde, Fennema, and Lamon (1990) represented that females outperformed males by only a negligible amount. An examination of age trends indicated that girls showed a slight superiority in computation in elementary school and middle school. There were no gender differences in problem solving in elementary or middle school; differences favouring males emerged in high school and college.

It would be very appropriate to divide the teaching of geometry into different levels according to the van Hiele theory. Secondary schools, with regard to their specialization, should determine what levels they want to achieve, and adapt teaching geometry to that goal. Van Hiele levels are equally suitable for both genders (Haviger and Vojkůvková, 2014), thus, gender difference is an important issue in mathematics.

State of the literature

- The geometry curriculum is developed and designed according to the van Hiele model of geometric thought.
- Several studies have been conducted to discover the implications of the theory for the current K-12 geometry curricula, and to validate aspects of the van Hiele model.
- Discovering the implications of the van Hiele theory for elementary school students is also very important.

Contribution of this paper to the literature

- It seems that the circular concept is the easiest for students; while the concept of the quadrilateral is the most difficult for students.
- Regarding the higher grades for all three figures (triangle, quadrilateral, and circle), The gender differences were significant differences.
- Based on these three basic figures (triangle, quadrilateral, circle), most students of grades 1 and 2 were at level 1, and grades 3 to 6 were at level 2. Only grades 5 and 6 could meet level 3.

The objectives of this paper are:

a) to determine the types and distributions of the first van Hiele levels of geometric thinking.
b) to determine the distributions of van Hiele levels of geometric thinking among 1st through 6th graders.
c) to determine the distributions of passing rates of each graders in van Hiele's geometric thinking.
d) to determine the differences of the passing rates of boys and girls in van Hiele's geometric thinking.

THEORICAL FRAMEWORK

In the late 1950s, two Dutch school teachers, Dina van Hiele-Geldof, and her husband, Pierre M. van Hiele, devised a model of geometric thought for helping students to learn geometry. They were concerned about their secondary school students' performances in geometry and were interested "in improving teaching outcomes" (van Hiele, 1986, 1999). The van Hieles' doctoral dissertations studied the complementary aspects of developing insight in geometry (Wu, 1994).

Adapted from Gestalt psychology, many of the ideas in the van Hiele model focus on the idea of a structure (Molina, 1990; van Hiele, 1986, 1999). P. M. van Hiele (1986) pointed out, "Most of the ideas of structure I have developed ... are borrowed from Gestalt theory" (p. 5). P. M. van Hiele adapted the concepts of levels, as derived from Jean Piaget, although he disagreed with Piaget on several points (Molina, 1990;

van Hiele, 1986). P. M. van Hiele (1986) claimed, "In any case, an important part of the roots of my work can be found in the theories of Piaget. It is important then, too, to emphasize the differences … " (p. 5). He pointed out six main differences between his theories and Piaget's theories. Pierre M. van Hiele "formulated the scheme and psychological principles; while D. van Hiele-Geld focused on the didactic experiment to raise students' thought levels" (Hoffer, 1983). Three major components were addressed in this model: (a) the nature of insight, (b) the levels of thought, and (c) the phases of learning (Wu, 1994). The focus of this study was on the levels of thought.

There are five levels of the van Hiele's geometric thought: "visual", "descriptive", "theoretical", "formal logic", and "the nature of logical laws" (van Hiele, 1986, p. 53). These five levels have been labelled in two different ways: Level 1 through Level 5 or level 0 through Level 4. Researchers have not yet come to a conclusion of which one to use. P. M. van Hiele (1986) said: "In the article of 1995, what was spoken of as the first level in now spoken of as the second level. So, in the continuation of the article, what was spoken of as a second level, we now speak of as a third level, and so on (p. 41)." In this study, these five levels were called Level 1 through Level 5 (adopted the recent claim of van Hiele), at the elementary level, students' geometry thinking level on Level 1 to level 3 (Škrbec and Čadež, 2015), thus, the focus of this study was on Level 1 to Level 3.

At the first level, students learned geometry through visualization (van Hiele, 1984). According to van Hiele (1986), "Figures are judged by their appearance. A child recognizes a rectangle by its form and a rectangle seems different to him than a square (p. 245)." At this first level students identify and operate on shapes (e.g., squares, triangles, etc.) and other geometric parts (e.g., lines, angles, grids, etc.) based on the appearance (van Hiele, 1984, 1986). Students recognize figures by their global appearance. They can say triangle, square, cube, etc., but they do not explicitly identify the properties of the figures (Hoffer, 1983).

At the second level, a student may realize that the opposite sides, and possibly even the diagonals of a rectangle, are congruent, but will not notice how rectangles relate to squares or right triangles (Hoffer, 1981). Students' analyse the properties of figures: "rectangles have equal diagonals" and "a rhombus has all sides equal," but they do not explicitly interrelate figures or properties. (Hoffer, 1983) Students analyse figures in terms of components and relationships between components, establish the properties of a class of figures empirically, and use the properties to solve problems. At this level, figures are bearers of their properties. That a figure is a rectangle means that it has four right angles, diagonals are equal, and opposite sides are equal. Figures are recognized by their properties. If one tells us that a figure drawn on a blackboard has four right angles, it is a rectangle, even if the figure is drawn badly. However, at this level properties are not yet ordered, and a square is not necessarily identified as being a rectangle (van Hiele, 1984, 1986).

At the third level, a student will understand why every square is a rectangle, but may not be able to explain, for example, why the diagonals of a rectangle are congruent (Hoffer, 1981). Students relate figures and their properties: "every square is a rectangle," but they do not organize sequences of statements to justify observations (Hoffer, 1983). At this level, properties are ordered, and are deduced one from another: one property precedes or follows another property. At this level the intrinsic meaning of deduction is not understood by students. The square is recognized as being a rectangle because, at this level, definitions of figure come into play (van Hiele, 1984, 1986).

The focus of this study was elementary school students and the first three van Hiele levels.

Patsiomitou and Emvalotis (2010) studied the students' concepts of geometric symmetry using software, and found that the geometric thoughts in line with Van Hiele, Erdogan, Akkaya, and Lebi Akkaya (2009) employed the geometric thinking level of Van Hiele to test the creative thinking of sixth graders, and found there are significant differences between the experimental group and the control group. Tutkun, and Ozturk (2013) employed the GEOGEBRA mathematics software to discuss the unit of "trigonometry and slope" of the second year of junior high school, and found there are significant differences in the level of understanding between the experimental group and the control group. Sharp and Zachary (2004) employed the theory of Van Hiele for the re-conceptualization and re-grouping of the teaching methods for engineering mechanics, which was helpful for the learning of college students.

METHODS AND PROCEDURES

Participants

The participants were 5,581 elementary school students, who were randomly selected from 25 elementary schools in 23 counties/cities in Taiwan. There were 2,717 girls and 2,864 boys. The numbers of participants, from 1st to 6th grades, were 910, 912, 917, 909, 920, and 1,013 students, respectively.

Instrument

The instrument used in this study, Wu's Geometry Test (WGT), was specifically designed for this project, as there were no suitable Chinese instruments available. This instrument was designed based on van Hiele level descriptors and sample responses, as identified by Fuys, Geddes, and Tischler (1988). There were 25 multiple-choice questions of the first van Hiele level; 20 in the second, and 25 in the third. The test focused on three basic geometric figures: triangle, quadrilateral, and circle.

Twenty-five questions at level one were characterized into nine types, as based on its geometric attributions. Questions 1, 2, and 3 are included in Type 1; questions 4, 5, and 6 are included in Type 2; questions 7, 8, and 9 are included in Type 3; questions 10, 11, and 12 are included in Type 4; questions 13, 14, and 15 are included in Type 5; questions 16 and 17 are included in Type 6; questions 18 and 19 are included in Type 7; questions 20, 11, and 22 are included in Type 8; and questions 23, 24, and 25 are included in Type 9.

The scoring criteria were based on the van Hiele Geometry Test (VHG), as developed by Usiskin (1982), in the project "van Hiele Levels and Achievement in Secondary School Geometry" (CDASSG Project). In the VHG test, each level has five questions. If the student answers three, four, or five first-level questions correctly, he/she has reached the first level. If the students (a) answered three questions or more correctly from the second level; (b) met the criteria of the first level; and (c) did not correctly answer three or more questions, from levels 3, 4, and 5, they were classified in the second level. Therefore, using the same criteria set by Usiskin (1982), the passing rate of this study was set at 60%. If the scores of the students did not follow the criteria, the cases were labelled "jump phenomenon" by the authors.

Validity and reliability of the instrument

The attempt to validate the instrument (WGT) involved the critiques of a validating team. The members of this team included five elementary school teachers, graduate students majoring in mathematics education, and six professors from the Mathematics Education Departments at several pre-service teacher preparation institutes. The team members were asked to review the instrument and provide feedback regarding whether each item was suitable. They also gave suggestions on how to make this test better.

In order to measure the reliability of the WGT, 289 elementary school students (from grades 1-6) were selected to take the WGT. These students were not participants in this study. The alpha reliability coefficient of the first van Hiele level of WGT was 0.67, 0.88 for level 2, and 0.94 for level 3, using SPSS® for Windows® Version 20.0.

Procedure

The class teachers of the participants administered the test in one mathematics class. The answers were graded by the project directors. The distribution of the questions is as shown in Table 1.

RESULTS

Total passing rate on basic figures

The passing numbers and passing rate for each type and each geometric shape at level 1 are reported in Table 1.

From the data of Table 1, the total passing rate was 72.30%. The overall passing rates of the triangle concept were 75.40%, 63.58% for quadrilateral, and 84.09% for circle. It seemed that the circle concept is the easiest for students, followed by the triangle concept, and the quadrilateral concept. The passing rates of each shape are a shown in Figure 1.

Table 1. The numbers passed and passing rate of each type and shape

	Triangle N=2848	**Quadrilateral N=2848**	**Circle N=2848**	**Total**
Type 1	71.21%	70.19%	75.81%	72.40%
Type 2	84.06%	7.44%	95.44%	62.31%
Type 3	94.14%	87.64%	95.79%	92.52%
Type 4	79.56%	65.91%	89.01%	78.16%
Type 5	58.60%	67.59%	78.62%	68.27%
Type 6	65.10%	43.75%		54.42%
Type 7	57.06%	59.52%		58.29%
Type 8	87.99%	83.46%	79.74%	83.73%
Type 9	80.86%	86.76%	74.23%	80.62%
Total	75.40%	63.58%	84.09%	72.30%

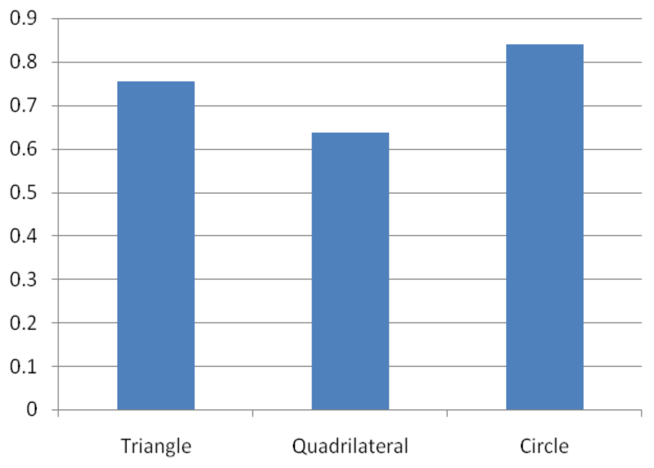

Figure 1. The passing rate of each shape

Overall performance on each type of level 1

The passing rates of each shape are as shown in Figure 2. The overall passing rates, from Type 1 to Type 9, were 72.40%, 62.31%, 92.52%, 78.16%, 68.27%, 54.42%, 58.29%, 83.73%, and 80.62%, respectively.

It seems that Type 3 is the easiest for students, followed by Type 8 and Type 9. Type 6 was the most difficult, followed by Type 7 and Type 2.

Type 1: Identification of Open and Closed Figures: The example of the Type 1 questions is as shown in Figure 3. The passing rates of the triangle concept were 71.21% and 70.19% for quadrilateral, and 75.81% for circle. It showed that students could easily identify open and closed figures in the circular concept, but had difficulties on quadrilateral.

Type 2: Identification of Convex and Concave Figures: The example of Type 2 questions is as shown in Figure 4. The passing rates of the triangle concept were 84.06% and 7.44% for quadrilateral, and 95.44% for circle. It showed that students

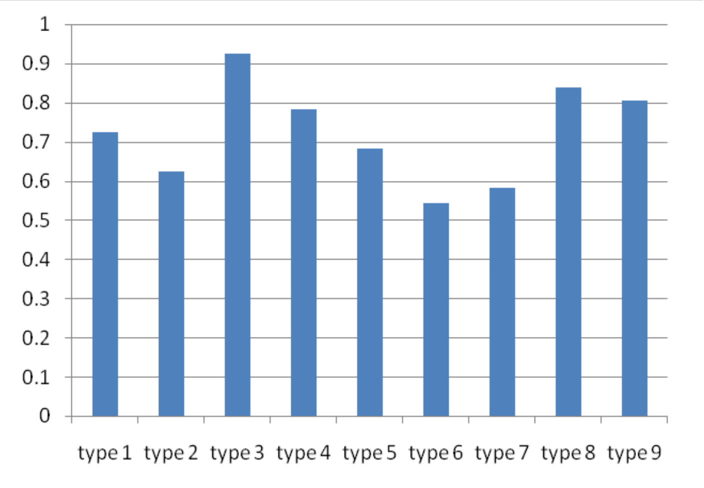

Figure 2. The passing rate of each shape

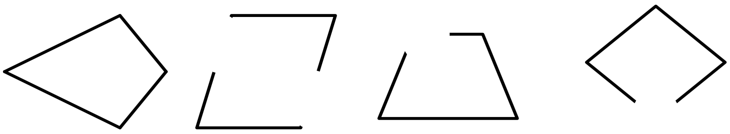

Figure 3. The identification of open and closed figure

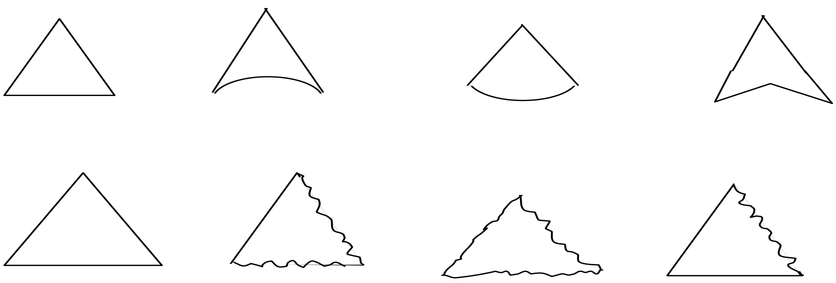

Figure 4. The identification of convex and concave figure, line and curve figure

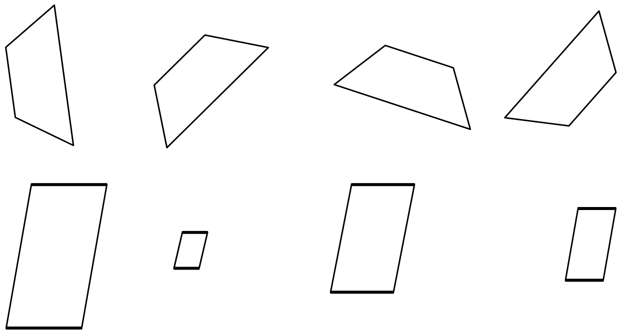

Figure 5. The identification of rotate figure, figures of different sizes

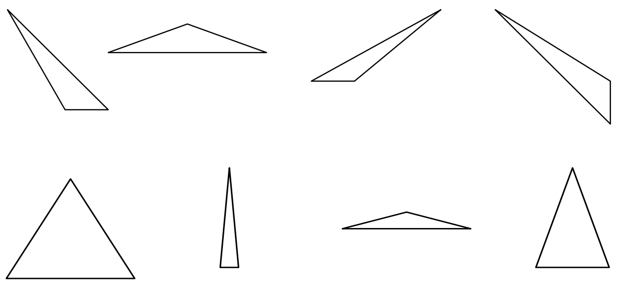

Figure 6. The identification of extremely obtuse figure, wide and narrow figure

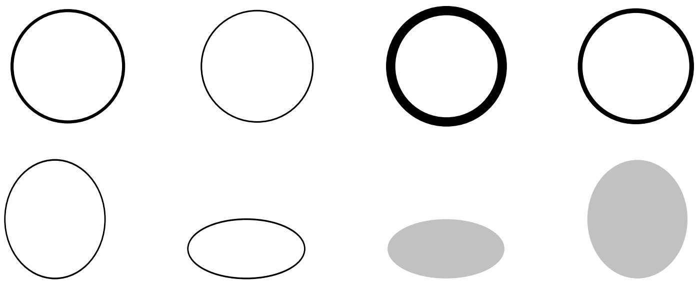

Figure 7. The identification of wide and contour figure, filled and hollow figure

could easily identify the convex and concave figures in the circular concept, but had difficulties on quadrilateral.

Type 3: Identification of Straight Line and Curve Line: The example of Type 3 questions is as shown in Figure 4. The passing rates of the triangle concept were 94.14% and 87.64% for quadrilateral, and 95.79% for circle. It showed that students could easily identify the straight line and curve lines in the circular concept, but had difficulties on quadrilateral.

Type 4: Identification of Rotate Figure: The example of Type 4 questions is as shown in Figure 5. The passing rates of the triangle concept were 79.56% and 65.91% for quadrilateral, and 89.01% for circle. It showed that students could easily identify the rotated figures in the circular concept, but had difficulties on quadrilateral.

Type 5: Identification of Figures of Different Sizes: The example of Type 5 questions is as shown in Figure 5. The passing rates of the triangle concept were 58.60% and 67.59% for quadrilateral, and 78.62% for circle. It showed that students could easily identify the figures of different sizes in the circular concept, but had difficulties on quadrilateral.

Type 6: Identification of Extremely Obtuse Figures: The example of Type 6 questions is as shown in Figure 6. The passing rates of the triangle concept were

65.10% and 43.75% for quadrilateral. It showed that students could easily identify the figures of extremely obtuse figures in the triangular concept, but had difficulties on quadrilateral.

Type 7: Identification of Wide and Narrow Figures: The example of Type 7 questions is as shown in Figure 6. The passing rates of the triangle concept were 57.06% and 59.52% for quadrilateral. It showed that students could easily identify the figures of wide and narrow figures in the quadrilateral concept, but had difficulties on triangular.

Type 8: Identification on Width of the Contour Line: The example of Type 8 questions is as shown in Figure 7. The passing rates of the triangle concept were 87.99% and 83.46% for quadrilateral, and 79.74% for circle. It showed that students could easily identify the width of the contour line in the triangular concept, but had difficulties on circle.

Type 9: Identification on Filled and Hollow Figures: The example of Type 9 questions is as shown in Figure 7. The passing rates of the triangle concept were 80.86% and 86.76% for quadrilateral, and 74.23% for circle. It showed that students could easily identify the filled and hollow figures in the quadrilateral concept, but had difficulties on circle.

Overall performance on triangle, quadrilateral, and circle

Based on the questions of triangle, 43.0% of the elementary school students were at van Hiele level 1, 28.0% at level 2, and 5.2% at Level 3. The students who were at level 1 of the questions of quadrilateral were 25.9%, 28.0% at level 2, and 5.5% at level 3. For the questions of circle, 35.7% of the elementary school students were at van Hiele level 1, 45.5% at level 2, and 7.7% at level 3 (See Table 2).

The percentage of students did NOT meet the criteria of level 1 (below level 1); for the triangle were 20.8%, 30.3% for quadrilateral, and 7.7% for circle. It seems that the circle concept is the easiest one for students, followed by the triangle concept, and the quadrilateral concept. It is worth mentioning that the percentage of students appeared “jump phenomenon”, for the triangle were 2.9%, 10.3% for the quadrilateral, and 3.3% for the circle.

The distributions of van Hiele levels of triangle concepts

The percentage of students appeared “jump phenomenon”, for the triangle were 2.9% (See Table 2). Thus, there were 5,419 (97.1%) students who could be assigned to levels 1 to 3. The distributions of van Hiele level of triangle concepts from grades 1 to 6 of each figure are as shown as Table 3.

Based on the questions of triangle, 48.1% of the grade 1 students were at van Hiele level 1, and 62.6% at grade 2. The grade 3 students at level 1 of the triangle concept were 55.9%, and 27.0% at level 2. The grade 4 students at level 1 of the triangle concept were 50.6%, and 40.6% at level 2. The grade 5 students at level 1 of the triangle concept were 29.9%, 51.3% at level 2, and 11.0% at level 3. The grade 6 students assigned at level 1 of the triangle concept were 19.8%, 53.9% at level 2, and 20.7% at level 3 (See Table 3).

Table 2. The overall distributions of levels 1 to 3

	Triangle		**Quadrilateral**		**Circle**	
	Frequency	**Percent**	**Frequency**	**Percent**	**Frequency**	**Percent**
Below level 1	1163	20.8	1691	30.3	431	7.7
Level 1	2402	43.0	1447	25.9	1995	35.7
Level 2	1561	28.0	1563	28.0	2538	45.5
Level 3	293	5.2	305	5.5	432	7.7
Jump	162	2.9	575	10.3	185	3.3
Total	5581	100.0	5581	100.0	5581	100.0

Table 3. The percentage analyzed by grades and levels based on triangle, quadrilateral and cycle

			Triangle				Total
			Below level 1	Level 1	Level 2	Level 3	
Grade	1	Count	472	438	0	0	910
		% within Grade	51.9%	48.1%	.0%	.0%	100.0%
	2	Count	341	571	0	0	912
		% within Grade	37.4%	62.6%	.0%	.0%	100.0%
	3	Count	151	494	239	0	884
		% within Grade	17.1%	55.9%	27.0%	.0%	100.0%
	4	Count	78	448	359	0	885
		% within Grade	8.8%	50.6%	40.6%	.0%	100.0%
	5	Count	68	262	449	96	875
		% within Grade	7.8%	29.9%	51.3%	11.0%	100.0%
	6	Count	53	189	514	197	953
		% within Grade	5.6%	19.8%	53.9%	20.7%	100.0%
Total		Count	1163	2402	1561	293	5419
		% within Grade	21.5%	44.3%	28.8%	5.4%	100.0%
			Quadrilateral				Total
			Below level 1	Level 1	Level 2	Level 3	
Grade	1	Count	627	283	0	0	910
		% within Grade	68.9%	31.1%	.0%	.0%	100.0%
	2	Count	488	424	0	0	912
		% within Grade	53.5%	46.5%	.0%	.0%	100.0%
	3	Count	288	330	225	0	843
		% within Grade	34.2%	39.1%	26.7%	.0%	100.0%
	4	Count	127	263	443	0	833
		% within Grade	15.2%	31.6%	53.2%	.0%	100.0%
	5	Count	86	84	441	113	724
		% within Grade	11.9%	11.6%	60.9%	15.6%	100.0%
	6	Count	75	63	454	192	784
		% within Grade	9.6%	8.0%	57.9%	24.5%	100.0%
Total		Count	1691	1447	1563	305	5006
		% within Grade	33.8%	28.9%	31.2%	6.1%	100.0%
			Circle				Total
			Below level 1	Level 1	Level 2	Level 3	
Grade	1	Count	220	690	0	0	910
		% within Grade	24.2%	75.8%	.0%	.0%	100.0%
	2	Count	97	815	0	0	912
		% within Grade	10.6%	89.4%	.0%	.0%	100.0%
	3	Count	58	285	546	1	890
		% within Grade	6.5%	32.0%	61.3%	.1%	100.0%
	4	Count	20	132	717	2	871
		% within Grade	2.3%	15.2%	82.3%	.2%	100.0%
	5	Count	22	35	644	159	860
		% within Grade	2.6%	4.1%	74.9%	18.5%	100.0%
	6	Count	14	38	631	270	953
		% within Grade	1.5%	4.0%	66.2%	28.3%	100.0%
Total		Count	431	1995	2538	432	5396
		% within Grade	8.0%	37.0%	47.0%	8.0%	100.0%

The distributions of van Hiele levels of quadrilateral concepts

The percentage of students appeared “jump phenomenon”, for the quadrilateral were 10.3% (See Table2). Thus, there were 5,006 (89.7%) students who could be assigned to levels 1 to 3. The distributions of van Hiele level of quadrilateral concepts from grades 1 to 6 for each figure are as shown in Table 3.

Based on the questions of quadrilateral, 31.1% of the grade 1 students were at van Hiele level 1, and 46.5% at grade 2. The grade 3 students at level 1 of the quadrilateral concept were 39.1%, and 26.7% at level 2. The grade 4 students at level 1 of the quadrilateral concept were 31.6%, and 53.2% at level 2. The grade 5 students at level 1 of the quadrilateral concept were 11.6%, 60.9% at level 2, and 15.6% at level 3. The grade 6 students who were assigned at level 1 of the quadrilateral concept were 8.0%, 57.9% at level 2, and 24.5% at level 3 (See Table 3).

The distributions of van Hiele levels of circle concepts

The percentage of students appeared "jump phenomenon" for the circle were 3.3% (See Table2). Thus, there were 5,396 (96.7%) students who were at levels 1 through 3. The distributions of van Hiele level of circle concepts from grades 1 to 6 for each figure are as shown in Table 3.

Based on the questions of circle, 75.8% of the grade 1 students were at van Hiele level 1, and 89.4% in grade 2. The grade 3 students at level 1 of the circle concept were 32.0%, 61.3% at level 2, and 0.1% at level 3. The grade 4 students at level 1 of the circle concept were 15.2%, 82.3% at level 2, and 0.2% at level 3. The grade 5 students at level 1 of the circle concept were 4.1%, 74.9% at level 2, and 18.5% at level 3. The grade 6 students who were assigned at level 1 of the circle concept were 4.0%, 66.2% at level 2, and 28.3% at level 3 (See Table 3).

This paper analysed the passing rate of the van Hiele geometric thinking of Taiwanese 1st to 6th graders, and studied the gender differences at different grades. The statement below shows the passed population and the passing rate.

The lower grades (first and second graders)

From the statistics in Table 4, we could determine the performances of different genders in level one. Regarding the first graders, both boys and girls reached the level proposed by Usiskin, that the passing rate of the circular concept is over 74%, while the passing rates are lower in both triangular (below 50%) and quadrilateral concepts (below 32%), which show that most students still cannot achieve Usiskin's criteria. It also shows that many students cannot achieve van Hiele's level one, which should be classified into level zero. This evidence shows that there exists a level zero, which is under the level one visual level.

Regarding the passing rates of boys and girls in van Hiele's level one, we concluded that the students' performances are at the level one of van Hiele geometric thinking.

Regarding the second graders, their performances in these three concepts are better than the first graders. The passing rates of triangular (about 62%) and quadrilateral (about 46%) concepts are over 45%, but both have over 89% in the circular concept.

In order to understand the differences between different genders of the lower grades in van Hiele's geometric thinking level, this research used one-way ANOVA to test the different genders in the lower grades regarding the differences among triangle, quadrilateral, and circle. The results are as shown in Table 5.

As shown in Table 5, the lower grades of the two genders have no significant differences in the three basic geometric shapes (triangle, quadrilateral, and circle) in level one (triangle: $p =.929$; quadrilateral: $p =.563$; circle: $p =.357$).

The middle grades (third and fourth grades)

From the statistics in Table 4, regarding the third graders, in level one, boys have the passing rate of 82.0% and girls have 78.2% in the triangular concept; boys have 60.7% and girls have 60.4% in the quadrilateral concept; boys have 92.2% and girls have 89.3% in the circular concept. It is shown in level one that the boys of the third grade do better than girls; however, the differences are not large. In level two, boys

Table 4. The passed population and ratio of different genders in the lower, middle and higher grades in van Hiele's geometric thinking level

			Triangular concept	Quadrilateral concept	Circular concept
The first grade	Boys	Passing rate%	49.8	30.2	77.2
	Girls	Passing rate%	46.7	31.9	74.6
The second grade	Boys	Passing rate%	61.7	47.1	89.5
	Girls	Passing rate%	63.5	45.9	89.3

			Triangular concept		Quadrilateral concept		Circular Concept	
			Level one	Level two	Level one	Level two	Level one	Level two
The third graders	Boys	Passing rate%	82.0	30.9	60.7	33.3	92.2	60.4
	Girls	Passing rate%	78.2	28.3	60.4	31.7	89.3	64.7
The fourth graders	Boys	Passing rate%	89.6	44.6	77.8	58.7	94.1	82.8
	Girls	Passing rate%	88.2	39.4	77.5	55.2	93.1	83.7

			Triangle concept			Quadrilateral concept			Circular Concept		
			Level one	Level two	Level three	Level one	Level two	Level three	Level one	Level two	Level three
The fifth graders	Boys	Passing rate%	90.5	64.2	5.6	72.3	82.0	6.1	90.5	94.6	19.8
	Girls	Passing rate%	88.9	59.5	3.8	85.1	78.2	5.7	92.6	91.6	17.2
The sixth graders	Boys	Passing rate%	93.6	77.0	10.7	71.7	88.1	14.2	94.0	96.5	32.0
	Girls	Passing rate%	89.9	69.6	7.0	69.9	82.5	9.5	92.0	92.8	24.3

Table 5. The differences of the geometric of all lower graders of different genders in level one

Level	Figures	Source of Variance	MS	F	p-value
Level one	Triangle	Between	.031	.008	.929
		Within	3.880		
	Quadrilateral	Between	1.392	.334	.563
		Within	4.169		
	Circle	Between	1.609	.848	.357
		Within	1.897		

***p < .01*

have the passing rate of 30.9% and girls have 28.3% in the triangular concept; boys have 33.3% and girls have 31.7% in the quadrilateral concept; boys have 60.4% and girls have 64.7% in the circular concept. We can obviously find that both genders reach 60% of Usiskin's criteria in the circular concept, which means that most boys and girls of the third grade can reach level two. Moreover, the passing rates of the boys seem better than girls in the other two figures (triangle and quadrilateral).

Regarding the fourth graders, their performance of each figure in level one is the same as the third graders. Although the passing rate is higher than the third graders, boys still do better than girls. It is noticeable that in level two, the grades reach Usiskin's criteria in the circular concept, but the girls have 83.7%, which is higher than the boys (82.8%).

To understand the difference between different genders of middle graders in van Hiele's geometric thinking level, this research used one-way ANOVA to discuss the differences between the two genders of the middle graders regarding the triangle, quadrilateral, and circle in levels one and two, as shown in Table 6.

From Table 6, the performance of middle graders in the three figures (triangle, quadrilateral and circle) in level one, and two figures (triangle and circle) in level two, has no large difference. Boys and girls of the middle grades have significant differences only in level two (p =.019) of the quadrilateral concept.

The higher grades (fifth and sixth graders)

From the statistics in Table 4 and Table 7, regarding the fifth graders, both passing rates of boys and girls are over 88%, which are 90.5% and 88.9%, respectively, which shows that there were no significant differences in the triangular concept in level one. In the quadrilateral concept, both passing rates of boys and girls are more than 72%, which are 72.3% and 85.1%, respectively, thus, the performances of boys and girls have no difference. In the circular concept, the passing rates of boys and girls are up to 90%, which are 90.5% and 92.6%, respectively, which shows that the two genders of the fifth level have no large differences in the circular concept of level one.

Table 6. The differences of the geometric of all middle grades of different genders in level one and two

Level	Figures	Source of Variance	MS	F	p-value
Level one	Triangle	Between	1.265	.435	.510
		Within	2.912		
	Quadrilateral	Between	.844	.228	.633
		Within	3.695		
	Circle	Between	2.500	2.003	.157
		Within	1.248		
Level two	Triangle	Between	5.094	2.674	.102
		Within	1.905		
	Quadrilateral	Between	11.181	5.488	.019*
		Within	2.037		
	Circle	Between	.404	.305	.581
		Within	1.323		

$^*p < .05$

Table 7. The differences of the geometric of all higher grades of different genders in level one, two and three

Level	Figures	Source of Variance	MS	F	p-value
Level one	Triangle	Between	10.110	4.645	.031*
		Within	2.177		
	Quadrilateral	Between	9.466	2.397	.122
		Within	3.949		
	Circle	Between	1.550	1.345	.246
		Within	1.153		
Level two	Triangle	Between	89.234	44.423	.000**
		Within	2.009		
	Quadrilateral	Between	35.162	20.413	.000**
		Within	1.723		
	Circle	Between	36.110	40.338	.000**
		Within	.895		
Level three	Triangle	Between	46.982	19.292	.000**
		Within	2.435		
	Quadrilateral	Between	112.835	39.293	.000**
		Within	2.872		
	Circle	Between	75.128	25.920	.000**
		Within	2.898		

$^*p < .05$ $^{**}p < .01$

In level two, the concepts of the quadrilateral and circle of fifth graders reach Usiskin's criteria, and the passing rate is around or over 78%. However, it is worth noticing that girls' performance in the triangular concept only has 59.5%, which shows that almost half of the grades cannot achieve Usiskin's criteria. On the other hand, the passing rate of boys is 64.2%, which reach the standard.

In level three, although the performance of triangular, quadrilateral, and circular concepts of most boys in the fifth grades do not reach level three, they do better than girls.

Regarding the sixth graders, no matter the concepts of triangle, quadrilateral, or circle, both boys and girls' performances reached level one.

In level two, the performances of triangular, quadrilateral, and circular concepts of boys and girls reached more than 69%, and there are no significant differences between them, as seen in Table 4.

In level three, both boys and girls' passing rates are lower, and most do not reach Usiskin's criteria, which is the same as scholar (Wu, 2003) raised: although the sixth graders can analyse the structures and elements of figures, they do not know how to explain them.

To understand the difference between different genders in the higher grades of van Hiele's geometric thinking level, this research used one-way ANOVA to test the different genders in triangle, quadrilateral, and circle in levels one, two, and three. Table 7 is as follows:

From Table 7, we can know that:

1. The results of one-way ANOVA show that in level one, the higher graders have significant differences in answering triangular questions ($p <.05$), but have no differences in answering quadrilateral questions ($p =.122$) or circular questions ($p =.246$).
2. In level two, all higher grades have significant differences in answering triangular questions ($p =.000$), quadrilateral questions ($p =.000$), and circular questions ($p =.000$).
3. All higher graders have significant differences in answering triangular questions ($p =.000$), quadrilateral questions ($p =.000$), and circular questions ($p =.000$) in level three.

Table 7 shows that all genders in the higher grades have no significant differences of two figures (quadrilateral and circle) in van Hiele's geometric thinking level one. However, there are significant differences in the triangle in level one, and three figures (triangle, quadrilateral, and circle) in levels two and three.

CONCLUSION

From the above discussions, the conclusions were drawn, as follows:

The passing rates

a) The results of this study found that the higher grades achieved higher passing rates.
b) Based on these three basic figures (triangle, quadrilateral, circle), most students of grades 1 and 2 were at level 1, and grades 3 to 6 were at level 2. Only grades 5 and 6 could meet level 3. This result is consistent with the research of Wu and Ma (2005a, 2006).
c) More than half of (up to 50%) grade 1 did NOT met the criteria of the first level (below level 1), as based on the triangle, about 69% on the quadrilateral, and about 23% on the circle. It seems that the circular concept is the easiest for students; while the concept of the quadrilateral is the most

difficult for students. This result is consistent with the research of Wu and Ma (2005a, 2006).

The gender differences

a) The lower graders of the two genders had no significant differences in the three basic geometric shapes (triangle, quadrilateral, and circle) in level one.
b) Regarding the performance of the middle grades in the three figures (triangle, quadrilateral, and circle) in level one, and two figures (triangle and circle) in level two, there were no significant differences. Boys and girls of the middle grades have significant differences only in level two ($p = .019$) of the quadrilateral concept. The passing rate of boys was 58.7% and 55.2% for girls.
c) Regarding the higher grades for all three figures (triangle, quadrilateral, and circle), in levels one, two, and three, there were significant differences, with the exception of two figures (quadrilateral and circle), in van Hiele's geometric thinking level one.

The implications of this study to math curriculum and teaching

a) The results of this study identified the easiest and most difficult concepts of basic figures for students. The authors of this study are interested to investigate why elementary students have difficulties in quadrilateral. One reason might be that quadrilaterals, with the exception of squares and rectangle, are rarely shown in textbooks or in their daily lives. The researchers of this study recommend that it is important to add more quadrilateral curriculum in textbooks.
b) The results of this study suggested that the higher grades had higher significant differences, thus, higher grades teachers should pay more attention to the learning situation of girls, and give them more practice.

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