A Tutorial Worksheet to Help Students Develop the Ability to Interpret the Dot Product as a Projection

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When we administered the Test of Understanding of Vectors (TUV) to students who were completing a physics university remedial course (which covers subjects of a traditional high school physics course), we observed that they had considerable difficulties in interpreting the dot product as a projection. As a result of this, we decided to design a tutorial worksheet to guide students through the development of this skill. The worksheet was implemented with 288 students of the same course in another semester. Upon using the TUV to evaluate the students' understanding, we confirmed that the tutorial worksheet facilitated their learning. This tutorial worksheet is presented in the Appendix and might be used by other physics instructors teaching this material in high schools, colleges or universities.

Keywords: curriculum development, dot product interpretation, instructional material, tutorial worksheet, vector concepts

INTRODUCTION

When we administered the Test of Understanding of Vectors (TUV) (Barniol & Zavala, 2014) to students completing a physics university remedial course, we observed that they had significant difficulties when interpreting the dot product as a projection. This result confirmed the need for new instructional material to foster students' understanding of this interpretation.

With this in mind, we referred to McDermott's study (McDermott, 2001), which outlines a process for developing new instructional material. The process has three steps: (1) conduct research on students' understanding, (2) use the findings to guide the development of instructional materials, and (3) carry out an assessment of the

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effectiveness of the new instructional materials, based on what the students have learned. "Tutorials in Introductory Physics" (McDermott & Shaffer, 2001) are well-known instructional resources designed following this process (Shaffer & McDermott, 1999).

Since the literature objectively reports that use of the Tutorials is one of the most effective teaching strategies for introductory physics (Finkelstein & Pollock, 2005), we decided to design a tutorial worksheet to increase students' understanding of the interpretation of dot product as a projection. The three objectives of this study are: (1) to present a tutorial worksheet with a design based on the analysis of students' difficulties when interpreting dot product as a projection; (2) present evidence of the need for this tutorial; and (3) evaluate its effectiveness.

PREVIOUS RESEARCH ON STUDENTS' DIFFICULTIES WHEN INTERPRETING DOT PRODUCT

Van Deventer (2008) interviewed students about the calculation of the dot product and found that some of them incorrectly considered the dot product of two vectors to be a vector between the two vectors. It was noted that these students were using vector addition tools to evaluate the dot product.

State of the literature

- Previous studies show that students have severe difficulties to interpret the dot product as a projection.
- Some of these studies present a complete classification of students' incorrect interpretations of the dot product.
- To date, no study has explored the relationship between the design of new instructional materials and student performance when interpreting the dot product as a projection.

Contribution of this paper to the literature

- In this article, we present a tutorial worksheet on the interpretation of the dot product as a projection. The strategy for its development was based on the design process described by McDermott and the analysis of students' difficulties.
- We evaluated the effectiveness of this tutorial and confirmed that it facilitates students' learning.
- We also present evidence of improved learning as a result of using this tutorial by analysing the answers of the students while using it in the classroom.

In previous studies, we focused specifically on students' difficulties in the interpretation of the dot product. First, we studied these difficulties in an open-ended problem that asked students to interpret the dot product of two vectors (Zavala & Barniol, 2010), and then, based on the results, we constructed a multiple-choice option problem and analyzed students' difficulties with it (Zavala & Barniol, 2013). In this latter study, it was found that very few of the students completing the last of three introductory university physics courses had chosen the correct option. The two most frequent errors were the interpretation of the dot product of the two vectors as the magnitude of a vector between the two vectors, and the interpretation of the dot product as the vector between the two vectors (as reported by Van Deventer). In both errors, students usually confused this product with the addition of vectors.

Subsequently, we designed the 20-item Test of Understanding of Vectors (TUV) and demonstrated it is a reliable assessment tool (Barniol & Zavala, 2014). The TUV evaluates ten vector concepts: direction, magnitude, components, unit vector, representation of a vector, addition, subtraction, scalar multiplication, dot product, and cross product. We incorporated the multiple-choice option problem presented in our previous study (Zavala & Barniol, 2013) as item 3 of the test. In the present article, we will show some results using the TUV and provide a detailed analysis of students' answers to item 3. Finally, it bears mentioning that in the last year a research article has been published (Heckler & Thomas, 2015) on the representation of vectors and its relation to operations; however, that article focuses on sum and subtraction and does not focus on instructional materials. Also, three research

studies have been published that present new instructional materials to help students learn direction, magnitude and addition of vectors (Wutchana, Bunrangsri & Emarat, 2015), component of vectors (Barniol & Zavala, 2015), and cross product (Mueanploy, 2015). That is, there have not been any research studies to date examining the relationship between the design of instructional materials and students' performance in interpreting the dot product as a projection.

THEORETICAL FRAMEWORK OF TUTORIAL WORKSHEETS

McDermott (2001) established that the tutorials provide experience in learning through guided inquiry and an emphasis on the constructing of concepts. The tutorial worksheets have two main aims: 1) to guide students in developing a conceptual framework of important topics that research shows are difficult for students, and 2) to address persistent conceptual difficulties. The worksheets contain questions that break down the reasoning process into steps to guide students in using scientific reasoning to build conceptual understanding. During the class session in which the tutorial worksheet is presented, students work collaboratively in groups of three or four. It is worth noting that the tutorials have been considered elsewhere as a reference to design new instructional material (for example, Wittmann, Steinberg & Redish, 2003; Loverude, Heron, & Kautz, 2009) and that this journal has published articles that present new instructional materials that, as the tutorials, foster students' active learning (Tandogan & Orhan, 2007; Bilgin, Senocak, & Sozbilir, 2009).

The implementation of the tutorial worksheets is carried out by graduate teaching assistants who work as tutorial instructors. McDermott (2001) mentions that tutorial instructors do not lecture or give answers, but rather help students to arrive at their own answers by posing questions that guide them through the necessary reasoning. The tutorials require training for tutorial instructors in both the subject matter and instructional method. This training seminar is carried out using the same material and techniques that the tutorial instructors are expected to use in class.

DESIGN OF THE TUTORIAL WORKSHEET

This section addresses the first objective of the study. The worksheet is included in the Appendix. Following the theoretical framework, this tutorial worksheet has two main goals: 1) to guide students in developing the ability to interpret the dot product as a projection, and 2) to address persistent conceptual difficulties in this interpretation. Both are described below.

The six different sections of the tutorial worksheet allow a step by step description of how students arrive at this interpretation when using the tutorial. In the first section, students develop the ability to interpret the dot product of two vectors as a projection, and then in the next four sections (2-5) students understand and use this interpretation in four possible situations: when the dot product is 1) positive, 2) zero, 3) negative and 4) equal to the negative value of the direct multiplication of the magnitude of the vectors (that is, when the angle between the two vectors is 180°). Finally, section 6 analyzes the degree to which the student has internalized the skill of interpretation, and it also looks at the most frequent errors made by students. A description of sections 1, 2 and 6 follows.

In the beginning of the tutorial worksheet, we present the definition of the dot product of two vectors (A and B) that form an angle θ as ABcosθ. Section 1 leads students through a geometrical analysis to understand the interpretation of two vectors, A and B, that form an angle θ as the projection of vector A onto vector B, multiplied by the magnitude of vector B. In this section, we first guide students to

sketch the projection of vector A onto vector B and then to calculate it as Acosθ. Next, we ask students to multiply this expression by the magnitude of vector B and to subsequently recognize that the new expression is equal to the definition of the dot product presented at the beginning of the tutorial. Finally, based on these results, we ask students to interpret the dot product.

Section 2 guides students to find the dot product of two vectors (which form an angle of 53.13° and are shown in a Cartesian plane, i.e., on a grid) by using two methods. In the first method, students are asked to find the dot product using the graphic interpretation learned previously. In the second method, students use the dot product definition ABCosθ. Finally, students are guided towards recognizing that both methods result in the same value. As we can see, this section is related to what is covered in section 1; however, in section 1, students work in an abstract approach with variables, and in section 2 they work in a concrete way with values to strengthen the geometrical interpretation.

As mentioned previously, students usually confuse the dot product with the addition of vectors. This general confusion is implicitly addressed throughout the entire tutorial worksheet process in which students are guided to make the correct interpretation. However, to address this error explicitly we decided to ask students to perform an explicit comparison between the addition of vectors and the dot product operation in section 6. Students have to synthesize and display this comparison in a table where they must establish the nature of the results obtained in each operation (vector or scalar), as well as the graphic method used in each of the operations.

METHODOLOGY

This research study was conducted at a large private Mexican university. All participants in this study were enrolled in the *Introduction to Physics* course, which is a remedial course taken by those students who enter the university without having scored a passing grade on the Physics Selection Test (PST). This course covers subjects of a traditional high school physics course: dimensional analysis, significant figures, density, trigonometry, kinematics in one dimension, and vectors.

The methodology used in this study calls for both a control group and an experimental group. For the control group, we analyzed students' results from the fall semester course of 2012. The textbook used by the students was "Introduction to University Physics" by Alarcon and Zavala (2005), and in class they also worked on collaborative activities presented in the "Activities Manual" by the same authors (Alarcon & Zavala, 2006). (Note that these activities were not specifically designed based on research of students' difficulties.) Within the topic of vectors, students worked with three different activities from the manual. The first one dealt with magnitude, direction, addition, subtraction and scalar multiplication of a vector; the second mainly addressed unit vectors and unit vector notation; and the third activity dealt with vector products.

For the experimental group, we analyzed students' results from the fall semester course of 2013. These students had the same instructors and used the same textbook as students from the previous year. They also worked with the first of the three collaborative activities. However, for this semester we implemented four new tutorial worksheets based on the research on students' most persistent difficulties: 1) calculation of components, 2) unit vector, 3) dot product, and 4) cross product. This article focuses only on the third tutorial worksheet. The implementation of these tutorials was carried out following the recommendations by McDermott (2001).

The PST was administered to all the participants before the beginning of the course. The PST is a multiple-choice test that evaluates students' knowledge of the
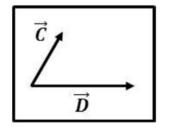
main topics of a traditional high school mechanics course. We used the PST to validate that both groups were comparable. To evaluate the participants' understanding of vector concepts once the course had ended, we used the TUV. We had 313 students in the control group and 288 students in the experimental group.

EVIDENCE OF THE NEED FOR A TUTORIAL WORKSHEET ON INTERPRETATION OF THE DOT PRODUCT

In order to present the evidence of the need for this tutorial (objective 2), we analyzed the scores from the TUV and the answers to item 3 of the students in the control group. The distribution of the scores was negatively skewed and the median was 13. It is noteworthy that the students who are on the median had difficulty answering correctly 7 out of 20 items.

Figure 1 presents item 3 of the TUV, the item that evaluates the interpretation of the dot product as a projection. We observed that only 47% of students chose the correct answer (option B). Moreover, this item was one of the seven most difficult items on the test. This is evidence of the need for a tutorial worksheet on the interpretation of the dot product as a projection. Furthermore, we noted two errors that have equal or higher percentages than 15%: options A and C. Each of these two errors refers to a vector between both vectors. 22% of students chose option A (interpreting the dot product as the magnitude of a vector between the two vectors), and 15% chose option C (interpreting it as a vector between the two vectors). This confirms the need to address this specific error in the tutorial.

3. The figure below shows vectors $\vec{C}$ and $\vec{D}$. Which option is the best interpretation of the dot product $(\vec{C} \cdot \vec{D})$?

(A) The magnitude of a vector between $\vec{C}$ and $\vec{D}$ pointing up to the right.
(B) The projection of vector $\vec{C}$ onto vector $\vec{D}$ multiplied by the magnitude of vector $\vec{D}$.
(C) A vector between $\vec{C}$ and $\vec{D}$ pointing up to the right.
(D) A vector perpendicular to both vectors.
(E) A vector in the direction of $\vec{D}$.

Figure 1. Item 3 of the Test Of Understanding of Vectors
Note: The item evaluates the interpretation of the dot product as a projection.

EFFECTIVENESS OF THE FOUR NEW TUTORIAL WORKSHEETS USED IN THE EXPERIMENTAL GROUP

To evaluate the effectiveness of the four new tutorial worksheets, we compared the TUV scores of the control and experimental groups. First, we evaluated whether both groups were comparable by analyzing the scores from the PST given before the course. Since both distributions of scores in the PST were not normal and their variances met the assumption of homogeneity of variance, we decided to use the no-parametric Mann-Whitney test (Field, 2013). This test indicated that the PST scores obtained by students in the control group (*Mdn*=44) did not differ significantly from those of students in the experimental group (*Mdn*=38) before the course, *U*=41360.5, *z*= -1.76, *p*=0.079. Based on this result, we can state that both groups were comparable.

To evaluate the effectiveness of the new tutorials, we compared the scores obtained on the TUV by both groups. Similar to the control group, we found that the experimental group had a negatively skewed distribution of scores in the TUV with a

median of 15. Since both distributions of scores in the TUV were not normal and their variances met the assumption of homogeneity of variance, we used the Mann-Whitney test. This test indicates that the scores obtained by students in the experimental group (*Mdn*=15) were significantly higher than those obtained by students in the control group (*Mdn*=13), *U*=55667.5, *z*= 5.001, *p*<0.01, *r*=0.2. This result shows that the newly designed tutorials helped students to increase their understanding of vector concepts. It's interesting to note that the items for which we found more differences between students' performance in the two groups were mostly those related to the tutorials. One of these items is the dot product interpretation item. We provide a detailed analysis below.

EFFECTIVENESS OF THE TUTORIAL WORKSHEET ON INTERPRETATION OF THE DOT PRODUCT

To evaluate the effectiveness of the tutorial worksheet (objective 3), we compared the control and the experimental groups' answers to item 3. To perform this comparison we first used the chi-square test (*p*<0.05) to determine whether there was a significant difference in the distribution of answers for both the control and experimental groups. We found a significant difference in the distribution of both groups' answers, χ^2(5, N = 601) = 21.17, *p*<0.01. Then, we determined which specific option was significantly different by using the chi-square test again following the procedure described by Sheskin (2007). We found significant differences in the selection of three options as shown in Table 1.

The most important difference found was in the selection of the correct answer: 63% of students who used the tutorial vs. 47% of students who did not. The increase in the proportion of students selecting the correct answer is evidence that the tutorial helps students to interpret correctly the dot product as a projection. Moreover, we also found a significant difference in the selection of the two most common incorrect answers: option A (13% with the tutorial vs. 22% without it) and option C (8% with the tutorial vs. 15% without it). These differences seem to be explained by the fact that these specific errors are directly addressed in the tutorial worksheet. These results show evidence of the effectiveness of the tutorial worksheet on guiding students to interpret the dot product as a projection.

EVIDENCE OF INCREASED STUDENT LEARNING AS A RESULT OF THE TUTORIAL WORKSHEET

To further evaluate the effectiveness of the tutorial worksheet, we decided to also present evidence of improved student learning as a result of using this tutorial, by analyzing students' answers when they completed it in the classroom. As mentioned previously, this worksheet has two main goals. The first is to foster the development of the interpretation of the dot product as a projection throughout the tutorial. There are two key points during the development of this interpretation. The first is in section 1, and leads students to arrive at this interpretation. The second is in

Table 1. Proportion of students of the control and experimental groups who chose each option in the item 3 of the Test of Understanding of Vectors

Option	Answer	Control group		Experimental group
A	Magnitude of a vector between both vectors	22%	↔	13%
B	Correct	47%	↔	63%
C	A vector between both vectors	15%	↔	8%
D	A perpendicular vector of both vectors	12%		12%
E	A horizontal vector	4%		3%

Note: The arrows indicate which options have significantly different results.

section 6 (the last section), in which students synthesize this interpretation in a comparative table. The second goal is to address the two most frequent errors. A comparative table in section 6 addresses the two errors. Below, we present evidence of the students' learning upon completing these sections.

Evidence of the interpretation of dot product as a projection developed by students

Here we present evidence of the kinds of answers given by students in sections 1 and 6. In section 1, the student correctly drew the projection of vector A onto vector B (question a). He next calculated it correctly as "*Acosθ*" (question b), and then multiplied this expression by the magnitude of vector B, thereby obtaining the correct expression "*(Acosθ)(B)*" (question c). Following that operation, he stated that this expression "*is equal*" to the definition of the dot product; and expressed the interpretation of the dot product as "*the projection of A onto B multiplied by the magnitude of B*" (question e), which is the main purpose of this section of the tutorial.

Also in section 6, students synthesize the underlying foundation of this interpretation. This same student pointed out the two main geometrical aspects of the dot product in the comparative table: first, that the result obtained in this operation is a "*scalar*". He also described the graphical method as follows: "*The projection of A onto B is obtained and then multiply these results by the magnitude of B.*" As we can see, these answers show evidence of the geometric interpretation of dot product as developed by students using the worksheet.

Evidence of how students discard the two most frequent errors

As mentioned earlier, the comparative table of section 6 addresses the two most frequent errors. In the previous subsection, we presented how a student obtains the dot product information and displays it in the table. Now we present how this student handled the addition of vectors in this section.

In section 6, the student correctly drew the addition of vectors A and B (question b), using the parallelogram method. Then, in the table, he stated that the result obtained in this operation is a "*vector*", and he described the parallelogram method to obtain the vector sum as: "*from the tail of B you draw vector A; the vector sum starts at the tail of B and ends at the tip of A.*" If we note that the student answered correctly the dot product and the addition of vectors, it is concluded that the student has developed a complete graphic interpretation of dot product and the addition of vectors and has been able to relate them in the comparative table. This answer presents evidence of how students compare both operations in the worksheet, and as mentioned earlier, this comparison seems to explain the decreased selection of the most frequent errors (options A & C) in item 3 of the TUV.

CONCLUSIONS

Many experienced physics teachers are aware of students' difficulties when interpreting dot product as a projection, and attempt to address these issues in their instruction. An important aspect of this study is that we have carried out systematic research to design and evaluate a tutorial worksheet on this subject, and we have shown evidence of the effectiveness of the tutorial. When using this tutorial, instructors do not teach to explicitly overcome these difficulties; rather, students achieve their own understanding using the tutorial worksheet. This is part of what constitutes active learning. The tutorial worksheet is presented in the Appendix. Physics instructors who teach this material in high schools, colleges or universities are welcome to use it.

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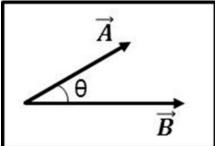
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Appendix: Tutorial worksheet on interpretation of the dot product

Definition: The dot product of vector $\vec{A}$ and $\vec{B}$ is written as $\vec{A} \cdot \vec{B}$. Mathematically, the definition of the dot product is $\vec{A} \cdot \vec{B} = AB\cos(\theta)$, where A and B are the magnitudes of the vectors and θ is the angle between the vectors.

1. Interpretation of dot product

a) The figure shows vectors $\vec{A}$ and $\vec{B}$. The angle between the vectors is θ. Sketch the projection of vector $\vec{A}$ onto vector $\vec{B}$.

To this end, draw a perpendicular line to vector $\vec{B}$ (from the tip of vector $\vec{A}$ until it touches vector $\vec{B}$). Note that a right triangle is formed. The projection of vector $\vec{A}$ onto vector $\vec{B}$ starts from the point where vector $\vec{A}$ begins until the drawn line touches vector $\vec{B}$. Name this projection as $Proj_{A \rightarrow B}$.

b) Calculation of the projection of vector $\vec{A}$ onto vector $\vec{B}$: If A is the magnitude of vector $\vec{A}$ and θ is the angle that vector $\vec{A}$ forms with $\vec{B}$, express, using the right triangle formed in the previous figure, the projection of vector $\vec{A}$ onto vector $\vec{B}$ depending on A and θ. Show your work.
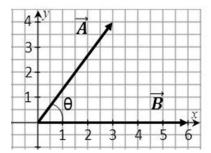
c) Interpretation of dot product: Multiply the projection of vector $\vec{A}$ onto vector $\vec{B}$, found in b), by the magnitude of vector $\vec{B}$.

d) Compare the expression found in c) with the definition of the dot product of any two vectors $\vec{A}$ and $\vec{B}$, as established in the beginning of this tutorial. Are they equal?

e) According to what you find in c), and your answer in d), how can you interpret geometrically the dot product of two vectors $\vec{A}$ and $\vec{B}$?

2. Calculation and interpretation of a positive dot product

Consider the vector shown in the figure. Vector $\vec{A}$ has a magnitude of 5 units; vector $\vec{B}$ has a magnitude of 6 units, and the angle θ is 53.13°. Next we will find the dot product $\vec{A} \cdot \vec{B}$ by using two different methods:

First method: Graphic method.

a) Observing the Cartesian plane and without performing calculations, what is the value of the projection of vector $\vec{A}$ onto vector $\vec{B}$?

b) Using this value, what is the value of the projection of vector $\vec{A}$ onto vector $\vec{B}$, multiplied by the magnitude of vector $\vec{B}$?
Recall that this value corresponds to the dot product $\vec{A} \cdot \vec{B}$.

Second method: Use the definition of the dot product stated at the beginning of the tutorial: $\vec{A} \cdot \vec{B} = AB \cos(\theta)$.
a) Now calculate the dot product of vectors $\vec{A}$ and $\vec{B}$ using this definition.

Is the value found with the graphic method equal to the value found using the definition? If not, look for the inconsistencies.
Note that in this case the grid and the given values help find the dot product when using the graphic method, but more often you will need to use the second method.
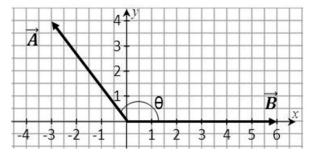
3. Geometric interpretation of dot product equal to zero
a) In the previous problem, what angle θ will result in a dot product equal to zero? Explain.

b) In this case, what is the value of the projection of vector $\vec{A}$ onto vector $\vec{B}$? Explain.

4. Calculation and interpretation of a negative dot product

Angle θ in the problem of section 2 is now 126.87° as shown in the figure. Recall that the magnitude of vector $\vec{A}$ is 5 and the magnitude of vector $\vec{B}$ is 6.

a) Calculate the dot product of these two vectors using the definition: $\vec{A} \cdot \vec{B} = AB \cos(\theta)$.

b) How does this result compare with the result obtained in section 2?

c) How do you interpret geometrically the fact that the absolute value of these two products is equal? (*Hint:* Note the value of the projection of vector $\vec{A}$ onto vector $\vec{B}$).

d) How do you interpret geometrically the negative sign in the previously calculated dot product? (*Hint:* Note the position that the projection of vector $\vec{A}$ onto vector $\vec{B}$ has with respect to vector $\vec{B}$).
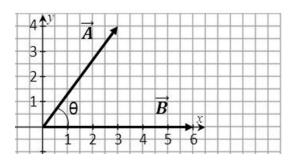
5. Geometric interpretation of dot product when the angle between the vectors is 180°

a) Consider the problem of section 4. If the angle between the two vectors θ is now 180°, what is the value of the projection of vector $\vec{A}$ onto vector $\vec{B}$? (*Hint*: Draw the described situation; note that it doesn't require calculations).

b) In this case, what is the value of the dot product $\vec{A} \cdot \vec{B}$?

6. Comparison between the addition of vectors and the dot product

Next we compare the addition of vectors and the dot product. Consider once again the two vectors in section 2 shown in the figure.

a) If we add the two vectors, is the result a vector or a scalar? Explain.

b) Draw the addition vector in the figure.
c) Is the dot product of these two vectors a vector or a scalar? Explain.

d) The following table will display the most important differences between the addition of two vectors and the dot product operation.

Operation	Is the result of this operation a scalar or a vector?	Describe the graphic method used to obtain the result in this operation. (*Hint*: For the addition, see the operation performed in 6 b) and for the dot product, see the graphic method in section 2).
Addition		
Dot product		

Note the differences between these two operations that you have learned thus far.