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An Appropriate Prompts System Based on the Polya Method for Mathematical Problem-Solving

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ABSTRACT

Current mathematics education emphasizes techniques, formulas, and procedures, neglecting the importance of understanding, presentation, and reasoning. This turns students into passive listeners that are well-practiced only in using formulas that they do not understand. We therefore adopted the Polya problem-solving method to provide students with a diversified foundation for problem-solving. Furthermore, giving students immediate feedback in the form of prompts can help them to find the answers on their own them, thereby helping them learn more effectively. The primary objective of this study was to investigate the influences of a teaching activity incorporating Polya's method and an appropriate prompt mechanism on the learning effectiveness of students. Research Subjects were two classes selected from an elementary school in Taiwan; one as the experimental group were taught with the providing appropriate prompts based on the Polya strategy of mathematical problems solving, the other one as the control group were treated by the conventional instructions. The results showed that: (1) there were significant differences between the experimental group and the control group in the learning effectiveness; (2) most of the students in the experimental group were satisfied by the proposed prompts system.

Keywords: Problem Solving, Prompt Applied in Teaching, Learning Achievement, Learning Attitude.

INTRODUCTION

Mathematics is the mother of all branches of science and the foundation of all scientific research, as the vast majority of scientific and engineering problems require mathematics to solve. It involves the use of abstraction and logical reasoning, the calculation of numbers, and the observation of how objects move. Mathematics can be described as a formal science that uses symbolic language to study concepts such as numbers, structure, variations, and space. Today, mathematics is used in various fields, from engineering to medicine, and is taught as a mandatory elementary school subject in many nations. Learning how to solve problems is essential to learning mathematics (Contreras, 2005; Felmer, Pehkonen & Kilpatrick, 2016). It is a teaching objective and has long been considered an essential issue in the school curriculum.

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State of the literature

- A mathematical problem-solving strategy refers to the ideas and methods that an individual generates when solving a problem and is crucial to problem-solving success.
- Many researchers' proposed approaches are similar for adopting or refining the Polya strategy of mathematical problems solving. The appropriate intervention was designed beforehand, and prompts were given based on the individual differences exhibited by students.
- Therefore, how to provide an appropriate mechanism to Polya's mathematical problem-solving strategies in the teaching activities in order to increase learning achievement of individual students becomes more important recently.

Contribution of this paper to the literature

- There proposes a new appropriate prompts system with four kinds of prompts for helping students to solve mathematical problems.
- After the teaching experiment, students with a moderate or low level of achievement in mathematics displayed significant differences in learning effectiveness.
- The students were satisfied with the teaching experiment and demonstrated a high level of acceptance of this teaching method.

Cunningham (2004) observed that current mathematics education does not focus on understanding, presentation, and reasoning but rather emphasizes techniques, formulas, and procedures. This turns students into passive listeners that are only well-practiced in using formulas that they do not understand. This study therefore sought to develop and test a teaching method which explicitly promotes problem-solving among elementary school students (Brown & Walter, 2005). Among the many theories proposed regarding problem-solving strategies, the problem-solving methods proposed by Polya in 1957 are the most comprehensive. In the book "How to Solve It", Polya suggests four steps, as shown in **Table 1**, for increasing motivation and the promotion of successful thinking habits in students: (1)

Table 1. Polya Problem-solving Strategy

Step	Problem-solving Strategy	Note
Step 1	Understanding the problem	Must clearly know what the question means, what are we looking for the answer. Need to first realize the key point and context of problem, and then be able to find the answer.
Step 2	Devising a plan	Clearly know the relationship between the points of problem, select a suitable approach and devise a plan for solving problem, which is most major task in the process of problem-solving.
Step 3	Carrying out the plan	Follow Steps 1 and 2, and practically calculate by yourself, and find the answer.
Step 4	Looking back	Look back the entire process of problem-solving; check the computation and the answer; discuss the meanings of the problem.

understand the problem, (2) devise a plan, (3) carry out the plan, and (4) review/extend (Felmer, Pehkonen & Kilpatrick, 2016; Polya, 1957).

Research into the differences between expert problem-solvers and novices has revealed that the two groups differ not in intelligence but in their ability to flexibly apply acquired knowledge and strategies (Chi, Bassok, Lewis, Reimann, & Glaser, 1989; Mayer, 1992; Schoenfeld, 1983). Specifically, giving students immediate feedback in the form of prompts as they solve a problem enables them to find answers on their own, which enhances the effectiveness of their learning and provides them with a greater confidence and sense of accomplishment. The research above therefore indicates that suitable prompting is also an important route to improving the problem-solving effectiveness of students.

At present, the majority of existing research (Craig, 2016; Devi, 2016; Han & Kim, 2016; Romiszowski, 2016; Rosli, et al., 2015) concerning Polya's methods focus on the development and assessment of relevant strategies and teaching applications. Few studies have included the topic of prompts. Therefore, how to design an appropriate mechanism to Polya's mathematical problem-solving strategies in the teaching activities becomes more important. Furthermore, we investigated the influence of the addition of appropriate prompts to Polya's methods on student learning effectiveness.

Based on the research background and motives above, the objective of this study was to examine the impact of teaching activities with Polya's problem solving methods and appropriate mechanisms on student learning effectiveness. The research questions that guided this study are as follows:

(1) Does the inclusion of appropriate prompts to teaching activities based on Polya's problem-solving methods have significant influence on the learning effectiveness of students?

(2) Does the inclusion of appropriate prompts to teaching activities based on Polya's problem-solving methods have significant influence on the learning effectiveness of students of varying degrees of achievement in mathematics?

THEORETICAL FRAMEWORK

To achieve our research aims, we designed a teaching activity aimed at elementary school students. The content of the included prompts was determined by the level of achievement in mathematics of the student. During the learning activity, the students completed exercises using the four steps suggested by Polya. In Steps 1 and 2, they had to understand the problem and find the way to solve it, which required them to clarify the nature of the problem before proceeding to the next step. In Step 3, they executed their plan and selected the correct answer, and in Step 4, they checked their work and then submitted their answer if they were comfortable that it was the best option. Polya indicated that teachers should take note of two important objectives when they provide prompts to their students: the first is to help the student solve the problem at hand, and the second is to develop the student's ability to solve the problem himself. Tsai (2009) mentioned that the key to the successful introduction of prompts is to provide those that are appropriate for the individual needs of student in question. We therefore developed an appropriate prompt for the first two steps. The prompt varies depending on whether the student exhibits high, moderate, or low achievement in mathematics.

Our investigation included the following independent, dependent, and control variables. The research structure is as follows:

(1) Independent variable:

A. experiment group: to provide adaptive prompts in the Polya problem-solving strategy in mathematics instructing activity.

B. control group: to perform a traditional mathematics instructing activity.

(2) Dependent variable:

Learning effectiveness: compared the scores of post-test and pre-test for the experiment group and the control group.

(3) Control variable:

A. same instructor;

B. same number of classes;

C. same content for teaching.

A teaching activity with Polya's methods and our appropriate prompts was adopted for the experiment group, and a conventional teaching strategy was used for the control group. Learning effectiveness was gauged using the difference between pre-test and post-test scores. A greater positive difference indicated better learning effectiveness. The control variables in this study included the teacher and the duration, content, and environment of teaching. Both classes were taught by the same mathematics teacher. In terms of duration, the teacher used the first 20 minutes of each class to perform the teaching activity.

METHODOLOGY

Participants

Due to constraints in human resources and time, and for the sake of coordination with the school's administration and convenience in experimentation and investigation, we selected two fifth-grade classes from an elementary school in Taiwan using random cluster sampling. In total, 58 students participated. The experiment group contained 29 students: 16 male students and 13 female students. The control group had the same learning experience as the experiment group, comprising 16 male students and 13 female students as well. Besides, there were no extra computer skills for all the participating students.

Prior to the experiment, we examined the prior knowledge and entry behaviours of the students in the two groups using an independent t-test of their pre-test scores. As shown in **Table 2**, a t-test produced t = -1.336 and p =.187 > .05, which indicates no significant differences in the scores of the two groups.

Table 2. The difference between the two groups, in terms of the results of the learning effectiveness pre-test

Pre-test	NO.	Mean	SD	t
Experimental group	29	14.51	5.93	-1.336
Control group	29	16.37	4.60	

Conceptual framework of the appropriate prompt system

This system was developed in a web environment using JavaScript and Html5 for front-end development. The learners can use an internet browser to log into the appropriate prompt system and begin the learning activities, where they access teaching materials in the database and then solve problems using Polya's methods. The back-end was established using the PHP programming language in conjunction with a MySQL database. The back-end server keeps records of the students' learning processes, which is accessible to the teacher. The function framework of this system includes the front-end, which is divided into the student end and the teacher end, and the back-end, which provides server management, as shown in **Figure 1**.

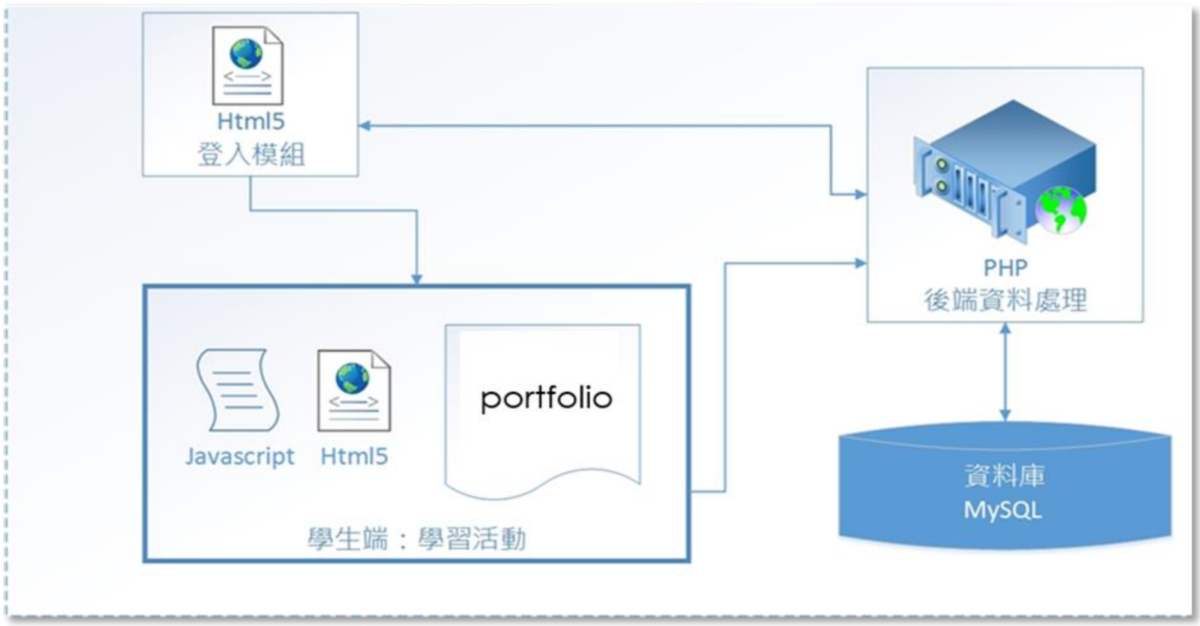

Figure 1. The system architecture of appropriate prompt system

The modules developed for the system include the front-end presentation module and the back-end data processing module:

(1) Front-end presentation module:

The front-end system is in charge of screen presentation, student interface operations, and learner history. The primary programming languages used are as follows:

(a) HTML5: This is the basic framework for websites. We used the Document Object Model (DOM), which comprises markup languages. Each markup tag coordinated with the Cascading Style Sheets (CSS) language to present different effects to users.

(b) JavaScript: This enabled basic web page control, which is used to control the basic question response logic in the front-end of the framework.

(2) Back-end data processing module:

(a) PHP: PHP stands for Hypertext Preprocessor. Its primary goal is to enable web developers to quickly write dynamic web pages and then effectively embed them into Html5. It also incorporates the language features of JavaScript and can be applied on various levels. We therefore chose PHP as the bridge to communicate with back-end data calculations.

(b) MySQL: MySQL itself is a relational database. MySQL databases used to quickly establish web pages in Windows, Apache, MySQL, and PHP have lower access speeds than object-oriented databases, but the advantage is that it is easy to get started.

Appropriate prompt design

If a student had trouble solving a problem in the experiment during Step 1 ("Understand the problem") or Step 2 ("Devise a plan"), we provided an appropriate prompt depending on the lesson unit and the learner's level of achievement in mathematics. The latter was determined based on the pre-test scores of the students: the high achievement group had the highest scores, the low achievement group had the lowest scores, and the remaining students were designated to the moderate achievement group. The three groups contained 8 students, 10 students, and 11 students, respectively. We designed varying prompts so as to guide students in finding answers. Depending on the level of achievement in mathematics of the student, the system provided the following prompts:

(1) Prompts for high achievement group:

(a) Keyword prompts: These prompts gave the students hints on keywords in the mathematical word problems and even on hints on where to start with unknown numbers so as to help students overcome initial obstacles in writing mathematical equations for mathematical word problems.

(b) Tip prompts: These prompts gave hints about the number of unknown numbers and corresponding relationships in order to help students understand the problem-solving procedure of writing mathematical equations for mathematical word problems.

(2) Prompts for moderate achievement group:

(a) Tip prompts: These prompts gave hints about the number of unknown numbers and corresponding relationships in order to help students understand the problem-solving procedure of writing mathematical equations for mathematical word problems.

(b) Pictorial representation prompts: Images can make the problem more concrete and guide the students in problem-solving and writing equations.

(c) Problem-solving instruction: This uses text descriptions to directly teach the students how to solve the problem.

(3) Prompts for low achievement group:

Problem-solving instruction: This uses text descriptions to directly teach the students how to solve the problem and may teach them from a different perspective than that given to students in the moderate achievement group.

Experiment procedure

For content, we chose Unit 9 in the second semester curriculum for fifth-graders “How to Write Equations”. In terms of the teaching environment, the first 20 minutes of each 40-minute period was used for conventional teaching, and the teaching activity with Polya’s methods and appropriate prompts took place during the last 20 minutes in a web environment. The schedule of four-class activities is described as follows.

1. To perform pre-test before teaching activities.

2. The first class:

The teacher shows the system interfaces and functions to the students participated in the teaching activities of the Polya mathematical problem-solving strategies. Then, students do an exercise about the topic “how to use symbols to express unknown variables” and perform the process of Polya solving-problem via the system. Finally, the teacher shows the right answers to the students and makes the students to self-criticism what they did wrong. An example is shown in **Figure 2**.

3. The second class:

Students do an exercise about the other topic “how to use symbols to express the equation with plus/minus unknown variables” and perform the process of Polya solving-problem via the system. Finally, the teacher shows the right answers to the students and makes the students to self-criticism what they did wrong. An example is shown in **Figure 3**.

4. The third class:

Students do an exercise about the third topic “how to use symbols to express the equation with multiplied/divided unknown variables” and perform the process of Polya solving-problem via the system. Finally, the teacher shows the right answers to the students and makes the students to self-criticism what they did wrong.

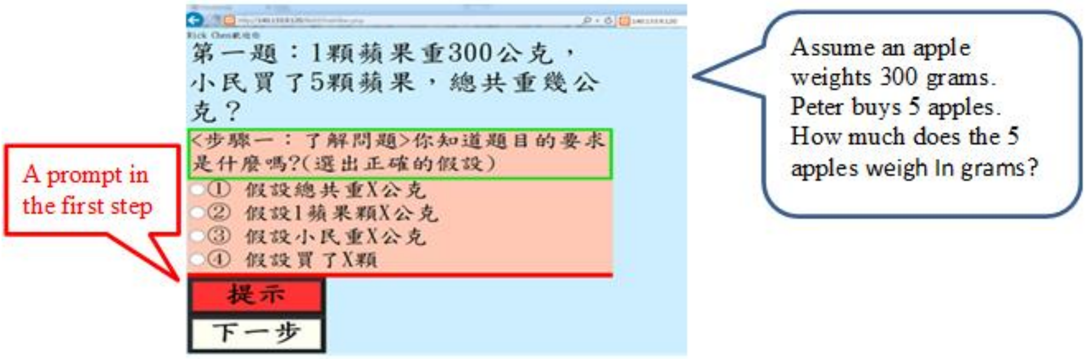

Figure 2. An appropriate prompt in the first step

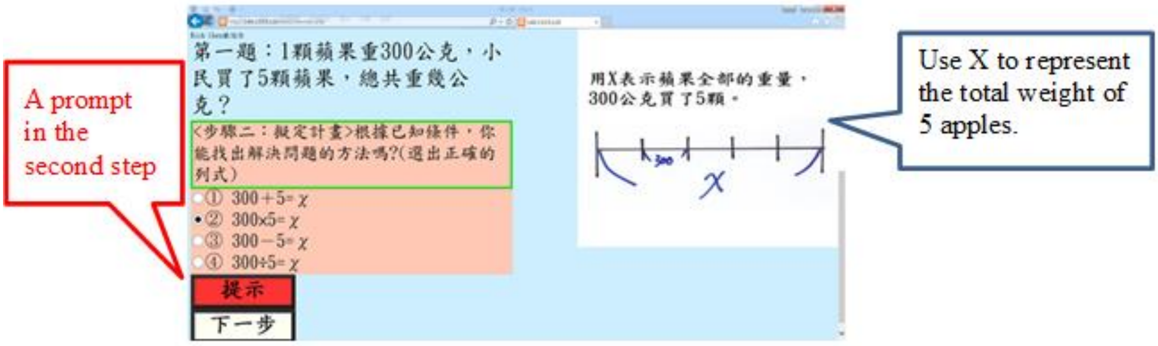

Figure 3. An appropriate prompt in the second step

5. The forth class:

Students do the last exercise about the topic "how to use symbols to express the equation with plus/minus/multiplied/divided unknown variables" and perform the process of Polya solving-problem via the system. Finally, the teacher shows the right answers to the students and makes the students to self-criticism what they did wrong.

6. To perform post-test after finishing the above four-week teaching activities.

Learning effectiveness test

The content of this test was based on the test used by Chen (2014). The content of the test was divided into three categories: identifying the unknown, writing the equation, and using the axioms of equality and parentheses. The test contained ten problems in all. The students scored from 0 to 2 points for each problem. Reliability analysis presented a Cronbach's α of 0.77, which meets standards suggested by experts, thereby showing good reliability.

User questionnaire

The purpose of this questionnaire was to seek the opinion of the students as to their interest and views regarding the system as shown in **Table 3**. The questionnaire was administered in a group setting. We used a five-point Likert scale (1 - strongly disagree to 5 - strongly agree) for Questions 1 through 25, and open-ended questions for Questions 26 through 32. To measure user-friendliness, we employed the System Usability Scale (SUS) developed by Brooke (1996). Questions 4, 5, 7, 15, and 21 were worded inversely. Content validity was analysed by expert evaluation, and reliability analysis presented a Cronbach's α of 0.912, thereby displaying good reliability as any score over 0.70 is considered acceptable (DeVellis, 2012; Nunnally, Bernstein & Berge, 1967).

Table 3. The questionnaire to student satisfaction

	Question Items
Q1	I think that there are some differences between online test system and paper test.
Q2	I think that using this system to learn makes math more fun.
Q3	I think that using this system to learn stimulates my interest in learning.
Q4	I think that I can also learn well if I do not use this system.
Q5	I think that I do not know what the major learning points are if I use such a problem-solving strategy.
Q6	I think that using this system helps me learn math.
Q7	I think that the prompts provided by the system would become a workload for learning.
Q8	I think that the step-by-step design of this system helps me learn math.
Q9	I think that the prompts helped me learn math.
Q10	I hope the teacher will give us similar systems to learn with in the future.
Q11	I think that learning this way makes me want to continue learning.
Q12	Operating this system was easy for me.
Q13	It took me only a short time to learn how to use the online test system.
Q14	The functions provided by the system are easy to use.
Q15	I do not know how to operate the system.
Q16	I hope that such a learning process can be applied to other courses.
Q17	I would be willing to learn online by myself at home.
Q18	I hope that online test system would replace paper test in the future.
Q19	I think that it could be better to add some animations in this system.
Q20	During operation, the button responses and problem presentations ran smoothly.
Q21	I have to study extra information in order to use the system.
Q22	Such a learning style makes me more accomplishments.
Q23	I understand the contents of prompts.
Q24	I can use the prompts to solve the problem and get the right answer.
Q25	I am able to read the contents of prompts.
Q26	Do I like the online test system?
Q27	How does the online test system help me?
Q28	Where does the online test system needed to be enhanced?
Q29	Which step do I spend more time for thinking?
Q30	Which step do I think needs prompts most?
Q31	Please write down some thoughts and suggestions for the prompts provided in the online test system.
Q32	Please write down some thoughts and suggestions for the online test system after finishing the work.

Design and implementation of teaching experiment

Step 1. The student logs into the system and obtains materials from the database.

Step 2. Each problem is structured according to the four steps of Polya's problem-solving methods. The first two steps proceed according to the student's level of achievement in mathematics, meaning that the prompts given to the student varies with the achievement group to which they have been allocated. If he submits an incorrect answer, a window will pop out to guide the student.

Step 3. The student must select an answer to proceed to Step 4.

Step 4. The system presents the answer previously chosen for the student to check his or her work. If there is a mistake, the student can return to Step 3 to correct it; otherwise, the student can submit his final answer. The system records the amount of time it takes the student to complete each step, prompt usage, and error rate.

Step5. After all tasks are completed, an achievement table is generated for the student, displaying the number of problems that he solved correctly.

RESULTS

Influence on learning effectiveness in mathematics

Table 4 displays the analysis results of the pre-test and post-test scores for the experiment group. A t-test produced $t = -4.479$ and $p < .05$, which indicates significant differences in the scores of the students in the experiment group before and after the experiment. **Table 5** shows the analysis results of the pre-test and post-test scores for the control group. A t-test produced $t = -1.996$ and $p = .056 > .05$, which indicates no significant differences in the scores of the students in the control group before and after the experiment. Therefore, applying Poyla's methods and appropriate prompts to the fifth-grade lesson (How to Write Equations) enhanced student learning effectiveness. This experiment therefore shows that learning with Poyla's mathematical problem-solving methods in conjunction with appropriate prompts benefits learning effectiveness more than conventional instruction.

Table 4. Paired samples t-test results of the pre-test and post-test for the experiment group

	Paired Differences		t	df	p
	Mean	SD			
Pre-test - Post-test	-4.03448	4.85124	-4.479	28	.000

Table 5. Paired samples t-test results of the pre-test and post-test for the control group

	Paired Differences		t	df	p
	Mean	SD			
Pre-test - Post-test	-1.34483	3.62802	-1.996	-1.996	.056

Influence on learning effectiveness of students with varying levels of achievement

(1) High achievement group

There are 8 students in high achievement group of the experiment group with the average score of post-test 19.87; and there are 9 students in high achievement group of the control group with the average score of post-test 19.66. From **Table 6** (Levene Statistics = 4.53, p = .05), there is no significant different between the high achievement students of the experiment group and the ones of the control group. **Table 7** presents the AVONA results (F = 0.95, p = .34 > .05), which indicate no significant differences in these two groups.

Table 6. Tests for homogeneity of the high achievement students of the experiment group and the ones of the control group

Levene Statistics	df1	df2	Sig.
4.53	1	15	.05

Table 7. ANOVA results of the high achievement students of the experiment group and the ones of the control group

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	0.18	1	0.18	0.95	0.34
Within Groups	2.87	15	0.19		
Total	3.05	16			

(2) Moderate achievement group

There are 10 students in moderate achievement group of the experiment group with the average score of post-test 19.3; and there are 10 students in moderate achievement group of the control group with the average score of post-test 17.8. From **Table 8** (Levene Statistics = 2.39, p = .13 > .05), there is no significant different between the moderate achievement students of the experiment group and the ones of the control group. **Table 9** presents the AVONA results (F = 5.67, p = .02 < .05), which indicate the learning effectiveness of students in moderate achievement group of the experiment group have significant better than the ones of the control group.

Table 8. Tests for homogeneity of the moderate achievement students of the experiment group and the ones of the control group

Levene Statistics	df1	df2	Sig.
2.39	1	18	.13

Table 9. ANOVA results of the moderate achievement students of the experiment group and the ones of the control group

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	11.25	1	11.25	5.67	0.02
Within Groups	35.7	18	1.98		
Total	46.95	19			

(3) Low achievement group

There are 11 students in low achievement group of the experiment group with the average score of post-test 17.72; and there are 10 students in low achievement group of the control group with the average score of post-test 14. From **Table 10** (Levene Statistics = 37.4, p = .06 > .05), there is no significant different between the low achievement students of the experiment group and the ones of the control group. **Table 11** presents the AVONA results (F = 4.48, p = .04 < .05), which indicate the learning effectiveness of students in low achievement group of the experiment group have significant better than the ones of the control group.

Table 10. Tests for homogeneity of the low achievement students of the experiment group and the ones of the control group

Levene Statistics	df1	df2	Sig.
3.74	1	19	0.06

Table 11. ANOVA results of the low achievement students of the experiment group and the ones of the control group

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	72.77	1	72.77	4.48	0.04
Within Groups	308.18	19	16.22		
Total	380.95	20			

The analysis above demonstrates that adding appropriate prompts to Polya's mathematical problem-solving methods in teaching activities exerts significant impact on the learning effectiveness of students with moderate or low levels of achievement in mathematics but has no significant influence on the learning effectiveness of students with a high level of achievement in mathematics.

Analysis of learning process

During our experiment, the proposed system recorded the progress of each student, including the amount of time it took the student to complete each step, prompt usage, and error rates. We analysed the relationships among the four steps. Table 12 exhibits the average amount of time the students in the experiment group spent on each of the four steps. As can be seen, the most amount of time was spent on Step 1. This corresponds to the results of Question 29 in the questionnaire, in which 33.3% of the students felt that more thinking time was needed for Step 1, the reasons mostly being the problem was difficult or required a lot of thought.

Table 12. The average amount of time spent in four steps

Step	Step 1	Step 2	Step 3	Step 4
Time (Sec.)	19.1	15.7	13.0	4.2

Comparison of the different achievement groups in the experiment group revealed that the students in the high achievement group spent less time in calculating each problem than those in the moderate or low achievement groups. The students in the low achievement group spent the most time thinking. In the amount of time spent on completing a problem, the three groups showed no significant differences, as shown in **Table 13**. The amount of time spent indicates the level of understanding that a student has with regard to the unit as well as his understanding of the system's operations and step procedure.

Table 13. The average amount of time spent for each group

The level of achievement	Average time(Sec.)
High achievement group	45.95
Moderate achievement group	50.19
Low achievement group	55.24

Table 14 displays the prompt usage in the various groups. As can be seen, the students in the low achievement group required more prompts on average than those in the moderate and high achievement groups. The students in the low achievement group also improved significantly more than those in the other groups, which indicates that the developed system is the most suitable for students with low learning effectiveness.

Table 14. The average amount of prompt usage for each group

The level of achievement	Average amount of prompt usage
High achievement group	9
Moderate achievement group	6
Low achievement group	36

Student satisfaction

As shown in **Table 3**, we distributed the questionnaire to students in the experiment group to gauge their satisfaction with and acceptance of the system as well as its usability. Containing 25 question items, the usability scale derived a Cronbach's α of 0.912. Most of the students felt that using this system increased their interest in learning mathematics. Moreover, they felt that the prompt design benefited their learning. The question items related to interest are shown in **Table 15**.

Table 15. Descriptive statistics of interest in using the proposed system

	a five-point Likert scale (%)				
	1	**2**	**3**	**4**	**5**
Q2. I think that using this system to learn makes math more fun.	0	6.9	10.3	10.3	72.4
Q3. I think that using this system to learn stimulates my interest in learning.	0	13.8	17.2	3.4	65.5
Q6. I think that using this system helps me learn math.	3.4	3.4	24.1	10.3	58.6
Q8. I think that the step-by-step design of this system helps me learn math.	10.3	3.4	17.2	20.7	48.3
Q9. I think that the prompts helped me learn math.	6.9	3.4	17.2	27.6	44.8
Q10. I hope the teacher will give us similar systems to learn with in the future.	10.3	3.4	10.3	10.3	65.5
Q11. I think that learning this way makes me want to continue learning.	10.3	0	6.9	17.2	65.5

There was a high level of acceptance of the system among the fifth-graders, and they furthermore reported that they found it easy to use. However, their responses regarding the integration and operation of the website functions were less positive, which reduced their willingness to engage in self-study. The relevant question items are as shown in **Table 16**.

DISCUSSION AND CONCLUSION

Influence on learning effectiveness

The results of the four-class teaching activity with Polya's problem-solving methods and appropriate prompts indicate that it had significant influence on the effectiveness of learning mathematics, creating a significant difference between the students of the experiment group and the control group.

Table 16. Descriptive statistics of system operation

	a five-point Likert scale (%)				
	1	**2**	**3**	**4**	**5**
Q12. Operating this system was easy for me.	3.4	10.3	6.9	17.2	62.1
Q13. It took me only a short time to learn how to use the online test system.	10.3	0	17.2	17.2	55.2
Q14. The functions provided by the system are easy to use.	6.9	6.9	10.3	17.2	58.6
Q17. I would be willing to learn online by myself at home.	10.3	10.3	13.8	6.9	58.6
Q20. During operation, the button responses and problem presentations ran smoothly.	6.9	13.8	20.7	27.6	31

Students have long been taught using a single method and often absorb information passively by using the previous strategies (Craig, 2016; Devi, 2016; Han & Kim, 2016). By making use of the appropriate prompt mechanism in this study, the students could effectively overcome obstacles in their learning process. This gave them more confidence and left a deeper impression. As a result, they were more willing to learning and participate in the process as a whole.

Influence on learning effectiveness of students with varying levels of achievement

The analysis results show that the teaching activity with Polya's problem-solving methods and appropriate prompts improved the learning effectiveness of students with a moderate or low level of achievement. The students in the low achievement group in particular showed the most improvement. In contrast, the students in the high achievement group did not show significant differences. Moreover, the study results show that the majority of the students in the high achievement group already presented above-par performance in the pre-test, which limited their scope for improvement and prevented any significant differences. The results of this study support the findings of existing studies proposing that employing Polya's methods can significantly improve the overall performance of students (Lai, 2010; Chang, 2012; Craig, 2016; Devi, 2016; Romiszowski, 2016).

Observations of the pre-test scores showed that the students with high achievement in mathematics already possessed a certain amount of prior knowledge, likely because they had already learned about the topic at cram school or had previewed the lesson themselves beforehand. Their scope for improvement was therefore limited.

Besides, as compared to Tsai (2009), the appropriate prompt mechanism in this study gives varying prompts to students depending on their level of achievement in mathematics, and the results show that the appropriate prompt mechanism was particularly effective in improving the learning effectiveness of students with a low level of achievement than that of students with a moderate or high level of achievement. The proposed approach is therefore an excellent option for remedial education.

Student satisfaction with appropriate prompt system

Analysis of the questionnaire results indicated that the students all had positive reactions to the combination of Polya's problem-solving methods and appropriate prompts. In terms of interest and attitude toward the system, the students all accepted this approach of teaching. They felt that when they encountered difficulties, the appropriate prompts helped them move forward autonomously. They reported that learning directly from the interactive prompts derived two benefits: it helped them to achieve significant improvement and to gain a deeper impression of the process through finding the answers themselves. The students also expressed that this method of learning increased their interest in learning math.

Suggestions for teaching applications

(1) The results show that the proposed teaching approach significantly improves the learning effectiveness of students in mathematics.

(2) The results indicate that the proposed teaching approach was most effective in improving the learning effectiveness of students with a low level of achievement. The proposed approach could therefore be effectively applied in remedial education. However, in this application the basic computer skills of the students should be taken into consideration. If needed, teachers could assist students as they familiarize themselves with the program and prompts.

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