MODESTUM
EURASIA Journal of Mathematics, Science and Technology Education, 2023, 19(1), em2207
ISSN:1305-8223 (online)
OPEN ACCESS
Research Paper
https://doi.org/10.29333/ejmste/12769

An empirically-based practical-realistic pedagogic mathematical model for teaching and learning of an antenna theory and design course

Vojo George Fasinu [1*], Nadaraj Govender [1], Pradeep Kumar [2]

[1] School of Education, University of KwaZulu-Natal, Durban, SOUTH AFRICA
[2] School of Engineering, University of KwaZulu-Natal, Durban, SOUTH AFRICA

Received 16 October 2022 ▪ Accepted 30 November 2022

Abstract

Despite the professional importance attached to the antenna theory and design course, most students and some academics still see the course as difficult and not easily explained via mathematical modelling (MM) despite some mathematical concepts integrated into the teaching and learning of an antenna theory and design. Due to this challenge, some students change their courses and opt for courses with less mathematical complexity. In view of this, this paper reports the review on the teaching and learning of an antenna theory and design using MM approach, relevant theoretical models reported by other researchers, with a comparative description of these theoretical frameworks. It also offers an empirical appraisal of a practical-realistic pedagogic mathematical model for teaching and learning an antenna theory and design course (PRPMM-TLATD) as a reliable model in the universities. In achieving this, data was gathered from four scholarly academics and 12 engineering students from a university in South Africa using qualitative approach. This finding generates the following stages as reported by the participants. And these stages include antenna validation by measurement, antenna validation by simulation, analysis of an antenna mathematically, personal conceptualization of the design work, total interpretation and validation of design problem, and problem resolution by mathematization. It also confirmed that the teaching and learning the design problem, antenna parameters modelling (mathematically), describing an antenna parameters mathematically, extra-mathematical working and prerequisite courses model were followed. The result of the study confirmed that the teaching and learning of an antenna theory and design could be classified into two domains, namely, paper-based design domain and a realistic domain as gathered from the data among engineering academics and students teaching and learning MM in a university in South Africa.

Keywords: mathematical, modelling, antenna, theory, design, realistic

INTRODUCTION AND REVIEW

Antenna technology is essential in the use of the current and varied communication devices in our daily life. It promotes the communication aspects of life in our technological society. However, for the advancement of the 4th industrial revolution in Africa, antenna and design courses are still not well planned and emphasized in academia (Balanis, 2016; Fasinu, 2021; Milligan, 2005). As a result of this, some electronics engineering students still describe the workings of an antenna as mysterious technology, which is difficult to understand due to the underlying complexity and plethora of mathematical concepts (Schantz, 2011; Schantz & Depierre, 2015). One of the main difficulty students experience is their inability to link mathematics concepts learnt as engineering students in prerequisite mathematical courses to the mathematical theories and concepts used as practicing electronics engineers, and this had led to a huge relevance problem in the industry (Bissell & Dillon, 2000; Fasinu et al., 2021; Karam, 2014; Pino-Fan et al., 2015). Against this background, this paper explores the theoretical framework for teaching and learning an antenna theory and design course and focuses on styles of teaching some basic prerequisite engineering courses that are related to an antenna theory and design. It also

✉ fasinu_george@yahoo.com **(*Correspondence)** ✉ govendern37@ukzn.ac.za ✉ kumarp@ukzn.ac.za

Contribution to the literature

- This study contributes to the existing literature on an empirically-based practical-realistic pedagogic mathematical model for teaching and learning of an antenna theory and design course using a naturalistic way of data analysis in gathering the participants' approaches of teaching and learning mathematical modelling in an engineering context.
- The uniqueness of the study relies on a qualitative approach using a variety of research instruments, offering an in-deep analysis of each of the engineering academics and students, regarding the initial and final perception on PRPMM-TLATD, as well as the outcomes.
- Findings reveal that a twelve-fold stage of teaching and learning antenna theory and design course will clear the air of confusion found among engineering academics' teaching, and students' learning on mathematical modelling in an antenna theory and design course.

offers an empirical appraisal of a practical pedagogic mathematical model for teaching antenna theory and design course (PRPMM-TLATD) as a reliable tool for teaching antenna design at universities. These initiatives were accomplished in order to resolve the deficiencies found the process of teaching and learning antenna theory and design courses. In achieving these outcomes, the study was carried out amongst 12 electronics engineering students and the four academics in a university in South Africa. The research question for this paper is, as follows:

What are the stages and empirically-based framework(s) involved in teaching and learning of mathematical modelling (MM) in antenna theory and design course and its related mathematical concepts?

The next section present the literature findings on the present teaching styles that are commonly used in the university where the study was undertaken. We also report on the previous models on MM with descriptions of their strengths and weaknesses regarding antenna theory and design and its prerequisite courses.

Review on the Teaching and Learning of an Antenna Theory and Design using Mathematical Modelling Approach

The modern teaching and learning approaches found among the academics these day has contributed to the increase in understanding of the students in our universities. However, there are limited understanding among some electronics engineering students in modelling their mathematical (MM) concepts that are common to the learning of an antenna theory and design course (Bissell & Dillion, 2000; Fasinu et. al., 2021; Pino-Fan et al., 2015; Redish & Kuo, 2015). In view of this some researchers improving the styles of teaching and learning among electronics engineering students will go a long way in increase students' academic performance in engineering related courses (Redish & Kuo, 2015). Karam (2014) similarly reported that the electronics engineering students commonly adopt limited skills when transferring their mathematical knowledge from an abstract situation to a practical reality. Some of the causes of the insufficiency in knowledge include inappropriate teaching approaches, poor learning materials, and poor assessment skills. Generally, the present instructional approaches for teaching MM has been found to be a weak one among the engineering students (Tuminaro & Redish, 2012). Indeed, the present teaching strategies used in electromagnetism and antenna related courses could be regarded as a source of the poor learning strategies evident among electronics engineering students (Bing & Redish, 2012; Tuminaro & Redish, 2007). In order to improve on this, these researchers advocate the use of mapping of meaning, a problem solving approach, epistemological framing and physical mapping, which should help the electronics engineering experts in modelling their mathematical ideas with fewer problems. On the other hand, some researchers suggest that the use of non-trivial approach in teaching antenna design will go a long way towards assisting the electronics engineering students in interpreting mathematical concepts for the physical reality (Karam, 20014; Mohammed, 2019).

The fact remains that the present MM approaches at some universities around the world depict mathematics as an educational problem that demands a quantitative approach in resolving it. It is on this note that researchers report that such an MM approach concentrates on the algorithmic manipulation, rote learning involving mainly memorization, superficial identification of mathematical meaning and using calculative tools (Karam, 2014; Redish & Kuo, 2015). By contrast, a future MM approach should be realistic in moving from an abstract learning style together with integrating practical learning (Gallegos, 2009; Hidayat et al., 2021). In support of this, it has been suggested that realistic modelling may generate a physical interpretation that justifies the use of mathematical concepts in interpreting the model. This will scaffold on theoretical knowledge and help in connecting physical assumptions through means such as logic, description of analogies for common mathematical concepts, increasing reasoning capacity and improving physical thought., interpretation and application. Therefore, this paper adds to the call for restructuring in the teaching of mathematics modelling in engineering departments in preparing students for future challenges in engineering practices (Karam, 2014). In as much as an engineering related courses are multidisciplinary in nature coupled with the goal of engineering education, which is to prepare the future engineers for realistic interpretation of mathematics, the process of teaching and learning of an antenna theory and design course with aim of modelling mathematics in a realistic manner will support this goal (Redish & Kuo, 2015). Thus, when MM is carefully practiced, it may assist the students' learning and support academics in increasing their interests in using MM during their teaching of engineering courses. They will need to refocus their teaching on the need to pay attention to the meaning of MM and an epistemological model of MM. Among other emphasis, they will also need to focus on the relationship between MM and the teaching of antenna theory and design, the reasoning behind teaching about antennae and their design in line with the physical world, together with a proper understanding of the laws, principles and explanations of physical reality (Gallegoes, 2009; Hidayat et al., 2021).

THEORETICAL REVIEW ON RELEVANT FRAMEWORKS

The review of the literature has confirmed that, several theoretical frameworks for teaching and learning of MM has been proposed such as Bing and Redish (2012), Blomhøj (2009), Blum and Leiß (2007), Ferri (2006), Goold (2012), Maaß (2006), Redish and Kuo (2015), and Redish and Smith (2008). Among which are, as follows:

(a) the *didactical model*, which gives room for modelling of mathematical concepts using the common traditional approach of teaching mathematical related problems (Ferri, 2006),

(b) the *realistic models* on its part take care of mathematical concepts in a realistic way and approach MM from a multidisciplinary perspective, as needed in engineering design, where the mathematics models are involved in translating from abstract form to real practical design of physical system, and

(c) didactical-realistic mathematical model (DRMM) as indicated in the previous papers, studies by Blomhøj (2009), Gallegos (2009), and Redish (2017) refer to some aspects of MM for a system design, but some required aspects of antenna theory and design are not fully addressed by them.

Therefore, DRMM gives room for the combination of both two models in order to resolve most discrepancies. However, none of these frameworks listed above has fully considered MM in an engineering context (Fasinu, 2021; Redish, 2017). In as much as the previous three models earlier published had not adequately addressed the problem of teaching and learning MM in an antenna theory and design course, therefore, it is necessary to propose model that enhanced the teaching and learning of an antenna theory and design using the empirical data.

METHODOLOGY

This study investigates an empirically-based practical-realistic pedagogic mathematical model for teaching and learning an antenna theory and design course (PRPMM-TLATD) among electronics engineering academics' and students from a university in South Africa. In getting the views of the participants, the researchers investigated different groups of participants, which include the academics teaching the prerequisite mathematics and the core antenna theory and design and the students in the course. A qualitative research approach was used as it was to obtain the experiences of all participants in naturalistic settings (Baxter & Jack, 2008; Burnard et al., 2008; Cohen et al., 2017; Creswell & Poth, 2018). Purposeful sampling procedure was used in selecting the appropriate group for the study (Creswell & Poth, 2018). In order to gather information from the participants, a survey questionnaire, interview guide and observation schedule were used as tools. The data analysis took several overall steps. Firstly, all the raw data collated and transcribed, separated into temporary categories, then themes were identified and validated by co-researchers. And the final result of the findings are reported and analyzed through coding and themes. Ethical permission was obtained from all participants and in accordance with the university requirements (Bloomberg & Volpe, 2018). For the purpose of collecting and reporting data analysis, 16 participants were invited for the study. And these include three engineering academics teaching the perquisite mathematics courses and an academic teaching the core antenna theory and design course. Furthermore, 12 engineering students, which these include the undergraduate and postgraduate students were equally adopted for the study. Accordingly, the data presented here is in line with the stages reported by the academics and students during the process of teaching and learning of an antenna theory and design. All these methods adopted has assisted the researchers in reporting the views of the engineering academics and students on the appropriate stages of teaching and learning mathematical modeling in antenna theory and design course, which is known as PRPMM-TLATD. This model above was generated from the combination of different theoretical and conceptual frameworks adopted at each stage of teaching and learning of an antenna theory and design as reported by the participants. And these are: Working mathematically model, understanding antenna parameters model mathematically, understanding design problem model, general mathematization model, Teaching and learning model for prerequisite courses among many others. All these steps have contributed to the stages and styles of teaching and learning antenna theory and design and prerequisite courses. From the views of the participants as stated in this study, the researchers argue that the discrepancy found in the teaching and learning of an antenna theory and design with the prerequisite mathematics courses if not reflected upon may lead to students' limited understanding of the appropriate learning styles. The detail result is hereby reported below.

PRESENTATION AND DISCUSSION

This study confirmed some emergent stages of teaching and learning of prerequisite mathematics courses and an antenna theory and design course that are offered in the school of electronics engineering. The findings and discussions were resolved in two sections after analysis of data: There are stages involved in teaching and learning of MM in antenna theory and design course and its related mathematics, and towards designing and reflecting on an PRPMM-TLATD for teaching and learning in an antenna theory and design course.

Stages Involved in Teaching and Learning of Mathematical Modelling in Antenna Theory and Design Course and Its Related Mathematics

12 stages were found to emerge after analysis from the discussion section reported by the four engineering academics teaching prerequisite mathematics courses and antenna theory and design course and are elaborated as followed with brief extracts of evidence from the survey questionnaires and interviews.

Stages one – Teaching and learning the prerequisite courses

This is a stage in teaching and learning an antenna theory and design, which reports the students' acquisition of some preliminary understandings of some mathematics and physics related concepts. Some of these mathematical concepts are linear programming, logic and Boolean equation, linear algebra, calculus with application of integration, Laplace transforms, linear ODEs and matrices, among many others. This list was gathered from the views of participants teaching and learning an antenna theory and design course. The academics teaching prerequisite mathematics, AC1, stressed that

> "Linear programming is one of them, set theory is another one, logic and Boolean algebra for engineering and computer engineering students, all these are very important for them. We can also use graph theory in solving some problem in engineering … there is another one called difference equation … we have differential equation and difference equation. Difference equation is the discrete part while differential equation is the continuous part. So, differential equation, difference equation, set theory, linear programing, and graph theory modules are very important for engineers" (interview).

All these prerequisite courses should enhance the students' understanding of the mathematical concepts as they will be required to model mathematically as well as present a sophisticated understanding of content knowledge of physics concepts that are needed for a deep understanding of antenna theory and design. Furthermore, another view reported by an academic teaching the module (AC4) stressed that

> "In antenna theory and analysis, we need to analyze the radiated fields i.e., electric fields and magnetic fields. With the help of electric fields and magnetic fields many radiation parameters such as antenna radiation patterns, gain of the antenna, directivity of the antenna, antenna polarization, three dB beamwidth, etc. are calculated, which are used to judge the performance of the designed antenna. Further, we need the knowledge of circuit theory, which is used to calculated antenna parameters at input terminals such as input impedance, bandwidth, voltage standing wave ratio, reflection coefficient, resonate frequency etc., which are the important antenna parameters at input terminals" (AC4, interview).

An undergraduate student (UG3) carefully stressed that

> "I can say all mathematics modules, mathematics 1A, 1B, 2A, and 2B since 3A have some link. Yes, mostly in calculations. Firstly, the integration and geometry we learn those sections in mathematics we are using them in some formulas /equations of antennas like the potential function equation, transformed to spherical unit vectors using the transformation matrix, equivalent circuit in transmitting mode, we also learn basic circuit from physics" (interview).

This view was supported by the handbook of the College of Agriculture, Engineering, and Science (CAES), which suggests that topics like radiation, and electromagnetism and related mathematics should be taught by the academics as prerequisite courses that are related before teaching them an antenna theory and design (Balanis, 2016; CAES, 2020). Therefore, one could argue that before teaching the antenna theory and design course, emphasis should be given to the topics of coordinate geometry, integration, Laplace transform, and electromagnetism, among many others. This introduction will ensure that teaching the prerequisite mathematics courses would go a long way towards enhancing students' understanding of the mathematical concepts to be successfully modelled in the antenna theory and design course.

Stage two – Working mathematically

This is a common style that is adopted when teaching mathematically related courses (Ferri, 2006). At this stage, the academics often focus on the results of the given question without considering the practical applications. In this case, different mathematical models could be found when calculating problem in a mathematical context, although this depends on the particular academic teaching the course. Here, different strategies could be used in arriving at the mathematical result. In achieving this, extra mathematical knowledge (EMK) and the students' prior learning needs to be considered and adopted before arriving at the required result of a given engineering problem. This was gathered from the view of an academic ACA1 who reported that

> "Differential equation is applied to any kind of problem … So, the one that goes along with it is difference equation that has to do with discrete values or discrete situations. When we are talking about discrete situations, then we use difference equation. But when we are talking about continuous situations we talk about differential equation. Those two things are very applicable in many problems. But people use more of differential equation than difference equation" (interview).

Therefore, concurring with the views of the participants, a highly developed pedagogical content knowledge (PCK) may be required of the academic for the best teaching outcome (Ferri, 2006). In support of this, Jensen (2007) reported that the use of applied mathematics alone does not resolve the issue of mathematics in the real-life situation of antenna design. Consequently, he suggested that the *transformation of a real-life situation into a mathematical problem* and this is regarded as being the core of a modelling method. This stage of modelling mainly caters for the prerequisite mathematics related courses.

Stage three – Understanding antennae parameters mathematically

The next stage of teaching antenna theory and design is where the academic teaches the fundamentals of antenna parameters. These antenna parameters include the radiation pattern of microstrip patch antenna (MPA) and its mathematical concepts, radiation power density of MPA and its mathematical concepts, beam width of an antenna and its mathematical concepts, radiation intensity of an antenna and its mathematical concepts and directivity of an antenna (Balanis, 2016). This was supported by the views of postgraduate student (PG1) and academic (AC4), as reported below.

PG1 argued that

> "I used the mathematical formulas that were used to calculate the parameter of the antenna" (interview).

Academic AC4 reported that

> "To start with, I do teach the grounded structure of an antenna, after which I move ahead in simplifying the whole components of an antenna in a clear way. I also go further to calculate and analyze the electric field and magnetic field of the small elements, using integration in calculating the electric field of the entire structure. At times, I use summation, and I also use integration to calculate as earlier said. Subsequently, I do calculate the substrate, which is the source of an antenna signal. Furthermore, I also try to integrate or adopt summation in order to have a total electric field and magnetic field indicated by the antenna. After which the electric and magnetic field results are being gotten and adopted as required. With these two components I can further calculate the variables parameters like: gain, radiation pattern of the antenna, the activities of the antenna, efficiency of the antenna and so on. I do calculate the electric field generated during the radiation of an antenna in space using a three dimensional coordinate system. Furthermore, in getting the differences between the coordinate system, like; a rectangular coordinate system or quadrilateral coordinate system, the use of three dimensional coordinates system remains valuable. For example, if I am dealing with a bi-antenna, I may need to adopt a cylindrical coordinate system in resolving its radiation pattern. And similarly, at some points, I need a point antenna in transferring a certain coordinate system to the principal nature of an antenna accordingly. During my teaching, I do emphasize on the calculation of an electric field in all the coordinates in each section, and with the help of these components I can calculate further by using that knowledge of the radiation of an antenna, having learnt some aspects of the power of an antenna, the gain, directivity and other a parameters of an antenna" (interview).

This presentation ties with the view of Balanis (2016) who suggested that when teaching and learning antenna theory and design, the academics teaching the course should include some major areas of antenna technology like radiation, intensity, the effective dielectric constant of an antenna, etc. in order to better facilitate students' understanding. Therefore, teaching and learning of some fundamentals of antenna parameters is an essential part of antenna theory and design. This was supported by statement from the academic teaching the course who confirmed that teaching antenna design involved teaching of the fundamentals of an antenna and it practical description using mathematical concepts Therefore, when teaching and learning antenna theory and design, the role of describing the antenna parameters mathematical cannot be overemphasized.

Stage four – Mathematically modelling of antennae parameters

After describing an antenna parameter to the students, the academic teaching the antenna theory and design course should see the necessity for teaching the application of the relevant mathematical concepts in a modelling manner. This was reported by the views of the academic AC4 who confirmed that

> "… To compute many antenna parameters, we apply many mathematics concepts hence we try to make it easy by teaching some mathematics in the beginning of the prerequisite courses and some topics wherever these are needed to teach so that students can understand the antenna analysis and design easily" (interview).

He further stressed that

> "… As I told you earlier, that in antenna course, we need analysis, therefore, analysis is very important. After which I do teach about the antenna theory and design in detail. More so, in antenna design course, there are some parameters of the antenna that I do teach, which aid the performance of the antenna. And these are the dB, resonant frequency, gain, directivity, and polarization among many others. Therefore, I need to analyze them because during my teaching process that is one of the major things I do teach. Further, I also teach my learners on the structure of an antenna coupled with the analysis of the antenna. For example, if I am teaching any part of antenna design course, I depend firmly on calculation of the electric and magnetic field in all types of antennas. In addition to this, the use of three dimensional coordinate in things to calculate the electric and magnetic field with the help of each element we consider the amplitude of an antenna array remain important. Similarly, when teaching a microstrip antenna, I do use transmission line model, cavity model, and method of moment in explaining some aspect of analysis in patch antenna. Therefore, the approach to be used in teaching a design problem depend on the topic, therefore, that is what I do teach. At my initial stage of teaching antenna theory, normally electromagnetic theory, which adopt three dimensional coordinate system remain important. More so, I do teach Chebyshev vector, coordinate system, and vector field fundamentals, which are used in the coordinate system. Vector field and magnetic field with other two concepts are calculated in excel. These three topics we do teach in details in electromagnetic field and field theory. More so, there is a need of teaching about a coordinate system, vector and scalar field and how to apply cross product and dot product, all these I teach in detail. Therefore, it depends on the type of antenna I am teaching, sometime during lecture I used what is required at a particular time. For instance, when I am teaching microstrip antenna, I do teach an appropriate function that could be introduced accordingly. I do explain to the students so that they can deal with the required concepts using the required mathematical concepts. For instance, if I am teaching an optical antenna, I need to explain an optical concepts like; reflection, deflection, therefore, I introduce every concepts as required" (interview).

This procedure corroborates with Redish and Kuo (2015) who argued that the engineering academics teaching antenna theory and design do not use mathematics theory as the mathematicians do, but they model them in applications and in a modelling manner. In support of this, the result further ties with the researchers that reported that at this point of teaching antenna theory and design, modelling some mathematical concepts, electromagnetism, and other physics related courses like optics among many others remain essential to the teaching of an antenna theory and design (Balanis, 2016; Milligan, 2005). On this note, the study data has confirmed that the modelling of some mathematical concepts is essential in teaching antenna related parameters.

Stage five – Understanding the design problem

During the teaching and learning of an antenna theory and design course, it has been shown that the process always begins with the academic posting a suggested design question. This was confirmed by AC4,

> "We use design analytical equations to design the antenna for a particular frequency. The dimensions of the antenna are calculated using design equations. If a novel antenna design is proposed then we try to give the analytical equations of the antenna so that these equations can be used for further design of the structure by other researchers" (interview).

More so, Karam (2014) and Mohammed (2019) supported this by confirming that the identifying and understanding of the design problem so the designer can arrive at an accurately designed device with little or no deficiency, which is clearly important when teaching and learning MM in antenna theory and design.

Therefore, in the case of an antenna design, any misunderstanding of a design problem can be resolved when an academic or the students adopt different mathematical ideas as learned from physics, engineering and mathematics.

Stage six – General mathematization of problem

Mathematization is a process of calculating the mathematical results that are relevant to the given design. Different mathematical models may be relevant when modelling a real-life problem, and the choice thus depends on the modeler. But nevertheless, different strategies must be evaluated alongside with the result– an acceptable mathematical formula will produce a fairly accurate result. Academic (AC1) added in this regard,

> "I teach with application to real world problem".

He further stressed that

> "in order for the students to understand better it is good to first bring the problem into their understanding. Like if you want to model or use mathematics for scheduling or production, you first talk about what is happening in the industry first. You will just bring the process … because they want to work in industries, some of them are already there because their fathers are engineers, and they are introducing them to those things already or they might have gone to work as volunteer workers at one industry or the other. Therefore, it is good to first bring to them what is happening, that will arouse their interest that is number one. Two, it opens their mind to what you want to do in mathematics. Then you will now bring mathematics in, by now saying that in order to solve this mathematics problem in engineering or in production company or in scheduling company or in any engineering company you need to bring them to variables" (interview).

This corroborates with Ferri (2006) who argued that the selected mathematical concepts that are being adopted by the students and academics when modelling them into the teaching and learning of an antenna theory and design may be inform of simple or complex equations, graphs, and theories among other may be suited given mathematical problem particularly when teaching an antenna theory and design course. Therefore, the role of mathematization using different mathematical tools cannot be overemphasized in resolving a general mathematical problem encountered when teaching an antenna teaching and design.

Stage seven – Interpretation and validation stage

It is an aspect that deals with proper and accurate interpretation of the design problem in order to meet the expectations of a real-life situation (Ferri, 2006). This stage assists the academics and students to imagine the result from the pseudo-concrete modelling domain. At this stage, it must be mentioned that the answers supplied by students based only on their knowledge of prerequisite mathematics may not be enough for them to think critically and resolve the pseudo-concrete issues of the design. This view was confirmed by an academic (AC1) who solved the mathematical problems, as shown in **Figure 1**.

The problem and solution in this study confirmed that the validation of results remains important for an antenna designer to structure their proposed design of a given problem. On this note, adopting different mathematical concepts for interpreting and validating the main result goes a long way towards achieving the design of an accurate antenna device.

Stage eight – Conceptual design

This is an imaginative representation stage that helps to interpret and validate the designed problem in a real-life situation. At this stage, students adopt creative mind in deciding the appropriate design approach that must be adopted when modelling the relevant concepts learned in the prerequisite mathematics course. This was confirmed by the view of a postgraduate student (PG1) who reported that

> "In term of the mathematics we use to calculate the width and length of the antenna that had already been simplified in the textbooks that guide us to find the given parameter of the antenna such as width, length and so on" (interview).

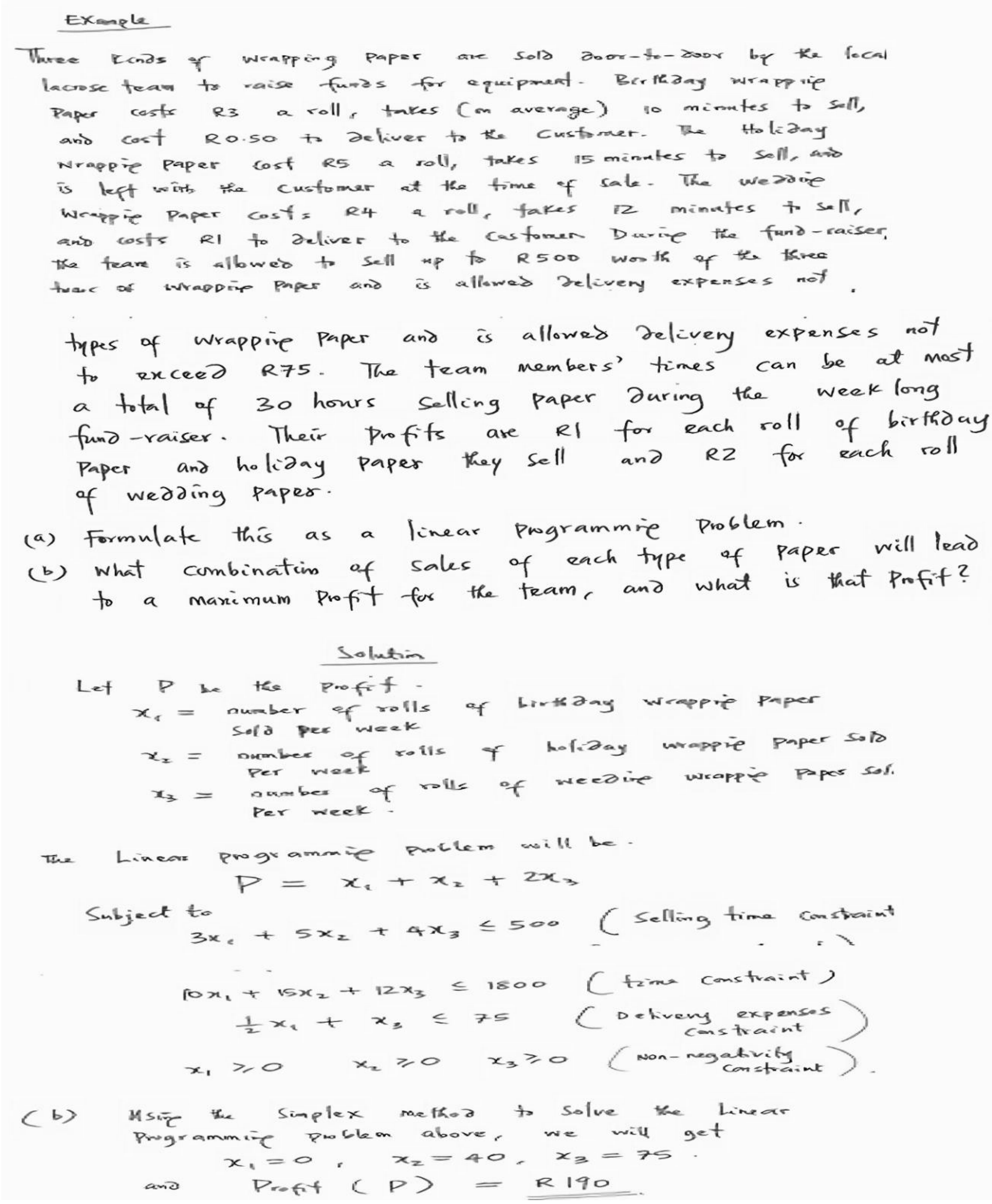
While an undergraduate student (UG1) stressed that

> "When I am looking at the diagram b/c of the antenna are such a profile structure of course they need geometry to get exact values and then I use series, and which also use integral gain and sometimes do everything as well to get final answer" (interview).

This corroborates with the views of some researchers such as Balanis (2016) who confirm that the step of translating a complex mathematical idea into a practical reality is a difficult task for many students. Therefore, the role of imaginative representations must be taken seriously in antenna design.

Stage nine – Mathematical analysis in practical antenna design

At this point, the engineering students, academics and experts take a reasonable time to resolve the given antenna design problem in a realistic way. Here, the engineering expert involved would adopt the relevant



Example

Three kinds of wrapping paper are sold door-to-door by the local lacrose team to raise funds for equipment. Birthday wrapping paper costs R3 a roll, takes (on average) 10 minutes to sell, and cost R0.50 to deliver to the customer. The Holiday wrapping paper cost R5 a roll, takes 15 minutes to sell, and is left with the customer at the time of sale. The wedding wrapping paper costs R4 a roll, takes 12 minutes to sell, and costs R1 to deliver to the customer. During the fund-raiser, the team is allowed to sell up to R500 worth of the three types of wrapping paper and is allowed delivery expenses not

types of wrapping paper and is allowed delivery expenses not to exceed R75. The team members' times can be at most a total of 30 hours selling paper during the week long fund-raiser. Their profits are R1 for each roll of birthday paper and holiday paper they sell and R2 for each roll of wedding paper.

(a) Formulate this as a linear programming problem.
(b) What combination of sales of each type of paper will lead to a maximum profit for the team, and what is that profit?

Solution

Let P be the profit.

x_1 = number of rolls of birthday wrapping paper sold per week

x_2 = number of rolls of holiday wrapping paper sold per week

x_3 = number of rolls of wedding wrapping paper sold per week.

The Linear programming problem will be.

$$P = x_1 + x_2 + 2x_3$$

Subject to

$3x_1 + 5x_2 + 4x_3 \leq 500$ (Selling time constraint)

$10x_1 + 15x_2 + 12x_3 \leq 1800$ (time constraint)

$\frac{1}{2}x_1 + x_3 \leq 75$ (Delivery expenses constraint)

$x_1 \geq 0 \quad x_2 \geq 0 \quad x_3 \geq 0$ (Non-negativity constraint).

(b) Using the Simplex method to solve the Linear Programming problem above, we will get

$x_1 = 0, \quad x_2 = 40, \quad x_3 = 75$.

and Profit (P) = R190

Figure 1. Example of mathematical problems and solutions (AC1)

mathematical concepts and antenna design equations to resolve some problem related to antenna design, which requires a high level of realistic MM. This was reported by academic (AC4) who stressed that

> "It is assumed that some required topics in antennas' analysis such as calculus, trigonometry, and coordinate geometry are the topics, which students coming for undergraduate program are familiar with coupled with adequate knowledge on these topics. However, if there is special or complex function among these topics, we explain it to students during the study of antenna analysis. We do not teach these topics as separate contents. Field theory and electromagnetic theory are the prerequisite modules for antenna theory, which is taught in selected topics in electronic engineering-microwaves. Some mathematical concepts such as vectors, coordinate systems, vector calculus in each coordinate systems, etc. are covered in the beginning of the module Filed theory as these are the frequently used topic in most of the antenna structures. Some special functions/topics such as sine integral, cosine integral, limits, etc. are explained wherever required" (interview).

Similarly, undergraduate student (UG3) stressed that

> "Yes, I actual selected topic 3 that focuses on the loop antennas. I used antenna theory by C. A. Balanis for understanding the topic. The other topics I learned them in class on notes, slides and tutorial from Moodle, which are done by other students. I also used the textbook for reference in this case. More so, mathematical concepts like; geometrical arrangement for the field analysis of a loop antenna, which is based on linear geometries from maths modules and Bessel function used under power density, radiation intensity, radiation resistance, and directivity, and other mathematics concepts we learned in maths were covered. I also used YouTube in learning some mathematical concepts that I had forgotten" (interview).

These views tie with the researchers who emphasized the importance of analytical equations for antenna design when resolving issues such as radiation pattern, effective dielectric field, and width of a patch, among many others (Balanis, 2016; Milligan, 2005).

Stage ten – Validation through simulation in practical design

This stage helps the academics and students in checking the accuracy of the antenna designed, due to the importance of accuracy. At this stage, the use of relevant software and mathematical concepts in testing and measuring the designed antenna cannot be overlooked. This was reported by an engineering undergraduate student (UG4) who reported that

> "We are learning various design techniques in the course work. I have basic knowledge on the CST studio software, which can be used to design antenna. More so, we are learning various design techniques in the course work. I have basic knowledge on the CST studio software, which can be used to design antenna. By knowing different physics of wave, reactions and yeah … we do adopt the wave theory and physics 1 and physics 2. All these knowledge help us in an antenna wave" (interview).

This stage ties with the researchers' views who argued that the use of software, which includes the computer simulation technology (CST) microwave studio software, and other simulation software, which were readily available at the university, along with Ansoft wave propagation and Hewlett Packard's EEsoft, which help the academics and experts in teaching about the strength and gain of an antenna (Balanis, 2016; Mohammed, 2019; Soliman, 2019). Therefore, teaching the application of software is necessary to allow leaners to understand the workability of their designed antenna.

Stage eleven – Checking the prototype of an antenna designed

It is a stage of teaching and learning an antenna theory and design, which help in describing the physically designed antenna model. It uses some major mathematical concepts that are basic to understand. Academics have found this stage to be it useful for increasing the students' motivation by learning through practical application of mathematical concepts. This was reported by undergraduate student (UG2) stressed that

> "Yes, my design project is based on the microstrip printed antenna" (interview).

He further reported that

> "To supplement the calculation of antenna design and to explain the proof of formulas" (interview).

At this stage, engineering academics would usually adopt the mathematical concepts of coordinate geometry, and calculus including integration along with some simpler mathematical concepts, while the emphasis at this stage is on the real antenna design and production. In all these cases the importance of the mathematical concepts cannot be overemphasized (Balanis, 2016; Constantine, 2005; Soliman, 2019). Nevertheless, the mathematical concepts cannot be adopted in isolation, as is often done by academics teaching prerequisite mathematics, because they have little understanding of how antennae work.

Stage twelve – Validation through measurement in designed antenna

This is a crucial stage in teaching practical antenna theory and design, which is done by measuring some important aspects in a designed antenna using different mathematical concepts. This stage is also known as the hands-on product because it concentrates on the use of mathematical concepts in various forms for measuring the components adopted (Soliman, 2019). This was supported by engineering student (UG1) who suggested that

> "because geometry [shape] can sometimes be complex, and because I have learnt necessary math module I can apply it in those modules to a given situation using the geometry I have learnt even though is a complex geometry" (interview).

This corroborates with Honchell and Miller (2001) who maintain that in as much as engineering students must manipulate the acquired mathematical knowledge into their designed products, the use of mathematical concepts during their component selection, measurement and other processes is vital (Honchell & Miller, 2001; Milligan, 2005). Therefore, it could be said that during the design of an antenna device, the

Table 1. Stages of teaching MM in antenna theory and design and guiding theoretical/conceptual framework(s)

	Stages of teaching & learning antenna theory & design	Guiding theoretical/conceptual framework(s) adopted
1	Teaching and learning model for prerequisite courses	Didactical model & PRPMM-ATD
2	Working mathematically model	Didactical model & PRPMM-TLATD
3	Understanding antenna parameters mathematically	Realistic model & PRPMM-TLATD
4	Understanding design problem model	PRPMM-TLATD
5	General mathematization model	Didactical model & PRPMM-TLATD
6	Interpretation and validation model	Realistic model & PRPMM-TLATD
7	Conceptual design model	PRPMM-TLATD
8	Mathematical analysis model in practical antenna design	Didactical model & PRPMM-TLATD
9	Modelling antenna parameters mathematically	Realistic model & PRPMM-TLATD
10	Validation through simulation model	DRMM & PRPMM-TLATD
11	A prototype model	DRMM & PRPMM-TLATD
12	Validation through measurement model	PRPMM-TLATD

validation of the design using mathematics concepts for the measurement of the width, length and breadth of the chosen patch is a necessary process.

Towards Designing and Reflecting on an Empirically-Based Practical-Realistic Pedagogic Mathematical Model for Teaching and Learning in an Antenna Theory and Design Course

The data, analysis and results of this study substantiates a practical MM procedure based on empirical data and is labelled the "practical-realistic pedagogic mathematical model" (PRPMM-TLATD) for teaching and learning in an antenna theory and design course. This model has possibilities of guiding the teaching and learning as adopted by engineering academics and students in this study. The theoretical and conceptual frameworks adopted in each stage of teaching and learning of an antenna theory and design are hereby stated in stages as follow: Teaching and learning model for prerequisite courses, working mathematically model, understanding antenna parameters model mathematically, understanding design problem model, general mathematization model, interpretation and validation model, conceptual design model, mathematical analysis model in practical antenna design, validation through simulation model, a prototype model, validation through measurement model and modelling antenna parameters mathematically (**Table 1**).

All these steps have contributed to the stages and styles of teaching and learning antenna theory and design and prerequisite courses. From the views of the participants as stated in this study, the researchers argue that the discrepancy found in the teaching and learning of an antenna theory and design with the prerequisite mathematics courses if not reflected upon may lead to students' limited understanding of the appropriate learning styles. **Table 1** analyzed below also gives detail information on the different models adopted by the academics and students when teaching and learning an antenna theory and design in different stages.

From the **Table 1**, a comparison of the four models shows that simple didactical mathematical model concentrates on the MM only in a theoretical way, while for its part, the realistic model gives more attention to the modelling of mathematical concepts in a realistic way. On the one hand, the combined DRMM gives room for both aspects: modelling of mathematics concepts in prerequisite mathematical courses and their modelling application in the core antenna theory and design course (**Figure 2**).

On the other hand, the PRPMM-TLATD provides a framework that gives attention to the prerequisite courses that should be linked to antenna theory and design. All these models deal with MM, but the PRPMM-TLATD has incorporated every aspect of modelling and teaching the concepts required for an antenna theory and design course (**Figure 2**). There are, however, some limitations in the PRPMM-TLATD due to the present structure of the core antenna theory and design curriculum and prerequisite courses. The present structure allows for little interaction among the engineering academics teaching prerequisite courses like physics or mathematics and the antenna theory and design course. Consequently, there may be discrepancies in their teaching approaches, which should be addressed to avoid confusion. Nevertheless, the proposed PRPMM-TLATD appears to be the most appropriate for teaching and learning of the antenna theory and design course and related its prerequisite courses and derived from this empirical study (see **Figure 2** for detail).

The 12 important ideas and stages that are core to the practical pedagogical mathematical model for teaching and learning of an antenna theory and design (PRPMM-TLATD) shown diagrammatically in **Figure 2** and listed in **Table 1** and are discussed in detail, as follows.

Prerequisite courses model

This is the first stage recommended by the department of engineering for engineering students and academics because it allows the students and academics to probe on prior learning and acquire some preliminary knowledge on relevant areas of antenna theory and design. It includes the teaching of some mathematical concepts as found in the departmental handbook (Balanis, 2016; CAES, 2020).

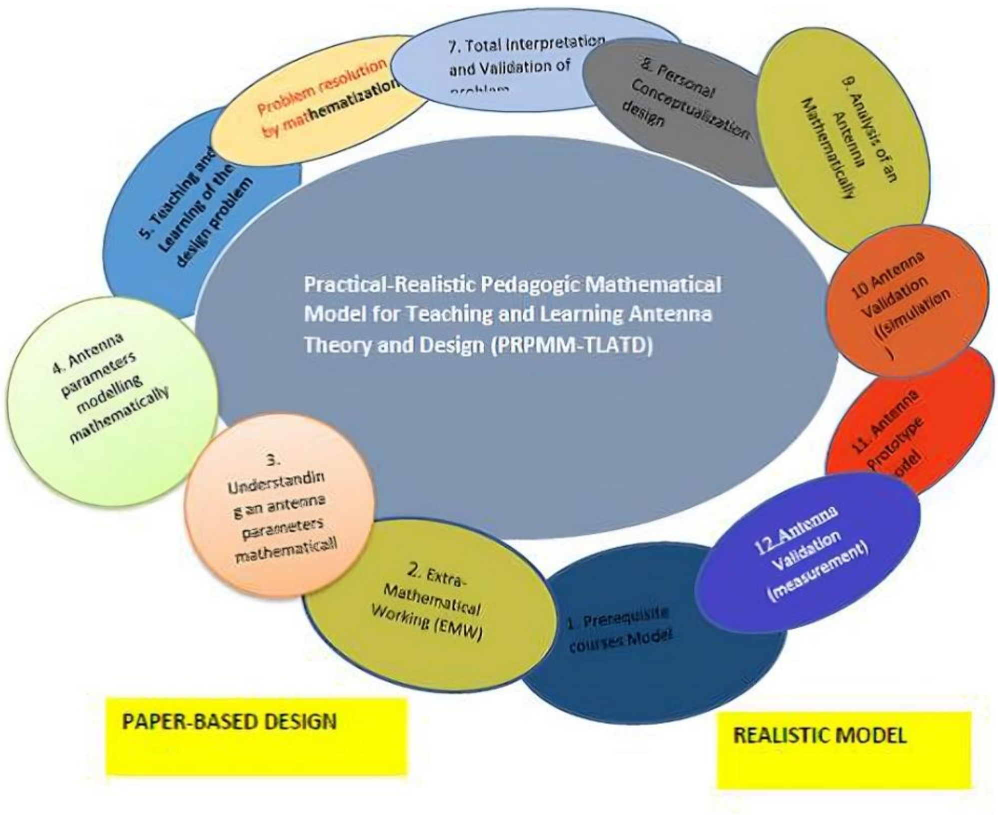


Figure 2. A practical-realistic pedagogical mathematical model for teaching & learning antenna theory & design (PRPMM-TLATD) (Fasinu, 2021)

Extra-mathematical working

It is another stage that adopts the adoption of different mathematical working styles in resolving the problem. This involves the use of by using different mathematical working styles with highly developed pedagogical content knowledge may be required of the academic for the best teaching outcome (Ferri, 2006).

Describing an antenna parameters mathematically

This is stage is common to an antenna theory and design stage, just because it assist the teaching and learning of an antenna theory and design, with the consideration of some key antenna concepts like radiation, intensity and the academic teaching the course should include some major areas of antenna technology like radiation, intensity other concepts that are related to antenna (Balanis, 2016; Milligan, 2005).

Antenna parameters modelling (mathematically)

This stage adopts a realistic teaching and learning style. It was confirmed by Balanis (2016), who suggested that when teaching antenna theory and design, the academic teaching the course should include some major areas of antenna technology like radiation, intensity, and the effective dielectric constant of an antenna in order to facilitate the students' understanding.

Teaching and learning the design problem

It is another stage of teaching antenna theory and design that entails the practical understanding of the design problem. Since, engineering academics teaching antenna theory and design do not use mathematics simply as the mathematicians do, but they model them in an application; that is in a modelling manner (Redish & Kuo, 2015).

Problem resolution by mathematization

This is a stage that can be actualized by calculating the mathematical related concepts with aim of getting relevant results on the given design. The stage allow the properly selected mathematical concepts to be used; it may be informing the equations, graphs, theories and any other mathematical tools, which would be most suited to the question to be addressed by MM, particularly in teaching an antenna theory and design course (Ferrio, 2005).

Total interpretation and validation of design problem

This stage is for the purpose of getting a proper and accurate interpretation of the problem to be designed aspect that deals with proper and accurate interpretation of the design problem in order to meet the expectations of a real-life situation (Ferri, 2005).

Personal conceptualization of the designed work

This stage involves the assistance of an imaginative representation of the interpreted and validated design solution for the real-life problem. The need for this stage is supported by Xiao et al. (2015), who reinforce the need to carefully identify and understand the design problem so the designer can arrive at an accurately designed device with little or no deficiency, which is clearly important when teaching and learning MM in antenna theory and design.

Analysis of an antenna mathematically

This stage is suggested in order to resolve the given engineering problem in a realistic way. It was also emphasized that the use of analytical equations for antenna design when resolving issues such as radiation pattern, effective dielectric field, and width of a patch, among many others (Balanis, 2016; Gómez-Tornero et al., 2011).

Antenna validation (simulation)

This another stage that assist in checking the accuracy of the antenna device designed using software. Engineering academics and students would make use of equipment such as an RF generator, a spectrum analyzer, a vector network analyzer, and RF power meter analyzer, among others (Balanis, 2016; Honchel & Miller, 2001; Mohammed, 2019; Soliman, 2019).

Antenna prototype model

This is the physically designed antenna device and found to be important when teaching and learning antenna theory and design course.

Antenna validation (measurement)

This stage involve the assessment, reflection and validation of the physically designed antenna using mathematical ideas Honchel and Miller, (2001), maintain that in as much as engineering students must manipulate the acquired mathematical knowledge into their designed products, there must be careful use of mathematical concepts during their component selection, measurement and other process.

CONCLUSION

This study has confirmed that when teaching and learning antenna theory and design, with the inclusion of prerequisite courses that are related with the course, the modelling of some mathematical concepts remain important. In addition to this, the researchers further confirmed that the integration of three models (didactic, realistic, and didactical-realistic mathematical model) into the 12 stages in the PRPMM-TLATD will support and guide the teaching and learning of antenna theory and design. The 12 stages are antenna validation by measurement, antenna validation by simulation, analysis of an antenna mathematically, personal conceptualization of the design work, total interpretation and validation of design problem, problem resolution by mathematization, teaching and learning the design problem, antenna parameters modelling (mathematically), describing an antenna parameters mathematically, extra-mathematical working, and prerequisite courses model. The stages stated were carefully structured under two sections as reported as paper-based design, which reports the theoretical aspect of MM and realistic design, which reports the practical aspect of MM. All these help to explicit and avoid the confusion that was initially reported on antenna as a complex device that cannot be easily explained mathematically. The suggested mathematical model derived PRPMM-TLATD from the empirical data can be modelled and assist other researchers into probing into the teaching of engineering related courses.

Author contributions: All authors have sufficiently contributed to the study and agreed with the results and conclusions.

Funding: No funding source is reported for this study.

Ethical statement: Ethical permission was obtained from all participants and in accordance with the university requirements. The ethical number for this study is HSSREC/00000238/2019.

Declaration of interest: No conflict of interest is declared by authors.

Data sharing statement: Data supporting the findings and conclusions are available upon request from the corresponding author.

REFERENCES

Balanis, C. A. (2016). *Antenna theory: Analysis and design*. Wiley & Sons.

Baxter, P., & Jack, S. (2008). Qualitative case study methodology: Study design and implementation for novice researchers. *The Qualitative Report, 13*(4), 544-559.

Bing, T. J., & Redish, E. F. (2012). Epistemic complexity and the journeyman-expert transition. *Physical Review Special Topics-Physics Education Research, 8*(1), 1-11. https://doi.org/10.1103/PhysRevSTPER.8.010105

Bissell, C., & Dillon, C. (2000). Telling tales: models, stories and meanings. *For the Learning of Mathematics, 20*(3), 3-11.

Blomhøj, M. (2009). Different perspectives in research on the teaching and learning mathematical modelling. In *Proceedings of the Mathematical Applications and Modelling in the Teaching and Learning of Mathematics* (pp. 1-17).

Bloomberg, L. D., & Volpe, M. (2018). *Completing your qualitative dissertation. A road map from beginning to end*. SAGE.

Blum, W., & Leiß, D. (2007). Deal with modelling problems. *Mathematical Modelling: Education, Engineering and Economics-ICTMA, 12*, 222. https://doi.org/10.1533/9780857099419.5.221

Burnard, P., Gill, P., Stewart, K., Treasure, E., & Chadwick, B. (2008). Analyzing and presenting qualitative data. *British Dental Journal, 204*(8), 429-432. https://doi.org/10.1038/sj.bdj.2008.292

CAES. (2020). *Handbook for College of Agriculture, Engineering and Science.* University of KwaZulu-Natal Printers.

Cohen, L., Manion, L., & Morrison, K. (2017). *Research methods in education*. Routledge. https://doi.org/10.4324/9781315456539

Constantine, A. B. (2005). *Antenna theory: Analysis and design*. Wiley-Interscience.

Creswell, J., & Poth, C. (2018). *Qualitative inquiry & research design*. SAGE.

Fasinu, V. G. (2021). *Engineering academics' teaching and students' learning of mathematical modelling approaches in antenna design course* [PhD thesis, University of KwaZulu-Natal].

Fasinu, V. G., Govender, N., & Kumar, P. (2021). Engineering academics' strategies of teaching mathematical modelling in antenna theory and design. *Universal Journal of Educational Research, 9*(11), 1857-1865. https://doi.org/10.13189/ujer.2021.091105

Ferri, R. B. (2006). Theoretical and empirical differentiations of phases in the modelling process. *ZDM, 38*(2), 86-95. https://doi.org/10.1007/BF02655883

Gallegos, R. R. (2009). Differential equations as a tool for mathematical modelling in physics and mathematics courses–A study of high school textbooks and the modelling processes of senior high students. In *Proceedings of the Mathematical Applications and Modelling in the Teaching and Learning of Mathematics* (pp. 19-34).

Gómez-Tornero, J. L., Cañete-Rebenaque, D., Quesada-Pereira, F. D., & Álvarez-Melcón, A. (2011). Interactive lab to learn radio astronomy, microwave & antenna engineering at the Technical University of Cartagena (Spain). *International Journal in Online Engineering, 7*(1), 10-18. https://doi.org/10.3991/ijoe.v7i1.1523

Goold, E. (2012). *The role of mathematics in engineering practice and in the formation of engineers* [Doctoral dissertation, National University of Ireland Maynooth].

Hidayat, R., Idris, W. I. W., Qudratuddarsi, H., & Rahman, M. N. A. (2021). Validation of the mathematical modeling attitude scale for Malaysian mathematics teachers. *EURASIA Journal of Mathematics, Science and Technology Education, 17*(12), em2047. https://doi.org/10.29333/ejmste/11375

Honchell, J. W., & Miller, A. L. (2001). Antenna design, simulation, fabrication and test tailored for engineering technology students. *Age, 6*, 1.

Jensen, T. H. (2007). Assessing mathematical modelling competency. In *Proceedings of the Mathematical Modeling: Education, Engineering and Economics* (pp. 141-148). https://doi.org/10.1533/9780857099419.3.141

Karam, R. (2014). Framing the structural role of mathematics in physics lectures: A case study on electromagnetism. *Physical Review Special Topics-Physics Education Research, 10*(1), 1-23. https://doi.org/10.1103/PhysRevSTPER.10.010119

Maaß, K. (2006). What are modelling competencies? *ZDM, 38*(2), 113-142. https://doi.org/10.1007/BF02655885

Milligan, T. A. (2005). *Modern antenna design*. Wiley & Sons. https://doi.org/10.1002/0471720615

Mohammed, M. I. (2019). Efficacy of antenna engineering research orientation among graduate students at Mekelle University Ethiopian Institute of Technology. *The International Journal of Electrical Engineering & Education, 56*(3), 222-237. https://doi.org/10.1177/0020720918796702

Pino-Fan, L. R., Assis, A., & Castro, W. F. (2015). Towards a methodology for the characterization of teachers' didactic-mathematical knowledge. *EURASIA Journal of Mathematics, Science and Technology Education, 11*(6), 1429-1456. https://doi.org/10.12973/eurasia.2015.1403a

Redish, E. F. (2017). Analyzing the competency of mathematical modelling in physics. In T. Greczyło, & E. Dębowska (Eds.), *Key competences in physics teaching and learning* (pp. 25-40). Springer. https://doi.org/10.1007/978-3-319-44887-9_3

Redish, E. F., & Kuo, E. (2015). Language of physics, language of math: Disciplinary culture and dynamic epistemology. *Science & Education, 24*(5), 561-590. https://doi.org/10.1007/s11191-015-9749-7

Redish, E. F., & Smith, K. A. (2008). Looking beyond content: Skill development for engineers. *Journal of Engineering Education, 97*(3), 295-307. https://doi.org/10.1002/j.2168-9830.2008.tb00980.x

Schantz, H. (2011). The art and science of ultra-wideband antennas. *Progress in Electromagnetics Research C, 18*, 57.

Schantz, H., & Depierre, R. E. (2015). Directive, electrically-small UWB antenna system and method. *US Patent No. 9,209,525, Patent and Trademark Press.*

Soliman, H. Y. (2019). Methodology of teaching and learning antenna theory at Port Said University. In *Proceedings of the 2019 IEEE Global Engineering Education Conference* (pp. 453-457). IEEE. https://doi.org/10.1109/EDUCON.2019.8725087

Tuminaro, J., & Redish, E. F. (2007). Elements of a cognitive model of physics problem solving: Epistemic games. *Physical Review Special Topics-Physics Education Research, 3*(2), 020101. https://doi.org/10.1103/PhysRevSTPER.3.020101

Xiao, X., Xu, H., & Xu, S. (2015). Using IBM SPSS modeler to improve undergraduate mathematical modelling competence. *Computer Applications in Engineering Education, 23*(4), 603-609. https://doi.org/10.1002/cae.21632