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An Unfair Option Game: The Effects of Asymmetric Government Subsidy to Innovators and Imitators

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ABSTRACT

This article develops a model of real option game to analyze effect of asymmetric government subsidy on the investment thresholds of private investors who compete with each other in the same market with uncertain price. The optimal investment thresholds of innovator and imitator are derived, and the effects of some key variables, such as inflation rate, interest rate, and degree of intellectual property protection, on those thresholds were analyzed. With the help of our model and a numerical example of sewage treatment industry, we demonstrate that subsidy to both innovator and imitator reduces optimal investment thresholds, while subsidy only to innovator raises the investment thresholds rather than reducing them. Based on the results of our study, we suggest that the government should offer subsidy to both of innovator and imitator indiscriminately, raise the overall level of IP protection, and reduce the financial cost for private investors in order to motivate them to invest in PPP projects.

Keywords: option game, real option, innovation public-private partnership

INTRODUCTION

Pollution is one of the major public concerns in China nowadays. Among various types of environmental pollution accompanied with rapid industrialization and urbanization of China, water pollution is an especially prominent one (Wang, 2016). In 2015, The Ministry of Water Resources of China released an investigation report showing that more than 80% of the shallow groundwater in the north and east mainland China had been polluted and some areas were contaminated with heavy metals and toxic organic compounds. According to the disclosed data of National Bureau of Statistics of People's Republic of China, the total discharge of sewage in 2014 reached 44.538 billion tons, and about 10 billion tons of sewage was untreated annually in the last 5 years. The recent 13th Five-Year Plan shows that China pledges to improve environmental standards and cut down on pollution. One of the measures is to motivate private funds to invest in sewage treatment industry. Among all the 1906Public-Private Partnerships (PPP)projects which National Development and Reform Commission (NDRC) promoted in the year 2015 and 2016, 249 projects are sewage treatment plants, taking up more than 13% of the total number of promoted projects. However, private investors seem more conservative than expected. The signing rate of these sewage treatment PPP projects has remained at a low level. The official data of NDRC shows that the signing rate of the promoted PPP projects is merely 31.5% in April 2016.

Some policy advisers argue that the government should subsidize the innovations of private investors of PPP projects to encourage the private investors to sign the PPP contracts (Zhu et al., 2017). For sewage treatment industry, there are many unprecedented technological challenges inherent in building and operating a new sewage treatment plant, and entailing costly R&D activities to overcome them. Because of the relatively low level of intellectual property (IP) protection compared with developed countries, innovators in sewage treatment industry in China face the possibility that imitators may copy the innovations at low cost, resulting in unfair competition and financial loss to innovators. By subsidizing the innovators of PPP projects, therefore, the government can boost the PPP signing rate. However, some other advisers believe that higher IP protection as result of innovation subsidies from government has negative effect on the diffusion of knowledge, which increases the investment thresholds of private investors and lowers the overall signing rate of PPP projects.

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Contribution of this paper to the literature

- Although subsidy to both innovator and imitator reduced optimal investment thresholds, subsidy only to the innovator raised the investment thresholds significantly rather than reducing them. Raising the degree of IP protection or reducing the financial cost resulted in lower optimal investment thresholds for both innovator and imitator.
- The Government should offer subsidy to both innovator and imitator indiscriminately, raise the overall level of IP protection, and reduce the financial cost for private investors to motivate them to invest PPP projects.
- The constant number of active firms. It would be worthwhile to apply the model in other market settings empirically and to simulate the effect of different policy interventions.

For private investor considering signing a PPP contracts, the PPP projects is an investment under uncertain conditions. Just as the purchaser of the call option in financial derivative market, the potential private investor of PPP project holds a real option. Since most contracts of PPP projects promoted do not contain clauses of exclusive licenses, the decision of the leader, the private investor entering the market first, has an effect on the followers' investment thresholds, and vice versa.

We construct a real option game between innovators and imitators with asymmetric government subsidies to analyze the dilemma of low signing rate of PPP projects, and use the data in sewage treatment industry to present a numerical example. Smets (1991) first presented the game theory of real option analysis. Based on Smets' work, Grenadier (1996) used option game to analyze the irrational real estate development. Pawlina and Kort (2006) analyzed the effect of cost asymmetry on the optimal real option game, and proposed that mature companies had less threat when a new company entered the market with a relatively high cost. Suttinon et al. (2012) adopted the framework of option games to evaluate trade-offs between flexibility and strategic commitment in infrastructure projects. Martzoukos and Zacharias (2008) demonstrated optimal exercise conditions for the real option to make costly strategic pre-investment R&D decisions in the presence of spillover effects with analytic tractability. You et al. (2014) studied the Poisson jump process based on traditional symmetric duopoly timing option game model, which describe the impact of radical technological innovation. By constructing an option game, Cerqueti et al. (2015) studied an interactive process of innovator and imitator.

Dynamic decision analysis of PPP projects is more complicated because of the involvement of government subsidies, tax deduction, and other factors not considered in classic models. Carbonara et al. (2014) added the length of concession period of PPP contracts into the conventional model. Kurniawan et al. (2015) expanded the financial models of PPP projects and presented the stakeholders' preference on financial indicators of PPP projects. Choi et al. (2010) provided a better understanding of investment decision of private funds in China's PPP market using water PPP projects as examples.

The contribution of this paper is two fold. First, although the interaction between innovators and imitators have been analyzed by many scholars, the effect of government innovation subsidies on the investment decisions of innovators and imitators competing in the same market has not been studied. We adopt real options approach and game theory to analyze the effects of asymmetric government subsidy on the optimal investment thresholds of innovators and imitators, and to explain how key variables, such as inflation rate, intellectual property (IP) protection level, interest rate, numbers of innovators and imitators affect the optimal investment thresholds. Second, we present a numerical example using real data from sewage treatment industry, explain the mechanism causing the dilemma of "low signing rate" plaguing PPP projects promoted by NDRC of China, and proposed some policy suggestions based on our findings.

We consider an industry in which firms can either be innovator or imitator. The firms compete with each other to enter the market. Neither innovator nor imitator receives government subsidy in the first case we presented, while both receive subsidy in the second case. In the third case, only the innovator is subsidized by government. We found that subsidy to both innovator and imitator reduced optimal investment thresholds, while subsidy only to innovator raised the investment thresholds significantly rather than reducing them.

The paper is organized as follows. Section 2 introduces the model set-up. In section 3, we derive the optimal investment thresholds of innovators and imitators and discuss the effects of key variables on the optimal investment thresholds. Section 4 presents a numerical example with realistic parameter specification, followed by policy suggestions. Section 5 concludes.

MODEL

Following the model settings originally proposed by Dixit and Pindyck (1994) and later adapted by Scandizzo and Ventura (2016) to analyze the interaction between innovators and imitators, we consider an industry composed of a given number of firms. Each firm can be either innovator or imitator. Firms in the industry are all risk neutral, and each of them has the capacity to produce the flow of one unit of output (product or service) without variable costs by paying an investment cost. The elasticity of demand is large enough so that firm which has paid the investment cost will produce at its capacity level. The uncertainty of each firm is idiosyncratic or independent across firms. The inverse demand function for each firm is given by

$$P_t = X_t D(Q_t), t > 0 \tag{1}$$

where P_t is the price for the output produced by the firm at time t. Q_t is the output flow of the industry at time t. Since each firm produces one unit of output,Q_t, which is treated as a continuous variable, equals the number of active firms which have paid the investment cost to entered market. $D(Q_t)$, is a non-stochastic decreasing function. $\{X_t\}_{t>0}$ is interpreted by Dixit and Pindyck (1994) as an idiosyncratic demand shock reflecting changes in relative tastes for the firms' products or services. $\{X_t\}_{t>0}$ is unique for each firm. In sewage treatment industry, for example, it can be interpreted as the random factor of sewage treatment price, which is affected by inflation, bargaining power of sewage treatment company, the relationship between the local government and the company, etc. Each firm gets an initial drawX_0 from a known distribution. $\{X_t\}_{t>0}$ follows a geometric Brownian motion that is firm-specific,

$$dX_t = \alpha X_t + \sigma X_t d\xi_t, \tag{2}$$

where α and σ are the drift and the diffusion coefficients of the stochastic process $\{X_t\}_{t>0}$, respectively, and $\{\xi_t\}_{t>0}$ is a standard Brownian motion.

By paying an investment cost, firms are activated and can thereafter produce the flow of one unit output. For an innovator, if$X$exceeds a critical value, or investment threshold, X_N, the innovator pays the investment cost, I, to become an active producer. If $X < X_N$, the innovator remains dormant until $X \geq X_N$. For an imitator, similarly, it pays investment cost, K $(K < I)$, to become active producer if X exceeds the imitator's investment threshold X_M, while it remains dormant and let X evolve until $X \geq X_M$.

We denote by N, M, respectively, the non-random stream of innovators and imitators, which will operate actively by paying investment costs. We introduce a Poisson death process at rate λ to ensure the long-term equilibrium of the industry. The number of firms get activated equals to the number of firm quit, or $N + M = \lambda Q$, so the number of active firms, Q, is constant in the long run. In other words, anon-random total volume of output can be produced by firms whose identities change through time but whose aggregate population distribution remains stationary. The assumption is made possible because uncertainty is firm-specific and independent across firms, and the law of large numbers ensures that industry aggregates are non-random.

Each firm pondering whether to pay the investment cost to operate actively is potential innovator or potential imitator, and can be regarded as holding a real option. The option value is the present value of the expected discounted returns. We present the value functions of potential innovator and imitator of three cases as follows.

Case1: Neither innovator nor imitator receives government subsidy

The value of potential innovator, $V_{N1}(X)$, and value of potential imitator, $V_{M1}(X)$, are given by

$$V_{N1}(X) = \sup_{\tau} E_X \left\{ e^{-\rho\tau} \left[\int_{\tau}^{\infty} e^{-\rho(t-\tau)} \big(X_t - g(X_t)\big) dt - C - I \right] \right\}, \tag{3a}$$

$$V_{M1}(X) = \sup_{\eta} E_X \left[e^{-\rho\eta} \left(\int_{\eta}^{\infty} e^{-\rho(t-\eta)} \frac{\gamma}{R} X_t dt - pC - K \right) \right], \tag{3b}$$

In Equation (3a), τ is the random time of activation of the potential innovator, ρ is the discount rate, $g(X_t)$ stands for the value of the imitation threat, I is the lump sum investment cost of the innovator, and C is the lump sum cost of the innovator's private IP protection against the imitators. The parameter I can be interpreted as the total cost consisting of fixed asset investment and R&D activity. The parameter C is a value to be determined and depends on the degree of public protection of intellectual property. Each potential imitator also has the option to operate actively. In Equation (3b), η is the random time of activation of the potential imitators, γ is the share of innovator's income which imitator plans to appropriate. The parameter γ measures the externality created by innovation and exploited by imitators. The parameter R is the ratio between the number of imitators and the number of innovators, or $R = M/N$. It can be interpreted as the fact that if the imitator finds an innovator to imitate, it shares its gains with the average number of imitators per innovator. The parameter p stands for the degree of IP protection, and measures the effectiveness of the private protection cost, C, paid by innovator to protect itself against imitator. The imitator takes C paid by the innovator as given. K is the investment cost for imitator. The parameter K can be interpreted as the total cost consisting of fixed asset investment, and searching and imitating an existing innovation.

Case2: Both innovator and imitator receive government subsidy

The value of potential innovator, $V_{N2}(X)$, and value of potential imitator, $V_{M2}(X)$, are given by

$$V_{N2}(X) = \sup_{\tau} E_X \left\{ e^{-\rho\tau} \left[\int_{\tau}^{\infty} e^{-\rho(t-\tau)} \big(X_t - g(X_t)\big) dt - C - I + S \right] \right\}, \tag{4a}$$

$$V_{M2}(X) = \sup_{\eta} E_X \left[e^{-\rho\eta} \left(\int_{\eta}^{\infty} e^{-\rho(t-\eta)} \frac{\gamma}{R} X_t dt - pC - K + S \right) \right], \tag{4b}$$

where S stands for government subsidy.

Case3: Only innovator receives government subsidy

The value of potential innovator, $V_{N3}(X)$, and value of potential imitator, $V_{M3}(X)$, are given by

$$V_{N3}(X) = \sup_{\tau} E_X \left\{ e^{-\rho\tau} \left[\int_{\tau}^{\infty} e^{-\rho(t-\tau)} \big(X_t - g(X_t)\big) dt - C - I + S \right] \right\}, \tag{5a}$$

$$V_{M3}(X) = \sup_{\eta} E_X \left[e^{-\rho\eta} \left(\int_{\eta}^{\infty} e^{-\rho(t-\eta)} \frac{\gamma}{R} X_t dt - pC - K \right) \right]. \tag{5b}$$

For all three cases above, $D(Q) = 1$, or $P_t = X_t$, since Q is assumed to be constant. τ and η are stochastic and not necessarily equal to each other.

In each case, innovator makes the investment decision first, and then imitator makes its decision. While making its own investment decision, the innovator takes the imitator's decision into consideration, and therefore we work backward to solve the two maximizing problems in each case.

OPTIMAL INVESTMENT THRESHOLDS

Case 1: Neither Innovator nor Imitator Receives Government Subsidy

Solving the imitator's problem

The solution to the problem in introduced in Section 2 is obtained by applying the dynamic programming technique following Dixit and Pindyck (1994), Pawlina and Kort (2006), and Chen et al. (2017). By solving the relevant Bellman equations, we derived the value of the option in the continuation and stopping regions.

Shortly, dynamic programming requires that

$$E\big(dV(X)\big) = \rho V(X). \tag{6}$$

By applying Ito's lemma, we can write the following Bellman equation:

$$(\sigma^2/2)V''X^2 + \alpha V'X - (\rho + \lambda) = 0, \tag{7}$$

The general solution of equation (7) is

$$V_{M1}(X) = A_1 X^{\beta_1} + A_2 X^{\beta_2}, \tag{8}$$

where β_1 and β_2 are the positive and negative root of the characteristic equation,

$$\frac{1}{2}\sigma^2\beta(\beta - 1) + \alpha\beta - (\rho + \lambda) = 0, \tag{9}$$

respectively. A_1 and A_2 in Equation (8) are two constants to be determined by the boundary conditions. Since the value of the investment option increases as the underlying asset or project increases and A_2 must equal to zero, and

$$V_{M1}(X) = A_1 X^{\beta_1}. \tag{10}$$

In the stopping region, the imitator takes the optimal action to exercise the option, and obtain the expected value of the investment, which is

$$V_{M1}(X) = \frac{\gamma}{R}\frac{X}{\delta} - pC - K, for\ X > X_{M1}. \tag{11}$$

In the continuation region, the imitator waits to exercise the option, and the worth of the value function equals to the discounted expected value of future gains,

$$V_{M1}(X) = E_t(e^{-\rho\eta}) \left(\frac{\gamma}{R}\frac{X}{\delta} - pC - K \right), for\ X \leq X_{M1}. \tag{12}$$

We can write the value-matching and the smooth-pasting as follows, respectively.

$$\begin{cases} A_1 X^{\beta_1} = \frac{\gamma}{R}\frac{X}{\delta} - pC - k \\ A_1 \beta_1 X^{\beta_1 - 1} = \frac{\gamma}{R}\frac{1}{\delta} \end{cases}. \tag{13}$$

By combining the condition and solving for X, we obtain the optimal stopping point

$$X_{M1} = \frac{\delta\beta_1}{\beta_1 - 1}\frac{R}{\gamma}(pC + K), \tag{14}$$

With Equation (11) and (12) the value of the option can be expressed as

$$V_{M1}(X) = \begin{cases} \frac{\gamma}{R}\frac{X}{\delta} - pC - K & if\ \ X > X_{M1} \\ \left(\frac{X}{X_{M1}}\right)^{\beta_1}\left(\frac{\gamma}{R}\frac{X_{M1}}{\delta} - pC - K\right) & if\ \ X \le X_{M1} \end{cases},$$

where the discount factor $e^{-\rho\eta} = (X/X_{M1})^{\beta_1}$, according to Dixit and Pindyck (1994).

Multiplying thresholdX_{M1} by $\mu > 1$, and solving for C, we obtain the protection cost needed to force the potential imitator remain in an inaction region

$$C_\mu = \frac{\mu}{p}\left[\frac{(\beta_1 - 1)}{\beta_1}\frac{\gamma}{R}\frac{X}{\delta}\right] - \frac{K}{p}. \tag{15}$$

Although C_μ reveals the innovator's capacity to keep the imitator at bay, it's not the optimally determined yet, from the innovator's viewpoint. We consider the objective function of the innovator to find the optimal value of C. Denote Π_{N1} as the innovator's discounted value upon activation for $X = X_{N1}$, and then

$$\prod_{N1}(C) = X_{N1}/\delta - C - G(X_{N1}) - I, \tag{16}$$

The total value of the imitation threat

$$G(X) = E_X\left\{e^{-\rho\tau}\left[\left[\int_\tau^\infty e^{-\rho(t-\tau)}\, g(X_t)dt\right]\right]\right\}. \tag{17}$$

The value of the threat$G(X)$ is modeled as given by the expression

$$G(X) = Rf[V_{M1}(X)], \tag{18}$$

where the value of the threat for the innovator is the expected net present value of the damage caused by imitator. From the perspective of innovator,

$$f[V_{M1}(X)] = \left(\frac{X}{X_{M1}}\right)^{\beta_1}\left(\frac{\gamma}{R}\frac{X_{M1}}{\delta}\right). \tag{19}$$

Combining equation (18) and (19), we can obtain

$$G(X) = \left(\frac{X}{X_{M1}}\right)^{\beta_1}\left(\frac{\gamma}{\delta}X_{M1}\right). \tag{20}$$

The innovator's optimal protection at activation therefore can be expressed as

$$\underset{C}{Max}\prod_N(C) = X_{N1}/\delta - C - G(X_{N1}) - I, \tag{21}$$

$$s.t. \qquad C > \frac{1}{p}\left[\frac{(\beta_1 - 1)}{\beta_1}\frac{\gamma}{R}\frac{X}{\delta}\right] - \frac{K}{p},$$

Since $\frac{d\Pi_N}{dC} = 0$ and $\frac{d^2\Pi_N}{(dX)^2} = 0$, we have that

$$\arg\max\prod_N(C) = C^*(X_{N1}) = \begin{cases} \frac{1}{p}\left[(\beta_1 Rp)^{\frac{1}{\beta_1}}\frac{\beta_1 - 1}{\beta_1}\frac{\gamma}{R}\frac{X_{N1}}{\delta}\right] - \frac{K}{p} & \text{if } \beta_1 Rp > 1 \\ 0 & \text{if } \beta_1 Rp \le 1 \end{cases}. \tag{22}$$

Combining the equation (14) and (22), we obtain that

$$X_{M1}^* = (\beta_1 Rp)^{1/\beta_1} X_{N1}, \tag{23}$$

where X_{M1}^*represents the investment threshold under the optimal protection cost.

Solving the innovator's problem

By combining equation (3), (17) and (22), the value of innovator is

$$V_{N1}(X) = \sup_{\tau} E_X \left\{ e^{-\rho\tau} \left[\int_{\tau}^{\infty} e^{-\rho(t-\tau)} X_t \left(1 - \gamma\beta_1(\beta_1 Rp)^{((1/\beta_1)-1)}\right) dt - I + K/p \right] \right\}. \tag{24}$$

Analogously, by applying the dynamic programming principle, the solution of equation is (24) as follows

$$V_{N1}(X) = \begin{cases} \frac{X}{\delta}\left[1 - \gamma\beta_1(\beta_1 Rp)^{\frac{1-\beta_1}{\beta_1}}\right] - \left(I - \frac{K}{p}\right) & \text{if } X > X_{N1} \\ \left(\frac{X}{X_{N1}}\right)^{\beta_1} \frac{X_{N1}}{\delta}\left[1 - \gamma\beta_1(\beta_1 Rp)^{\frac{1-\beta_1}{\beta_1}}\right] - \left(I - \frac{K}{p}\right) & \text{if } X \le X_{N1} \end{cases}, \tag{25}$$

According to the value-matching and the smooth-pasting conditions, we have the optimal investment threshold of innovator

$$X^*_{N1} = \frac{\delta\beta_1}{\beta_1 - 1}\left[\frac{I - K/p}{1 - \gamma\beta_1(\beta_1 Rp)^{((1-\beta_1)/\beta_1)}}\right], \tag{26}$$

And the optimal investment threshold of imitator is

$$X^*_{M1} = (\beta_1 Rp)^{1/\beta_1}\frac{\delta\beta_1}{\beta_1 - 1}\left[\frac{I - K/p}{1 - \gamma\beta_1(\beta_1 Rp)^{((1-\beta_1)/\beta_1)}}\right]. \tag{27}$$

The effects of key variables on X^*_{N1} and X^*_{M1}

Proposition1: *The innovator's optimal investment threshold, X^*_{N1}, and that of the imitator, X^*_{M1}, are functions of price volatility, σ. An increase in σ increases X^*_{N1} and X^*_{M1} if $\log(\beta_1 Rp) < 1$ (see **Appendix I** for the proof).*

There are two price parameters, diffusion rate, σ, and drift rate, α, which can be interpreted as price volatility and inflation rate respectively, in our model. Proposition 1 shows that an increase of price volatility, σ, will result in the increase of investment thresholds of innovator and imitator if $log(\beta_1 Rp) < 1$, encouraging them to invest in the PPP projects, while the effects are uncertain if $log(\beta_1 Rp) \ge 1$. The effect of inflation rate, α, on the investment thresholds depends on the specific values of parameters in our model (see ***Appendix I***).

Proposition 2: *The innovator's optimal investment threshold, X^*_{N1}, and that of the imitator, X^*_{M1}, are functions of the degree of IP protection, p. An increase in p decreases X^*_{N1} and X^*_{M1}, respectively (see Appendix II for the proof).*

Proposition 2 explains the effects of degree of IP protection on the investment thresholds of innovator and imitator. The market infrastructure parameters in our model include the degree of IP protection, p, and discount factor, ρ, which is usually interpreted as prevailing interest rate in financial market. Proposition 2 suggests that raising degree of IP protection will reduce the investment of threshold of both innovator and imitator. The effect of interest rate on investment thresholds is uncertain and depends on the specific values of parameters in our model (see Appendix I).

Proposition 3: *The innovator's optimal investment threshold, X^*_{N1}, and that of the imitator, X^*_{M1}, are functions of the number of innovators, N, and the number of imitators, M. An increase in the former (latter) decreases (increases) X^*_{N1} and X^*_{M1}.*

Proof: Because $\partial X^*_{N1}/\partial \mathrm{R} > 0$ and $\partial X^*_{M1}/\partial \mathrm{R} > 0$ (*see* ***Appendix II*** *for the proof*), and because $\partial R/\partial N < 0$and $\partial R/\partial M > 0$, we have

$$\frac{\partial X^*_{N1}}{\partial N} = \frac{\partial X^*_{N1}}{\partial R}\frac{\partial R}{\partial N} < 0, \text{ and } \frac{\partial X^*_{N1}}{\partial M} = \frac{\partial X^*_{N1}}{\partial R}\frac{\partial R}{\partial M} > 0;$$

$$\frac{\partial X^*_{M1}}{\partial N} = \frac{\partial X^*_{M1}}{\partial R}\frac{\partial R}{\partial N} < 0, \text{ and } \frac{\partial X^*_{M1}}{\partial M} = \frac{\partial X^*_{M1}}{\partial R}\frac{\partial R}{\partial M} > 0.$$

According to proposition 3, a market that consists of large number of innovators and small number of imitators has relatively lower investment thresholds for both innovator and imitator.

Case 2: Both Innovator and Imitator Receive Government Subsidy

Solving the imitator's problem

The option value of imitator in Case 2 is as follows.

$$V_{M2}(X)=\begin{cases}\frac{\gamma}{R}\frac{X}{\delta}-pC-K+S & \text{if } X>X_{M2}\\ \left(\frac{X}{X_{M2}}\right)^{\beta_1}\left(\frac{\gamma}{R}\frac{X_{M2}}{\delta}-pC-K+S\right) & \text{if } X\le X_{M2}\end{cases}. \tag{28}$$

The imitator's investment threshold is

$$X_{M2}=\frac{\delta\beta_1}{\beta_1-1}\frac{R}{\gamma}(pC+K-S). \tag{29}$$

Solving the innovator's problem

The value function of innovator is

$$V_{N2}(X)=\sup_{\tau}E_X\left\{e^{-\rho\tau}\left[\int_{\tau}^{\infty}e^{-\rho(t-\tau)}X_t\left(1-\gamma\beta_1(\beta_1Rp)^{(1/\beta_1)^{-1}}\right)dt-I+(K-S)/p+S\right]\right\}. \tag{30}$$

By applying dynamic programming principle, the solution to Equation (30) is

$$V_{N2}(X)=\begin{cases}\frac{X}{\delta}\left[1-\gamma\beta_1(\beta_1Rp)^{\frac{1-\beta_1}{\beta_1}}\right]-\left(I-\frac{K-S}{p}-S\right) & \text{if } X>X_{N2}\\ \left(\frac{X}{X_{N2}}\right)^{\beta_1}\frac{X_{N2}}{\delta}\left[1-\gamma\beta_1(\beta_1Rp)^{\frac{1-\beta_1}{\beta_1}}\right]-\left(I-\frac{K-S}{p}-S\right) & \text{if } X\le X_{N2}\end{cases}. \tag{31}$$

According to value-matching and smooth-pasting conditions, we obtain the innovator's optimal investment threshold

$$X_{N2}^{*}=\frac{\delta\beta_1}{\beta_1-1}\left[\frac{I-(K-S)/p-S}{1-\gamma\beta_1(\beta_1Rp)^{((1-\beta_1)/\beta_1)}}\right]. \tag{32}$$

The optimal investment threshold of imitator is therefore

$$X_{M2}^{*}=(\beta_1Rp)^{1/\beta_1}\frac{\delta\beta_1}{\beta_1-1}\left[\frac{I-(K-S)/p-S}{1-\gamma\beta_1(\beta_1Rp)^{((1-\beta_1)/\beta_1)}}\right]. \tag{33}$$

The effects of key variables on X_{N2}^{*} and X_{M2}^{*}

The effects of the price parameters, α and σ, the market infrastructure parameters, p and ρ, and the number of innovators and imitators, M and N, on optimal investment thresholds $\boldsymbol{X_{N2}^{*}}$ *and* $\boldsymbol{X_{M2}^{*}}$ in Case 2 are the same as those on $\boldsymbol{X_{N1}^{*}}$ and $\boldsymbol{X_{M1}^{*}}$ in Case 1.

Proposition 4: *An increase of government subsidy, S, decreases the innovator's and imitator's optimal investment thresholds, if both innovator and imitator receive subsidy.*

Proof: Because $1-\gamma\beta_1(R\beta_1p)^{(1-\beta_1)/\beta_1}$ is negative according to Scandizzo and Ventura (2016), and we have β_1-1 is positive and $0<p<1$, X_{N2}^{*} decreases with the increase of S according to Equation (32). The proof of the effect of S on $X_{M2}^{*}$is similar to that of X_{N2}^{*}.

Proposition 4 suggests that government can encourage both innovator and imitator to invest in the PPP projects by increasing the amount of subsidy, if the government offer subsidy indiscriminately (without discriminating innovator or imitator).

Case 3: Only Innovator Receives Government Subsidy

Solving the imitator's problem

The option value of imitator in Case 3 is

$$V_{M3}(X)=\begin{cases}\frac{\gamma}{R}\frac{X}{\delta}-pC-K & \text{if } X>X_{M3}\\ \left(\frac{X}{X_{M3}}\right)^{\beta_1}\left(\frac{\gamma}{R}\frac{X_{M3}}{\delta}-pC-K\right) & \text{if } X\le X_{M3}\end{cases}. \tag{34}$$

The imitator's investment threshold is

$$X_{M3} = \frac{\delta\beta_1}{\beta_1 - 1}\frac{R}{\gamma}(pC + K). \tag{35}$$

Solving the innovator's problem

The value function of innovator is given by

$$V_{N3}(X) = \sup_{\tau} E_X\left\{e^{-\rho\tau}\left[\left[\int_{\tau}^{\infty} e^{-\rho(t-\tau)} X_t\left(1 - \gamma\beta_1(\beta_1 Rp)^{((1/\beta_1)-1)}\right)dt - I + K/p + S\right]\right]\right\}. \tag{36}$$

By applying the dynamic programming principle, we obtain the solution to Equation (36),

$$V_{N3}(X) = \begin{cases} \frac{X}{\delta}\left[1 - \gamma\beta_1(\beta_1 Rp)^{\frac{1-\beta_1}{\beta_1}}\right] - \left(I - \frac{K}{p} - S\right) & \text{if } X > X_{N3} \\ \left(\frac{X}{X_{N3}}\right)^{\beta_1}\frac{X_{N3}}{\delta}\left[1 - \gamma\beta_1(\beta_1 Rp)^{\frac{1-\beta_1}{\beta_1}}\right] - \left(I - \frac{K}{p} - S\right) & \text{if } X \leq X_{N3} \end{cases}. \tag{37}$$

According to value-matching and smooth-pasting conditions, we obtain the innovator's optimal investment threshold,

$$X^*_{N3} = \frac{\delta\beta_1}{\beta_1 - 1}\left[\frac{I - K/p - S}{1 - \gamma\beta_1(\beta_1 Rp)^{((1-\beta_1)/\beta)}}\right], \tag{38}$$

and imitator's optimal investment threshold,

$$X^*_{M3} = (\beta_1 Rp)^{1/\beta_1}\frac{\delta\beta_1}{\beta_1 - 1}\left[\frac{I - K/p - S}{1 - \gamma\beta_1(\beta_1 Rp)^{((1-\beta_1)/\beta)}}\right]. \tag{39}$$

The effects of key variables on X^*_{N3} and X^*_{M3}

The effects of the price parameters, α and σ, the market infrastructure parameters, p and ρ, and the number of innovators and imitators, M and N, on optimal investment thresholds $\boldsymbol{X^*_{N3}}$ and $\boldsymbol{X^*_{M3}}$ in Case 3 are the same as those on $\boldsymbol{X^*_{N1}}$ and $\boldsymbol{X^*_{M1}}$ in Case 1.

Proposition 5: *An increase of government subsidy, S, increases both the innovator's and imitator's optimal investment thresholds, if only the innovator receive subsidy from government.*

Proof: Because $1 - \gamma\beta_1(R\beta_1 p)^{(1-\beta_1)/\beta_1}$ is negative according to Scandizzo and Ventura (2016), and because $\beta_1 - 1$ is positive,X^*_{N3} is increase with the increase of S according to Equation (38). The proof of the effect of S on X^*_{M3} is similar to that of X^*_{N3}.

Proposition 5 suggests that increasing the government subsidy will raise the investment thresholds of both innovator and imitator, if the subsidy is given only to the innovator. The explanation to this counterintuitive result lies in the fact that asymmetric government subsidy reduces the threat of imitator for innovator. Without subsidy, the imitator's optimal investment threshold is much higher, and the imitator is more reluctant to invest, posing less threat to innovator. The innovator has now higher option value and tends to wait for higher price for underlying assets or projects, hence the higher optimal investment threshold. Therefore, instead of encouraging private investors to invest projects, an increase of asymmetric government subsidy will make both innovator and imitator more reluctant to enter the market.

NUMERICAL EXAMPLE

In Section 2 and Section 3, we developed a model which enables us to analyze the optimal investment thresholds of private investors under conditions of symmetric and asymmetric government subsidy to innovator and imitator. We also discussed the effects of price parameters, market infrastructure parameters, and population structure of innovators and imitators on the optimal investment thresholds.

In this section, we use data from China's sewage treatment industry to specify parameters in our model, and use the numerical example to examine the effects of the key variables on the investment thresholds of innovators and imitators.

Parameter Specification

Table 1 shows the parameter specification for our models under the background of China's sewage treatment industry. The price parameters include drift rate, α, and diffusion rate, σ, which are interpreted as the price trend,

Table 1. Parameter specifications for the sewage treatment industry in China

Parameter	Variables	Value	Unit
Drift rate	α	0.02	Per year
Diffusion rate	σ	0.30	Per year
Discount rate	ρ	0.04	Per year
Degree of IP protection	p	0.43	--
M/N	R	2.00	--
Imitator's benefit rate	γ	2.00	--
Poisson death rate	λ	0.06	Per year
Innovator's investment	I	400	10^6 Yuan
Imitator's investment	K	320	10^6 Yuan
Government subsidy	S	100	10^6 Yuan

Table 2. Optimal investment thresholds of innovator and imitator

Threshold	Value (Yuan/ton)
X'_{N1}	1.2898
X'_{M1}	1.5194
X'_{N2}	0.9586
X'_{M2}	1.0998
X'_{N3}	1.5396
X'_{M3}	1.8359

and price variation, respectively. Based on the monthly inflation rate, published by National Bureau of Statistics of P. R. China (NBS), the Consumer Price Index (CPI) in 2016 is calculated as 2.1%, and therefore we set the drift rate as 0.02. Following Suttinon et al. (2012), we use the average standard deviation of annual returns of 22 listed companies in the sewage treatment industry in China as proxy for diffusion rate. The data of annual returns of listed companies from 2010 to 2016 can be obtained from public sources. The arithmetic average of standard deviation is 30.02%, and therefore we set the diffusion rate as 0.3.

The market infrastructure parameters include degree of IP protection, p, and discount rate, ρ. Following the method of parameter specification of Scandizzo and Ventura (2016), we quote the U.S. Chamber International IP Index published in 2016, and set p as 0.43, the score of degree of IP protection of P. R. China. The one year time deposit interest rate in China in 2016 is around 4%, we therefore set $\rho = 0.04$.

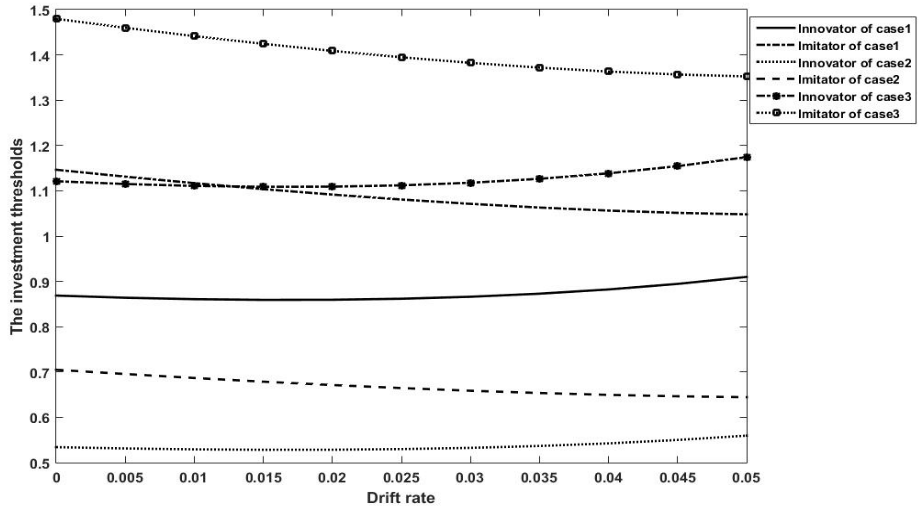
The ratio of the number of imitators to number of innovators, M/N, or R, in sewage treatment industry can be calculated based on the latest statistics disclosed by Intellectual Property Office of P. R. China (2014). The ratio is 1.994, and we therefore set R to 2. The Poisson death rate is set as 0.058, which is the death rate of companies in China in 2016according to State Administration for Industry and Commerce of P. R. China. Since government does not discriminate innovators or imitators when negotiating price with sewage treatment companies in practice, we set$\gamma = R$. The average fixed asset investment of 249 sewage treatment projects promoted by NDRC is around RMB 400 million Yuan. By studying cases of two sewage treatment projects and interviewing 7 experts in sewage treatment industry, we found that the sewage treatment plant of the investment scale of 400 million Yuan usually has the capacity of treating 100 thousand tons of sewage per day, and that the present value of total government subsidies for the sewage treatment plants of the investment scale of 400 million is around 25% of the total fixed asset investment. Furthermore, imitator can usually save 20% investment cost by copying existing innovation according to the experts whom we interviewed. Therefore, we set innovator's fixed asset investment, I, to 400, imitator's fixed asset investment, K, to 320, and government subsidy, S, to 100.

Optimal Investment Thresholds

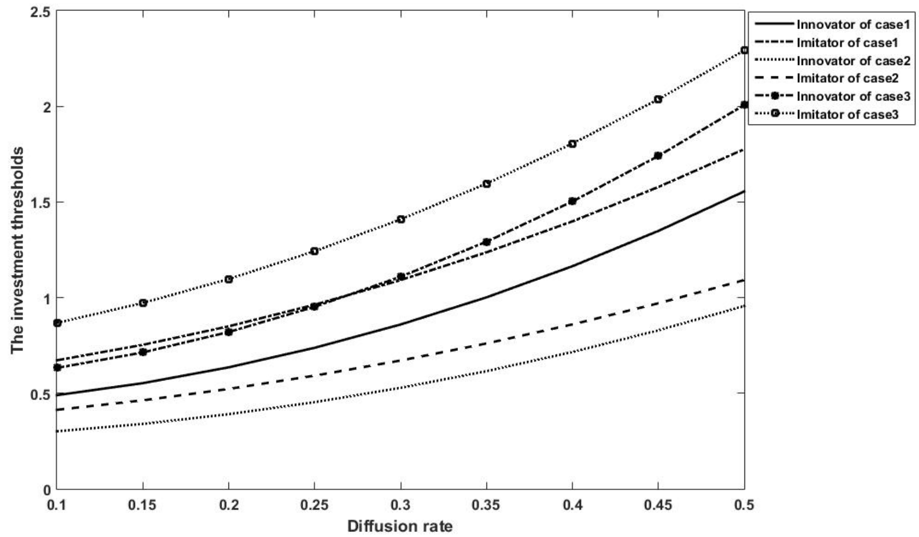
Table 2 demonstrates the optimal investment thresholds per ton of sewage for innovator and imitator in our three cases. The values in **Table 2** are calculated by applying parameter specifications of **Table 1** in our models. Take X'_{N1} for example. First, we obtained the value of innovator's optimal investment threshold (per annum), X^*_{N1}, by plugging in the values in **Table 1**. Second, we divided X^*_{N1} by 36.5 million tons (365 days, 100 thousand tons of sewage water each day explained in Section 4.1). Third, we add 0.43 to the resulting value to get the value of innovator's optimal investment threshold per ton of sewage, $X^{'}_{N1}$, which conforms to the quoting practice in sewage treatment industry, where "price per ton" is usually used. The reason we add 0.43 lies in the fact that we omit the operating costs in our model, but there is averagely RMB 0.43 Yuan per ton of operating costs in practice according to Pei (2008).

Table 2 shows that the optimal investment thresholds of the imitator are higher than those of the innovator in all three cases, and that the thresholds in case 2 are the lowest, while thresholds in case 3 are the highest. The results show that subsidy to both innovator and imitator reduces investment thresholds significantly, suggesting that government should subsidize both innovator and imitator in order to encourage private investors to invest the sewage treatment PPP projects. However, asymmetric subsidy only to the innovator, with the intention to reduce financial burden of the innovator and therefore to boost the innovator's willingness to invest and overall innovation level of the industry at the same time, raises the optimal investment thresholds of both innovator and imitator significantly as a result. The reason lies in the fact that asymmetric government subsidy reduces the threat of imitators for innovators, as explained in section 3.3.

The value in **Table 2** also explains the low willingness for private funds to invest sewage treatment PPP projects. According to the disclosed data of NDRC, the average price for sewage treatment in 2015 is RMB 0.977 Yuan per ton, very close to the thresholds in Case 2 and much lower than the thresholds in Case 1, suggesting that private investors will not invest in sewage treatment projects without considerable government subsidy.



(a)



(b)

Figure 1. The optimal investment thresholds on the price parameters, (a) drift rate, α, and (b) diffusion rate, σ, with other parameter specifications listed in **Table 1**

Price Parameters

Figure 1(a) shows that an increase in inflation has different effects on the optimal investment thresholds of innovator and of imitator in sewage treatment industry. In all three cases, an increase in inflation rate raises the optimal investment thresholds for innovator, while an increase in inflation rate reduces the thresholds for imitator. **Figure 1(b)** demonstrates the effect of price volatility on the optimal investment thresholds. In all three cases, the optimal investment thresholds of both innovator and imitator increase as diffusion rate, σ, increases, consistent with Proposition 1in Section 3. High degree of uncertainty raises the value of real option for both innovator and imitator, pushing up the investment thresholds. In both graphs of **Figure 1**, Case 2 has the lowest optimal investment thresholds while Case 3 has the highest thresholds, consistent with the results in **Table 2**, suggesting the necessity of indiscriminative government subsidy.

Market Infrastructure Parameters

Figure 2 demonstrates the effects of market infrastructure parameters on the optimal investment thresholds of private investors. **Figure 2(a)** shows that an increase in degree of IP protection, p, results in decrease of optimal

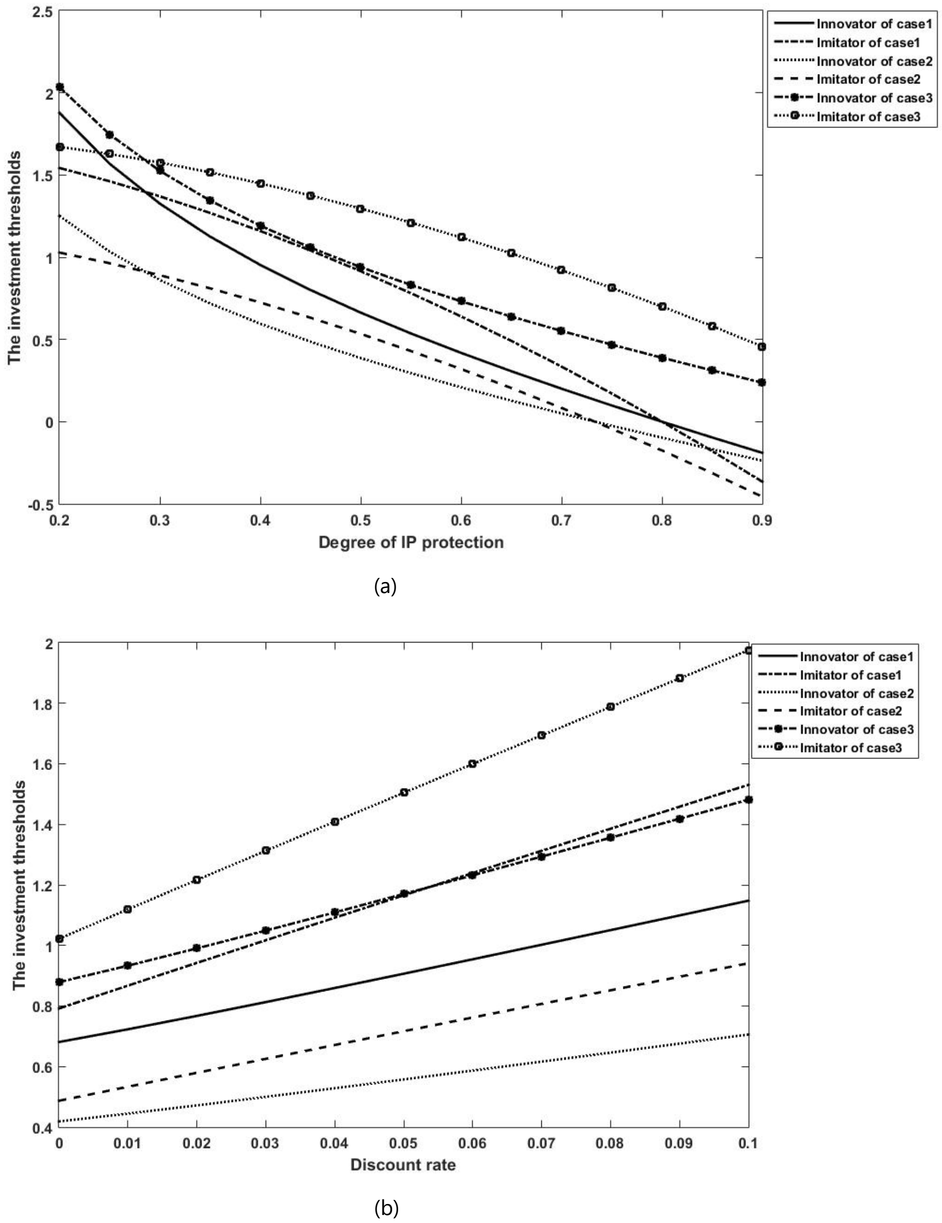


Figure 2. The optimal investment thresholds on the market infrastructure parameters, (a) degree of IP protection, p, (b) discount rate, ρ, with other parameter specifications listed in **Table 1**

investment thresholds for innovator and imitator in all three cases. The finding is consistent with Proposition 2 in Section 3. The higher the degree of IP protection is, the higher the effectiveness of private IP protection is. With lower private IP protection cost, C, it is possible for the innovator to have good IP protection effect with high degree of IP protection, decreasing the total cost for the innovator. Therefore, the high degree of IP protection reduces the optimal investment thresholds of innovators, and those of imitators as well according to Equation (23). Moreover, the threshold curves of the innovator in all three cases are convex to the origin, while those of the imitator are concave, suggesting the existence of marginal diminishing effect for threshold reduction of the innovator. If the degree of IP protection is low, a small increase in the degree of IP protection will result in large amount of reduction of threshold for innovators, while the effect is not so significant if the value of p is high.

Figure 2(b) shows that optimal investment thresholds of both innovator and imitator increase with the increasing discount rate, ρ, in all three cases. With higher discount rate, or higher prevailing interest rate in capital market, the present value of project shrinks. The innovators and imitators need higher price for the service or product to offset the effect of high discount rate, hence the higher optimal investment thresholds.

Subsidy Parameter

Figure 3 illustrates the effect of government subsidy on optimal investment thresholds in Case 2 and Case 3, since there is no subsidy involved in Case 1. The thresholds of innovator and imitator are higher in the case where only innovator receives subsidy than those in the case where both innovator and imitator receive subsidy. Furthermore, the effect of government subsidy is different in the two cases. An increase in subsidy reduces the thresholds if both parties receive subsidy, while the higher the subsidy is, the higher the thresholds are if only the innovator receive subsidy. The results are consistent with Proposition 4 and 5 introduced in section 3.

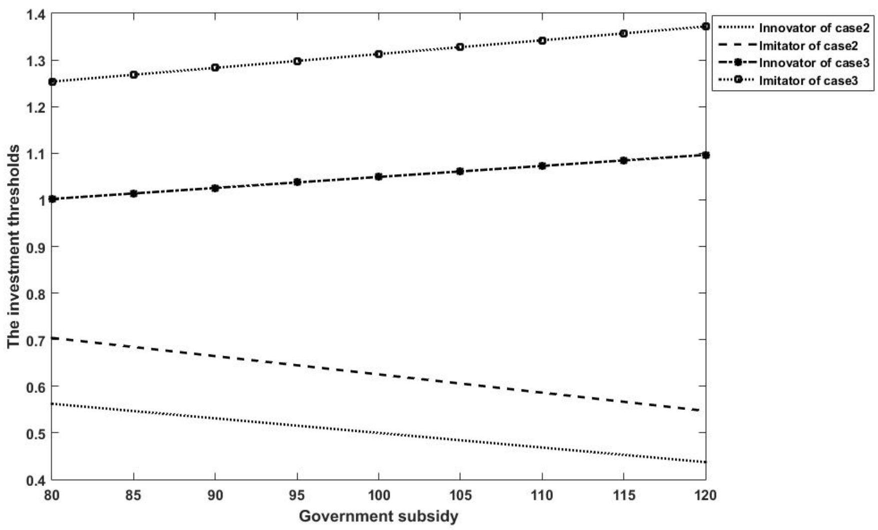
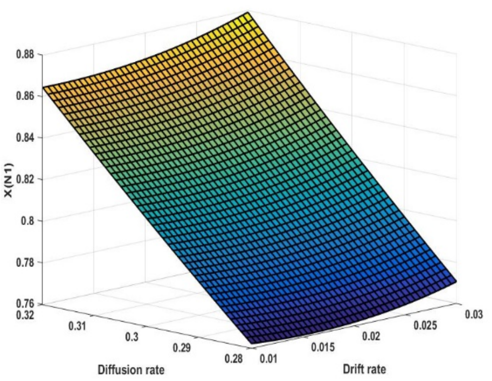


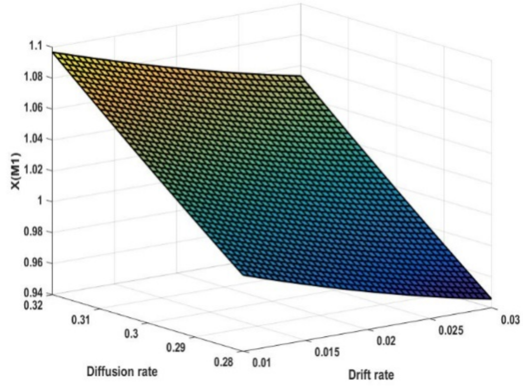
Figure 3. The optimal investment thresholds on the subsidy parameters, S, with other parameter specifications listed in **Table 1**

Comparative Analysis

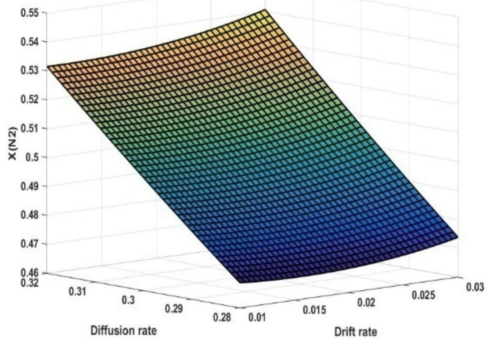
Figure 4 illustrates the optimal investment thresholds of innovator and imitator on the market parameters, inflation, α, and price volatility, σ. From the six graphs in **Figure 4**, it can be concluded that the optimal investment thresholds of both innovator and imitator are more sensitive to the variation of price volatility than that of inflation. **Figure 5** shows the optimal investment thresholds on the market infrastructure parameters, degree of IP protection, p, and discount rate, ρ. Compared with the effect of the discount rate, or interest rate prevailing in capital market, on the investment threshold, the effect of degree of IP protection is much stronger, suggesting that raising the overall level of IP protection, although more difficult, is much more effective than reducing interest rate, in order to motivate private investors to invest in PPP projects.



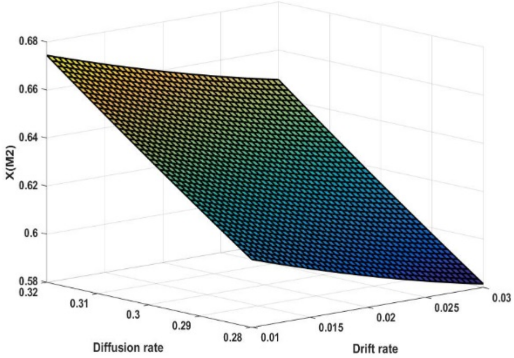
(a)



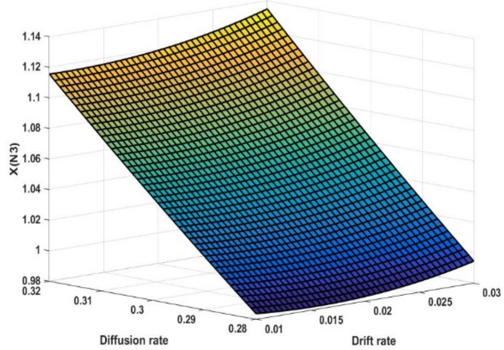
(b)



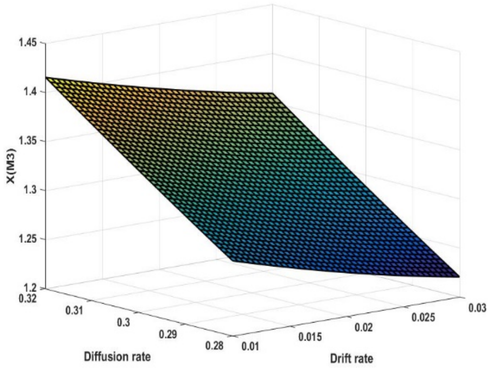
(c)



(d)



(e)



(f)

Figure 4. The optimal investment thresholds on the market parameters, inflation, α, and price volatility, σ, with other parameter specifications listed in **Table 1**

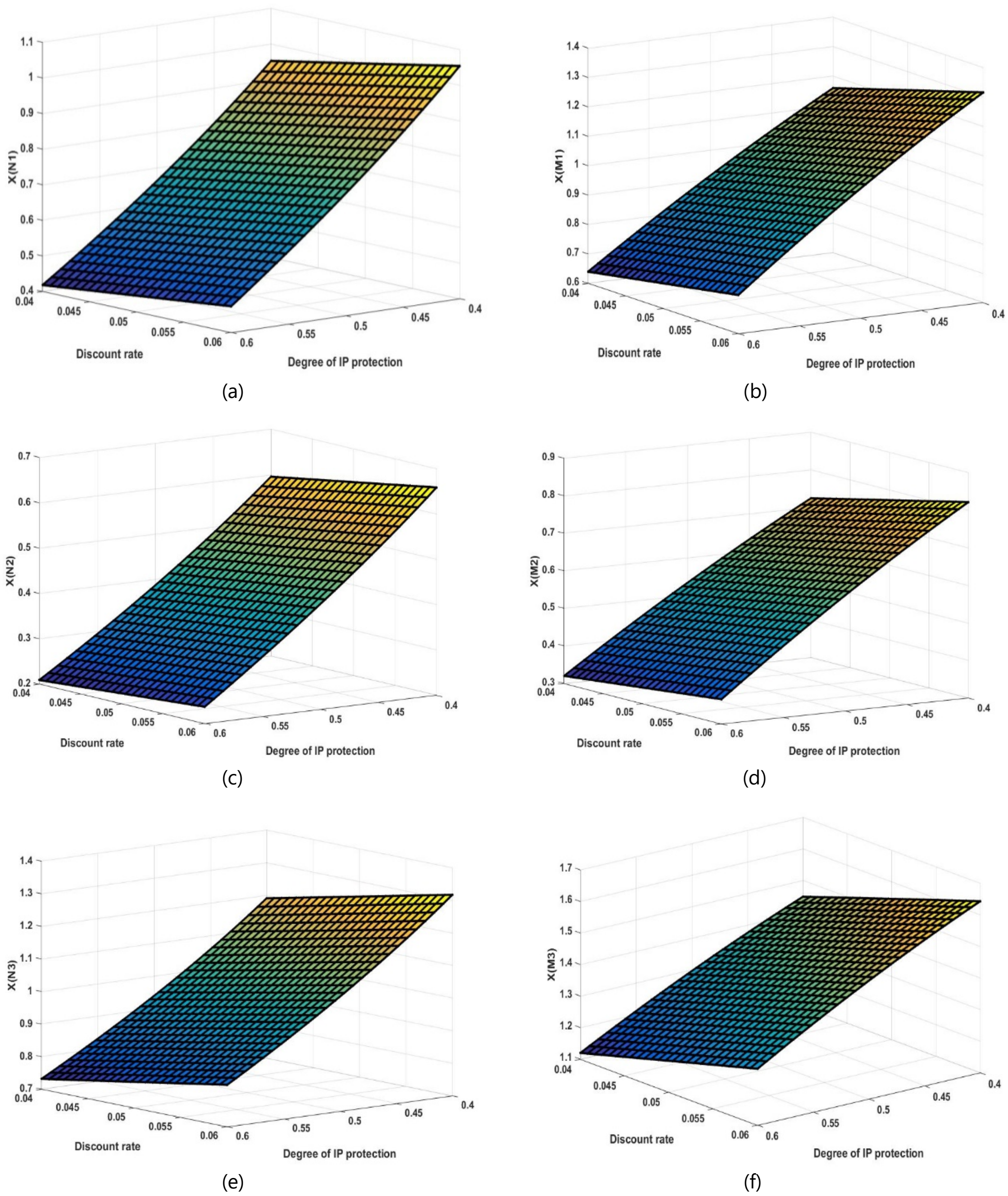


Figure 5. The optimal investment thresholds on the market infrastructure parameters, degree of IP protection, p, and discount rate, ρ, with other parameter specifications listed in **Table 1**

Policy Implications

According to our model and numerical example, it is necessary to offer government subsidy to private investors in order to encourage them to invest sewage treatment PPP projects. As shown in Section 4.2, the average price for sewage treatment without government subsidy is RMB 0.977 Yuan per ton in mainland China, while the optimal investment thresholds are 1.29 Yuan per ton and 1.52 Yuan per ton for the innovator and imitator respectively. Without government subsidy, the private investors are reluctant to enter the market. Even by receiving the subsidy, the optimal investment thresholds are 0.96 Yuan per ton and 1.10 Yuan per ton for the innovator and imitator respectively, very close to the average price of sewage treatment. The result sheds some light on the reasons behind the low signing rate of the promoted sewage treatment PPP projects.

We find that subsidy only to innovator increases the optimal investment thresholds for both innovator and imitator, rather than reducing them. The thresholds of the innovator and imitator are 1.54 Yuan and 1.84 Yuan respectively, much higher than the prices where both parties receive subsidy, and even higher than the original prices where no one receives subsidy. The explanation lies in the fact that asymmetric government subsidy reduces the threat of imitators for innovators. With less threat posed by imitators, the innovators tend to wait for higher price for sewage treatment, resulting in higher optimal investment thresholds for both parties. Therefore, the policy suggestion, that the government should subsidize the innovations of private investors of PPP projects to encourage the private funds to sign the PPP contracts, is not supported by our findings. From results of our study, we suggest that the government should offer subsidy to both of the innovator and imitator indiscriminately in order to encourage the private investors to sign the sewage treatment PPP projects.

The results of our study show that raising the degree of IP protection helps to reduce the optimal investment thresholds of the innovator and imitator dramatically, indicating the importance of IP protection in the effort to encourage private funds to invest PPP projects. Moreover, our results show that reducing discount rate also helps to reduce the optimal investment thresholds, implying that lowering financial cost is another way to boost the signing rate of PPP projects. Therefore, we suggest that the government should raise the overall level of IP protection and issue policies to reduce the financial cost of private investors in order to motivate them to invest in the sewage treatment PPP projects.

CONCLUSIONS

In this paper, we developed a model which enable us to analyze the optimal investment thresholds of innovators and imitators who compete with each other in the same market. We applied our model in three cases to analyze the effect of symmetric and asymmetric government subsidies. In our first case, which served as the benchmark for comparison, neither innovator nor imitator received government subsidy, while both parties received subsidy in the second case and only the innovator received subsidy in the third case. In all three cases, the optimal investment thresholds of the innovator and imitator were derived and presented, and the effects of some key variables, such as inflation rate, price volatility, interest rate, and degree of IP protection, on those thresholds were analyzed. Using numerical example, in which parameters were specified in the background of sewage treatment industry of China, we demonstrated that although subsidy to both innovator and imitator reduced optimal investment thresholds, subsidy only to the innovator raised the investment thresholds significantly rather than reducing them. We also showed that raising the degree of IP protection or reducing the financial cost resulted in lower optimal investment thresholds for both innovator and imitator. Based on our findings, we suggested that the government should offer subsidy to both innovator and imitator indiscriminately, raise the overall level of IP protection, and reduce the financial cost for private investors to motivate them to invest PPP projects. Possible extensions of our study include the endogenization of the investment thresholds, and relaxation of some assumptions, such as the constant number of active firms. It would be worthwhile to apply the model in other market settings empirically and to simulate the effect of different policy interventions.

ACKNOWLEDGEMENT

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Appendix I

The proof of the effect of α, σ and ρ on innovator's investment threshold, X_{N1}^*, is as follows.

In Case 1, we have

$$X_{N1}^* = \frac{\delta\beta_1}{\beta_1 - 1}\left[\frac{I - K/p}{1 - \gamma\beta_1(\beta_1 Rp)^{((1-\beta_1)/\beta_1)}}\right],$$

where $\delta = \rho - \alpha + \lambda$. Because λ is positive, and because nominal interest rate is higher than inflation rate in equilibrium, or $\rho > \alpha$, according to Fisher's Effect, we have $\delta > 0$.

β_1 is the positive root of the characteristic equation (9), and

$$\beta_1 = \left((\sigma^2/2) - \alpha\right) + \sqrt{\left(\alpha - (\sigma^2/2)\right)^2 + 2\sigma^2(\rho + \lambda)}/\sigma^2.$$

According to Dixit and Pindyck (2015), $\partial\beta_1/\partial\alpha < 0$, $\partial\beta_1/\partial\sigma < 0$, $\partial\beta_1/\partial\rho > 0$.

We take the derivative of X_{N1}^* with respect to α. According to Scandizzo and Ventura (2015), in order to have practical meaning, we need that $\gamma\beta_1(R\beta_1 p)^{(1-\beta_1)/\beta_1} > 1$ and $K/p > I$. Therefore,

$$\frac{\partial X_{N1}^*}{\partial\alpha} = \underbrace{\left(I - \frac{K}{p}\right)\frac{-\beta_1}{\beta_1 - 1}\left[\frac{1}{1 - \gamma\beta_1(R\beta_1 p)^{((1-\beta_1)/\beta_1)}}\right]}_{(-)} +$$

$$\left\{\underbrace{\delta\left(I - \frac{K}{p}\right)\frac{\partial\beta_1}{\partial\alpha}}_{(+)}\left\{\underbrace{\left[\frac{-1/(\beta_1 - 1)^2}{1 - \gamma\beta_1(R\beta_1 p)^{(1-\beta_1)/\beta_1}}\right]}_{(+)} + \underbrace{\left(\frac{1}{\beta_1 - 1}\right)\left[\frac{\gamma(R\beta_1 p)^{(1-\beta_1)/\beta_1}}{[1 - (\gamma\beta_1(R\beta_1 p)^{(1-\beta_1)/\beta_1})]^2}\right]}_{(+)}[1 - log(R\beta_1 p)]\right\}\right\}.$$

Hence, if

$$1 - \log(R\beta_1 p) < \frac{[1 - \gamma\beta_1(R\beta_1 p)^{((1-\beta_1)/\beta_1)}]\beta_1}{\delta\gamma(R\beta_1 p)^{((1-\beta_1)/\beta_1)}}\frac{1}{\partial\beta_1/\partial\alpha} + \frac{1 - \gamma\beta_1(R\beta_1 p)^{((1-\beta_1)/\beta_1)}}{(\beta_1 - 1)\gamma(R\beta_1 p)^{((1-\beta_1)/\beta_1)}},$$

we have $\boldsymbol{\partial X_{N1}^*/\partial\alpha > 0}$; otherwise, $\boldsymbol{\partial X_{N1}^*/\partial\alpha < 0}$.

We take the derivative of X_{N1}^* with respect to σ, and

$$\partial X_{N1}^*/\partial\sigma = (\partial X_{N1}^*/\partial\beta_1)(\partial\beta_1/\partial\sigma).$$

The derivative of X_{N1}^* with respect to β_1 is

$$\frac{\partial X_{N1}^*}{\partial\beta_1} = \delta\underbrace{\left(I - \frac{K}{p}\right)}_{(-)}\left\{\underbrace{\left[\frac{-1/(\beta_1 - 1)^2}{1 - \gamma\beta_1(R\beta_1 p)^{(1-\beta_1)/\beta_1}}\right]}_{(+)} + \underbrace{\left(\frac{1}{\beta_1 - 1}\right)\left[\frac{\gamma(R\beta_1 p)^{(1-\beta_1)/\beta_1}}{[1 - (\gamma\beta_1(R\beta_1 p)^{(1-\beta_1)/\beta_1})]^2}\right]}_{(+)}[1 - \log(R\beta_1 p)]\right\}.$$

If $\log(\beta_1 Rp) < 1$, $\partial X_{N1}^*/\partial\beta_1$ is negative, and $\boldsymbol{\partial X_{N1}^*/\partial\sigma > 0}$**,** because $\partial\beta_1/\partial\sigma < 0$. If $log(\beta_1 Rp) \geq 1$, $\partial X_{N1}^*/\partial\beta_1$ can be either positive or negative, depending on the specific values of parameters.

We take the derivative of $X_{N1}^*$with respect to ρ, and

$$\frac{\partial X_{N1}^*}{\partial\rho} = \underbrace{\left(I - \frac{K}{p}\right)\frac{\beta_1}{\beta_1 - 1}\left[\frac{1}{1 - \gamma\beta_1\left(R\beta_1 p\right)^{((1-\beta_1)/\beta_1)}}\right]}_{(+)} +$$

$$\left\{\underbrace{\delta\left(I - \frac{K}{p}\right)\frac{\partial\beta_1}{\partial\rho}}_{(-)}\left\{\underbrace{\left[\frac{-1/\left(\beta_1 - 1\right)^2}{1 - \gamma\beta_1\left(R\beta_1 p\right)^{(1-\beta_1)/\beta_1}}\right]}_{(+)} + \underbrace{\left(\frac{1}{\beta_1 - 1}\right)\left[\frac{\gamma\left(R\beta_1 p\right)^{(1-\beta_1)/\beta_1}}{\left[1 - \left(\gamma\beta_1\left(R\beta_1 p\right)^{(1-\beta_1)/\beta_1}\right)\right]^2}\right]}_{(+)}\left[1 - \log\left(R\beta_1 p\right)\right]\right\}\right\}.$$

$$\frac{\partial X_{N1}^*}{\partial\rho} = \underbrace{\left(I - \frac{K}{p}\right)\frac{\beta_1}{\beta_1 - 1}\left[\frac{1}{1 - \gamma\beta_1(R\beta_1 p)^{((1-\beta_1)/\beta_1)}}\right]}_{(+)} +$$

$$\left\{\underbrace{\delta\left(I - \frac{K}{p}\right)\frac{\partial\beta_1}{\partial\rho}}_{(-)}\left\{\underbrace{\left[\frac{-1/(\beta_1 - 1)^2}{1 - \gamma\beta_1(R\beta_1 p)^{(1-\beta_1)/\beta_1}}\right]}_{(+)} + \underbrace{\left(\frac{1}{\beta_1 - 1}\right)\left[\frac{\gamma(R\beta_1 p)^{(1-\beta_1)/\beta_1}}{[1 - (\gamma\beta_1(R\beta_1 p)^{(1-\beta_1)/\beta_1})]^2}\right]}_{(+)}[1 - log(R\beta_1 p)]\right\}\right\}.$$

Hence, if

$$1-\log(R\beta_1 p)<\frac{1-\gamma\beta_1(R\beta_1 p)^{((1-\beta_1)/\beta_1)}}{(\beta_1-1)\gamma(R\beta_1 p)^{((1-\beta_1)/\beta_1)}}-\frac{\left[1-\gamma\beta_1(R\beta_1 p)^{((1-\beta_1)/\beta_1)}\right]\beta_1}{\delta\gamma(R\beta_1 p)^{((1-\beta_1)/\beta_1)}}\frac{1}{\partial\beta_1/\partial\rho},$$

we have $\boldsymbol{\partial X^*_{N1}/\partial\rho>0}$; otherwise, $\boldsymbol{\partial X^*_{N1}/\partial\rho<0}$.

The proof of the effect of α, σ and ρ on imitator's investment threshold, X^*_{M1}, is as follows.

We have

$$X^*_{M1}=(\beta_1 Rp)^{1/\beta_1}\frac{\delta\beta_1}{\beta_1-1}\left[\frac{I-K/p}{1-\gamma\beta_1(\beta_1 Rp)^{((1-\beta_1)/\beta_1)}}\right].$$

We take the derivative of X^*_{M1} with respect to α, and

$$\frac{\partial X^*_{M1}}{\partial\alpha}=\underbrace{\left(I-\frac{K}{p}\right)\frac{-\beta_1}{\beta_1-1}\left[\frac{(R\beta_1 p)^{1/\beta_1}}{1-\gamma\beta_1(R\beta_1 p)^{((1-\beta_1)/\beta_1)}}\right]}_{(-)}+\underbrace{\delta\left(I-\frac{K}{p}\right)\frac{\partial\beta_1}{\partial\alpha}\left[\frac{[-1/(\beta_1-1)^2](R\beta_1 p)^{1/\beta_1}}{1-\gamma\beta_1(R\beta_1 p)^{(1-\beta_1)/\beta_1}}\right]}_{(+)}+$$
$$\underbrace{\delta\left(I-\frac{K}{p}\right)\frac{\beta_1}{\beta_1-1}\frac{\partial\beta_1}{\partial\alpha}\left[\frac{(1/\beta_1)^2(R\beta_1 p)^{1/\beta_1}}{[1-(\gamma\beta_1(R\beta_1 p)^{(1-\beta_1)/\beta_1})]^2}\right]}_{(+)}[1-\log(R\beta_1 p)]$$

Hence, if

$$1-\log(R\beta_1 p)<\frac{\left[1-\gamma\beta_1(R\beta_1 p)^{((1-\beta_1)/\beta_1)}\right](R\beta_1 p)^{(1/\beta_1)}}{\delta[(1/\beta_1)^2(R\beta_1 p)^{1/\beta_1}]}\frac{1}{\partial\beta_1/\partial\alpha}+\frac{\left[1-\gamma\beta_1(R\beta_1 p)^{((1-\beta_1)/\beta_1)}\right](R\beta_1 p)^{(1/\beta_1)}}{\beta_1(\beta_1-1)[(1/\beta_1)^2(R\beta_1 p)^{1/\beta_1}]},$$

we have $\boldsymbol{\partial X^*_{M1}/\partial\alpha>0}$; otherwise, $\boldsymbol{\partial X^*_{M1}/\partial\alpha<0}$.

We take the derivative of X^*_{M1} with respect to σ, and we have

$$\partial X^*_{M1}/\partial\sigma=(\partial X^*_{M1}/\partial\beta_1)(\partial\beta_1/\partial\sigma).$$

The derivative of X^*_{M1} with respect to β_1 is

$$\frac{\partial X^*_{M1}}{\partial\beta_1}=\delta\underbrace{\left(I-\frac{K}{p}\right)\left[\frac{[-1/(\beta_1-1)^2](R\beta_1 p)^{1/\beta_1}}{1-\gamma\beta_1(R\beta_1 p)^{(1-\beta_1)/\beta_1}}\right]}_{(-)}+\delta\frac{\beta_1}{\beta_1-1}\underbrace{\left(I-\frac{K}{p}\right)\left[\frac{(1/\beta_1)^2(R\beta_1 p)^{1/\beta_1}}{[1-(\gamma\beta_1(R\beta_1 p)^{(1-\beta_1)/\beta_1})]^2}\right]}_{(-)}[1-\log(R\beta_1 p)]$$

If $\log(\beta_1 Rp)<1$, $\partial X^*_{M1}/\partial\beta_1$ is negative, and $\boldsymbol{\partial X^*_{M1}/\partial\sigma}>0$. If $\log(\beta_1 Rp)\geq 1$, $\partial X^*_{M1}/\partial\beta_1$ can be either positive or negative, depending on the specific values of parameters.

We take the derivative of X^*_{M1} with respect to, ρ, and we have

$$\frac{\partial X^*_{M1}}{\partial\rho}=\underbrace{\left(I-\frac{K}{p}\right)\frac{\beta_1}{\beta_1-1}\left[\frac{(R\beta_1 p)^{1/\beta_1}}{1-\gamma\beta_1(R\beta_1 p)^{(1-\beta_1/\beta_1)}}\right]}_{(+)}+\underbrace{\delta\left(I-\frac{K}{p}\right)\frac{\partial\beta_1}{\partial\rho}\left[\frac{[-1/(\beta_1-1)^2](R\beta_1 p)^{1/\beta_1}}{1-\gamma\beta_1(R\beta_1 p)^{(1-\beta_1)/\beta_1}}\right]}_{(-)}+$$
$$\underbrace{\delta\left(I-\frac{K}{p}\right)\frac{\beta_1}{\beta_1-1}\frac{\partial\beta_1}{\partial\rho}\left[\frac{(1/\beta_1)^2(R\beta_1 p)^{1/\beta_1}}{\left[1-(\gamma\beta_1(R\beta_1 p)^{(1-\beta_1)/\beta_1})\right]^2}\right]}_{(-)}[1-\log(R\beta_1 p)].$$

Hence, if

$$1-\log(R\beta_1 p)<-\frac{\left[1-\gamma\beta_1(R\beta_1 p)^{((1-\beta_1)/\beta_1)}\right](R\beta_1 p)^{(1/\beta_1)}}{\delta[(1/\beta_1)^2(R\beta_1 p)^{1/\beta_1}]}\frac{1}{\partial\beta_1/\partial\rho}+\frac{\left[1-\gamma\beta_1(R\beta_1 p)^{((1-\beta_1)/\beta_1)}\right](R\beta_1 p)^{(1/\beta_1)}}{\beta_1(\beta_1-1)[(1/\beta_1)^2(R\beta_1 p)^{1/\beta_1}]},$$

we have $\boldsymbol{\partial X^*_{M1}/\partial\rho>0}$; otherwise, $\boldsymbol{\partial X^*_{M1}/\partial\rho<0}$.

Appendix II

The proof of the effect of R on innovator's investment threshold, X^*_{N1}, is as follows.

We have

$$X^*_{N1} = \frac{\delta\beta_1}{\beta_1 - 1}\left[\frac{I - K/p}{1 - \gamma\beta_1(\beta_1 Rp)^{((1-\beta_1)/\beta_1)}}\right].$$

We take the derivative of X^*_{N1} with respect to R, and

$$\frac{\partial X^*_{N1}}{\partial R} = \left(\frac{\delta\beta_1}{\beta_1 - 1}\right)\underbrace{\left(I - \frac{K}{p}\right)}_{(-)}\left[\frac{\overbrace{\gamma\beta_1 p(1-\beta_1)(R\beta_1 p)^{(1-2\beta_1)/\beta_1}}^{(-)}}{[1 - \gamma\beta_1(R\beta_1 p)^{(1-\beta_1)/\beta_1}]^2}\right].$$

Therefore, $\boldsymbol{\partial X^*_{N1}/\partial R > 0}$.

The proof of the effect of R on imitator's investment threshold, X^*_{M1}, is as follows.

We have

$$X^*_{M1} = (\beta_1 Rp)^{1/\beta_1}\frac{\delta\beta_1}{\beta_1 - 1}\left[\frac{I - K/p}{1 - \gamma\beta_1(\beta_1 Rp)^{((1-\beta_1)/\beta_1)}}\right].$$

We take the derivative of X^*_{M1} with respect to R, and

$$\frac{\partial X^*_{M1}}{\partial R} = \left(\frac{\delta\beta_1}{\beta_1 - 1}\right)\underbrace{\left(I - \frac{K}{p}\right)}_{(-)}\left\{\frac{\overbrace{p(R\beta_1 p)^{(1-\beta_1)/\beta_1}[1 - \gamma\beta_1(R\beta_1 p)^{(1-\beta_1)/\beta_1}]}^{(-)} + \overbrace{\gamma\beta_1 p(1-\beta_1)(R\beta_1 p)^{2(1-\beta_1)/\beta_1}}^{(-)}}{[1 - \gamma\beta_1(R\beta_1 p)^{(1-\beta_1)/\beta_1}]^2}\right\}.$$

We have therefore $\boldsymbol{\partial X^*_{M1}/\partial R > 0}$.

The proof of the effect of p on innovator's investment threshold, X^*_{N1}, is as follows.

We have

$$X^*_{N1} = \frac{\delta\beta_1}{\beta_1 - 1}\left[\frac{I - K/p}{1 - \gamma\beta_1(\beta_1 Rp)^{((1-\beta_1)/\beta_1)}}\right].$$

We take the derivative of $X^*_{N1}$with respect to$p$, and

$$\frac{\partial X^*_{N1}}{\partial p} = \left(\frac{\delta\beta_1}{\beta_1 - 1}\right)\left[\frac{\overbrace{(K/p^2)[1 - \gamma\beta_1(R\beta_1 p)^{(1-\beta_1)/\beta_1}]}^{(-)} + \overbrace{\gamma\beta_1 R(I - K/p)(1-\beta_1)(R\beta_1 p)^{(1-2\beta_1)/\beta_1}}^{(+)}}{[1 - \gamma\beta_1(R\beta_1 p)^{(1-\beta_1)/\beta_1}]^2}\right].$$

Therefore, if

$$p > \frac{K^2\left[1 - \gamma\beta_1(R\beta_1 p)^{(1-\beta_1)/\beta_1}\right]}{\gamma\beta_1 R(\beta_1 - 1)(R\beta_1 p)^{(1-2\beta_1)/\beta_1} I} + \frac{K}{I},$$

we have $\partial X^*_{N1}/\partial p < 0$. It can be proven that the right hand side of the inequality, $K^2\left[1 - \gamma\beta_1(R\beta_1 p)^{(1-\beta_1)/\beta_1}\right]/\gamma\beta_1 R(\beta_1 - 1)\,(R\beta_1 p)^{(1-2\beta_1)/\beta_1} + K/I$, is negative. Since $p > 0$, we have $\boldsymbol{\partial X^*_{N1}/\partial p < 0}$.

The proof of the effect of p on imitator's investment threshold, X^*_{M1}, is as follows.

We have

$$X^*_{M1} = (\beta_1 Rp)^{1/\beta_1}\frac{\delta\beta_1}{\beta_1 - 1}\left[\frac{I - K/p}{1 - \gamma\beta_1(\beta_1 Rp)^{((1-\beta_1)/\beta_1)}}\right].$$

We take the derivative of $X^*_{M1}$with respect to, p, and we have

$$\frac{\partial X^*_{M1}}{\partial p} = \left(\frac{\delta\beta_1}{\beta_1 - 1}\right)\left\{\frac{\left[\left((R\beta_1 p)^{(1-\beta_1)/\beta_1} R(I - K/p) + (K/p^2)(R\beta_1 p)^{1/\beta_1}\right)\right]\left[1 - \gamma\beta_1(R\beta_1 p)^{(1-\beta_1)/\beta_1}\right] + \overbrace{(I - K/p)\gamma\beta_1 R(1-\beta_1)(R\beta_1 p)^{(1-2\beta_1)/\beta_1}}^{(-)}}{[1 - \gamma\beta_1(R\beta_1 p)^{(1-\beta_1)/\beta_1}]^2}\right\}$$

Because $0 < p < 1$, we have $p^2 I - pK + Kp\beta_1 > 0$. Using this result, it can be proven that

$$(R\beta_1 p)^{(1-\beta_1)/\beta_1} R(I - K/p) + (K/p^2)(R\beta_1 p)^{1/\beta_1} > 0,$$

We therefore have $\boldsymbol{\partial X^*_{M1}/\partial p < 0}$.