# Analyzing Pre-Service Primary Teachers' Fraction Knowledge Structures through Problem Posing

Cigdem Kilic
*Mersin University, TURKEY*

In this study it was aimed to determine pre-service primary teachers' knowledge structures of fraction through problem posing activities. A total of 90 pre-service primary teachers participated in this study. A problem posing test consisting of two questions was used and the participants were asked to generate as many as problems based on the following conditions: i) using both $\frac{1}{2}$ and $\frac{3}{4}$ fractions, and ii) using either $\frac{1}{2}$ or $\frac{3}{4}$ fractions. Data were analyzed using both semantic and constant comperative analysis techniques. The results of the study showed that there was substantial diversity in the problems posed by pre-service primary teachers. Moreover, participants preferred to pose story and symbolic equations in first task and story equation in second task. Furthermore, the participants faced some issues such as not realizing $\frac{1}{2} + \frac{3}{4}$ situation is more than 1, missing data, choosing wrong number, using different fractions and posing non-fraction problem.

*Keywords*: fraction, knowledge structure, primary pre-service teacher, problem posing, teacher education

## INTRODUCTION

Effective teaching is one of the most important aims of teacher education. Student teachers should be well educated in terms of all topics that they will teach when they become teachers. Tichá & Hošpesová (2012) indicated that pre-service teacher education students enter university with naive ideas about the nature of mathematics and mathematics education. Thus, there is room for adding new information to pre-service teachers' knowledge structures that they gain in high school. If they are educated well, then they can teach and arrange mathematics in accordance with school mathematics curriculum activities. It is obvious that developing pre-service teachers' mathematical knowledge structures during their education tenure is very important for their professional development, and it is possible to achieve this goal through courses in teacher education programs. In Turkey, pre-service primary teachers learn how to teach mathematics in primary

Correspondence: Cigdem Kilic,
Mathematics Education Department, Education Faculty, Mersin University, Mersin, Turkey.
E-mail: ckilic6@gmail.com

teacher education programs through method courses such as Teaching Mathematics I and Teaching Mathematics II. One of the topics they will subsequently teach in primary schools is problem posing including many mathematical concept such as fraction.

## What is problem posing and its benefits?

Problem posing is the creation of new problems or the reformulation of a given problem (Tichá & Hošpesová, 2009), which is a similar definition that was presented by Stoyanova and Ellerton (1996), who defined it as "as the process by which, on the basis of mathematical experience, students construct personal interpretations of concrete situations and formulate them as meaningful mathematical problems."

Problem posing involves several skills, such as formulating problems from every day and mathematical situations, using a proper approach for posing problems for the mathematical situations, and recognizing relationships among different topics in mathematics (Abu-Elwan, 1999). Problem posing can be used to benefit both students and teachers, and helps the students to expand their understanding of mathematics and explore problems *and* solutions together, rather than focusing only on finding solutions (Stoyanova, 2003). It is also a tool that can be used to develop and strengthen the students' critical thinking skills (Nixon-Ponder, 1995), which can be used as an indicator of a deeper understanding of a concept (Rizvi, 2004). Problem posing also has many benefits for pre-service primary school teachers, such as helping to improve their pedagogical content knowledge of mathematics education (Tichá & Hošpesová, 2009) and influencing their views about what it means to understand mathematics (Toluk-Uçar, 2009), which might help to improve their mathematical knowledge (Kılıç, 2013). Contreras (2007) asserted that pre-service teachers who engage actively and reflectively in problem posing processes generate non-trivial, productive, and well-posed mathematical problems. Lavy and Bershadsky (2003) encouraged pre-service teachers to implement problem posing activities in their future classes; however, they must pose problems correctly in order to promote a classroom situation in which creative problem solving is the central focus (Abu-Elwan, 1999).

## Classification of problem posing framework

Based on the literature, there are different classification frameworks related to problem posing situations (Stoyanova & Ellerton, 1996; Silver & Cai, 1996; Christou et al., 2005). Stoyanova and Ellerton (1996) offered a framework that consists of free, semi-structured, and structured problem posing situations. This framework is explained in more detail below:

- In free problem posing, students are asked to pose a problem on the basis of a natural situation, such as "make up a difficult problem" or "a money problem".
- In semi-structured problem posing situations, students are given an open situation and are invited to explore the structure or to finish it. Posing

***State of the literature***

- Problem posing is very important mathematical activity.
- Problem posing has many contributions on students, pre-service teachers and teachers.
- Thanks to problem posing knowledge structure of pre-service teachers related to mathematical topics can be measured.

***Contribution of this paper to the literature***

- At the beginning of the primary school students are expected to pose problems in many topics, fraction is one of these.
- Considering the behavior of teachers affect students' learning it is important to educate them through teacher trainee program.
- Analyzing pre-service teachers' knowledge of fractions by means of problem posing will help teacher educators to develop their knowledge structure of fraction. Doing that will contribute to pre-service teachers are well equipped regarding problem posing.

problems based on pictures or equations are examples of semi-structured problem posing situations.

- Structured problem posing situations occur when a well-structured problem or problem situation is given and the task is to construct new problems (Stoyanova, 2003).

## Problem posing and fraction in Turkish Mathematics Curriculum

When a pre-service primary school teacher becomes a teacher in Turkey, they will have to follow a standard, national mathematics curriculum, which, since 2006, has contained problem posing applications, especially with regard to the curriculum for first through eighth grades (MEB, 2009). In the curriculum, some of the learning objectives are related to posing fraction problems. The Turkish mathematics curriculum and mathematics documents emphasize that pupils should understand fraction concepts and the meaning of fraction operations (NCTM, 2000; MEB, 2009). In the first three grades, students learn the basic components of fractions, such as comparing and ordering, while in the fourth and fifth grades, they begin to learn operations with fractions, such as addition, subtraction, and multiplication, and start to experience posing problems (MEB, 2009).

## The meaning of fractions and operations with fractions

There are five meanings of fractions, namely *operator* (Cathcart et al., 2003; Charalambous & Pitta-Pantazi, 2007), *part-whole*, *ratio*, *quotient or division* (Holmes, 1995; Reys et al., 1998; Cathcart et al., 2003; Charalambous & Pitta-Pantazi, 2007), *measure* (Charalambous & Pitta-Pantazi, 2007) and four operations, namely addition, subtraction, multiplication, and division (Holmes, 1995; Reys et al., 1998; Cathcart et al., 2003). The ratio meaning of a fraction is based on the idea that a fraction can represent a ratio between two quantities, whereas the quotient meaning expresses a division, and the operator includes an operation (Cathcart et al., 2003) measure meaning for example, $\frac{3}{4}$ corresponds to the distance of 3 (1/4-units) from a given point (Charalambous & Pitta-Pantazi, 2007). In addition, the part whole meaning of a fraction involves the concept of dividing a unit into equal-sized parts and comparing a part or parts to the unit (Holmes, 1995). The meanings of fractions can be explained in the example of $\frac{3}{5}$ , which indicates that a whole has been divided into five equal parts and that three of those parts are taken (part-whole). Quotient means three cookies are given to five people equally, and ratio meaning is where there are three boys for every five girls in the classroom. Moreover, the problem "In a restaurant, there are 20 people and $\frac{3}{5}$ of them are women. How many women are there in the restaurant?" is an example of the operator meaning of fractions. For measure meaning the same example as indicated before can be given such as the distance of 3 (1/5-units) from a given point. Importantly, an understanding of fraction meanings helps to understand the operations of fractions and problem solving (Charalambous & Pitta-Pantazi, 2007, citied in Behr et al., 1983).

Operations with fractions, such as addition and subtraction, have the same meanings as they do with whole numbers. Adding with fractions involves *joining or combining* ideas, while subtracting with fractions involves *separating or comparing* ideas (Holmes, 1995; Cathcart et al., 2003). Some examples of problems that include addition and subtraction operations with fractions are following; "Tuana ate $\frac{1}{8}$ of a pizza for a snack and $\frac{5}{8}$ for lunch. How much of the pizza did she eat?" (joining meaning of addition), "Melik rode his bicycle $\frac{7}{10}$ of a kilometer to school. That is $\frac{5}{10}$ of a kilometer farther than his sister Nazlı. How far did Nazlı ride?"(comparing

meaning of subtraction), "İsmet had $\frac{11}{12}$ of a yard of ribbon for wrapping a birthday gift for his wife. He used $\frac{8}{12}$ of a yard for around the box. How much ribbon was left to make the bow?" (separating meaning of subtraction). Multiplication with fractions involves *repeated additions* as it does with whole numbers ($5 \times \frac{1}{2}$) and the *"of" meaning or a part of a part* ($\frac{1}{2} \times \frac{1}{5}$). Division with fractions involves *measurement and equal-sharing* situations (Holmes, 1995; Cathcart et al., 2003). Measurement (or repeated subtractions) involves the question "How many times is X in Y?", while the partition interpretation involves an equal-sharing situation (Holmes, 1995). Multiplication and division with fractions also have the same operations as those with whole numbers do (Cathcart et al., 2003). Problem examples that include meanings of multiplication and division operations with fractions are listed is following; "How much cake is a share 5 times as large as two-thirds?" (repeated addition meaning of multiplication), "Bedrinaz had $\frac{1}{3}$ of a pizza left after lunch. She ate $\frac{1}{2}$ of what was left before she went to bed. How much of the pizza did she eat?" ("of" meaning of multiplication), "There is a $\frac{1}{2}$ of a bottle of milk and it will be poured into a $\frac{1}{4}$ of a bottle. How many bottles are needed for this division?" (measurement meaning of division) and "There is $\frac{1}{3}$ of a pizza on the table. Two friends will share it. How much did they get?" (partition meaning of division).

## Recent studies conducted with pre-service teachers/teachers about problem posing

Recent studies have shown that pre-service teachers (Rizvi, 2004; Goodson-Espy, 2009; Luo, 2009; Toluk-Uçar, 2009; Işık, 2011; Işık & Kar, 2012; Tichá & Hošpesová, 2012; Kılıç, 2013) and in-service teachers (Koichu et al.,2012) have some issues in problem posing activities related to fractions. Posing problems requiring four arithmetical operations with fractions is a problematic activity for both pre-service and in-service teachers. Teachers and pre-service teachers have issues with problem posing, including operations with fractions and especially those featuring multiplication (Goodson-Espy, 2009; Luo, 2009; Işık, 2011) and division (Rizvi, 2004; Işık, 2011;Işık& Kar, 2012; Koichu et al., 2012).

In regard to the benefits gained by teachers, problem posing tasks can help them gain insight into the way in which students construct their mathematical understanding and can be a useful assessment tool (Lin, 2004). Stoyanova (2003) found that problem posing develops the students' understanding of mathematics and their ability to understand is dependent upon the teachers' ability to incorporate problem posing activities in mathematics classrooms. Lowrie (2002) indicated that problem posing actions of students can be nurtured by teachers' actions. Based on these previous findings, it was thought that subject matter content knowledge, which is content knowledge for teaching (Shulman, 1986) of pre-service teachers' fraction structures, can be evaluated through problem posing and can be improvable through courses during teacher education. As indicated by Ellerton (2013) pre-service teacher education students bring considerable mathematical and pedagogical insight into their involvement with problem posing in mathematics content classes. For that reason to develop their content knowledge is very important for mathematics education. In summary, while a number of studies have investigated pre-service teachers' knowledge of problem posing related to operations with fractions however analyzing pre-service teachers' knowledge structures of fractions including "knowing all meanings of fractions and four arithmetical operations with fractions" by means of problem posing was not studied

at all. Considering fraction knowledge structures of pre-service teachers is important for both mathematical learning and teaching, in that study it was aimed to present this gap in the literature by examining the ability of pre-service primary teachers' fraction knowledge structures through problem posing. It also provides insights into the similarities and differences of pre-service primary teachers' posed problem types. In this study, the following research question is addressed:

What kinds of problems will be posed by participants using $\frac{1}{2}$ and/or $\frac{3}{4}$ fractions regarding meanings of fractions and operations with fractions?

## METHODS

In this study, main data were collected through a problem posing task, and clinical interviewing was subsequently used as a qualitative research method (Ginsburg, 1997).

### Participants

The participants of the study consisted of pre-service primary school teachers who were in their sixth academic semester. All participants had enrolled in the Mathematics Teaching I method course and attending a Mathematics Teaching II method course during their education. This group was selected because when they become a primary teacher, they have to teach problem posing related to fractions as well as additional operations with fractions and the meaning of fractions to their students. Participants were selected using a two-step sampling process in order to prevent bias. In the first sampling process, 90 pre-service primary teachers participated in the study. Of these participants, 53 of were female and 37 of were male and their ages ranged between 20- and 21-years-old. In the second sampling among these participants, 6 (3 female and 3 male) volunteered for purposeful sampling (Gay, et al., 2006) using the criterion sampling technique (Patton, 1990) for clinical interview in order to understand their fraction knowledge structures in greater depth. Taking the Mathematics Method Course I and being introduced to the different problems in this study were the criteria for choosing participants. The real names of the participants were kept confidential (Patton, 2002) and nicknames were used. The code "I" was used for the researcher=interviewer and $PPT_1$, $PPT_2$, $PPT_3$, $PPT_4$, $PPT_5$, and $PPT_6$ were used for pre-service primary teachers conducting the interviews.

### Data collection

Posing problems related to mathematical topics is a good way to reveal pre-service teachers' knowledge structures of these topics. Therefore, in this study I aimed to determine pre-service teachers' knowledge structures (meanings of fractions and operations with fractions) through problem posing activity. Pre-service teachers could reflect their (mis)conception related to being one of the important topic fractions. Furthermore, participants' problem conceptions and issues they encountered while posing problems could be assessed. In this study semi-structured problem posing application being one of the problem posing situation (Stoyanova & Ellerton, 1996), which can allow to researcher to determine participants fraction knowledge structures in a certain limits was considered. In first step of data collection, participants were asked to pose as many problems as possible using $\frac{1}{2}$ and/or $\frac{3}{4}$ fractions in a problem posing task. The reason for selecting these numbers was to diagnose the participants' fraction knowledge

structures regarding meanings of fractions and operations with fractions. The problem posing task is following:

- Task 1: Pose as many different (not being similar to each other) problems as you can using the $\frac{1}{2}$ and $\frac{3}{4}$ fractions. (This task is asked to reveal participants' fraction structures related to operations with fractions).

- Task 2: Pose as many different (not being similar to each other) problems as you can using $\frac{1}{2}$ or $\frac{3}{4}$ fractions. (This task is asked to reveal participants' fraction structures related to meanings of fractions). Forty minutes were allowed for participants to generate/pose problems based on the problem posing task.

In second step of data collection process, clinical interviews were conducted with the 6 volunteer participants. The study by diSessa (2007, p.525) explained clinical interviewing as follows: "Typically, a clinical interview is a one-on-one encounter between an interviewer, who has a particular research agenda, and a subject. The interviewer proposes usually problematic situations or issues to think about and the interviewee is encouraged to engage these as best he/she can. The focal issue may be a problem to solve, something to explain, or merely something to think about". In order to understand clinical interview questions' conformity, validity, and reliability, a pilot study was conducted with one pre-service teacher. As a result of the pilot study, the questions were revised in order to reveal mathematical misconceptions and uncertainties as well as unexpected situations (Goldin, 2000). Questions that were used in the clinical interviews were open-ended and allowed for assessment of participants' thinking processes (Hunting, 1997). During the interviews, questions such as *"Do you think that this situation is problem?" "What do you think about your posed problem? Is it suitable for problem posing situation?"Why? Can you explain?"* and *"Could you pose any other problems?"* were asked. The interviews with participants took between 20-25 minutes and were tape recorded.

## Data analysis

The data obtained from the study were analyzed using semantic (Tichá & Hošpesová, 2012) and constant comparative analysis (Glaser & Strauss, 1967) techniques. The reason for choosing semantic analysis was because analyzing structures of posed problems semantically and constant comparative analysis was preferred by the researcher for coding data under a category combined with the previous incidents in the same and different groups coded in the same category. Produced problems/statements were first listed and classified according to their semantic structures. Every generated problems/mathematical statements were coded under the meaning of fractions, operations with fractions, or combining those with other issues. A situation such as $\frac{10}{3}$ of a number is 150, so what is the number?" was coded as a story equation and $\frac{1}{2} + \frac{3}{4} + \frac{1}{8}$ =? was coded as a symbolic equation (Kılıç, 2013). In the last step of the data analysis, frequency and percentage of the produced problems/statements were calculated. For analyzing the clinical interviews, data from the transcripts were first transcribed verbatim by the researcher (Bogdan & Biklen, 1998) and coded based upon categories and sub-categories. These categories and sub-categories were developed by the researcher based on previous studies of fraction meanings (ratio, part-whole, quotient, operator and measure) and operations with fractions (addition (joining), subtraction (comparison and separating), multiplication (repeated addition and of meaning), division (measurement and partition meaning)).

### Validity and reliability

In order to increase the reliability and validity of the study, member checks and prolonged engagement techniques were used as suggested by Lincoln and Guba (1985). Furthermore, the researcher asked for the opinion and assessment of one colleague who is blinded to the data and unbiased regarding the code list and research findings. In order to examine inter-rater reliability, the colleague independently classified the posed problems. The formula of Miles and Huberman (1994) was used to calculate inter-rater reliability and was determined to be 95% for first task and 93% for the second task. The pilot study also contributed the validity and reliability of the problem posing task in this study.

### RESULTS

In this section the categories that emerged from the study in accordance with Task 1 and Task 2 will be presented separately in tables.

### Semantic structures of posed problems using $\frac{1}{2}$ and $\frac{3}{4}$ fractions

The problems posed by participants using $\frac{1}{2}$ and $\frac{3}{4}$ fractions are provided in Table 1 as the frequency and percentage. Furthermore, the issues that were encountered in this task are provided at the end of the Table 1.

As shown in Table 1, participants produced a total of 267 situations that were either a problem or not a problem. These situations include operations with

**Table 1**. Semantic structures of posed problems by participants using $\frac{1}{2}$ and $\frac{3}{4}$ fractions

| Using $\frac{1}{2}$ and $\frac{3}{4}$ fractions together in a problem | | Frequency | Percentage |
|---|---|---|---|
| **Addition** | | 17 | 6.36 |
| **Subtraction** | | | |
| | Comparison | 4 | 1.49 |
| | Separating | 2 | 0.74 |
| **Multiplication** | | | |
| | Repeated addition | 5 | 1.87 |
| | Of meaning | 5 | 1.87 |
| **Division** | Partition | 2 | 0.74 |
| | Measurement | 4 | 1.49 |
| **Combining meanings of fractions** | | | |
| | Operator+operator | 81 | 30.33 |
| **Meanings of fractions+ operations with fractions** | | 18 | 6.74 |
| **Symbolic equation** | | | |
| | Addition | 13 | 4.86 |
| | Subtraction | 9 | 3.37 |
| | Multiplication | 10 | 3.74 |
| | Division | 9 | 3.37 |
| **Story equation** | | | |
| | Operations with fractions | 25 | 9.36 |
| | Meaning of fractions + operations with fractions | 18 | 6.74 |
| **Issues** | | | |
| | $\frac{1}{2}+\frac{3}{4}>1$ | 36 | 13.48 |
| | Missing data | 7 | 2.62 |
| | Choosing wrong natural number | 2 | 0.74 |
| **Total** | | 267 | |

fractions, such as addition, subtraction, multiplication, and division, combining the meaning of fractions, meaning of fractions + operations with fractions, symbolic equations, or story equations. Furthermore, some participants had issues while posing problems using $\frac{1}{2}$ and $\frac{3}{4}$ fractions together in a problem. Among operations with fractions, participants mostly posed problems including the addition operation (6.36%) with fractions as well as multiplication (repeated addition 1.87%, of meaning 1.87%), subtraction (comparison 1.49%, separating 0.74%) and division (measurement 1.49% and partition 0.74%). Examples of posed problems related to operations with fractions are following; One example for addition with fractions posed a problem by participant $PPT_5$ such as "Ayse gave $\frac{1}{2}$ of her cake to Ali and Ahmet gave $\frac{3}{4}$ of his cake to Ali. How much cake does Ali have in total? During interview participant declared that he posed problems requiring operations with fractions, such as addition and tried to pay attention to gathering pieces of cake. $PPT_5$ considered joining meaning of addition with fractions and only used $\frac{3}{4}$ and $\frac{1}{2}$ fractions in a problem. An example for comparing meaning of subtraction with fractions was posed by the $PPT_3$ participant, such as "Eylül and Miray would like to share their same size apples. Eylül wants $\frac{1}{2}$ of an apple and Miray was $\frac{3}{4}$ of an apple. Under which situation will Eylül be more profitable?". Participant $PPT_3$ declared that she tried to pose a different problem and tried to ask a problem that compares $\frac{1}{2}$ and $\frac{3}{4}$ fractions. In this example the $PPT_3$ participant considered a comparison meaning of subtraction with the $\frac{1}{2}$ and $\frac{3}{4}$ fractions. An example for separating meaning of subtraction with fractions was posed by the $PPT_1$ participant, such as "Çiğdem has $\frac{3}{4}$ of an apple, she ate $\frac{1}{2}$ of an apple. How much apple was left?". She asserted that she focused on the fractions and operations with fractions while posing that problem. A posed problem representing the "of" meaning of the multiplication with fractions such as "I ate $\frac{3}{4}$ of $\frac{1}{2}$ of an apple, so how much apple did I eat?". During interview participant $PPT_5$ indicated that he tried to pose a different problem that what was posed in the first and tried to pose a problem that can be solved by operations.

One example for measurement meaning of division with fractions posed by the $PPT_6$ participant was "If I take $\frac{3}{4}$ of an apple, how many times do I eat $\frac{1}{2}$ of an apple?". $PPT_6$ declared that he considered how many times $\frac{3}{4}$ went into $\frac{1}{2}$ while posing the problem and considered two fractions in a problem. As shown in Table 1, most of the participants preferred to pose problems (30.33%) that combined the meaning of fractions using both $\frac{1}{2}$ and $\frac{3}{4}$ fractions, such as using two times the operator meaning of fractions. An example of $PPT_3$'s written work was "Ali and Ayse would like to share walnuts and pears that they gathered from the garden. Ali would like to take $\frac{1}{2}$ of 12 walnuts and Ayse would like take $\frac{3}{4}$ of 16 pears. How many walnuts and pears does each take?" An interview conducted with $PPT_3$ is shown below:

    I:Is it an appropriate problem for a problem posing situation?

    PPT3: Yes and there is more. For example, I needed to use the numbers 12 and 16, because it is limited to $\frac{1}{2}$ and $\frac{3}{4}$ fractions.

    I: Why?

    PPT3: Posing problems using $\frac{1}{2}$ and $\frac{3}{4}$ fractions is limited, and therefore I used other numbers such as 12 and 16.

In this problem, participant $PPT_3$ used an operator meaning of fractions two times. The participant considered $\frac{1}{2}$ of 12 and $\frac{3}{4}$ of 16 in posing a problem. 6.74% of the responses regarded the meaning of fractions + operations with fractions using $\frac{3}{4}$ and $\frac{1}{2}$ fractions. One example for including the meaning of fractions + operations with fractions posed by $PPT_2$ using $\frac{3}{4}$ and $\frac{1}{2}$ together was as follows: "The mother of Ali divided a cake into two parts and gave $\frac{1}{2}$ of that cake to Ali. Later, Ali took $\frac{3}{4}$ of $\frac{1}{2}$ of the cake. How much cake did Ali eat in total? During interview participant $PPT_2$ declared that she posed a problem considering the given situation in given problem posing situation. There is a limit in the situation. In this example part-whole meaning of the fraction and of meaning of multiplication with fractions was used by $PPT_2$ participant while posing the problem. Participants also preferred to pose symbolic equations using $\frac{3}{4}$ and $\frac{1}{2}$ operations with fractions. (4.86%) of the responses were addition, (3.37%) were subtraction, (3.74%) were multiplication, and (3.37%) were division. Examples for symbolic equations like $\frac{3}{4} + \frac{1}{2} =?$, $\frac{3}{4} - \frac{1}{2}=?$, $\frac{3}{4}$ x $\frac{1}{2}$ and $\frac{3}{4} : \frac{1}{2}=?$ can be given. In those examples, participants only focused on mathematical exercises. An interview with participant $PPT_4$ about that category is shown below:

> I: Is it a problem that you wrote? (showing $\frac{3}{4} + \frac{1}{2} =?$)
> PPT4: It can be a problem in a specific context, but it is a type of four arithmetical operations with fractions.
> I: What should it be? Can you pose a problem that considers what you said about this situation?
> PPT4: If I wrote it involving an event, such as sharing bread or comparing it to something else, comparing it can be a problem.

Participant $PPT_4$ realized that he can generate a mathematical problem using fractions in a mathematical context. Furthermore, some participants produced story equations using $\frac{3}{4}$ and $\frac{1}{2}$ fractions. 9.36% of responses regarded operations with fractions and 6.74% included the meaning of fractions + operations with fractions. The example of a story equation that included the meaning of fractions + operations with fractions produced by participant $PPT_6$ as follows: "Ezgi first spent $\frac{1}{2}$ of her money and then spent $\frac{3}{4}$ of the rest. She has 12 liras remaining. How much money did she have at the beginning?" She/he declared that she/he posed the problem using $\frac{1}{2}$ and $\frac{3}{4}$ with 12 in the problem.

Some participants had issues while posing problems. Most of the issues that participants encountered were not being aware of $\frac{1}{2} + \frac{3}{4} > 1$. Of the participants, 13.48% experienced this problem. An example for this situation is as follows: "Tuana has 24 walnuts. She gave $\frac{1}{2}$ to her brother and $\frac{3}{4}$ to her cousin. How many walnuts does Tuana have now?" An interview excerpt from participant $PPT_1$ about this problem is shown below:

> I: Do you think that the problem that you posed is a problem?
> PPT1: Yes it is a problem.
> I: Why?
> PPT1: There are givens and a need to find the result.
> I: Do you think that this problem is solvable or not?
> PPT1: It is solvable.
> I: Can you solve it right now?
> PPT1: One moment. The result is wrong.

I: Why?

PPT1: There are 24 walnuts. $\frac{1}{2}$ of the walnuts is 12 and $\frac{3}{4}$ of the walnuts is 18. The total is 30, and 30 is more than 24. It is not a problem in that context.

I: Do you think that the problem that you posed is in accordance with the posing situation?

PPT1: It is in accordance with the formal problem but it is wrong logically because it is 30.

I: Could you pose other problems that consider this problem posing situation?

PPT1: I could pose a problem but I can do the same default. I wrote that without thinking about the outcome of the problem.

In this posed problem, participant PPT$_1$ did not consider that the $\frac{1}{2} + \frac{3}{4}$ situation is more than 1 (total). She focused on the only fractions and not on reality. Another issue emerged in the study using missing data (2.62%). An example for this issue posed by PPT$_4$ was "One person tiles 600 m² in the first day. On the second day, how much should he tile?" In this example, participant PPT$_4$ posed a situation using missing data (not using $\frac{1}{2}$ and $\frac{3}{4}$). The interview conducted with PPT$_4$ is shown below:

I: What do you think about this problem that you posed?

PPT4:I am not sure it is a problem that includes fractions.

I: Why?

PPT4: I should add fractions as asked in the task (showing $\frac{1}{2}$ and $\frac{3}{4}$ fractions)

I:If I ask right now to pose a problem including $\frac{1}{2}$ and $\frac{3}{4}$ fractions. What would like to pose?

PPT4: I can say for example: "In the first day, he used $\frac{1}{2}$ of 600 and on the second day he used $\frac{3}{4}$ of the rest. How much did he tile in total over the two days?

During the clinical interview, the participant PPT$_4$ realized that he should use $\frac{1}{2}$ and $\frac{3}{4}$ fractions while posing the problem. A few responses were about choosing the wrong natural number (0.74%). Participant PPT$_3$ posed a problem like "There are 30 students in a classroom. $\frac{1}{2}$ of those are boys and $\frac{3}{4}$ of the boys are blond. How many blond boys are there in the classroom?" During the clinical interview, PPT$_3$ declared that she focused only on numbers that were not in the context of the problem and did not think about whether it was solvable or not. If the participant preferred 40 or any number which can be divided 8 instead of 30, that problem could have been solved.

## Semantic structures of posed problems using $\frac{1}{2}$ or $\frac{3}{4}$ fractions

The posed problems by participants using $\frac{1}{2}$ or $\frac{3}{4}$ fractions are shown in Table 2. In this task, participants posed problems that included meanings of fractions, combining the meanings of fractions, meanings of fractions + operations with natural numbers, and story equations that included the meanings of fractions and meanings of fractions + operations with fractions. Moreover, some issues were encountered by participants, such as missing data, using different fractions, non-fraction problems, and choosing wrong number.

As seen from Table 2, participants produced 400 situations using $\frac{1}{2}$ or $\frac{3}{4}$ fractions either a problem or being not a problem. Participants posed problems including

**Table 2.** Semantic structures of posed problems by participants using $\frac{1}{2}$ or $\frac{3}{4}$ fractions

| Using $\frac{1}{2}$ or $\frac{3}{4}$ | Frequency | Percentage |
|---|---|---|
| **Meanings of fractions** | | |
| Part-whole | 69 | 17.25 |
| Operator | 153 | 38.25 |
| Ratio | 16 | 4 |
| Quotient | 8 | 2 |
| **Combining meanings of fractions** | 11 | 2.75 |
| Operator+ Operator ($\frac{1}{2}$ and $\frac{1}{2}$ or $\frac{3}{4}$ and $\frac{3}{4}$ ) | | |
| **Meanings of fractions + operations with natural numbers** | 22 | 5.5 |
| **Story equation** | | |
| Meanings of fractions | 74 | 18.5 |
| Meanings of fractions+operations with fractions | 18 | 4.5 |
| **Issues** | | |
| Missing data | 9 | 2.25 |
| Using different fractions | 11 | 2.75 |
| Non-fraction problems | 8 | 2 |
| Choosing wrong number | 1 | 0.25 |
| **Total** | 400 | |

meaning of fractions such as part-whole (17.25%), operator (38.25%), ratio (4%), and quotient (2%). An example for the part-whole meaning of the $\frac{3}{4}$ is: "I divided bread into four equal parts and I ate 3 parts of that bread. How much bread did I eat?" The interview with PPT$_5$ is shown below:

> I: What do you think. Is that problem related to a problem posing situation?
> PPT5: Yes it is appropriate for a problem posing situation.
> I: Why?
> PPT5: There is a whole here and I tried to consider the part-whole. I divided a bread into equal parts and I ate some equal parts of a bread. I asked how much I ate in total in the problem.

Participant PPT$_5$ considered a whole and its equal parts. An example for the operator meaning of a fraction, such as PPT$_3$, posed a problem like "I read $\frac{3}{4}$ of a 100-page book yesterday. How many pages did I read?" PPT$_3$ declared that she preferred 100 with $\frac{3}{4}$. An example for ratio meaning of a fraction problem was posed by participant PPT$_2$ such as "Find the ratio between 3 unit cubes and 4 unit cubes." Participant PPT$_2$ declared that she considered the ratio while posing the problem. In this example, the participant considered a comparison of two quantities of the unit cubes, which are same type. An example of quotient was "There are 3 pieces of bread and 4 people who will share them equally. How much bread does everybody take?" posed by participant PPT$_4$. Participant indicated that he used $\frac{3}{4}$ dividing meaning like 3 to 4.

Some of the responses posed by participants included combining the meanings of fractions, such as operator + operator (2.75%). Participants used operator meanings of fractions together ( $\frac{1}{2}$ and $\frac{1}{2}$ or $\frac{3}{4}$ and $\frac{3}{4}$ ) while posing problems. Participant PPT$_3$ posed a problem related to this category such as "The course fee is 1000 liras and I earned $\frac{1}{2}$ of 1000 discount and later $\frac{1}{2}$ of the discounted fee discount. How much money should I pay?" Participant PPT$_3$ indicated that she posed a problem using two

times$\frac{1}{2}$. 5.5% of the responses were posed by participants using meanings of fractions + operations with natural numbers. The example for this situation posed by participant PPT$_6$ was as follows: "In a class, there are 32 students. $\frac{3}{4}$ of those students passed the exam. Two students did not attend the exam. How many students failed in exam?". Participant PPT$_6$ declared that he focused on $\frac{3}{4}$ and needed other numbers 32 and 2. In this example, this participant used the operator meaning of fraction and operations with natural numbers. In posing a problem related to $\frac{3}{4}$, participant PPT$_6$ used $\frac{3}{4}$ as an operator meaning and 2 as a natural number. Participants also posed story equation, including the meanings of fractions (18.5%) and the meanings of fractions + operations with fractions (4.5%). The example posed problem by participant PPT$_2$ for the meanings of fractions was "I gave $\frac{1}{2}$ of my pencils to my friend. I now have 9 pencils lest. What was the number of pencils I had before?" The participant declared that she/he posed the problem using 9 and $\frac{1}{2}$ in the problem. Participant PPT$_2$ posed a story equation including the part-whole meaning of fractions. An example for the meanings of fractions + operations with fractions was posed by participant PPT$_5$ as follows "One runner ran $\frac{1}{2}$ of the length of a road, and later he ran $\frac{3}{4}$ of the rest road, so how many meters did he run?" PPT$_5$ considered part-whole meaning of fraction and multiplication with fractions (of meaning of multiplication).

Participants had issues while posing problems using $\frac{1}{2}$ or $\frac{3}{4}$ fractions. 2.25% of responses posed using missing data and 2.75% problems were posed using different fractions, 2% were non-fraction problems and one response were posed choosing the wrong number. PPT$_5$ used different fractions while posing the problem. He used the $\frac{4}{5}$ fraction, which was not mentioned in the problem posing task and declared that he wrote it wrong because he would typically say something different. While posing the problem he made a mistake. Participant PPT$_5$ used a different fraction that was not given in the task. An example for a non-fraction problem can be given such as "In a school there are 180 students. The number of boys is two times more than girls. How many boys are there in this school?" posed by participant PPT$_1$. During the clinical interview with the participant, she realized that she/he had not pose a fraction problem using fractions mentioned in the task. Participant PPT$_2$ posed a problem using wrong number such as " In a classroom there are 25 students and $\frac{3}{4}$ of them got high scores in mathematics. So how many students did take high scores in that classroom?". During the interview she mentioned that she focused only the fraction not the number.

## CONCLUSION AND DISCUSSION

Problem posing is a very important topic because of its benefits. Considering its importance and the fact that teachers will have to teach this topic in schools, it is important to educate them during their training to become a teacher. Problem posing is a good way to analyze fraction content knowledge structures of both students (Işık & Kar, 2012) and pre-service teachers (Toluk-Uçar, 2009; Luo, 2009; Işık, 2011; Kılıç, 2013) and is a useful assessment instrument, such as a diagnostic tool (Tichá & Hošpesová, 2012). In order to evaluate pre-service teachers' mathematics knowledge, problem posing tasks are recommended (Luo, 2009). In Turkish mathematics curriculum, teachers are expected to teach problem posing to their students in the context of fractions. Therefore, this study sought to determine

the pre-service teachers' fraction knowledge structures using problem posing applications during mathematics methods courses' sessions. Understanding pre-service teachers' problem posing performance in the context of meaning of fractions and operations with fractions should have an important contribution to the improvement of future teacher education programs. Additionally, if they will be educated well in fraction and problem posing knowledge, they will teach those topics effectively when they become a teacher.

The results of the study showed that pre-service primary teachers posed different problems using $\frac{1}{2}$ or/and $\frac{3}{4}$ fractions. Participants considered operations with fractions, meanings of fractions, a combination of both, story equation and story equation. In the first task they posed problems mainly combining meanings of fractions (operator + operator), rather than operations with fractions. Posing problem using two times the operator meaning of a fraction is a good result because as indicated in the study of Charalambous and Pitta-Pantazi (2007), understanding fractions as operators enhances the comprehension of the multiplication of fractions, particularly determining $\frac{3}{4}$ of $\frac{1}{2}$ (taking a part of a part of the whole). Furthermore, in first task participants posed problems related to addition and multiplication operations with fractions much more than other operations with fractions. In the second task, the participants preferred to use the operator meaning of a fraction. Moreover they did not pose any problems related to measure meaning of fraction. In both tasks, participants preferred to use the operator meaning of a fraction. When participants posed problems, including the operator meaning of a fraction, participants used natural numbers besides fractions in nature. This finding is consistent with the findings of the study by Kılıç (2013) related to posing fraction problems in a free problem posing activity. In that study, participants did not focus on operations with fractions as much, which needs to be used in fractions in this context. Based on this study, it was found that pre-service primary teachers generated story and symbolic equations instead of posing story problems.

In this study, pre-service primary teachers had some issues in both tasks. In the first task, many of the participants posed problems ignoring $\frac{1}{2} + \frac{3}{4} > 1$. They were not aware that $\frac{1}{2} + \frac{3}{4}$ is more than 1. After the clinical interview, some participants realized that they did not pose a problem in some cases. They only focused on numbers rather than reality. This situation can be explained based on the fact that in the academic background (fraction knowledge structures and problem posing) of pre-service primary teachers in Turkey. In Turkey, problem posing applications were placed in mathematics curriculums (MEB, 2009) starting in 2006, but were not placed in teacher education courses. Therefore, this topic is still new for pre-service teachers. As indicated in the study of Ellerton (2013) they need to have more opportunities to pose problems as they will need these skills as classroom teachers. Moreover, issues such as using missing data in posing a problem, using different fractions, being non-fraction problems, and choosing natural numbers emerged in the study. Tichá and Hošpesová (2012) asserted that one possible reason for many misunderstandings of the fraction can be due to the different meanings that are used. Thus, it can be concluded that participants have some issues regarding fractions and problem solving. The problem perception by participants is also problematic, as indicated in the study of Kılıç (2013). Furthermore, in that study it was found that some participants have abundant subject matter knowledge on meanings of fractions and operations with fractions. Having issues in generating problems may be, as Rizvi (2004) indicated, because pre-service primary teachers had never tried and never been asked to pose problems related to fractions before. In previous studies of this subject, pre-service teachers have had difficulties in problem posing situations that require operations with fractions (Rizvi, 2004; Luo,

2009; Toluk-Uçar, 2009; Işık, 2011) or did not prefer so much while problem posing (Kılıç, 2013). Therefore, pre-service teachers may have preferred to generate the problem with the meaning of fractions, rather than operations with fractions in this study. Chapman (2012) indicated that prospective elementary school teachers' sense-making of problem posing was dependent on their mathematical knowledge, imagination or creativity, and past experience with problem solving. Furthermore, posing different problems can be explained by participants' previous educational experience, such as secondary education (Tichá & Hošpesová, 2012).

Considering that problem posing is a good way to analyze mathematical knowledge structures and reveal (mis)conceptions of the participants during the pre-service teacher education period, many problem posing activities can be performed that include several mathematical topics. Previous studies have asserted that pre-service teachers should participate in problem posing activities (Abu-Elwan, 1999; Lavy & Bershadsky, 2003; Contreras, 2007; Toluk-Uçar, 2009; Luo, 2009; Işık, 2011; Kılıç, 2013) and problem posing should be a main activity in teacher education courses (Abu-Elwan, 1999; Rizvi, 2004; Barlow & Cates, 2006; Korkmaz & Gür, 2006; Tichá & Hošpesová, 2012; Kılıç, 2013). Barlow and Cates (2006) indicated that courses in teacher trainee programs must be designed to provide opportunities to revise unexamined knowledge and beliefs about the subject matter. Korkmaz and Gür (2006) also advised that a course related to problem posing should be placed in teacher education programs and these skills should be enhanced (Abu-Elwan, 1999). Considering that subject matter content knowledge includes the structure of subject matter, including ways in which the basic concepts and principles are organized (Shulman, 1986), it becomes important to intervene during the training of teachers. As previously stated, knowledge of the subject matter by pre-service teachers is important (Luo, 2009) and also teacher education was also one of the factors that influenced the understanding of good teaching (Leong, 2013).

In this study, different from the previous studies, pre-service primary teachers' fraction knowledge structures were analyzed through studying semi-structured problem posing activities which gives a picture of pre-service teachers' fraction knowledge structures in the context of problem posing. Future studies could involve a structured problem posing activity with pre-service primary teachers giving structured fraction mathematical sentences, such as $\frac{1}{2} + \frac{3}{4}$ , $\frac{3}{4} - \frac{1}{2}$ and etc. or fractions in order to improve their knowledge and ability related to operations with fractions and meanings of fractions. This study was conducted at the beginning of the Mathematics Teaching II course, so at the end of the course a similar study can be conducted with the same sample after problem solving as well as fraction and problem posing topics are explained by the educator. In this sense, problem solving, problem posing, and fraction knowledge structures of participants could be assessed affectively. Different problem posing tasks, including digital image photos (Nicol& Bragg, 2009), computer programs, or spreadsheets (Abramovich & Cho, 2008), can be applied during teacher education programs to assess and develop pre-service teachers' problem posing performance and conceptual changes in fraction knowledge structures. Nicol & Bragg (2009) indicated that using digital images during a problem posing activity enables the participants to develop strategies for problem posing and to become more critically aware of the mathematical potential within their environment. As indicated in the study of Chen et al. (2011), intervention studies wherein one would compare the effects of various instructional approaches on pre-service teachers' topic-related cognitions and beliefs can be investigated. In future studies, posed problems of pre-service primary teachers could be analyzed in terms of linguistic and mathematical complexity (Silver & Cai, 1996) and monothematic nature (Tichá & Hošpesová, 2012). Moreover, the

relationship between pre-service primary teachers' problem posing performance regarding fraction knowledge structures and other factors such as cognitive and emotional can be investigated.

## REFERENCES

Abramovich, S.& Cho, E. K. (2008). On mathematical problem posing by elementary pre-teachers: The case of spreadsheets. *Spreadsheets in Education (eJSiE), 3*(1), 1-19.

Abu-Elwan, R. (1999). The development of mathematical problem posing skills for prospective middle school teachers. In Mina, F.& Rogerson, A. (Eds.), *Paper presented at the* p*roceedings of the International Conference on Mathematical Education into the 21st Century,* (pp.1-8 ). Cairo, Egypt

Barlow, A. T. & Cates, J. M.  (2006). The impact of problem posing on elementary teachers' beliefs about mathematics and mathematics teaching. *School Science and Mathematics, 106*(2), 64-73.

Bogdan, R.  C. & Biklen,  S. K.  (1998).  *Qualitative research for education: An introduction to theory and methods.* (3rd Ed.). Boston: Allyn & Bacon.

Cathcart, W. G., Pothier, Y. M., Vance, J. H & Bezuk, N. S. (2003). *Learning mathematics in elementary and middle schools.* (3rd Ed. ). N.J. : Merrill/Prentice Hall.

Chapman, O. (2012). Prospective elementary school teachers' ways of making sense of mathematical problem posing. *PNA, 6*(4), 135-146.

Charalambous, C. Y & Pitta-Pantazi, D. (2007). Drawing on a theoretical model to study students' understandings of fractions. *Educational Studies in Mathematics*, *64*, 293–316.

Chen, L., Dooren,W., V. Chen, Q & Verschaffel,L. (2011). An investigation on Chinese teachers' realistic problem posing and problem solving ability and beliefs. *International Journal of Science and Mathematics Education*, *9*, 919-948.

Christou, C., Mousoulides, N., Pittalis M., Pitta-Pantazi, D.&Sriraman, B. (2005). An empirical taxonomy of problem posing process. *ZDM, 37*(3), 149-158.

Contreras, J. (2007). Unraveling the mystery of the origin of mathematical problems: Using a problem-posing framework with prospective mathematics teachers. *The Mathematics Educator, 17*(2), 15-23.

diSessa, A. A. (2007): An interactional analysis of clinical interviewing. *Cognition and Instruction, 25*(4), 523-565.

Ellerton, N. F. (2013).  Engaging pre-service middle-school teacher-education students in mathematical problem posing: development of an active learning framework. *Educational Studies in Mathematics,* DOI 10.1007/s10649-012-9449-z.

Gay, L. R., Mills, G. E. & Airasian, P. (2006). *Educational research: Competencies for analysis and applications.*(8th Ed.). New Jersey: Pearson Prentice Hall.

Ginsburg, H. P. (1997). *Entering the child's mind: The clinical interview in psychological research and  Practice.* New York: Cambridge University Press.

Glaser, B.G & Strauss, A. L. (1967). *The discovery of grounded theory: Strategies for qualitative research.*  Chicago, IL: Aldine.

Goodson-Espy, T. (2009). An exploration of preservice teachers' creation and analysis of fraction multiplication word problems.  In Swars, S. L., Stinson, D. W., & Lemons-Smith, S. (Eds.),  *Proceedings of the 31st annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education.*  Atlanta  (pp. 1330-1338). GA: Georgia State University

Goldin, G. (2000). A scientific perspectives on structured, task-based interviews in mathematics education research. In A.E. Kelly & R.A. Lesh (Eds.), *Handbook of research design in mathematics and science education* (pp. 517-546). Mahwah, NJ: Lawrence Erlbaum Associates.

Holmes, E. E. (1995). *New directions in elementary school mathematics: Interactive teaching and learning.*  Englewood Cliffs, N.J. : Merrill.

Hunting, R. P. (1997). Clinical interview methods in mathematics education research and practice. *Journal of Mathematical Behavior, 16*(2), 145-165.

Işık, C. (2011). Conceptual analysis of multiplication and division in fractions posed by pre-service elementary mathematics teachers. *Hacettepe University  Journal of Education Faculty, 41*, 231-243.

Işık, C. & Kar, T. (2012). An error analysis in division problems in fractions posed by pre-service elementary mathematics teachers. *Educational Sciences: Theory & Practice*, *12*(3), 2303-2309.

Kılıç, Ç. (2013). Pre-service primary teachers' free problem posing performances in the context of fractions: An example from Turkey *The Asia Pacific Education Researcher* DOI: 10.1007/s40299-013-0073-1.

Koichu, B. Harel, G. & Manaster, A. (2012). Ways of thinking associated with mathematics teachers' problem posing in the context of division of fractions. *Instructional Science*, 1-18.

Korkmaz, E.& Gür, H. (2006). Öğretmen adaylarının problem kurma becerilerinin belirlenmesi. [Determining of prospective teachers' problem posing skills].*Balıkesir Üniversitesi Fen Bilimleri Enstitüsü Dergisi, 8*(1), 64-74.

Lavy, I.& Bershadsky, I. (2003). Problem posing via "what if not?" strategy in solid geometry- a case study. *Journal of Mathematical Behavior*, *22*, 369-387.

Leong, K.E. (2013). Factors that Influence the understanding of good mathematics teaching. *Eurasia Journal of Mathematics, Science & Technology Education*, *9*(3), 319-328.

Lin, P. J. (2004). Supporting teachers on designing Problem Posing tasks as a tool of assessment to understand students' mathematical learning. In M. J. Hoines & A. B. Fuglestad (Eds.), *Proceedings of the 28th Conference of the International Group f or the Psychology of Mathematics Education* (pp. 257-264). Norway: Bergen University College.

Lincoln, Y.S.& Guba, E. G. (1985). *Naturalistic inquiry.* California, New Burry Park: Sage Publication.

Lowrie, T. (2002). Designing a framework for problem posing: young children generating open-ended tasks. *Contemporary Issues in Early Childhood, 3*(3),354-64.

Luo, F. (2009). Evaluating the effectiveness and insights of pre-service elementary teachers' abilities to construct word problems for fraction multiplication. *Journal of Mathematics Education, 2*(1), 83-98.

MEB (2009). *İlköğretim matematik dersi 1-5.sınıflar öğretim programı*. [Primary School Mathematics Curriculum (1.-5 grades)]. Ankara Devlet Kitapları Basımevi.

Miles, M.B. & Huberman, A.M. (1994). *An expanded sourcebook qualitative data analysis* (2 nd Ed.).California: Sage Publications.

National Council of Teachers of Mathematics (2000). *Principles and standards for school mathematics.* Reston, VA: NCTM.

Nicol, C. & Bragg,L. (2009) Designing problems : what kinds of open-ended problems do preservice teachers pose?*. Proceedings of 33rd Annual Meeting of the International Group for the Psychology of Mathematics Education*, PME, (pp. 4-225-4-232). Thessaloniki,Greece.

Nixon-Ponder, S. (1995). Using problem posing dialogue in adult literacy education. Teacher to teacher. *Adult Learning, 7*(2), 10-12.

Patton, M. Q. (1990). *Qualitative evaluation and research methods.* (2nd Ed.). California: Sage Publication.

Patton, M. Q. (2002). *Qualitative research and evaluation methods*. (3rd Ed.). California: Sage Publication.

Reys, R. E., Suydam, M. N., Lindquist, M. M. &Smith, N. L. (1998). *Helping children learn mathematics.* (5th Ed.). Needham Heights: Allyn&Bacon

Rizvi, N. F. (2004). Prospective teachers' ability to pose word problems. *International Journal for Mathematics Teaching and Learning, 12*, 1-22.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15*(2), 4–14.

Silver, E. A. & Cai, J. (1996). An analysis of arithmetic problem posing by middle school students. *Journal for Research in Mathematics Education, 27*(5), 521-539.

Stoyanova, E. & Ellerton, N. F. (1996). A framework for research into students' problem posing in school mathematics. In P. Clarkson (Ed.), *Technology in Mathematics Education* (pp.518–525). Melbourne: Mathematics Education Research Group of Australasia.

Stoyanova, E. (2003). Extending students' understanding of mathematics via problem posing. *The Australian Mathematics Teacher, 59(*2), 32-40.

Tichá, M.& Hošpesová, A. (2009). Problem posing and development of pedagogical content knowledge in pre-service teacher training. *Proceedings of CERME 6* (pp.1941-1950). Lyon, France.

Tichá, M.& Hošpesová, A. (2012). Developing teachers' subject didactic competence through problem posing. *Educational Studies in Mathematics,* 1-11.

Toluk- Uçar, Z. (2009). Developing pre-service teachers understanding of fractions through problem posing. *Teaching and Teacher Education, 25*(1), 166-175.

❖ ❖ ❖