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Application of Grey Relational Analysis to Decision-Making during Product Development

Shih-Wen Hsiao
National Cheng Kung University, TAIWAN
Hsin-Hung Lin
National Cheng Kung University, TAIWAN
Ya-Chuan Ko
Asia University & China Medical University, TAIWAN

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ABSTRACT
A multi- attribute decision-making (MADM) approach was proposed in this study as a prediction method that differs from the conventional production and design methods for a product. When a client has different dimensional requirements, this approach can quickly provide a company with design decisions for each product. The production factors of a product are usually complex and difficult to predict. A focus group that comprises seven professional product designers was formed in this study to determine those exact assessment criteria based on their practical experiences of pneumatic door closer designs. These criteria include operability, manufacturability, style, creativity, and cost. We recommend using grey-level designs to assess and resolve the product design and production planning problems. A case study on pneumatic door closers was conducted and a weighted value was assigned to each of the assessment criteria. New product series were created for the verification of the proposed design approach. In a grey-level design assessment, the design ideas of clients and designers are represented by grey levels. After that, a grey relational analysis is used to determine those factors that are valued by clients for predicting the priority of each of the design elements for a product series. The proposed approach can assist designers in predicting a product's design quality and recommend the optimal alternative within a product series.
Keywords: demand forecast, production management, Grey relational analysis, product development, product design

INTRODUCTION

A product design process is consisted of integrations and analyses. A product design usually originates from the clients' requirements. During the process of identifying detailed product design ideas, the complexity of related problems gradually enhances. Each of the design phases requires assessments before a decision can be made. Desai (2010) proposed the concept of combined designs which assist design personnel in designing a product system that can be

Correspondence: Shih-Wen Hsiao, *Department of Industrial Design, National Cheng Kung University, Tainan City, Taiwan, ROC, No. 1 University Road, 70101 Tainan, Taiwan*
swhsiao@mail.ncku.edu.tw

State of the literature

- The concept of design manufacturability has apparently integrated the design functions of an enterprise in other aspects so that the information related to costs can be precisely provided for design purpose. Any inadequate design might possibly increase the cost due to the redesigning efforts and this leads to the delay in product realization.
- In comparison with single design schemes, the assessment of serialized product design schemes is more difficult and complicated. When making a decision on a product by different approaches, there is always a certain portion of incomplete grey area that exists during the process.
- A decision-making method for the assessment of designs based on the grey concept was proposed in this study by integrating linguistic variables.

Contribution of this paper to the literature

- The main contribution of this study is to obtain a reference priority for the design and development of products. The optimal alternative can be found by resolving the practical problems of a design scheme.
- The validity of the product decision-making model that was proposed in this study has been verified by the case study of serialized designs. The results indicated that this approach can effectively identify the priority of serialized design schemes under the condition of design serialization.
- With the integration of industries and enterprises, this study proposed an approach that can sustain product serialization by adopting sales information on the market. By filtering various product design strategies, different improvement resolutions can be prioritized for the manufacturability assessment of single-type products for further selection and comparison.

easily assembled. This approach reduces the time required during the process of assembling products and reduces the cycle time as well. A few assessment criteria can be used to verify the effect of this approach and they have to comply with the limitation of each stage. An ideal assessment or decision needs to meet each of the design requirements and this process forms a MADM problem. The main objective of the MADM approach is to accurately forecast issues such as production volumes. This method can support production lines and produce a variety of products with a high efficiency (Hibino, et al., 2014; HSIAO, et al., 2016; Huang, et al., 2016).

MADM is an essential part of evaluation and decision-making because it focuses on solving the arrangement problems of multiple attribute design solutions. Because of decision-makers' experiences and the uncertainty of various requirements, the messages acquired are usually undefined. Thus, MADM is also classified to groups of decision-making under uncertainty. In order to resolve the problem of decision-making under uncertainty, the concept of fuzzy set theory could be integrated by using fuzzy decisions to determine design solutions (Hong and Choi, 2000). MADM uses various types of assessment approaches based on the fuzzy set theory to determine the importance of group decision framework. When the fuzzy set theory is used for different ratings and comparisons of risk endurance, the group decision framework can help enterprises determine the priority of design requirements so as to enhance client satisfaction (Liu, 2008; Zhong, 2016; Lo, 2016). It could also be combined with fuzzy synthetic decision models by integrating decision-making models to resolve problems (Zha, et al., 2008). As for the high similarity of evaluation and decision-making among design concepts, Szmidt and Kacprzyk (2002) proposed a method: using intuitionistic fuzzy preference relations such as intuitionistic fuzzy core and consensus winner etc. to help make decisions. For the certainty of attribute weight, Atanassov, Pasi, and Yager (2005) suggested using an algorithm to provide accurate attribute numbers. Li (2005) thought that we should investigate vague data first, and then build linear programming models so as to get the optimal weight attribute. Another way to clearly label alternative attribute was to use vague group to represent how satisfied the alternatives are to the alternative features (Liu and Wang, 2007). It is known from the above-mentioned references that MADM has several evident results. However, typical decision-making approaches still focus mainly on the evaluation among different design solutions while other related researches remain uncertain when it comes to product series evaluation of specific design solutions. Serialized product consists of several similar products which might be in different sizes but have the same appearances and features, or in the same size but with different functions and colors. Generally speaking, each product series is usually diversified in order to meet various market requirements (Alizon, et al., 2007; Hsueh 2016). For example, for a series of wheel set products, they are similar in appearance but different in size. Cars in the same series might have the same size and appearance but different interior equipment or colors. The concept of design manufacturability has apparently integrated the design functions of an enterprise in other aspects so that the information related to costs can be precisely provided for design purpose. Any inadequate design might possibly increase the cost due to the redesigning efforts and this leads to the delay in product realization. The purpose of the three procedures in this study is to reduce the cost for designing a product. The design units are examined based on the difference between the optimized cost and the functional cost. After the ranking of the design parameters of a unit is obtained, the optimal design can be determined based on the hierarchical configuration (Lee, 2011; Lee, 2016; Chan, 2015). The result of the study by Mortensen et al. (2010) proposed an overall framework for planning product series. This framework supports the verification of the structure of customized products and services provided by a company. The framework has five consecutive aspects, which include the market, products, supply and production, organizations, and operation process. The collaborative innovation and opportunities can help strengthen the network between organizations for learning together, resource distribution and organizational innovation for sales so as to generate positive influences (Tsai, 2016; Tu, 2016). In a way of integration and subject concepts, this approach can resolve the problems that are related to engineering designs and develop the capability of various types of tasks for managing information and techniques (Meng, 2014). Ramanujam (2011) found a relationship between the cutting velocity, feed rate, and cutting depth and the specific power along with the surface roughness. The grey relational analysis was used to determine the optimal machining parameter for identifying the experimental results and the comparison between the studies of cutting tool wear analyses. The factors of innovative product development models affect customers' intention and they include the perceived usefulness, perceived accessibility, and the use of danger factors and variables (Hsu, 2016; Weng, 2016). Modeling assignments have been used to stimulate participants' recollection and to construct the ideas that are related to the change of concepts (Sahin, 2015). An evaluation is related to subjective inaccuracy and fuzziness, and the multi-criteria group decision model. A selection depends on the evaluation target and this explained that apparent structural relationship of adequate evaluation indices exists between the evaluation indices and the target (Chang, 2016; Ulusoy, 2014).

So far, the frequently used decision-making methods in the design field include the evaluation matrix, AHP, grey, fuzzy, TOPSIS, etc. The evaluation matrix is one of MADM approaches. It is mainly used for determining the mutual relationship between undecided solutions, competitive solutions, and the relevant design factors. By using quantitative decision-making analysis to filter out the optimal solution, the strength and weakness could be compared to the actual products in the market (Pugh, 1991; Hsueh, 2016). Other methods such as AHP, Grey, Fuzzy, and TOPSIS have been proved to effectively be used in making decisions from various solutions (Aguilar-Lasserre, et al., 2009; Hsiao and Tsai, 2005). Nonetheless, if the factors such as the return on investment and humanities need to be considered during the decision-making process of a design, the entire evaluation process would result in insufficient and incomplete information. However, the currently existing decision-making methods are hard to make up for this drawback (Liu, 2008; Lo, 2016).

Grey Theory uses the concept of "grey" to describe data features in reality. If the data is complete, it's called "white." On the contrary, the data is called "black" when it's insufficient, and it's called "grey" when the data is incomplete. Grey is namely between black and white. We could use the degree of grey color to indicate the completion of data in any grey event. The so called "incomplete data" means (1) unsure system factors (2) unclear factor relations (3) unclean system structures (4) unknown system principles. The problem of making a decision usually has to take multi-attributes into account including the mutual influence between various factors. Decision-makers usually have to make an optimal decision which could fit in with the current situation under the circumstances of limited resources and insufficient data. And Grey Theory is effective in dealing with data (Li et al., 2007). Besides considering multi-attributes in product design, users' feelings and their linguistic expressions are also involved. Linguistic variable can be used to evaluate the importance of users' requirements and to select the level of new products (Karsak, 2004; Lai, et al., 2006). Through Grey number to convey linguistic variable has thought to be closer to uses' feelings.

Zhang et al. (2005) proposed Grey relational analysis in which the uncertainty of multi-attributes could be reflected by interval. It's usually used in industrial project system assessment. Zhai et al. (2009) used the grey relational analysis for the evaluation of design concepts. Hsu et al. (2012) modified the original grey relational analysis by using the analytical network process (ANP) in order to assist suppliers in making decisions. In the meanwhile, Li et al. (2007) and Lin et al. (2008) proposed a method to calculate the grey probability that is expressed by linguistic grey numbers. The method was also successfully utilized to assist

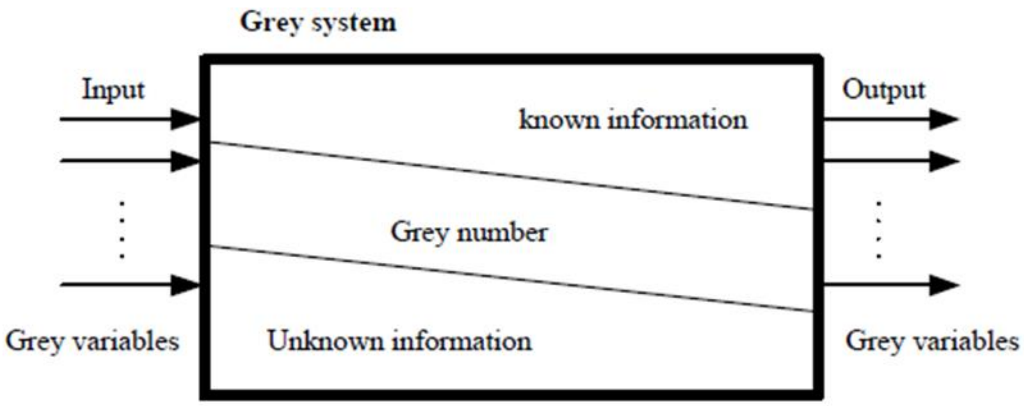

Figure 1. The concept of grey system

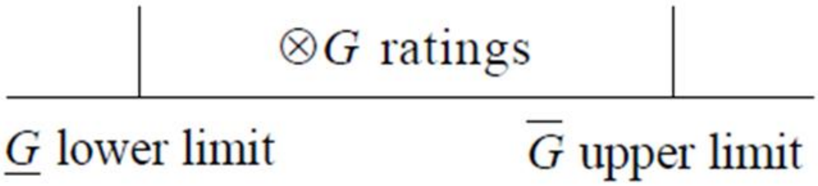

Figure 2. The concept of grey system

suppliers in making decisions. García-Lapresta (2006) used grey numbers to represent linguistic variables and to make decisions by majority. Xu (2004) suggested listing the priority among solutions by linguistic variables during a group decision-making process. The preferences of decision-makers should also be considered as the first priority during the design process. For the assessment of product series designs, those which are more difficult and complex should be assessed firstly. When making decisions, it is known that each product has many portions that are incomplete grey areas with a certain degree of similarity. Therefore, based on the criteria of assessing grey concepts, this study proposed an approach which integrates linguistic variables when making design decisions. It resolved the problem of making decisions when assessing a series of product designs. The design of pneumatic door closers was selected as a case study for further experiments.

GREY NUMBER

The concept of a grey system, which is a system consisted of grey numbers and grey variables, includes unknown information as shown in **Figure 1**.

After the uncertain region is limited by the upper and the lower limit, the size of the grey number region is determined by Eq. (1).

$$\otimes G = [\underline{G}, \overline{G}] \tag{1}$$

Four fundamental calculations of the grey numbers $\otimes G_1 = [\underline{G_1}, \overline{G_1}]$ and $\otimes G_2 = [\underline{G_2}, \overline{G_2}]$ are defined as Eq. (2)~(5).

$$⊗G_1+⊗G_2 = [\underline{G}_1 + \underline{G}_2, \overline{G}_1 + \overline{G}_2] \tag{2}$$

$$⊗G_1-⊗G_2 = [\underline{G}_1, \overline{G}_2 - \overline{G}_1, \underline{G}_2] \tag{3}$$

$$⊗G_1\times⊗G_2 = \left[\min\left(\underline{G}_1\underline{G}_2, \underline{G}_1\overline{G}_2, \overline{G}_1\underline{G}_2, \overline{G}_1\overline{G}_2\right), \max\left(\underline{G}_1\underline{G}_2, \underline{G}_1\overline{G}_2, \overline{G}_1\underline{G}_2, \overline{G}_1\overline{G}_2\right)\right] \tag{4}$$

$$⊗G_1\div⊗G_2 = [\underline{G}_1, \overline{G}_1]\times\left[\frac{1}{\underline{G}_2}, \frac{1}{\overline{G}_2}\right] \tag{5}$$

Grey number $⊗G=[\underline{G}, \overline{G}]$ on the upper and the lower limits is a continuous linear function. Rewrite it as Eq. (6).

$$f(t) = \underline{G} + \left(\overline{G} - \underline{G}\right)t, 0 \le t \le 1 \tag{6}$$

When t=0, $f(t)$ is equal to the $⊗G$ lower limit value. Moreover, when t=1, $f(t)$ is equal to the $⊗G$ upper limit. Rewrite Eq. (6) as Eq. (7).

$$f(t) = p + rt, 0 \le t \le 1 \tag{7}$$

Assuming a function set m_0, and for $\mathrm{g} \in m_0$, $f(t) = p_1 + r_1 t$. $\mathrm{g}(t) = p_2 + r_2 t, 0 \le t \le 1$. Therefore, the related addition operation is expressed as Eq. (8).

$$f + \mathrm{g} = (p_1 + p_2\ )\ +\ (r_1 + r_2)t, 0 \le t \le 1 \tag{8}$$

For the subtraction of two functions, the addition function can be presented as + (-g). Moreover, $-\mathrm{g} = -(p_2 + r_2\ )\ +\ r_2 t$, thus the subtraction of two functions can be presented as Eq. (9).

$$f - \mathrm{g} = p_1 - (p_2 + r_2\ )\ +\ (r_2 + r_2)t, 0 \le t \le 1 \tag{9}$$

The interval of the grey number $⊗G$ is defined by Eq. (10).

$$L(⊗\ G) = [\overline{G} - \underline{G}] \tag{10}$$

The grey number whitening function as shown in Eq. (11) presents the whitening value from the grey number.

$$⊗(x) = \alpha\times\underline{G} + \beta\times\overline{G} \tag{11}$$

and $\alpha, \beta \in [0,1]$.

GREY NUMBER DESIGN EVALUATION

This study applies grey related analysis method to the evaluation of a series of pneumatic door closer designs. The evaluation criteria, weighted values, and linguistic variables need to be identified prior to the evaluation. The goal of this evaluation is to find the factors that can effectively satisfy series product design project in multiple attributes decision-making and prioritize the design options.

First, the target project for evaluation is denoted by S. A series of product design cases have a total of m designing solutions and it is denoted by S_m as shown in Eq. (12).

Table 1. The scale of attribute weights $\otimes w$

Scale	$\otimes w$
(7) Very high	[0.9,1.0]
(6) High	[0.6,0.9]
(5) Medium high	[0.5,0.6]
(4) Medium	[0.4,0.5]
(3) Medium low	[0.3,0.4]
(2) Low	[0.1,0.3]
(1) Very low	[0.0,0.1]

Table 2. The scale of attribute rating $\otimes G$

Scale	$\otimes G$
(1) Very poor	[0, 1]
(2) Poor	[1, 3]
(3) Medium poor	[3, 4]
(4) Fair	[4, 5]
(5) Medium good	[5, 6]
(6) Good	[6, 9]
(7) Very good	[9, 10]

$$S = \{S_1, S_2, \ldots, S_m\} \tag{12}$$

Under the S_m design cases, there is number n of single product solutions, which are denoted by S_{mn} as shown in Eq. (13).

$$S_m = \{S_{m1}, S_{m2}, \ldots, S_m\} \tag{13}$$

The evaluation criteria of a product designing case were determined by the focused group members who are designers, related manufacturers, or marketing salespeople. It is mainly used for evaluating the advantageous and disadvantageous levels of the designing solutions. The set of design solutions is denoted by Q_a as shown in Eq. (14).

$$Q = \{Q_1, Q_2, \ldots, Q_a\} \tag{14}$$

Within our case study, each evaluation criterion has its own weight. The weight of each evaluation criterion is within the scale of 1 to 7. The weighting in the studying case is related to user feelings and linguistic expressional questions. Therefore, each scale of the weight is denoted by $\otimes w$. As shown in **Table 1**. There is a set of evaluation criteria and the weight of each criterion for evaluation is denoted by $\otimes w_a$ as shown in Eq. (15).

$$\otimes w = \{\otimes w_1, \otimes w_2, \ldots, \otimes w_a\} \tag{15}$$

The evaluation result of each evaluation criterion is denoted by a value in the range of scale 1 to 7. The grey number of each scale is denoted by $\otimes G$ as shown in **Table 2**.

The sales volume of a product might have some difference within the product solution of each design case S. A higher sales volume means quicker return on investment and related sales information can be obtained from the sales department. As for the investment priority weight in each single item of products, there are n single product solutions and they are denoted by W_n as shown in Eq. (16).

$$w = \{w_1, w_2, \dots, w_n\} \tag{16}$$

Step 1

First, calculate the weight of each evaluation criterion for the design case. The weight of an evaluation criterion is evaluated and defined by the focused group members. Assuming there are K focused group members to provide a number of j evaluation criteria on weight ranking, the evaluation of person No. K to the j criterion is $\otimes w_j^k = [\underline{w}_j^K, \overline{w}_j^K]$. Each criterion has $\otimes w_j^k = (j = 1,2, \dots\dots, a)$ points in total. We used Eq. (17) to represent the equation of weight $\otimes w_j$ of item. Q_j.

$$\otimes w_j = \frac{1}{K}[\otimes w_j^1 + \otimes w_j^2 + \ldots + \otimes w_j^K] \tag{17}$$

Step 2

Calculate and obtain the member of K within single product solution S_{mn}. Obtain the grey numbers from a number of evaluation criteria as shown in Eq. (18).

$$S_{mn} = \begin{bmatrix} \otimes G_{11} & \otimes G_{12} & \cdots & \otimes G_{1a} \\ \otimes G_{21} & \otimes G_{22} & \cdots & \otimes G_{2a} \\ \vdots & \vdots & \ddots & \vdots \\ \otimes G_{K1} & \otimes G_{K2} & \cdots & \otimes G_{Ka} \end{bmatrix} \tag{18}$$

Grey number $\otimes G^a$ from S_{mn} evaluation criteria can be simplified as Eq. (19). Moreover, rewrite Eq. (18) as Eq. (20).

$$\otimes G^a = \frac{1}{K}[\otimes G_{1a} + \otimes G_{2a} + \ldots + \otimes G_{Ka}] \tag{19}$$

$$S_{mn} = [\otimes G^1 \ \otimes G^2 \ \cdots \otimes G^a] \tag{20}$$

In addition to the weight of the evaluation criteria $\otimes w_j$, Eq. (21) can be obtained as well.

$$S^*{}_{mn} = [\otimes G^1 \ \otimes G^2 \ \cdots \otimes G^a] \times \begin{bmatrix} \otimes w_1 \\ \otimes w_2 \\ \vdots \\ \otimes w_a \end{bmatrix} \tag{21}$$

Rewrite the equation as Eq. (22).

$$S^*{}_{mn} = [\otimes G^*_{mn}] \tag{22}$$

Step 3

The grey evaluation matrix S for a series of product solutions is denoted by Eq. (23).

$$S^{**} = \begin{bmatrix} S_{11}^{*} & S_{12}^{*} & \cdots & S_{1n}^{*} \\ S_{21}^{*} & S_{21}^{*} & \cdots & S_{2n}^{*} \\ \vdots & \vdots & \ddots & \vdots \\ S_{m1}^{*} & S_{m1}^{*} & \cdots & S_{mn}^{*} \end{bmatrix} \tag{23}$$

Step 4

To normalize the series product solution S and the grey evaluation matrix of the evaluation criteria, each factor from the matrix is normalized and denoted by Eq. (24).

$$\otimes G^{**}{}_{mn} = \left[\frac{\underline{G}_{mn}^{*}}{G_n^{*max}}, \frac{\overline{G}_{mn}^{*}}{G_n^{*max}}\right] \tag{24}$$

Assuming $G_n^{*max} = max_{1 \le i \le m}\left\{\overline{G}^{*}{}_{in}\right\}$, the normalization leads to grey numbers in the range of [0, 1]. The normalized grey decision matrix S** can be obtained and denoted by Eq. (25).

$$S^{**} = \begin{bmatrix} \otimes G_{11}^{*} & \otimes G_{12}^{*} & \cdots & \otimes G_{1n}^{*} \\ \otimes G_{21}^{*} & \otimes G_{22}^{*} & \cdots & \otimes G_{2n}^{*} \\ \vdots & \vdots & \ddots & \vdots \\ \otimes G_{m1}^{*} & \otimes G_{m2}^{*} & \cdots & \otimes G_{mn}^{*} \end{bmatrix} \tag{25}$$

Step 5

Calculate the investment priority weight W from normalized evaluation criteria matrix to obtain the adjusted evaluation matrix $\otimes D$ as Eq. (26).

$$\otimes \mathrm{D} = \begin{bmatrix} \otimes V_{11} & \otimes V_{12} & \cdots & \otimes V_{1n} \\ \otimes V_{21} & \otimes V_{22} & \cdots & \otimes V_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \otimes V_{m1} & \otimes V_{m1} & \cdots & \otimes V_{mn} \end{bmatrix} \tag{26}$$

Here, $\otimes V_{ij} = \otimes G_{ij} \times W_j$. By using Eq. (11) that is the grey whitening function and assuming α=β=0.5, the advanced grey numbers by weight whitening, $V_{ij} = \otimes (V_{ij})$ resulted in Eq. (27).

$$\mathrm{D} = \begin{bmatrix} V_{11} & V_{12} & \cdots & V_{1n} \\ V_{21} & V_{22} & \cdots & V_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ V_{m1} & V_{m1} & \cdots & V_{mn} \end{bmatrix} \tag{27}$$

The highest decision value of each single solution is the optimal decision for each item as $V_n^{max} = \max_{1 \le i \le m}\{V_{in}\}$ and the size of ranking is the advantage and disadvantage ordering of V_{in} each single solution.

GREY RELATIONAL ANALYSIS

Step 1: Determine the ideal optimal target series

Carry out the grey related analysis of the series of product solutions so as to obtain each solution's decision vale V_n^{max}. The ideal optimal target series is denoted by $S0$ as shown in Eq. (28).

$$S_0 = \{V_1^{max}, V_2^{max}, \ldots, V_n^{max}\} \quad (28)$$

Step 2: Calculate grey relational coefficients

Initialize the value of the sequence in each solution. Calculate the grey relational coefficients within the sequence by Eq. (29).

$$\xi_i\,(k) = \frac{\min_i \min_k |S_0(k) - S_i(k)| + \rho \max_i \max_k |S_0(k) - S_i(k)|}{|S_0(k) - S_i(k)| + \rho \max_i \max_k |S_0(k) - S_i(k)|} \quad (29)$$

where ρ is identified coefficient and $0 \le \rho \le 1$. Here we take $\rho = 0.5$.

Step 3: Calculate Grey relation

Calculate the grey relation between each evaluation solution $S_i = \{S_1, S2, \ldots, S_m\}$ and the ideal target solution S_0.

Calculate the grey relation between each solution by Eq. (30) which is the average of grey relational coefficients $\xi_i\,(k)$.

$$r_i = \frac{1}{n}\sum_{k-1}^{m} \xi_i(k),\ i = 1,2 \ldots, n \quad (30)$$

Step 4: Prioritize the results

Finally, we prioritized the results from big to small according to the grey relations between each solution and the optimal solution S0. After that, we obtained the decision-making ranking of all solutions. A higher grey relation value indicates the solution is closer to the ideal one.

CASE STUDY

The purpose of the case study is to investigate the effects of the multi-attribute assessments on products of single-type designs. Among the design options to be evaluated, the overall assessment of each design option should be completed before the ranking of product design options. Pneumatic door closers were selected as the target product for the investigation in this study. The pneumatic door closers that are available on the market can be classified by five design criteria. Moreover, the anticipated sales volume of each type may possibly be different. For a company, the products to be manufactured are usually prioritized based on their sales volume projections and the anticipated return on investment. As a result, the assessment of a series of product design options should be based on the real demands for

Table 3. Design options of pneumatic door closers

Reaction force (20℃) of gas springs at the maximum length of 10mm ((N denotes the required reaction force (during elongation)					
	263(N) S_{m1}	**147(N) S_{m2}**	**88(N) S_{m3}**	**79 (N) S_{m4}**	**69 (N) S_{m5}**
S1					
S2					
S3					

the company. After further investigation, the target pneumatic door closers are classified into five types which include 263(N), 147(N), 88(N), 79 (N), and 69 (N).

Determine grey numbers for design evaluation

Step 1: The design options and decision criteria of the pneumatic door closers in this study can be classified into three types which are denoted by $S = \{S_1, S_2, S_3\}$ as shown in Table 3. Each type includes five product criteria which are denoted by $S_m = \{S_{m1}, S_{m2}, S_{m3}, S_{m4}, S_{m5}\}$.

Table 4. Weights of each evaluation criterion

Product designer	P1	P2	P3	P4	P5	P6	P7	Weight $\otimes w_j$
Mobility	7	6	7	6	5	5	6	[0.642,0.800]
Manufacturability	6	6	5	7	5	6	7	[0.657, 0.843]
Product form	7	6	5	6	5	7	4	[0.629, 0.785]
Creativity	6	5	6	7	6	5	4	[0.585, 0.771]
Cost	5	4	4	6	6	7	5	[0.557, 0.714]

Seven professional product designers in the evaluation team screened out the design assessment criteria. Each of the seven professional product designers has more than ten years of design experiences. The evaluation team was formed by these seven senior product designers with abundant experiences. They considered various types of customer requirements and ranked the design options based on their practical sales experience. These five assessment criteria include the mobility, manufacturability, product form, creativity, and cost which are denoted by $Q = \{Q_1, Q_2, Q_3, Q_4, Q_5\}$. The assessment matrix was generated and the grey relational analysis was conducted to determine the priority of product series as shown in **Table 3**.

Step 2: Calculate the grey weights of the assessment criteria. The same professional product designers conducted assessments on the five criteria including mobility, manufacturability, product form, creativity, and cost so as to determine the individual weight. Calculate the current weights by Eq. (17) along with the assessment criteria $\otimes w_j$ as shown in **Table 4**.

Step 3: Determine the grey assessment results, i.e., $S = \{S_1, S_2, S_3\}$ according to the results obtained from the evaluation conducted by those seven professional product designers. The results are shown in **Table 5**.

Step 4: Calculate the grey assessment results of the target product series. An example of the S_{11} 263(N)-Model 1 is shown in **Table 6**, which is based on the ranking conducted by seven professional product designers according to the scale as shown in **Table 2**. For the product series S_1 No.11, the assessment criteria include five categories and the results obtained by following Step 1 are shown in **Table 6**.

The resulting grey numbers are (6.000, 7.286), (5.571, 6.571), (5.142, 6.428), (5.857, 7.714), and (4.857, 6.142) respectively. With the addition of the design evaluation criteria weight $\otimes w_j$, we can obtain S_{11}^* =[27.427, 34.141].

Step 5: Build the Grey evaluation matrix of product series solutions in each of the product solutions. Calculate the scores of every single product. Build the grey evaluation matrix S^* of product series solutions as shown in **Table 7**.

Table 5. Ranking of the evaluation criteria for five design options

Product designer		Mobility					Manufacturability					Styling					Creativity					Cost				
		M1	M2	M3	M4	M5	M1	M2	M3	M4	M5	M1	M2	M3	M4	M5	M1	M2	M3	M4	M5	M1	M2	M3	M4	M5
S_1	K1	6	6	6	7	5	4	5	5	7	6	4	5	6	4	4	5	3	4	5	7	5	5	6	6	5
	K2	5	4	7	6	6	7	5	5	6	5	3	6	5	7	6	4	5	6	6	5	4	5	5	6	4
	K3	7	5	6	5	6	7	5	4	5	5	7	6	4	5	6	5	5	4	3	5	4	5	7	6	4
	K4	5	4	6	5	5	4	6	5	3	6	5	6	7	5	5	6	4	5	6	7	5	6	4	7	5
	K5	5	6	6	6	5	5	6	7	5	6	6	5	4	4	7	6	5	7	5	7	6	5	5	6	6
	K6	5	6	7	5	6	4	5	7	6	6	5	6	6	3	6	7	6	3	5	4	5	6	6	6	5
	K7	6	6	7	6	6	4	6	5	4	5	4	5	5	4	5	6	5	7	6	5	5	5	6	7	6
S_2	K1	4	5	7	3	3	5	3	6	5	6	6	7	7	6	6	5	6	6	7	5	6	5	4	5	6
	K2	6	5	6	4	3	7	5	6	6	5	4	5	5	4	5	4	5	5	7	6	5	6	7	6	5
	K3	6	5	3	6	5	5	3	6	6	3	4	5	6	5	6	4	7	7	6	7	5	6	5	6	6
	K4	6	7	5	6	6	5	4	5	6	5	3	6	6	6	5	6	5	5	5	7	6	5	7	5	6
	K5	5	7	6	6	5	3	6	5	6	5	6	6	7	5	5	5	6	6	7	5	6	5	7	6	5
	K6	5	7	6	5	7	5	6	6	5	4	5	5	6	5	6	3	5	5	4	3	5	4	3	6	5
	K7	6	6	7	5	5	6	6	5	4	6	5	3	6	5	4	4	6	4	5	5	4	7	5	6	6
S_3	K1	5	5	7	6	6	6	5	6	6	6	7	5	6	6	7	6	6	5	7	6	5	6	3	6	6
	K2	5	4	6	5	4	5	4	5	5	5	7	6	6	5	5	6	7	7	5	3	4	5	4	5	6
	K3	6	6	6	7	6	5	4	5	6	5	4	6	3	6	5	5	6	5	7	6	6	3	7	5	6
	K4	5	5	5	7	4	5	4	6	6	6	6	6	5	6	6	5	4	6	5	4	5	5	5	3	6
	K5	5	4	6	5	5	4	3	5	4	5	5	5	3	6	6	5	6	7	6	5	6	6	5	4	5
	K6	6	6	6	6	4	5	5	4	7	6	5	4	4	5	6	5	6	4	6	6	5	5	3	6	5
	K7	5	5	5	6	5	7	6	5	6	7	6	6	5	6	6	6	5	7	6	5	7	5	6	7	5

Table 6. Grey numbers obtained by five evaluation criteria for S_1 263(N)-Model 1

Product designer	K1	K2	K3	K4	K5	K6	K7	$\otimes G^a$
Mobility	[6, 9]	[5, 6]	[9,10]	[5, 6]	[5, 6]	[5, 6]	[6, 9]	[6.000, 7.286]
Manufacturability	[4, 5]	[9,10]	[9,10]	[4, 5]	[5, 6]	[4, 5]	[4, 5]	[5.571, 6.571]
Product form	[4, 5]	[3, 4]	[9,10]	[5, 6]	[6, 9]	[5, 6]	[4, 5]	[5.142, 6.428]
Creativity	[5, 6]	[4, 5]	[5, 6]	[6, 9]	[6, 9]	[9,10]	[6, 9]	[5.857, 7.714]
Cost	[5, 6]	[4, 5]	[4, 5]	[5, 6]	[6, 9]	[5, 6]	[5, 6]	[4.857, 6.142]

Table 7. Grey evaluation criteria matrix S^* of product series

	1	2	3	4	5
S_1	[27.427, 34.141]	[26.285, 35.285]	[30.428, 38.285]	[28.285, 37.000]	[28.571,37.285]
S_2	[25.285,33.428]	[32.000, 41.000]	[30.571,39.285]	[28.857, 38.571]	[27.714, 36.285]
S_3	[28.285, 36.142]	[25.857, 34.571]	[27.571, 35.428]	[30.285, 40.142]	[27.571, 37.142]

Step 6: Normalize the grey evaluation matrix in **Table 7**. Normalize the factors in the matrix and build the grey evaluation matrix ** S of normalized product series solutions as shown in **Table 8**.

Table 8. Grey evaluation matrix S^{**} of normalized product series

	1	2	3	4	5
S_1	[0.759, 0.945]	[0.641, 0.861]	[0.775, 0.975]	[0.705, 0.922]	[0.766, 1.000]
S_2	[0.700, 0.925]	[0.780, 1.000]	[0.778, 1.000]	[0.719, 0.961]	[0.743, 0.973]
S_3	[0.783, 1.000]	[0.631, 0.843]	[0.702, 0.902]	[0.754, 1.000]	[0.739, 0.996]

Table 9. Weights of five design models

	1	2	3	4	5
Sales volume ratio	1	1	2	4	8
Weight	W_1	W_2	W_3	W_4	W_5

Table 10. Grey evaluation matrix $\otimes D$ with the prioritized weights according to the sales volume of each model.

	1	2	3	4	5
S_1	[0.759, 0.945]	[0.641, 0.861]	[1.550, 1.950]	[2.820, 3.688]	[6.128, 8.000]
S_2	[0.700, 0.925]	[0.780, 1.000]	[1.556, 2.000]	[2.876,3.844]	[5.944,7.784]
S_3	[0.783, 1.000]	[0.631, 0.843]	[1.404, 1.804]	[3.016, 4.000]	[5.912,7.968]

Table 11. Evaluation matrix D of the whitened product series

	1	2	3	4	5
S_1	0.852	0.751	1.750	3.254	7.064
S_2	0.812	0.890	1.778	3.360	6.864
S_3	0.891	0.737	1.604	3.508	6.940

Step 7: Specify the weight of each of the design combinations. This is the respective sales volume ratio of each of the pneumatic door closers. As shown in **Table 9**, the ratio of the smallest sales volume is 1, followed by an increasing geometric series.

Step 8: Apply the weights in **Table 9** to adjust the values in **Table 7**. First, the values in **Table 9** were transformed by the scales in **Table 1**. After that, add the adjusted weights into the normalized evaluation matrix in **Table 7**. This results in a modified evaluation matrix $\otimes D$ as shown in **Table 10**.

Step 9: Take the whitening procedure to calculate the grey numbers and put the results into **Table 11**, which is the whitened evaluation matrix D.

Step 10: Obtain the resulting grey numbers after design evaluation

The evaluation scores of each product series solution can be obtained from the whitened evaluation matrix in **Table 11**. After that, the ideal results of all solutions are listed in **Table 12**.

Table 12. Advantageous and disadvantageous ranking of each product series

Item	Order	Highest score
1	$S_{31} > S_{11} > S_{21}$	0.891
2	$S_{22} > S_{12} > S_{32}$	0.890
3	$S_{23} > S_{13} > S_{33}$	1.778
4	$S_{34} > S_{24} > S_{14}$	3.508
5	$S_{15} > S_{35} > S_{25}$	7.064

Table 13. Initialized values of each solution series

	1	2	3	4	5
S_0	1.000	1.000	1.000	1.000	1.000
S_1	0.956	0.844	0.984	0.928	1.000
S_2	0.911	1.000	1.000	0.958	0.972
S_3	1.000	0.828	0.902	1.000	0.982

Table 14. Variances between the ideal optimal targets in product series

	1	2	3	4	5	$\min_{k} \mid \Delta_i (k) \mid$	$\max_{k} \mid \Delta_i (k) \mid$
$\Delta_1 (k)$	0.044	0.156	0.016	0.072	0.000	0	0.156
$\Delta_2 (k)$	0.089	0.000	0.000	0.042	0.028	0	0.089
$\Delta_3 (k)$	0.000	0.172	0.098	0.000	0.018	0	0.172
$\min_{i} \min_{k} \mid \Delta_i (k) \mid$						0	
$\max_{i} \max_{k} \mid \Delta_i (k) \mid$							0.172

Grey relational analysis

Step 1: Generate the ideal optimal target ranking

After analyzing product series solution by the grey relational analysis, the highest scores of the product series V_n^{max} can be obtained after the evaluation as shown in **Table 12**. The scores can be combined so as to generate the ideal optimal target ranking as $S_0 = \{0.891, 0.890, 1.778, 3.508, 7.064\}$.

Step 2: Calculate grey relations

Calculate the grey relations between each of the evaluation solutions $S_i = \{S_1, S_2, \ldots, S_m\}$ and the ideal optimal target alternative S_0. First of all, initialize the values of the alternative sequence as in **Table 13**.

Step 3: Determine the variances between the ideal optimal targets.

This step is to calculate the variances between the ideal optimal targets as shown in **Table 14**.

Step 4: Calculate the grey relational coefficient $\xi_i (k)$ within the sequences and the grey relation r_i

Table 15. $\xi_i\ (k)$ and r_i between product series

	1	2	3	4	5	r_i
ξ_1	0.163	0.355	0.843	0.544	1.000	0.5810
ξ_2	0.491	1.000	1.000	0.672	0.754	0.7834
ξ_3	1.000	0.333	0.467	1.000	0.827	0.7254

Calculate the grey relational coefficient $\xi_i\ (k)$ in Eq. 29 within the sequences and the grey relation r_i in the solutions. Therefore, we can determine the average of the grey relational coefficient $\xi_i\ (k)$, which are $r_1 = 0.5810, r_2 = 0.7834, r_3 = 0.7254$ respectively.

Step 5: Propose the result of Grey relational analysis

Finally, we rank the solution models in product series in a sequence from the lowest to the highest priority according to its grey relation between each solution and the optimal target solution S_0. A higher value of grey relation indicates the solution model is closer to the ideal solution.

The results of the evaluation on the series of pneumatic door closer models indicated that $\mathrm{r}_2 > \mathrm{r}_1 > \mathrm{r}_3$. That is to say, $S_2 > S_1 > S_3$.

CONCLUSIONS AND SUGGESTIONS

This study proposed the application of grey design to the assessment of product designs. The approach of conducting the grey relational analysis of variables by assessment criteria can be realized by grey number calculations. The priority of product designs can be obtained as a reference for further development. Solution S_2 is the optimal alternative in real sales. After the verification of the pneumatic door closer product series, it is known that the optimal ranking is $S_{22} > S_{23} > S_{25} > S_{24} > S_{21}$ for mass production and this can also be verified from the assessment matrix of the product series as shown in **Table 11**. For the production of single product type, Model S_2 is the optimal solution, followed by Model S_3. The contribution of this study to product design optimization is as follows.

1) Build a design assessment model by grey numbers

Determining the optimal design among product series is more complicated than the design optimization of single product. With a case study on pneumatic door closer series designs, this study proposed an approach of design assessment based on grey numbers and the grey relational analysis was used to determine the optimal design among product series.

2) Confirm the validity of design solution optimization

Under the condition of serialized designs, the approach that was proposed in this study can be used to effectively verify the priority of alternative design solutions. In addition, the priority of all of the design models within a complete solution can be identified effectively.

3) Propose design weights to be referenced by sales improvement on the diversified market

For product marketing and sales on a real market, the conventional approach of assessing design alternatives is no longer enough without eliminating the design alternatives with a lower weight. In this study, we proposed an approach for scholars to collaborate with enterprises in screening out product designs for modification based on the sales volume on the market. The results of this study indicated that a design can better meet real market requirements if the design weights are taken into consideration.

On the basis of this study, researchers are recommended to conduct further studies on the topics that are described as follows.

1. From the aspect of grey system, this study utilized only the grey statistics, grey decision, and grey relational analysis. Other approaches such as grey prediction, grey modeling, grey clustering, and grey entropy weighting approaches are not included in this study. For future developments, it is recommended to incorporate these approaches into the design and development process and designers will benefit a lot from those approaches when carrying out analyses for design decisions.

2. The sample size required by the grey system theory is relatively smaller and thus this approach is the most suitable one for any design assessment which contains a smaller amount of samples. Researchers are recommended to code the entropy weighting approach and the grey relational analysis in program in future research by further incorporating these theories into related analyses.

3. For the future scope of work, it is recommended to include the grey relational analysis into the evaluation models of consumers and different types of products can be further investigated as an extension to this work. Moreover, the multi-criteria decision-making (MCDM) method can be applied in combination with other methods so as to facilitate the relevant research.

REFERENCES

Aguilar-Lasserre, A. A., Bautista Bautista, M. A., Ponsich, A., & González Huerta, M. A. (2009). An AHP-based decision-making tool for the solution of multiproduct batch plant design problem under imprecise demand. *Computers and Operations Research*, *36*(3), 711-736.

Alizon, F., Shooter, S. B., & Simpson, T. W. (2007). Improving an existing product family based on commonality/diversity, modularity, and cost. *Design studies*, *28*(4), 387-409.

Anoop, D., & Anil, M. (2010). Facilitating design for assembly through the adoption of a comprehensive design methodology, *International Journal of Industrial Engineering*, *17*(2), 92-102.

Atanassov, K., Pasi, G., & Yager, R. (2005). Intuitionistic fuzzy interpretations of multi-attribute multi-person and multi-measurement tool decision making. *International Journal of Systems Science*, *36*(14), 859-868.

Chan, K. K. (2015). Salient Beliefs of Secondary School Mathematics Teachers Using Dynamic Geometry Software. *Eurasia Journal of Mathematics, Science & Technology Education, 11*(1), 139-148.

Chang, T. C., & Wang, H. (2016). A Multi Criteria Group Decision-making Model for Teacher Evaluation in Higher Education Based on Cloud Model and Decision Tree. *Eurasia Journal of Mathematics, Science & Technology Education, 12*(5), 1243-1262.

Changjun, L., Wenlong, J., Liu, E., & Wu, X. (2012). A multi-hierarchy grey relational analysis model for natural gas pipeline operation schemes comprehensive evaluation. *International Journal of Industrial Engineering*, *19*(6).

García-Lapresta, J. L. (2006). A general class of simple majority decision rules based on linguistic opinions. *Information Sciences*, *176*(4), 352-365.

Hibino, H., Tanaka, K., Umezawa, Y., & Fukuda, Y. (2014). Method for production forecast in demand-synchronized production. *Journal of Advanced Mechanical Design Systems and Manufacturing*, *8* (6):JAMDSM0076.

Hong, D. H., & Choi, C. H. (2000). Multicriteria fuzzy decision-making problems based on vague set theory. *Fuzzy Sets and Systems*, *114*(1), 103-113.

Hsiao, S. W., & Tsai, H. C. (2005). Applying a hybrid approach based on fuzzy neural network and genetic algorithm to product form design. *International journal of industrial ergonomics*, *35*(5), 411-428.

Hsiao, S. W., Lin, M. H. & Hsiao, H. H. (2016). A product manufactures scheduling method based on the grey evaluation. *Journal of Advanced Mechanical Design, Systems, and Manufacturing, 10*(1).

Hsu, C. C., Liou, J. J. H., & Chuang, Y. C. (2012). Integrating DANP and modified grey relation theory for the selection of an outsourcing provider. *Expert Systems with Applications*, *40*(6), 2297-2304.

Hsu, M. W. (2016). An Analysis of Intention to Use in Innovative Product Development Model through TAM Model. *Eurasia Journal of Mathematics, Science & Technology Education*, *12*(3), 487-501.

Hsueh, S. L., & Kuo, C. H. (2016). Factors in an Interdisciplinary Curriculum for the Students of Industrial Design Designing Multifunctional Products. *Eurasia Journal of Mathematics, Science & Technology Education*, *12*(4), 1075-1089.

Huang, Y. C., Tu, J. C., & Hung, S. J. (2016). Developing a Decision Model of Sustainable Product Design and Development from Product Servicizing in Taiwan. *Eurasia Journal of Mathematics, Science & Technology Education*, *12*(5), 1285-1302.

Karsak, E. E. (2004). Fuzzy multiple objective programming framework to prioritize design requirements in quality function deployment. *Computers and Industrial Engineering*, *47*(2-3), 149-163.

Lai, H. H., Lin, Y. C., Yeh, C. H., & Wei, C. H. (2006). User-oriented design for the optimal combination on product design. *International Journal of Production Economics*, *100*(2), 253-267.

Lee, D. K. (2016). Analyzing Team Based Engineering Design Process in Computer Supported Collaborative Learning. *Eurasia Journal of Mathematics*, *Science & Technology Education*, *12*(4), 767-782.

Lee, H., & Lee, J. (2011). Optimal cost estimation for improvement of product design. *International Journal of Industrial Engineering, 18*(5), 232-243.

Li, D. F. (2005). Multiattribute decision making models and methods using intuitionistic fuzzy sets. *Journal of Computer and System Sciences*, *70*(1), 73-85.

Li, G. D., Yamaguchi, D., & Nagai, M. A. (2007). Grey-based decision-making approach to the supplier selection problem. *Mathematical and Computer Modelling*, *46*(3-4), 573-581.

Lin, Y. H., Lee, P. C., & Ting, H. I. (2008). Dynamic multi-attribute decision making model with grey number evaluations. *Expert Systems with Applications*, *35*(4), 1638-1644.

Liu, C. H. (2008). A Group Decision-Making Framework with Various Rating Attitudes Using Fuzzy Set Theory to Prioritize Design Requirements in Quality Function Deployment. *International Journal of Industrial Engineering*, *15*(3), 294-303.

Liu, H. W., & Wang, G. J. (2007). Multi-criteria decision-making methods based on intuitionistic fuzzy sets, *European Journal of Operational Research*, *179*(1), 220-233.

Lo, C. H. (2016). Building a Relationship between Elements of Product Form Features and Vocabulary Assessment Models. *Eurasia Journal of Mathematics, Science & Technology Education*, *12*(5), 1399-1423.

Meng, C. C. (2014). Secondary Students' Perceptions of Assessments in Science, Technology, Engineering, and Mathematics (STEM). *Eurasia Journal of Mathematics, Science & Technology Education*, *10*(3), 219-227.

Mortensen, N. H., Hvam, L., Haug, A., Boelskifte, P., Lindschou, C., & Frobenius, S. (2010). Making product customization profitable. *International Journal of Industrial Engineering*, *17*(1), 25-35.

Pugh, S. (1991). *Total Design: Integrated Methods for Successful Product Engineering*. Wokingham: Addison-Wesley.

Ramanujam, R., Muthukrishna, N., & Raju, R. (2011). Optimization of Cutting Parameters for Turning Al-SiC (10p) MMC Using ANOVA and Grey Relational Analysis. *International Journal of Precision Engineering and Manufacturing, 12*(4), 651-656.

Sahin, Z., Yenmez, A. A., & Erbas, A. K. (2015). Relational Understanding of the Derivative Concept through Mathematical Modeling: A Case Study. *Eurasia Journal of Mathematics, Science & Technology Education, 11*(1), 177-188.

Szmidt, E., & Kacprzyk, J. (2002). Using intuitionistic fuzzy sets in group decision making. *Control and Cybernetics*, *31*(4), 1037-1053.

Tsai, I. C., & Lei, H. S. (2016). The Importance and Satisfaction of Collaborative Innovation for Strategic Entrepreneurship. *Eurasia Journal of Mathematics, Science & Technology Education, 12*(3), 569-582.

Tu, J. C., & Chiang, Y. H. (2016). The Influence of Design Strategy of Peer Learning on 3-D Software Learning. *Eurasia Journal of Mathematics, Science & Technology Education*, *12*(5), 1263-1271.

Ulusoy, F. M., & Onen, A. S. (2014). A Research on the Generative Learning Model Supported by Context-Based Learning. *Eurasia Journal of Mathematics, Science & Technology Education*, *10*(6), 537-546.

Wu, Y. W., Weng, K. H., & Young, L. M. (2016). A Concept Transformation Learning Model for Architectural Design Learning Process. *Eurasia Journal of Mathematics, Science & Technology Education*, *12*(5), 1189-1197.

Xu, Z. (2004). A method based on linguistic aggregation operators for group decision making with linguistic preference relations. *Information Sciences*, *166*(1-4), 19-30.

Zha, X. F., Sriram, R. D., Fernandez, M. G., & Mistree, F. (2008). Knowledge-intensive collaborative decision support for design processes: A hybrid decision support model and agent. *Computers in Industry*, *59*(9), 905-922.

Zhai, L. Y., Khoo, L. P., & Zhong, Z. W. (2009). Design concept evaluation in product development using rough sets and grey relation analysis. *Expert Systems with Applications*, *36*(3), 7072-7079.

Zhang, J., Wu, D., & Olson, D. L. (2005). The method of grey related analysis to multiple attribute decision making problems with interval numbers. *Mathematical and Computer Modelling*, *42*(9-10), 991-998.

Zhong, X. M., & Fan, K. K. (2016). A New Perspective on Design Education: A "Creative Production-Manufacturing Model" in "The Maker Movement" Context. *Eurasia Journal of Mathematics, Science & Technology Education*, *12*(5), 1389-1398.

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