"Away from the Textbook," Metacognitive Strategies in Mathematics: A Qualitative Study on Saudi Students' Motivation to Learn Mathematics

Khalid S. Alzahrani [1*]

[1] Imam Abdulrahman bin Feisal University, Dammam, SAUDI ARABIA

Abstract

This study explores Saudi students' perceptions of their motivation to learn Mathematics using metacognitive strategies. Data were collected using a qualitative case study design among 13 Saudi secondary school students. The findings from the data collected through semi-structured interviews show that several motives encourage the participants to learn metacognitively. For instance, the findings demonstrate that the desire to experiment with a new concept helped to capture the interest of the learners and stimulate their curiosity to learn. Likewise, the desire to improve methods of thought which can meet the personal needs and goals of the learner may foster a positive attitude in learners and therefore contribute to their motivation. In addition, it is argued that students' confidence in their skills may help them feel they can successfully control their learning. Finally, it was shown that the desire of students to improve their thinking strategies in their lives reinforced their achievements with internal rewards and therefore helped them to be satisfied with their learning.

Keywords: metacognition, motivation, mathematics, IMPROVE program, cognitive skills

INTRODUCTION

Flavell (1979) demonstrated that young children have intellectual disabilities. Thus, according to the study of Torres and Ash (2007), the cognitive regulation and cognitive monitoring is essential to encourage this type of actions for both children and adults. It is because, the progressive reformation in mental procedures results to biological development with experiences from environment. Flavell's (1979) term "metacognition" arose out of this area of research. The use of the term "metacognition", according to Keller (1987), began in empirical evidence of psychology in two different areas of research: information about cognition and cognitive regulation. The first relates to information related to thought processes, and the second relates to guiding and observing reasoning. In addition, Clayton et al. (2010) argued that research identifying with metacognition is dependent on the recognition of one's own understanding of discernment and the underlying cycles of understanding. The foregoing refers to one's own understanding of the components of one's own perception and the perception of others, although the latter refers to observing the intellectual movement, its application and its impact on the methodology of critical thinking, despite the guidelines of recognition.

Relation of Motivation with Metacognition

It is crucial issue for instructors to motivate students in class and tend to aware cognitive development in learning. It is based on comprehension of students that can become difficult in case they lack motivation. They could be engaged with knowledge construction. If, this relation is not addressed, it leads to lack in motivation those results to weak educational performance. The other important factor is impact on understanding and the performance of the students. The process of metacognition has been referred to awareness of individual with critical approach to analyse it. The crucial determiner to performance of student is better explained through cognition and understanding in classroom context.

Education in Saudi Arabia

The Kingdom of Saudi Arabia possesses a centralised education system (Alfares, 2014; Alnesyan, 2012; Alsaeed, 2012) in which the Ministry of Education

Contribution to the literature
- This study takes different accounts of basic facts that tend to combine relation of teaching and learning of Mathematics with meta-cognitive processes.
- This study can prove as a contribution for future researchers in context of Saudi Arabia to design a framework that could highlight genuine relation of teaching Mathematics and its learning based on use of meta-cognitive processes.
- This study can be taken as a source to view the opinions of teachers of Mathematics and students that they employ metacognitive process in classroom.

oversees education policy for the entire country. It manages the construction and equipping of educational facilities, along with the content and distribution of all textbooks, which are standardized throughout the Kingdom. The education system is divided into five levels, with kindergarten stage non-compulsory, six years of primary school, middle and high schools of three years each (secondary spans from 15 to 18 years old), and a separate tertiary education system. All five levels are overseen by the Ministry. The academic year tends to consist of two 18-week terms, with two weeks set aside for examinations. Each class period usually lasts 45 minutes, with the total number of periods weekly ranging from 26 to 33 periods, depending on grade and subject emphasis. Mathematics is a key subject whereby students are obliged to study the subject for five periods per week. While the education system is sex-segregated, both genders receive the same quality of education, with almost identical subjects and school stages, although there may be slight contrasts based on differing needs.

The goals and policies of education in the Kingdom of Saudi Arabia were built on a group of foundations, as published on the Ministry of Education's website. Among these was the enabling of the student to possess the skills of continuous learning. In order to achieve such goals, the Ministry seeks to improve academic curricula, teaching methods and evaluation processes, which will reflect positively on students' learning. One of the most important practical steps taken to achieve educational goals in the Kingdom is the King Abdullah Project for the Development of Education – named 'Tatweer' ('Development') in Arabic (TATWEER). This project began in 2008 and seeks to present educational services through projects and programs to elevate the educational process and to develop and improve pedagogy.

Aim of Study

This review aims to discover the impact of making a change to include metacognition very deliberately, in mathematics teaching and learning.

Research Questions

RQ1: How is the use of metacognition central to the teaching and developing motivation to study mathematics, and why?

RQ2: What really encouraging signs and challenges are evident for the students and their teachers willing to work on their mathematical exhibition through metacognition that motivates the students?

RQ3: What attributes seem to improve the constructive outcomes of using metacognitive loops while disrupting the positive effects of motivational learning in students?

Importance of Study

Educators must first be taught how to train students so that they can fully engage their students in mastering to learn mathematics. Maehr (1984) drew attention to the lack of research examining the metacognitive prerequisites for educators seeking to improve this type of experience among their students. This non-attendance is troubling given various studies showing that a teacher's conceptual view of students' learning mathematics affects classroom discussion. In terms of subjective-explicit metacognition, Marulis and Nelson (2020) confirmed that interaction is associated with the idea of a core job along with the specific abilities required for explicit subjects. Subsequently, the use of metacognition, especially in teaching mathematics, will remain a wide range of inquiries requiring more researches. Taking into account several of these fundamental aspects in relation to the idea of a connection between metacognition and mathematics, given the veracity of the learning of mathematics and teaching in Saudi Arabia, this review, which is largely absent in the instructive environment of this country, has attempted to distinguish between the views of professors and their students' use of metacognition in their classrooms. It is crucial reason behind that tends to focus teaching of mathematics and metacognition in current study. The importance in research is conducted to motivate in meta-cognition phase of mathematical education.

LITERATURE REVIEW

Background of Study

In spite of these principles, the introduction of a concrete meaning for the idea of metacognition is still difficult in light of its interdisciplinary nature. To this problem of definition is added another problem - the distinction of perception and metacognition. Korpershoek and van der Werf (2015) highlighted this, explaining that the distinction between perception and metacognition is a test, and that a wide range of metacognitive functions will show that no system is doing its job in portraying them. By adopting a more precise strategy, Efklides (2011) uncovered what discernment means and explained it by offering an understanding that includes understanding and knowing the world and how he or she operates in this unique situation. As they point out, the process of understanding involves the acquisition, improvement and double play of information and intellectual ability. As for itself, it consists of memories that have been formed through the control and inclusion of "raw input"- or rather, data processed by one of the five abilities or arising from intellectual abilities for instance, experiences, learning, thought, recall and reasoning.

According to Rosen et al. (2011), the simplest human activity is completely dependent on intellectual action. This action is manifested in various abilities to organize, control and adequately use it to perform intellectual tasks. Identified test of determination, since intellectual abilities cannot be distinguished from each other, they can be covered. Subsequently, comprehension was divided into broader intellectual abilities such as instance, experiences, learning, thought, recall, reasoning, and perception, the contrast between the metacognitive and intellectual cycles, as noted by (Pintrich & DeGroot, 1990). He went on to explain that cognitive cycles are about doing, and metacognitive cycles are about choosing and ordering what is required and testing what is being done.

Given this mass of controversy, the introduction of a definition of metacognition does not mean that there is a consistent understanding of the boundaries of an idea. This is because in the long term, the degree of definition has evolved along with metacognition, which has evolved into a diverse idea (Ambrose et al., 2010). Despite this, the requirement for hypothetical clarity is clearly present. This will include refined definitions and images of different parts of the idea (Elliot & McGregor, 2001). Thus, we can conclude that metacognition, from an educational point of view, hints at information about a person, as well as testing and controlling one's own organised cognitive activities, which requires certain metacognitive abilities, such as ordering and evaluation. With regard to the study of the idea of metacognition, an important question remains the definition of the main subject of the idea of metacognition. Moreover, Ozcan and Gumus (2019) referred to the fact that the idea of the technique of self-observation and control is fundamental in the evolving field of metacognition, and vein, McInerney and Ali (2006) point of view is, "the subject of metacognition is regulation of one's own information processing". There are several fundamental aspects of the idea of the connection between metacognition and Mathematics, which thus give this survey an important colloquial identity.

First of all, the findings from the critical literature analysis show that students see problems in mathematics and critical thinking activities because they ignore a wide range of intellectual or metacognitive cycles (Jansen & Middleton, 2011; Keller 1987). In any case, this may lead to the assumption that aspiring students lack significant metacognition (Ocak & Yamac, 2013). Moreover, many researches have stated that numeric representation in general and largely depends on the use of metacognitive procedures (Clayton et al. 2010; Korpershoek & van der Werf, 2015). Consequently, metacognition is central to the learning system, which ultimately has a significant impact on the demonstration of student's learning in school and on their number (Efklides, 2011; Pintrich & DeGroot, 1990; Rosen et al. 2011). Third, and this is all the more obvious, the inability of the students to reproduce the necessary cycle of observation and control in the learning process is an indicator of the low efficiency of science, in contrast to the lack of numerical information (DeGroot, 1990; Efklides, 2011; Ibrahim et al., 2017; Kurtz & Borkowski, 1984; Ocak & Yamac, 2013). Thus, the viability of the critical thinking skills will increase when the students becomes fit to test and manage their own learning processes (Ambrose et al., 2010; Elliot & McGregor, 2001; McInerney & Ali, 2006; Ozcan & Gumus, 2019). Fourth, the students can be trained to work on numbers using metacognitive abilities such as regulation or monitoring (George, 2012; Jansen & Middleton 2011; Maehr, 1984; Marulis & Nelson, 2020; Ocak & Yamac, 2013).

Metacognition

The project's philosophy is based on a group of principles such as student centric learning, cooperative learning, active learning based on discovery and investigation, developing thinking skills, developing decision-making skills, and linking learning with real-life contexts.

Flavell (1979), Kluwe (1982), and Brown's (1987) definitions should be considered to clarify the concept of metacognition in this regard. There were three reasons behind choosing these definitions, which in turn assisted in the undertaking of this study. Firstly, they present a theoretical framework for metacognition instead of other definitions which focus on specific parts of metacognition (Gama, 2004). Secondly, they distinguish between different aspects of metacognition - knowledge

and regulation of cognition (Gama, 2004). Thirdly, they have the most relevance for education (Alzahrani, 2017). Flavell (1979, p. 1232) defines metacognition as

> "one's knowledge concerning one's own cognitive processes and products or anything related to them" and as "the active monitoring and consequent regulation and orchestration of these processes in relation to the cognitive objects or data on which they bear, usually in the service of some concrete goal or objective."

Likewise, Brown (1987) views this concept as one's knowledge and control of one's own cognitive process. This idea was stressed by Kluwe (1982, p. 202) who claimed that metacognitive activities involve knowledge about one's "own thinking and that of others" in addition to monitoring and regulating "the course of [one's] own thinking".

Based on the above definitions, it can be argued that the idea of metacognition is composed of two fundamental notions: (1) knowledge of cognition, and (2) monitoring and regulating cognition, to which Kluwe (1982) referred as "executive processes". More specifically, the notion of knowledge relates to the knowledge or beliefs about the characteristics of one's own cognition and about how information is processed. In addition, the monitoring and regulation of the cognitive process is a central element of metacognitive strategies as it relates to the executive process of cognition (Flavell, 1979). This notion of executive process, according to Kluwe (1982), comprises four elements that can be understood in the forms of simple questions: identifying ("what am I doing?"), checking ("did I succeed? did I make progress?"), evaluating ("is my plan good? are there better alternatives?"), and predicting ("what could I do? what will the result be?"). In this respect, Brown (1987) adds that this aspect of metacognition involves various activities that include planning, monitoring and evaluating. Planning activities involve predicting the results, planning strategies or choosing alternative ways while the aspect of monitoring involves testing, rescheduling or revising learning strategies. Although Bakkaloglu (2020) asserted that the literature displays that there are diverse aspects of metacognition, it can be said, according to Alzahrani (2017, p. 525), that

> "metacognition mainly relates to one's knowledge, monitoring and control of one's own systematic cognitive process, which requires specific skills including planning and evaluation."

It should be added, however, that there is no consensus among scholars about the temporal and spatial limits of the notion of metacognition; therefore, this notion has increasingly become a multifaceted concept (Buratti & Allwood, 2015).

Motivation

Motivation refers to the personal investment that an individual has in reaching a desired state or outcome (Ambrose et al., 2010). According to Elliot and McGregor (2001), in general students seek more than one goal in different learning situations, and they have different ways to reach their goals. The authors focus on both mastery and performance goal orientations. While mastery goals are concerned with internal competence, performance goals are concerned with normative competence. Korpershoek and van der Werf (2015) claim that extrinsic goal orientations can be referred to as performance goal orientations, but they are different; extrinsic goal orientation involves the desire to be praised or rewarded by others, while performance goal orientation is the desire for one's performance to be compared with the others' socially. In this regard, Ozcan and Gumus (2019) explain that motivation is the tendency to behave in a specific direction and has two main dimensions: intrinsic and extrinsic. Extrinsically motivated students seek out external rewards for their behaviour in the form of high grades, academic honours, scores on tests, and awards from parents or teachers. Intrinsically motivated students, on the other hand, engage in learning activities to satisfy their interest or curiosity. Thus, this type of motivation reflects students' intrinsic interests in the content, materials, or tasks. According to Korpershoek and van der Werf (2015), intrinsic motivation depends on the desire of the student to do something or to be involved in making decisions simply because he/she enjoys doing these things, which leads the student to be more flexible and organized. In contrast, extrinsic motivation has to do with being forced to participate or perform by others.

In the same vein, McInerney and Ali (2006) suggested taking into consideration extrinsic and social goals; they came up with this idea from Maehr's Theory of Personal Investment. Maehr (1984) claims that achievement goals are four: task goals or mastery, ego goals or performance, extrinsic rewards, and social unity goals. Korpershoek and van der Werf (2015) asserted that extrinsic and social goal orientations are both used, in addition to the more commonly used mastery and performance goal orientations. According to Clayton et al. (2010), if students who get involved in the process of making decisions feel confident, their performance improves, and their sense of belonging to the group becomes stronger. They even become more self-motivated to comprehend new information, learn, and improve new plans. Korpershoek and van der Werf (2015) asserted that intrinsic motivation can easily be affected by the positive feelings that the students get when they, for instance, complete a task successfully. These positive emotions function as an internal stimulating motivation which leads to more accomplishments and even better performance.

Motivation and Metacognition in Mathematics Learning

The current study seeks to show how we can make Mathematics learning more desirable when teachers and students take into consideration the fact that metacognition and motivation to learn are connected to each other, and this makes using metacognitive learning more practical. This is the reason behind focusing on mathematics specifically for the present study. A significant body of empirical research has been conducted on motivation and metacognition in the context of mathematics education. For example, studies have shed light on the positive correlation between the use of metacognition and students' motivation to achieve (Kurtz & Borkowski, 1984), between intrinsic motivation and metacognition (DeGroot, 1990; Ibrahim et al. 2017), between internal motivation and self-efficacy in mathematics which could significantly predict metacognitive skills (Ocak & Yamac, 2013) and also between metacognitive knowledge and motivation (Carr et al., 1994). Marulis and Nelson (2020) also mentioned that, to different extents, both declarative and procedural metacognition were connected in a positive way to motivation. Metacognition even predicted motivation indicatively. According to Schoenfeld (2016), the field of mathematics have been inherently a social based activity. The community need to be trained that includes scientists of mathematics engaged for solving different scientific patterns. The attempts of this system are based on observations, studies and the experimentations. It determines principles and regularities within systems. It tends to axiomatically even theoretically made models in systems abstracted in real world. The main tools in mathematics are representation, manipulation and abstraction.

Previous Contributions

More specifically, supporting the above research, intrinsic motivation has been found to have a strong relationship with metacognitive strategies (Efklides, 2011; Pintrich & DeGroot, 1990) as intrinsically motivated students tend to be more academically involved and employ productive and meaningful metacognitive strategies compared to their non-intrinsically motivated peers. Hence, according to Pintrich and DeGroot (1990), intrinsic motivation plays a significant role in their achievement and the type of metacognitive strategies used. In addition, for Printrich and DeGroot (1990) the magnitude of a student's intrinsic motivation on academic tasks influences the selection of an appropriate metacognitive strategy to solve academic problems. By contrast, Desoete et al. (2019) claimed that poor mathematics performers tend to be less intrinsically motivated and less metacognitively accurate. However, in their study motivation was operationalized by a single question ("Did you like these exercises?"), which might be only a small part of intrinsic

motivation. Hence, more research is needed to confirm these results, including a broader assessment of metacognition and motivation as well as other context variables. In addition, Ibrahim et al. (2017) found a reciprocal correlation between intrinsic motivation and metacognition. In this instance, intrinsic motivation and metacognition were found to interact in a more mutually influential fashion, in which the intrinsic motivation influenced the quality of metacognition while the metacognition simultaneously affected the quality of intrinsic motivation. This means that, when a learning activity is able to arouse students' interest, both cognitive and metacognitive strategies will be used, mental resources will be maximally utilized, and information processing and imagination will be expanded and operated. Similarly, Korpershoek and van der Werf (2015) mentioned that most of the students in their study preferred learning via case discussions and practical activities, which is consistent with mastery goal orientation. These findings, according to Korpershoek and van der Werf (2015), match the findings in the available literature about metacognition which support its relationship with better strategies of learning, academic performance and success. Jansen and Middleton (2011) provide teachers with an adequate and accessible review of research on motivation. Based on this, three critical aspects appear to be relevant to us here. First, it is assumed that "all students are motivated"; this suggests that students are always motivated to do something or to behave in a certain way. Our concern, therefore, is whether their motivations align with our teaching goals. Second, "motivation is adaptive", which means that students' past and present experiences can shape their motivations. Thus, we can reasonably assume teachers can have a positive influence on student motivation. Finally, "success matters"; that is, if students feel successful and in control, they tend to have a more positive relationship with the material they are working on and with their learning. Following on these assumptions, George (2012) suggests that the instructor's main role is, therefore, to share enthusiasm, provide a supportive and enticing environment, and leave the rest to the learner.

Karaali (2015) mentioned that research on motivation and metacognition has focused on the impact of motivation on metacognition. In his study, the emphasis is, instead, on the impact of metacognition on motivation. Hence, one central aspect of this study is to investigate the possibility of employing metacognitive strategies in mathematics to enhance students' motivation in learning. Therefore, the central question posed in this paper is: can metacognition be used to direct motivation? In other words, in this article we focus on how metacognitive strategies in mathematics teaching and learning may influence motivational beliefs, as it is conceivable that "motivational beliefs are both a cause and an effect of a student's efforts to learn

metacognitively" (Zimmerman & Moylan, 2009, p. 305). One central premise of this study is that motivational beliefs are both a cause and an effect of a student's efforts to learn metacognitively. In support of this claim, Zimmerman and Moylan (2009) argue that most self-regulation intervention studies produced not only gains in students' academic performance but also improvements in their strategic behaviour and motivation. Clearly, exploring the intersection of metacognition and motivation has opened new windows to our understanding of how students self-regulate and self-sustain their learning.

METHODOLOGY

Based on the above theoretical premises, this study aims to explore students' perceptions of their motivation to learn Mathematics metacognitively. The main aim of this study was to gain a better understanding of the nature of the relationship between metacognition and motivation in the context of mathematics learning in Saudi Arabia. A qualitative approach was preferred as it enables a deep understanding of how people make sense of, and experience, their social realities and context (Merriam, 1998). Hence, given the aims of this research, this study adopts an explanatory approach. Data were collected through individual semi-structured interviews.

Participants

Given the qualitative nature of this research, generalisation was not an objective of this study. Instead, given the aim of this study, a small number of participants was preferred employing a purposive sampling strategy (Merriam, 1998). It is because small number of students was enrolled in each classroom. As a result, thirteen students took part in the study since this is the total of each class in the school. They were all 17 years old and lived in the same area of the city of Taif, Saudi Arabia. For ethical reasons and to preserve their anonymity, they were attributed the following pseudonyms: Ahmed, Hasan, Khalid, Ali, Saleh, Fahad, Salem, Bader, Rami, Aziz, Salman, Ibrahim, and Amar. The sample size is normal in the qualitative approach (Fraenkel & Wallen, 2006), thereby the researcher does not aim to improve through a specific strategy or to enhance students' achievement but to understand how students make sense of the relationship between metacognition and motivation. All participants are males since the educational system in Saudi Arabia is based on single-sex education especially at secondary schools. They belonged to undergraduate level at University. The subject of calculus is taught is taken for study. These students were chosen through coordination with their mathematics teachers in order to determine which students were best able to express their opinions and feelings, with these students being of various

educational achievement levels in the same class. The study was conducted in a secondary school in Taif city which was deemed a suitable site for three main reasons: the number of students were 30 and they belonged to same class in order to facilitate using metacognitive teaching, the mathematics teachers were cooperative and welcomed the idea of implementing metacognitive teaching strategies, and teachers and students had some experience with cooperative learning in mathematics. These aspects were essential to implement the design framed around the IMPROVE program based on cooperative learning.

The IMPROVE Program

The IMPROVE program was designed by Mevarech and Kramarski (1997) and comprises three interrelated components:

1. Facilitating both strategy acquisition and metacognitive processes,

2. Learning in cooperative team[s] of four students with different prior knowledge: one high, two middle, and one low-achieving student, and

3. Provision of feedback-corrective-enrichment that focuses on lower and higher cognitive processes (Mevarech & Kramarski, 1997, p. 369).

The term, IMPROVE is an acronym for the following steps: *Introducing* new concepts, *Metacognitive* questioning, *Practising*, *Reviewing* and reducing difficulties, *Obtaining* mastery, *Verification*, and *Enrichment*. This framework was initially designed to be implemented in small groups of four students of different abilities, particularly after a new concept had been introduced to students. Using this framework, learners are encouraged to raise three types of metacognitive questions:

1. Comprehension question: "What's in the problem?"

2. Connection question: "What are the differences between the problem you are working on and the previous problems?"

3. Strategic question: "What is the strategy/tactic/principle appropriate for solving the problem?"

The IMPROVE program was preferred for use in this study because it uses metacognitive perceptions and how they can be implemented in mathematics teaching. In addition, it has been proven to have a positive effect on mathematics performance at numerous ages (Cetin et al., 2014; Grizzle-Martin, 2014; Kramarski & Michalsky, 2013). Nevertheless, the aim of the current study is not to investigate a specific strategy or to assess students' knowledge but to understand how students make sense of the connection between metacognition and motivation.

Reasons to use IMPROVE program in study than other programs

The reason behind employment of improved program was to acknowledge the engagement of employee with the turnover reduction phase in coming years. The employees can actively take part towards better of company. It can gain pride with the accomplishment of different task. It can lead towards great sense with few reasons behind leaving organization. It is well organized program to identify different opportunities in improvement for help in organization and meet goals that could increase profits and reduce costs with acceleration of innovation in coming years.

Procedures

Before implementing the program and commencing data collection, the IMPROVE program was discussed with the teacher of the chosen class. During this discussion, the teacher raised questions about the way to implement the program.

As the IMPROVE program is based on a cooperative learning model with peer interaction and corrective feedback, students were arranged in small groups of four. They worked cooperatively using materials designed by the teacher and took turns in asking and answering three sorts of metacognitive questions using the following prompts: comprehension, connection and strategy questions. In case of disagreement within a cooperative group, members were to resolve their disagreement through discussion and write down the agreed solution. Students were asked to write the final solution as well as their mathematical workings in addition to a sample of metacognitive responses worded in the form: "this is a problem about ...", "the difference between this problem and the previous problem is ...", "the mathematical principle appropriate for solving the problem is . . . because . . ..". Following the activities, the teacher clarified certain issues that he observed within the groups.

As example of these steps, the title lesson is Rational Expressions (see Appendix A),

Example: Reduce the rational expressions like in the following to lowest terms:

$$\frac{x^2 - 25}{x^2 - 7x + 10}$$

1. Understand and categorize the problem
   a. Given: A rational expression, as follows:
   $$\frac{x^2 - 25}{x^2 - 7x + 10}$$
   b. Demand: Reduce the rational expressions to the lowest terms.
   c. Categorize: Basic Algebra.

2. Solving strategy: Factoring the numerator and the denominator of the expressions than simply common factors.
3. The solution
   a. Factor the numerator:
   $$x^2 - 25 = (x - 5)(x + 5)$$
   b. Factor the denominator:
   $$x^2 - 7x + 10 = (x - 2)(x - 5)$$
   c. Simplify common factors:
   $$\frac{25 - x^2}{x^2 - 7x + 10} = \frac{(x - 5)(x + 5)}{(x - 2)(x - 5)}$$
   d. Write the rational expression in lowest terms:
   $$\frac{(x + 5)}{(x - 2)}$$
   e. Finally, find the solution:
   $$(x^2 - 25)(x - 2) = (x^2 - 7x + 10)(x + 5)$$

The teacher decided when to apply the IMPROVE program according to the lesson content and the readiness of the students. As a result, this approach was applied in eight sessions over seven weeks. This duration came with the suggestion of Schraw et al.'s (2015) study, which showed that programs extending from six weeks to several months tended to be more efficient. This is because longer-term procedures allowed students to become more familiar with what they were being schooled in. The researcher attended with the teacher to be sure that his teaching was based on the framework. Interviews were conducted with individual students during the last two weeks of this period to explore their perceptions of their motivation to learn mathematics using metacognitive strategies. Each interview lasted 30 minutes. The interview involved questions such as: How do you find metacognitive learning? Do you motivate yourself to become a metacognitive learner? Why and how? How do you perceive the relationship of metacognition with motivation in mathematics learning?

Research Instrument

The interview protocol was employed in this study. It was to analyse the in-depth knowledge of participants about meta-cognitive strategies that motivate them to learn mathematics. The interview protocol is semi-structured, where the participants can state about their interpretations on meta-cognitive strategies that motivate them to learn mathematics. The interview protocol was administered to them after the session of IMPROVE program. The participants were individually administered to state their responses.

Interview protocol

The five questions were referred to the participants. The questions asked wanted to know about the concept of meta-cognition, the ways to improve meta-cognitive

skills in learning of mathematics, the metacognitive skills that could increase their confidence and how metacognition is part of their extracurricular motive. These are represented in separate headings in section of findings and analysis.

DATA ANALYSIS

The method employed in this study involved just elements of theoretical thematic analysis. After acquainting myself with the data and having formulated some general ideas about the notable features within it, I then began to generate preliminary coding by assigning a 'code' to specific content using a software called MAXQDA. I had a long list of codes that were assigned to extracts. I then examined each coded extract and organised these codes into groupings that I called 'categories'. These categories were checked by a colleague (who has a doctoral degree in education) who agreed with the logical aspect of these groupings after extensive discussion. This phase involved sorting these different codes into potential categories, and collating all the relevant coded data extracts within these categories using the software. I then read through the 'code system' (as it is called in the software) and pondered how much each code agreed with the category. Then I created themes that were inferred based on the links between the different categories.

Coding

The coding is a first step in analysis after the collection of data. The codes to particular context are processed through MAXQDA.

Categories

The codes are categorized further for further analysis of data that is collected.

FINDINGS AND DISCUSSION

Metacognition as a New Concept

The interview findings suggest that the desire to experiment with a new concept was a strong motivation for participants. This was explained by Bader, who felt curious about experimenting with a new learning method:

> "I have the motivation to learn with metacognition, and at the very least it is something I am curious about."

Ahmed also explained that he had a positive experience of learning mathematics with metacognition because he "love[s] to adapt to new things in [his] learning". These views from Ahmed and Bader are indicative of a wider belief among the participants; the data suggest that these learners were motivated by the

idea of experimenting with a new concept such as metacognition. It is worth noting here that this is generally considered an important element that tends to sustain the student's attention in the learning process. In this respect, Keller (1987) emphasised the importance of motivation in maintaining and sustaining attention. To sustain learners' attention, Keller (1987) argued curiosity ought to be increased through designing learning activities centred around problems to be solved by learners engaged, for example, in experiential situations and through questioning strategies, as included in the IMPROVE program.

Metacognition to Improve Thinking Skills

The desire to improve their thinking skills proved to be another source of motivation for the participants to implement metacognitive strategies in their Mathematics learning. For example, Salem asserted:

> "I have the motivation to learn through this method because it helps me to understand mathematics learning to a greater extent, which makes it easier to deal with Mathematics problems."

Talking about his motivation to implement metacognitive strategies in mathematics problem solving, Bader made the link between enthusiasms and thinking skills; he explained:

> "I feel that it calls for enthusiasm, as it relates to my method of thinking and clarifies to me where exactly the problem lies in the course of my thinking when dealing with Mathematics problems."

Ahmed also explained that his motivation to use metacognition was increased by this belief that it could also enhance his thinking skills. For him, it was an incentive to learn, as he explained:

> "I have the incentive to learn metacognitively, as it will develop my thinking. Also, when everyone thinks in a certain way, they believe that this is the correct way to think, but if they are able to identify the positives and negatives in their thought, they would be enabled to correct and develop it."

Adding to that, Ali explained that he felt the motivation to learn mathematics using metacognitive strategies as it could "make [him] independent in [his] thinking, away from the textbook." A similar idea was developed by another student, Fahad, who, while reflecting on his metacognitive learning experience, expressed the view that his motivation was increased because he found this strategy:

> "…better for learning Mathematics because it develops a student in his thinking to become a

logical thinker, and the role of the student is no longer in copying and pasting mathematics concepts."

Another participant, Hasan, stated that:

"learning mathematics metacognitively is useful because it brings you to better ways of thinking"

while Ali explained that his motivation was driven by the idea that metacognition could "help [him] improve [his] method of thinking." This is commensurate with Kim et al. (2014), when they found that motivation positively affected the students' effort regulation in terms of academic performance. Furthermore, while students, who were highly encouraged to study, learn, and improve their academic performance, showed high levels of academic independence, those who were less motivated tended to be more dependent on the controlling strategies (Soenens et al., 2012).

Metacognition as a Source of Confidence

In addition to improving thinking skills, participants found another source of motivation for learning mathematics metacognitively. Their sense of independence and control, as explained above, was also found to be linked to a perceived increase in confidence. For example, several learners admitted feeling more confident about their learning, which was a source of motivation to apply metacognitive strategies in their learning. For instance, Salem said:

"Learning through metacognition is very important because it changes my method of thinking. I feel like my method of thinking improved last year and that my way of dealing with Mathematics is different. I found myself to be better in my method of answering questions, and this pushes me to learn metacognitively."

Rami commented on his experience of metacognition in the mathematics class, as follows:

"Metacognitive learning is very useful because it organized how I dealt with the problem and made me more confident in following a systematic method to solve it. The solution will be closer to the correct one because this methodology gives me a good start in my solving, and this is what motivated me to learn metacognitively."

Similarly, Aziz said:

"Metacognitive learning is useful, despite the benefits at the beginning of implementation not being tangible. However, we tried to apply it until we felt that it was not hard anymore, then we noticed its benefit as a solving methodology, even

if we did not arrive at the solution. This is what motivated me to pursue metacognitive learning."

What stands out from the findings is that the participants had a sense of success. The learners interviewed attached importance to success (Jansen & Middleton, 2011) which seemed to result from their feeling of control over the learning process. By taking control over their learning and getting a sense of success, in problem solving for instance, metacognitive learning created a positive relationship with the content and the material. As noted by Keller (1987), in addition to relevance and attention, confidence plays a dominant role in learners' motivation. According to Clayton et al. (2010), when students, who get involved in the process of making decisions, feel confident, their performance improves. They even become more self-motivated to comprehend new information, learn, and improve on their plans. This is in line with the findings of Korpershoek and van der Werf (2015) that intrinsic motivation can easily be affected by the positive feelings that the students get when they, for instance, complete a task successfully. These positive emotions function as an internal stimulating motivation which leads to more accomplishments and even better performance. For instance, it is likely that learners may have genuine fears or uncertainty about a task, which could prevent them from completing it effectively. Therefore, it is critical to design learning tasks and activities that create the right level of confidence. Moreover, the learning environment and the instructions should also contribute to maintaining an appropriate level of confidence in learners. This is because learners' confidence directly relates to their "expectancy for success". As explained earlier, the notion of control is, in this regard, critical as learners need to understand that this success is primarily based upon their own efforts, which is one major finding from this research.

Metacognition as an Extracurricular Motive

As seen above, a noticeable aspect of the data is the fact that students felt in control of their learning and were motivated to learn "away from the textbook" beyond the constraint of what they had previously been used to in the mathematics class in their context. More emphatically, several participants also extended the benefits of metacognition beyond the formal context of learning and mathematics education. Indeed, they explained that more than improving their thinking skills to solve mathematical problems, what also motivated them to implement metacognition was that it could improve their way of thinking in their lives. In this respect, participants expressed that metacognition was an extracurricular motive, Bader articulated this as follows:

"It seems to me that metacognition is related in a sense to lifestyles, as it tells us how to overcome

problems and this is followed in steps. So, I expect that its benefits will extend from Mathematics problems to social life."

Ahmed added that

"metacognition being part of a student's culture is useful in general and not specifically for the learning of mathematics."

In a similar manner, Fahad stated,

"I expect that if I changed my thinking, it wouldn't only benefit me in my mathematics learning, but also in my daily life".

Ahmed also mentioned that it could have a long term impact on his life; hence his desire and motivation to use metacognition, he said:

"I have the motive to learn with this methodology, and if I were able to do so for a longer period of time then I am sure it would change my method of thinking generally in life skills. This is the reason that I am pushed to learn metacognitively. I really feel that I benefited from it because maths questions are problems, and daily life has problems too, and the method of thinking to solve them is like this method."

Fahad added:

"Metacognition is better for solving Mathematics problems and can be used to approach the solution to more general problems."

Hence, it can be understood from the interviews conducted in this study that these Saudi learners' intrinsic motivation was significantly related to their experience of metacognitive strategies in mathematics learning, as has been suggested in other studies as well (e.g. Efklides, 2011; Pintrich & DeGroot, 1990). This is because intrinsically motivated students have been found to be generally more engaged in their learning and to implement effective metacognitive strategies in their learning. Hence, to sustain learners' desire to learn it is essential that they gain a sense of satisfaction not only with the process of learning, that is the strategies in place to achieve a learning goal, but also with the product of learning, that is the knowledge and skills they gain through this process (Keller, 1987). This finding is consistent with the study of Rosen et al. (2011), which found that metacognitive strategies gave students a chance, through different resources which function as planning, monitoring, and evaluation tools, to observe their learning process. Consequently, using educational models which adopt these strategies is important due to their effectiveness in reaching academic goals by linking learning to everyday situations and raising awareness of the restrictions when reasoning, thinking, and solving

problems. This sense of satisfaction can also be fostered by a strong sense of self-confidence and achievement as a result of positive interactions with the materials, peers or adult experts. Hence, giving learners the opportunity to voice concerns and opinions about their learning experience is, in this respect, particularly critical.

Through examining the collected data from interviews with participating students, it was found that students perceived many aspects of motivation that encouraged them to learn mathematics through metacognition. The desire to experiment with a new concept tended to help capture their interest thereby stimulating their curiosity to learn. In addition, the desire to improve methods of thought and thinking skills to meet their personal needs and goals had a positive impact on their attitude towards metacognition. Likewise, the study showed that students felt they were gaining confidence in terms of learning skills, which was another important motivating factor that helped them feel in control of their own learning and success. Finally, students were not only motivated by the benefits of metacognition in the context of academic learning but also felt it could also improve their way of thinking in their lives more broadly; this also gave them motivation to learn through metacognition as it gave them a sense of satisfaction.

Interestingly, these four motivational strategies closely relate to the notions of *Attention*, *Relevance*, *Confidence*, and *Satisfaction* which are essential components of Keller's (1987) ARCS Model. This typology is particularly helpful to curriculum and syllabus designers and teachers as it can help organise knowledge about learner motivation as well as design practical activities to foster these four elements.

Based on the above findings, this study argues that motivation and metacognition are connected. This is supported by several studies on motivation and metacognition in learning (e.g. DeGroot, 1990; Efklides, 2011; Ibrahim et al., 2017; Korpershoek & van der Werf, 2015; Kurtz & Borkowski, 1984; Ocak & Yamac, 2013).

CONCLUSIONS

Based on theoretical notions of metacognition and motivation, this study qualitatively explored Saudi secondary school students' perceptions of their motivation to learn mathematics metacognitively. Through examining the collected data from interviews with participating students, it was found that numerous motives were seen by learners as encouraging them to learn through metacognition. First, the desire to experiment with a new concept was perceived as helpful in capturing their interest and stimulating their curiosity to learn. Second, students felt that the desire to improve methods of thought which could meet their personal needs and goals fostered a positive attitude. Third, students expressed being confident in their skills, which

was another important motivating factor which enhanced their sense of success and control. Finally, the desire of students to improve their way of thinking in their lives was also a motive to learn through metacognition, which was perceived as a way to reinforce their sense of accomplishment and satisfaction. Hence, the connection of metacognition and motivation has clearly opened new windows to our understanding of how students self-sustain their learning. Therefore, it is valuable to conclude with Korpershoek and van der Werf's (2015) assertion that, to improve academic outcomes and make learning more desirable, educators should take into consideration the fact that metacognitive awareness and motivation to learn are connected to each other, which makes employing self-regulated learning strategies a productive approach. Finally, it is recommended that more studies be conducted in different contexts related to motivation and metacognition strategies.

Recommendations

The outcomes of the study emphasise the necessity of students being more conscious of their learning through metacognitive awareness. This is done to offer such pupils with a more conducive creative environment, to allow them to openly communicate their workable solutions without shame, and to encourage the four primary metacognitive skills: planning, management, monitoring, and assessment. These abilities, in turn, assist pupils in improving their performance in mathematics class. The paper proposes a fundamental practise model of metacognitive mathematics learning, which comprises the actions that a learner should take when dealing with mathematical issues. Metacognitive concerns are portrayed and connected to comprehending, classifying, and assessing the topic inside group conversations in student work groups.

According to the conclusion of the research, there is a lack of foresight and retraining for teaching via metacognition, whether at university or throughout a teacher's employment in education. This is seen as one of the difficulties that teaching faces as a result of metacognition. These findings suggest the possibility of a future collaboration between school administrators and research institutions to establish a conducive environment with the future adoption of metacognition. It is also proposed that entities be established tasked with developing innovative teaching approaches. The instructor would therefore be able to speak with such bodies in order to improve their performance in using the approach.

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APPENDIX A

Worksheet based on the IMPROVE programme

Group Members 1. 2. 3. 4.	Worksheet [............]	
	Lesson Title: Rational Expressions	

EX: Reduce the rational expressions $\frac{x^2-25}{x^2-7x+10}$ **to lowest terms**

1. **Understanding and categorizing the problem**

Given: a rational expression $\frac{x^2-25}{x^2-7x+10}$

Demand: Reduce the rational expressions to lowest terms

Categorising the problem: Basic Algebra

2. **Solving strategy**

Factoring the numerator and the denominator of the expressions than simply common factors

3. **The solution**

A Rational Expression	Factor the numerator	Factor the denominator	Simplify common factors	Write the rational expression in lowest terms
$\frac{x^2-25}{x^2-7x+10}$	x^2-25 $=(x-5)(x+5)$ (difference of squares)	$x^2-7x+10$ $=(x-2)(x-5)$ (product-sum method)	$\frac{25-x^2}{x^2-7x+10}=$ $\frac{(x-5)(x+5)}{(x-2)(x-5)}$	$\frac{x+5}{x-2}$

4. **Check the solution:** $(x^2-25)(x-2)=(x^2-7x+10)(x+5)$

5. **Compare with other problems**

6. **Summarize the main difficulties in solving the problem**

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