Eurasia Journal of Mathematics, Science & Technology Education, 2016, **12**(9), 2503-2515
doi: 10.12973/eurasia.2016.1306a

OPEN ACCESS

Cognitive Activities in Solving Mathematical Tasks: The role of a Cognitive Obstacle

Radovan Antonijević
University of Belgrade, Faculty of Philosophy, SERBIA

•*Received 24 November 2015*•*Revised 25 February 2016* •*Accepted 12 March 2016*

In the process of learning mathematics, students practice various forms of thinking activities aimed to substantially contribute to the development of their different cognitive structures. In this paper, the subject matter is a "cognitive obstacle", a phenomenon that occurs in the procedures of solving mathematical tasks. Each task in mathematics teaching should potentially be designed so that it contains requests that should be performed. Based on that, a cognitive obstacle that students face at the thought plan is created, as well as a cognitive challenge. In the process of "overcoming a cognitive obstacle" in solving the assigned task in mathematics teaching, it is necessary for a student to make an adequate cognitive effort and to optimally engage reference part of the actual cognitive capacity. The process takes place through exercising different thinking activities (thinking operations), using the previous knowledge and experience in solving certain groups of tasks. A system of cognitive obstacles should be grounded in the teaching mathematics, in order to enable an adequate level of thought activization of students and the development of various mathematical cognitive micro-structures (abilities, skills, knowledge, etc.). It also enables students to develop and improve their capacity of mathematical thinking.

Keywords: mathematical task, cognitive obstacle, cognitive challenge, cognitive effort, developmental transformation.

INTRODUCTION

The nature of students' activities in mathematics education is determined by the need that implementation of various thinking activities should be, among other things, *a means of development and improvement of mathematical thinking and of different cognitive structures*, that represent the basis for functioning of mathematical thinking in students. It is based on the fact that thinking activities of students is one of the essential factors of cognitive development.

Any general cognitive structures in students (ability, skill, knowledge, experience) can be developed only on the basis of exercising adequate thinking operations. In the case of mathematics, solving mathematical tasks serves as a basis for initiating and practicing different activities of mathematical thinking. Various thinking operations

Correspondence: Radovan Antonijević,
University of Belgrade, Faculty of Philosophy, Department of Pedagogy and Andragogy
Čika Ljubina 18-20, 11000 Belgrade, Serbia.
E-mail: aa_radovan@yahoo.com

are carried out in solving mathematical problems and understanding mathematical contents, and thinking operations that occur as part of the conceptual understanding and understanding of procedures are of special significance (Swanson & Williams, 2014). These activities enable the development and improvement of various *mathematical cognitive structures*. These specific structures operate under the auspices of mathematical thinking and in this paper are referred to as *cognitive micro-structures*. They represent specific abilities, skills, knowledge and experiences in operating with different mathematical structures, such as measures, algebraic structures (groups, fields, etc.), topologies, metric structures (geometries), orders, events, equivalence relations, and others. For example, particular cognitive micro-structures are the following: an ability of operating with algebraic structures, a skill of operating with topologies, knowledge of operating with measures, and an experience of operating with arrays, and others.

As a key factor of impact on the cognitive development of students, as well as the development of cognitive micro-structures, mathematics teaching can be seen as an *organized system of mathematical contents and tasks that enable creating and overcoming various cognitive obstacles*. This definition is the construct serving as the base for generating an appropriate explanation of function of the cognitive obstacle in solving a mathematical task. It represents a kind of working hypothesis in this work. The function of cognitive obstacles in the process of learning taking place in mathematics teaching can be seen in this context.

State of the literature

- Since there is difficulty in understanding certain mathematical content, there is also difficulty in solving certain mathematical tasks.
- The developed capacities of conceptual understanding in students make the basis for successful solving of mathematical problems.
- Solving problematic tasks in teaching mathematics enables a student to reach a higher level and quality of educational achievement.

Contribution of this paper to the literature

- The method of qualitative analysis of the role of phenomena that occur in the process of learning mathematics is applied in this paper, referring to understanding of the process of solving mathematical tasks.
- Solving mathematical tasks is carried out through the process of overcoming a cognitive obstacle, which is created in students at the cognitive level, representing a kind of "bridge" between the known and unknown in the task.
- The function of overcoming cognitive obstacles in solving mathematical problems is related to the possibility of accomplishing the improvement of certain cognitive micro-structures in students, based on exercising the thinking activities.

The role of thinking activities of students in mathematics teaching can be explained through analysis of specificities in the mechanism of elementary contribution of a certain activitiy to the development of particular cognitive micro-structures in students. This contribution is accomplished in each situation of solving the mathematical tasks, which represent a means of initiating thinking activities of students. This requires analysis in order to discover which effect a student's particular thinking activity has on development of certain cognitive micro-structures in students. In order for this influence mechanism to be deeper comprehended, we need to consider the role and importance of a cognitive obstacle in teaching and learning mathematics.

ON THE CONSTRUCT OF "COGNITIVE OBSTACLE"

In this paper, the construct of "cognitive obstacle" in the process of learning mathematics is considered as a thought phenomenon that occurs when students are faced with the specific mathematical content or problem. The very nature of the cognitive obstacle can be seen in situations of solving any kind of mathematical task, particularly in problematic types of tasks.

In each mathematical task there is a structure in which the given set of elements occurs. It represents the setting of the task as well as what is known in the context of the task. At the same time, there are requirements to be met, that is what is required in the task, or what is *unknown* in advance or *incomprehensible* in the context of the task, that can be reached by solving the task. The concepts of "unknowns" and "representing unknowns" are also mentioned by other authors (for example, Van Dooren, Vershcaffel & Onghena, 2002). The relationship between the known and the unknown parts of the task is defined as a task complexity. If a known part of the task is of broader scope than the unknown one (there are more known elements in a task than unknown ones), then it is a task of a low level of complexity (an "easy" task), and vice versa. The term "unknown" in this sense is not the same to unknown as a variable in algebra (e.g. *x* and *y* as the unknowns in an equation), and these categories should not be mixed. If we take a mathematical task, the relationship between known and unknown parts of the task varies from student to student, and it's caused by a student's individual cognitive capacity in facing the task. On the other hand, the unknown as the variables *x* and *y* has the fixed scope in the structure of an equation.

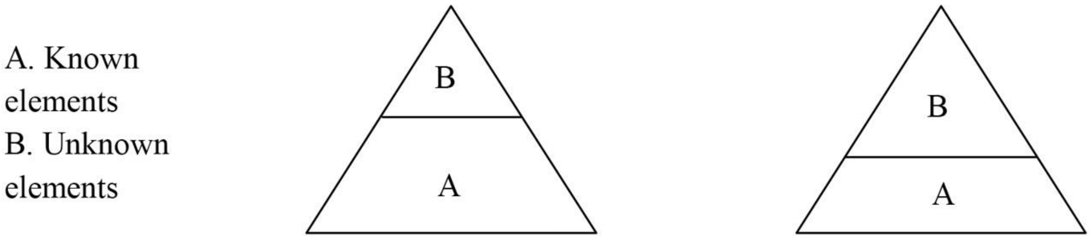

Figure 1. Task complexity: relation between known and unknown elements

At the cognitive level, the thought model of a mathematical task structure is created. On that basis, *a cognitive obstacle is created at the cognitive level, as a cognitive construct that expresses a barrier in transition between the known and unknown parts in the task*, that is, the given and required elements in the context of a task. Therefore, overcoming a cognitive obstacle in learning mathematics is such that it appears as a certain kind of "bridge" towards something that is *new*, *unknown*, *incomprehensible*, as a *general or situational deficit* (gaps, lack or weakness) in functioning of certain cognitive micro-structures (ability, skill, knowledge, experience). There are different situations when "something is not understandable" for a student, in a mathematical content or task. For example, Ormond (2012) states that a weakness occurs in fractional understanding (understanding of fractional numbers), as a problem in solving different algebraic tasks, and notes that it is part of a broader context of algebraic understanding. Understanding is a cognitive capacity that is related to various aspects of the task and its solving, such as the following: task setting, procedures in solving the task, various elements in the task, relationship between the given elements and required transformation to be done, and so on. These are potential points for emerging of a cognitive obstacle, so that one or more cognitive obstacles can be created on the basis of a single task.

A situation in teaching mathematics when a student solving mathematical task has a problem understanding something, can be resolved in several ways. The way of resolving depends on the level of understanding (Eraslan, 2005). In some cases it can be resolved by a student's individual discovery of elements which are crucial for increasing the level of understanding. In other cases a student cannot make a breakthrough individually, but only with the teacher's or another student' help. The teacher can help a student in different ways, such as suggestion, additional explanation, giving similar task respectively, etc.

Several authors discuss the concept related to a student's understanding of mathematical contents, such as "understanding", "misunderstanding", "difficulty in understanding", "lack of understanding", "cognitive conflict", and others. These concepts describe characteristics of the functioning of thought when the phenomenon defined as cognitive obstacle occurs, although these authors do not use the term "cognitive obstacle". For example, Ormond (2012) uses the term "misunderstanding", emphasizing that it appears in the situation of solving algebraic tasks in the lower grades of primary school, which is noted as "misunderstanding of the meaning of the equal sign". Sahin, Yelmaz and Airbus (2015) discuss difficulties that arise in solving tasks in the area of derivatives (secondary school mathematics teaching), which they qualify as "difficulties in understanding". In discussing certain problems that occur in understanding of mathematical contents, Schoenfeld (1994) uses the concept of "incorrect understanding". Tall (1977) uses the concepts of "cognitive conflict" and "lack of understanding", in an effort to describe situations in which a student "explains" a concept or content, based on the cognitive schemas they posses. These are the situations in which "lack of understanding" appears (the term "schema" is used in Piagetian sense). The concept of "cognitive conflict" is used by Meisner (1986), to describe the situation when obstacle arises in solving mathematical tasks. Hizarci, Ilgun and Kucuk (2014) use the concept "epistemological obstacle", which defines a kind of difficulty in understanding of certain mathematical contents. Herscovics (1989) discusses "cognitive obstacles" as difficulties that emerge in solving some categories of mathematical tasks. The concept "cognitive obstacle" is also used by Yoshida and Sawano (2002), in relation to solving mathematical tasks in the area of fractions.

Different kinds of cognitive obstacles appear in teaching mathematics. Most of them represent a situation with the current lack of understanding of the relations existing in mathematical tasks. For example, Yoshida and Sawano (2002) describe the nature of such a cognitive obstacle. They state that a situation when students work with fractions makes a cognitive obstacle in students, i.e. when a student does not understand that a fraction represents both a part of the whole and the whole itself that could be divided. This type of a cognitive obstacle appears in tasks with dividing fractions, and for their successful solution it is necessary to achieve a student's understanding of the relationship between the parts and the whole.

When a cognitive obstacle is created, the thinking activities intended towards its overcoming are initiated. They are, in the case of solving a problematic task, different thinking operations in the context of a problematic situation, such as the following: identification and analysis of elements of the task structure, discovering relationships that exist between the elements, testing possibilities for performing different transformations, searching for a procedure that leads to solution, and others (Sweller, 1988; Abdullah, Halim & Zakaria, 2014). All these thinking operations are undertaken in order for the missing key element (one or more) to be discovered. Therefore, cognitive obstacle arising in solving problematic task is always associated with some kind of discovery that takes place in the process of solving these types of tasks.

Any situation where a student is faced with the mathematical content containing the known and unknown parts of the content constitutes the cognitive obstacle. This is a situation in which the unknown part should be discovered on the basis of the given and known part of the task, i.e. the transition towards the unknown should be made. This takes place with any mathematical content area incomprehensible for a student, which he/she needs to understand. Swanson and Williams (2014) suggest that the transition also happens in a situation when a student needs to make a certain abstract content of a mathematical concept concrete, in terms of understanding the features the concept expresses.

In the following text, we intend to explain different characteristics of a cognitive obstacle, through phenomena such as following: (1) *relationship between a cognitive obstacle and a student's cognitive potential – the adjustment of cognitive obstacle*, (2) *cognitive challenge*, and (3) *cognitive effort*. It can be supposed that these characteristics are the ones that are crucial for deeper understanding of the function of a cognitive obstacle in solving mathematical task. The orientation is to describe adequately the role of a cognitive obstacle through these phenomena, mainly as a kind of hypothesizing and proposing what this role needs to be in order to fulfill its essential functions in solving mathematical tasks.

ADJUSTMENT OF COGNITIVE OBSTACLE, COGNITIVE CHALLENGE AND COGNITIVE EFFORT

A student's *actual cognitive capacity* makes the basis for solving mathematical tasks. It is necessary for the difficulty of the task to be individualized, and adjusted to that capacity, starting from the fact that students of the same age differ in terms of capacity for solving mathematical tasks of a certain level of difficulty. Actual cognitive capacity means the totality of cognitive potential a student possesses, in an area that is important for the successful problem-solving, and thinking in general as well as mathematical thinking operate on the basis of that potential. This capacity involves certain general and specific cognitive abilities and skills, previous knowledge, experience of solving different mathematical tasks, developed interests in mathematics, and others. The abilities, skills, knowledge and experiences have been organized in the aforementioned general cognitive structures and mathematical cognitive micro-structures. Actual cognitive capacity comprises cognitive structures that have already been developed (*developed cognitive potential*), and cognitive structures that are still in the phase of intensive development (*incompletely developed cognitive potential*).

Actual cognitive capacity differs from student to student, so it is necessary to adjust individually the difficulty of mathematical tasks. This is necessary because the same cognitive obstacle in the learning process, created on the basis of a mathematical task, is not of the same difficulty for all the students. The need for the individual adjustments in the process of learning mathematics means that the level of difficulty of mathematical tasks that a student solves needs to be adjusted to the general and specific cognitive micro-structures that the student possesses. Based on that, a student can solve tasks which are at optimal levels of cognitive demands (Stein, Grover & Henningsen, 1996). In that sense, there is a problem of the relationship between *the level of difficulty of the task and the level of difficulty of a cognitive obstacle* (for example, levels of unknown and incomprehensible in the content of cognitive obstacle), and *the actual cognitive capacity of a student*. Therefore, the position of a cognitive obstacle in relation to the actual cognitive capacity of a student in a particular area is important for the effective solving of the mathematical task.

Mathematical tasks for learning fulfill their function to a greater extent if they enable improvement of a student's cognitive micro-structures that are not completely developed, by activating the appropriate thinking operations. For any cognitive structure that develops in a student there is an area of optimal sensitivity for its development. This is an area of *incompletely developed cognitive potential*. This means that it is optimum if a student practices thinking operations in mathematics, basically containing incompletely developed (undeveloped) cognitive structures in a certain area (abilities, skills, knowledge and experiences). It is a role of tasks that require a student's high levels of mathematical thinking and reasoning (Stein & Lane, 1996) This attitude stems from the logic of the internal features of the process of cognitive development, and it implies acceptance of the attitude that the development of certain cognitive structure in students can be systematically influenced by organized teaching and learning (Swanson & Williams, 2014). The impact on the development of a certain cognitive structure is realized in the case when a created cognitive obstacle is positioned in the point of *optimum sensitivity* for development of the structure. This kind of impact is necessary to be ensured in the teaching and learning of mathematics, where the influence on development of different cognitive micro-structures occurs.

What does an optimum level of difficulty of a cognitive obstacle for a student in the process of solving a mathematical task depend on (Figure 2: Position 3)? There are several different factors that determine this kind of optimization and coordination in the learning process. It is understood that solving mathematical task of a certain difficulty requires a system of structured thinking activities (procedures, operations), which are an integral part of the process of solving the task. When assigning mathematical tasks, a teacher should take care to set them in ways that authentically engage a student in the thinking processes of mathematics (Otten, 2010). However, there is a question – how can a teacher be sure that a student can authentically be engaged in solving a task? It needs to be based on a teacher's estimation of a student's actual cognitive capacity to solve a task (Antonijević, 2007). If a teacher fails to estimate and assigns a more difficult task to a student, which cannot be solved by a student's individual work, it can be solved through help from a teacher or cooperative activities with another student. When considering an age of students, the same mathematical task is not equally "difficult" for all the students. When the mathematical task is of optimum level of difficulty for an average student, it is at the same time "easy" for advanced students (Position 2) and "difficult" for weak students (Position 3). Therefore, the relationship between the difficulty of a mathematical task and the actual cognitive capacity of a student determines whether or not the task being solved by a student will enable creating a cognitive obstacle of the optimum difficulty.

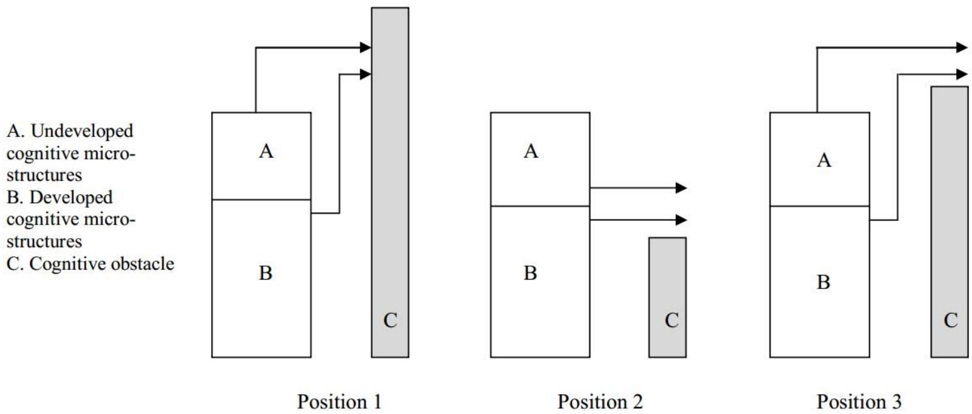

Figure 2. Positions of cognitive obstacles in solving mathematical tasks

When students work together to solve mathematical problems (pair work, group work), they have a higher level of common cognitive capacity, and therefore can solve more complex tasks than they individually could. Collaborative learning is realized through the interaction and co-action in such options of solving tasks (Francisco, 2013; Martin *et al*., 2006), which contributes to a more complete understanding of the content which must be mastered. These capacities become mutually complementary in situations of cooperative mathematical tasks solving, that requires a higher level of cognitive effort.

Cognitive obstacle as part of the process of solving a task in mathematics learning is an obstacle in the true sense of the word, in a situation when it creates a kind of a *cognitive challenge* in a student. Therefore, a cognitive obstacle in the process of solving a mathematical task should serve as a means of a cognitive challenge initiation, and allow the student to start a series of thinking activities that will lead to the solution of the task. What are characteristics of a phenomenon named "cognitive challenge"? Such a situation can be created by a new task in teaching mathematics, a task the student has not previously encountered. Generally, in any situation in the process of learning mathematics, the anticipation of a new and unknown represents a kind of cognitive challenge, which occurs as a result of natural curiosity related to the internal cognitive need to make "the unknown to be known" and "the incomprehensible to be comprehensible". This kind of inner cognitive need for mastering the new and unknown that originally exists in the mind of each person is the basis for creating a situation of cognitive challenge in teaching mathematics.

In order to overcome a cognitive obstacle, students need to make an adequate cognitive effort to solve the task. The *cognitive effort* can be defined as the level of engagement of a set of different thinking operations, aimed at overcoming a cognitive obstacle. Stein and Smith (1998) point to the fact that there are tasks with lower-level demands and higher-level demands. Accordingly, they cause different expected student's responses and different levels of cognitive efforts. In the case of tasks with higher-level demands, thinking activities on the conceptual level are required from a student. Cognitive effort which is made depends on the relationship that is established between the following: (1) characteristics of the complexity and difficulty of the task, and (2) characteristics of the reference part of actual cognitive capacity of a student. This relationship is specific and differs from student to student. Adjustment of a cognitive obstacle means that a certain cognitive obstacle functions optimally in the procedure of solving a mathematical task, in which a student, using all available cognitive means (abilities, skills, knowledge, experiences, etc.) solves a formulated task, making an optimum level of cognitive effort. Therefore, a task that a student solves in mathematics should be neither too difficult nor too easy. The optimum level of cognitive effort depends on several factors, such as the level of understanding or misunderstanding parts and elements of the task content, the level of previous knowledge in relation to solving that kind of a task, the used form for solving task (individually, cooperatively), the presence of external help (from a teacher or another student), etc. Naturally, a student should accomplish a higher level of cognitive effort the problem is the lack of understanding the task content, as opposed to the situation when there is the lack of knowledge, necessary for successful solving the task through the required way.

If a created cognitive obstacle is positioned within an area of ability, skill and knowledge that has already been developed in a student (in the case of an easy task) then the cognitive obstacle would not have its developmental impact. It would not allow further improvement of the cognitive micro-structure and would not enable the progressive transformation into a new advanced state of its development. On the other hand, if a cognitive obstacle is significantly above the actual capabilities of a student, judging by the level of its difficulty (in the case of difficult task), then a student would not be able to independently solve a formulated mathematical task. Henningsen and Stein (1997) and Otten and Herbel-Eisenmann (2009) refer to that kind of mathematical task as a task written with a high level of cognitive demand. Solving such tasks in the option of individual work of a student has no effect on the improvement of any cognitive micro-structure, in cases when they cannot be solved individually. However, these tasks can be applied only through models of cooperative and group work, because these models of work enable solving this kind of mathematical tasks owing to their complexity and a level of cognitive demand.

STAGES IN SOLVING MATHEMATICAL TASKS

Solving a mathematical problem is a process that consists of several stages, in terms of creating a cognitive obstacle at the thought plan, through interaction between student and the task content. That is the way a cognitive obstacle performs its essential function in the process. They are the following stages: (1) *understanding the task content*, (2) *creating a cognitive obstacle*, and (3) *overcoming a cognitive obstacle*. As a result of a student's thinking activities in the process of solving a mathematical task *progressive changing of certain cognitive characteristics* in a student occurs. This outcome is a result of exercising thinking activities in all three stages in a task solving. These stages are realized in a series of interconnected and conditioned sequences in the process of solving the task, having both a procedural and casual character.

Understanding task content

The first step in the process of solving a task is introducing oneself to what is given in the task. A student does it by applying the analysis of the structure, content, and basic elements of the task as well as their interrelationships. In cases of solving difficult tasks it is necessary to give assistance to a student, through guidance, suggestions, giving additional instructions and information, and more. When in that sense a student masters content of a task, we can say that he or she "understands" it. To what extent a student will be effective in understanding the content of a task depends on the basic characteristics of the task, as well as on previous experience of a student in solving certain types of tasks. Based on study results, Clarke & Sanders (2009) point out that certain types of mathematical tasks contribute to the development of a student's capacity of understanding more complex tasks. The authors emphasize that in the lower grades, in the area of learning fractions, tasks with graphics represent one of the key tools for understanding fractions. The role of graphics in solving mathematical tasks successfully is stressed by Loong (2014), who notes them as "virtual manipulatives".

The next step in the process of solving a task, which can go hand in hand with the first step, is *understanding demand* set in the task. Within task content there are certain elements aimed for a student to conceptualize the essence of the demand set in a task at the cognitive level. When this happens, it can be said that a student understands what is requested in the task. In the case of problematic tasks, the student at one moment discovers the essence of the demand that is made in the task (Sweller, 1988). It is also the basis which should allow a student to discover ways (procedures) that should be applied in solving the task. The demand in the task needs to be clearly formulated, as it has the function of directing a student to exercise thinking operations that lead to the solution of a task.

Creating a cognitive obstacle

The process of creating a cognitive obstacle takes place at the cognitive level as a *thinking process* that consists of several interrelated procedures. Some of these procedures also appear in the preceding stage of solving a task. Creating a cognitive obstacle is not an independent process, but a part of a wider process of solving the task.

A cognitive obstacle is created in the immediate dependency of the basic characteristics of a task. In a way, a cognitive obstacle is the thought equivalent of a relationship that occurs between the given and requested in a task. Therefore, the essential characteristics of a cognitive obstacle are adequate and correspond to the characteristics of the task. It is a kind of "mental representation" that expresses the relationship between the given and requested in a task. The moment of the creation of a cognitive obstacle is the moment of forming the *internal thought model* (Sweller, 1988), which is adequate to characteristics of the relationships in a task, that is, a "moment" when a student has essentially understood these relationships.

The cognitive obstacle characteristics are also determined by the characteristics of the actual cognitive capacity of a student. Therefore, a cognitive obstacle may not always be a "true copy" of its external equivalent, so it is also constituted in accordance with the logic of a student (De Corte, 2000), i.e. depending on the characteristics of thought at a certain age. Success in overcoming a cognitive obstacle also depends on the form in which it is constituted at the cognitive level, and on the student's perception of the relationship of the given and required in a task, as well as on the reference part of the actual cognitive capacity that a student possesses, which is important for the successful solving of a task.

As it is the case with a mathematical task, a cognitive obstacle can, to a certain extent, represent something *new* and *unknown* for a student. The level of the new and unknown depends on the student's previous experience in solving specific categories of tasks. When these experiences do not exist and when a student is faced with a completely new kind of a mathematical task, then a cognitive obstacle represents something completely new and unknown to him. Those are the situations when a student in the process of teaching solves more difficult and complex tasks, as well as tasks in new areas. It is necessary to put in significantly higher level of cognitive effort in solving such tasks.

The process of creating a cognitive obstacle can be followed by modification of a mathematical task in the learning process (Stein & Smith, 1998). This refers to the transformation of a task, which goes through the following stages: (1) the task as it appears in curricular materials, (2) task as set up by a teacher, and (3) task as implemented by a student. This modification implies the need of giving additional explanations to a student, suggestions and assistance, which should provide an understanding of the content of the task.

In each particular case of solving a mathematical task, it is of crucial importance for its successful solution what type of the relationship will be established between the created cognitive obstacle, on the one hand, and the actual cognitive capacity of a student, on the other, as it has already been described as a position of a cognitive obstacle.

Overcoming a cognitive obstacle

Student activities initiated by a task directly refer to overcoming a created cognitive obstacle. Its overcoming takes place through an interactive relationship between a student and the content of a task. In addition, if two or more students work together on a task, both a parallel interpersonal interaction and interaction with the content are carried out. In this case, each student creates a specific internal cognitive obstacle, which differs from student to student, and is created in accordance with the actual cognitive capacity that each student possesses.

A student’s thinking activities oriented to seeking the solution of a task start from the moment of encountering a task, and they are intensified after the creation of a cognitive obstacle. The activities are focused on understanding a task as the wider context in relation to the constituted cognitive obstacle (Yoshida & Sawano, 2002). It is usually needed to analyze a task, to observe the basic elements that appear in a task, to discover the basic connections and relationships, as well as to discover a way to solve a task. All this is accompanied by a series of thinking operations, which may be different in shape, intensity, level of organization and systematics. This series of operations can be in the range from a set of relatively unorganized activities, to the organized systematic set of activities. In some cases of solving a mathematical task in teaching, a series of thinking operations takes place through *algorithm of activities* (Henningsen & Stein,1997), which appears as an organized and systematic set of activities leading to the solution of the assigned task.

A cognitive obstacle has a *guiding function* in relation to the activities of thinking that a student will exercise in the process of solving the task. It determines the level of cognitive challenge and cognitive effort, as well as the forms and intensity of thinking operations necessary for solving a task. A student will exercise the thinking operations that lead to overcoming a cognitive obstacle and solving a task.

PROGRESSIVE CHANGING OF COGNITIVE MICRO-STRUCTURES

As a cognitive output of the process of overcoming a cognitive obstacle, progressive changing of certain cognitive micro-structures in students occurs, in terms of its development and improvement. Practicing various thinking operations in situations when a student tries to solve the assigned mathematical task, may potentially have as a result the *improvement of one or more cognitive characteristics of students* (abilities, skills, experiences of learning). The basic condition for the realization of this causal relationship is the creation of a cognitive obstacle in students, which is at an optimum level of difficulty. Such cognitive obstacle optimally activates their *actual cognitive capacity* and *incompletely developed cognitive potential*.

What is the essence of improving a cognitive micro-structure? Whenever there is a situation of successful overcoming a cognitive obstacle in the process of learning, *a progressive change* of the previous state in the process of developing of a cognitive micro-structure is achieved. From this previous state everything that emerged as a lack, incompleteness and the like is at least partially removed. For example, when it comes to the level of understanding a mathematical content, it means that successful solution of certain types of mathematical tasks can remove everything that was unknown or insufficiently understood in the state. Therefore, a cognitive obstacle represents a *means to achieve a progressive change and improvement* in the process of some cognitive micro-structures development.

Progressive change of some cognitive micro-structures that occurs on the basis of overcoming a cognitive obstacle can be different in its nature, quality, level and importance. This change depends on many different factors, which are related to the actual cognitive capacity of students, as well as to the nature and basic characteristics of a cognitive obstacle. At the same time, the crucial importance may be given to the level of *cognitive effort* which is put (Chandler & Sweller, 1991), and which is necessary for overcoming the obstacle, and which the quality of the achieved progressive change depends on.

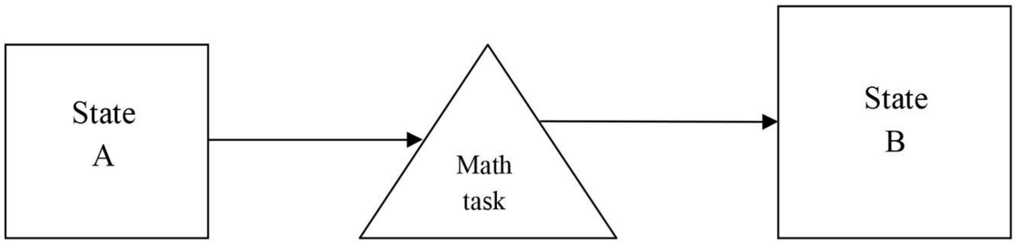

Figure 3. A→B transformation of a cognitive micro-structure

In order to illustrate what kind of outcome occurs when thinking operations for overcoming a cognitive obstacle are exercised, we will describe *a model of transformation of some micro-structures*, which we will symbolically describe as $A \rightarrow B$ (Figure 3). Supposing that a certain cognitive micro-structure in a student progressively changes from the state *A* to state *B*, on the basis of solving mathematical tasks that allow the exercise of various thinking operations at an optimum level of intensity. Here the term "state" means a set of elements present in a cognitive micro-structure, which can be improved by solving a mathematical task. It is the set appearing at a point of a micro-structure's developmental line. The state *A*, with certain characteristics, appears before starting solving a mathematical task. If the task allows a student putting intensive effort of thought, it may be assumed that the *cognitive effort* of the student can lead to improvement of the activated cognitive abilities or skills. This improvement leads to the constitution of the new state *B* in the development of a certain cognitive micro-structure, i.e. the transformation "from-*A*-to-*B*" ($A \rightarrow B$) is carried out.

When does this kind of improvement occur? For example, solving mathematical tasks in the area of multiplication of fractions enables a student to improve different elements of a micro-structure, which can be denoted as a "skill of multiplication of fractions". Some of these elements imply understanding of using operation of multiplication in different cases, and understanding of a relationship between different forms of fractions as parts of this micro-structure, which can be denoted as micro-substructures. By each following task, the level of these two kinds of understanding can be improved, if it enables a student to make a cognitive effort of the level of intensity which is higher than one that he/she made in solving a previous task.

Whenever there is at least a *minimum change* in the quality of a certain cognitive micro-structure, on the basis of solving a mathematical task, we can talk about creation of the new state *B* in its development. The newly established state *B* is a *more advanced state*, regarding to the state *A* that precedes it. In the new state *B* there is at least a minimum element of the qualitative new, based on the exercise of thinking operations in solving a mathematical task.

Described transformation in the process of learning mathematics in its essence is an *elementary change* that is going on in a certain cognitive micro-structure in a student. In some cases it can occur as a minimum improvement, while in some other cases it appears as a significant improvement. In solving mathematical tasks a parallel improvement of several cognitive micro-structures in a student can potentially occur, depending on the complexity and difficulty of the tasks (Kaput, 2008). It is essential that wholeness of teaching and learning of mathematics consists of a series (system) of such elementary progressive changes (improvements), which essentially determine, direct and guide the flow and nature of the whole process of cognitive development for each student.

The importance of the achieved progressive change in the development of a certain cognitive micro-structure by the model $A \rightarrow B$, is reflected in the various segments of progress in the capacity of solving mathematical tasks in teaching, such as the following: (1) improvement of the capacity of solving new tasks of similar difficulty levels, with less cognitive effort; (2) improvement of the capacity of solving tasks of greater difficulty, with similar level of cognitive effort. Based on the realized improvements, a student has at least a minimally increased capacity for solving complex tasks in each of the mentioned situations.

CONCLUSION

In the process of learning mathematics a student's encounter with a certain task that needs to be solved, as well as creation of a cognitive obstacle need to create a situation of a *cognitive challenge* for a student and subsequently an appropriate *cognitive effort* necessary for its overcoming. That is what a more complete function of a cognitive obstacle in learning of mathematics consists of. Also, it is necessary for the difficulty of mathematical tasks and cognitive obstacles in the process of solving these tasks to be adjusted to the actual cognitive capacity of a student in the learning process (possessing a certain level of ability, skills, knowledge, learning experiences, etc.). In such a situation a student practices certain thinking activities directed toward overcoming the cognitive obstacles that are adequate to basic characteristics of a cognitive obstacle a student is faced with.

Each cognitive obstacle in the process of learning mathematics should have a *guiding function* that should trace the direction of the process of solving a mathematical task. It should serve as a *model for initiating the system of thinking* *operations* in a student. On the other hand, that system of a student's activities represent a means of development of particular cognitive micro-structures.

Progressive change in the development of a cognitive micro-structure has a retroactive effect, in relation to each following situation of solving mathematical tasks in teaching. It can be expected that each new advancement of this type enables a student to solve more complex tasks, tasks of a higher level of difficulty, or tasks that require a higher level of cognitive effort. Based on that, in the series of tasks which are solved one after the other, a *gradual increase of the tasks difficulty level* can be achieved, so that each subsequent task that a student solves can be to a certain extent more difficult, when compared to the previous task. Such orientation allows realizing of an optimum impact on the development of certain specific cognitive micro-structures in a student, through the process of learning mathematics. In a situation when the task is solved in a way of cooperative learning, each following task can be on a higher level comparing to the situation when a student solves the task individually, since two or more students cooperatively can solve more difficult tasks.

Intellectual education in mathematics appears as a complex process of influence on the cognitive development of a student. As an integral part of the whole of this process, there is an organized and systematic series of cognitive obstacles, which serve as a means of initiating a complex series of organized thinking activities of a student. That is how the process of teaching and learning mathematics enables an immediate impact on development and improvement of various cognitive micro-structures in students, and subsequently an impact on their cognitive development at each particular age group of students.

REFERENCES

Abdullah, N., Halim, L. & Zakaria, E. (2014). VStops: A thinking strategy and visual representation approach in mathematical word problem solving toward enhancing STEM literacy, *Eurasia Journal of Mathematics, Science & Technology Education*, *10*(3), 165-174.

Antonijević, R. (2007). Differences in teaching and learning mathematics in relation to students' mathematics achievement in TIMSS 2003, *The Second IEA Research Conference: Proceedings of the IRC-2006 – Volume One* (269-281). Amsterdam: IEA.

Chandler, P. & Sweller, J. (1991). Cognitive load theory and the format of instruction, *Cognition and Instruction*, *8*(4), 293–332.

Clarke, B. & Sanders, P. (2009). Task involving models, tools and representations: Making the mathematics explicit as we build tasks into lessons, *Australian Primary Mathematics Classroom*, *14*(2), 10-14.

De Corte, E. (2000). Marrying theory building and improvement of school practice: A permanent challenge for instructional psychology, *Learning and Instruction*, 10, 249-266.

Eraslan, A. (2005). *A qualitative study: Algebra honor students' cognitive obstacles as they explore concepts of quadratic functions*. Electronic Theses, Treatises and Dissertations. Paper 557.

Francisco, J.M. (2013). Learning in collaborative settings: Students building on each other's ideas to promote their mathematical understanding, *Educational Studies in Mathematics*, *82*, 417-438.

Henningsen, M. & Stein, M.K. (1997). Mathematical tasks and student cognition: Classroom-based factors that support and inhibit high-level mathematical thinking and reasoning, *Journal for Research in Mathematics Education*, *28*(5), 524-549.

Herscovics, N. (1989). Cognitive obstacles encountered in the learning of algebra. In S. Wagner & C. Kieran (Eds.): *Research issues in the learning and teaching of algebra* (60-86). Hillsdale, NJ: Lawrence Erlbaum.

Hizarci, S., Ilgun, S. & Kucuk, S. (2014). Epistemological obstacles in mathematical understanding: A function example, *International Journal of Academic Research Part A*, *6*(5), 92-95.

Kaput, J. (2008). What is algebra? What is algebraic thinking? In J. Kaput, D. Caraher, & M. Blanton (Eds), *Algebra in the early grades* (5–18). Mahwah, NJ: Lawrence Erlbaum.

Loong, E.Y.K. (2014). Fostering mathematical understanding through physical and virtual manipulatives, *Australian Mathematics Teacher*, *70*(4), 3-10.

Martin, L., Towers, J., & Pirie, S. (2006). Collective mathematical understanding as improvisation. *Mathematical Thinking and Learning*, *8*(2), 149–183.

Meisner, H. (1986). Cognitive conflicts in mathematics learning, *European Journal of Psychology of Education*, *1*(2), 7-15.

Ormond, C. (2012). Developing "algebraic thinking": two key ways to establish some early algebraic ideas in primary classroom, *Australian Primary Mathematics Classroom*, *17*(4), 13-21.

Otten, S. (2010). Conclusions within mathematical task enactments: A new phase of analysis. In P. Brosnan, D.B. Erchick & L. Flevares (Eds.): *Proceedings of the 32nd annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (661-669)*.* Columbus, OH: The Ohio State University.

Otten, S., & Herbel-Eisenmann, B.A. (2009). Multiple meanings in mathematics: Beneath the surface of area. In S.L. Swars, D.W. Stinson & S. Lemons-Smith (Eds.): *Proceedings of the 31st annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (296-303). Atlanta, GA: Georgia State University.

Sahin, Z., Yemez, A.A. & Erbas, A.K. (2015). Relational understanding of the derivative concept through mathematical modeling: A case study, *Eurasia Journal of Mathematics, Science & Technology Education*, *11*(1), 177-188.

Schoenfeld, A.H. (1994). Reflections on doing and teaching mathematics. In A.H. Schoenfeld (Ed.): *Mathematical thinking and problem solving* (53-70). Hillsdale, NJ: Erlbaum.

Stein, M.K., Grover, B.W. & Henningsen, M. (1996). Building student capacity for mathematical thinking and reasoning: An analysis of mathematical tasks used in reform classrooms, *American Educational Research Journal*, *33*, 455-488.

Stein, M.K. & Lane, S. (1996). Instructional tasks and the development of student capacity to think and reason: An analysis of the relationship between teaching and learning in a reform mathematics project, *Educational Research and Evaluation*, 2, 50-80.

Stein, M.K. & Smith, M.S. (1998). Mathematical tasks as a framework for reflection: From research to practice, *Mathematics Teaching in the Middle School*, 3, 268-275.

Swanson, D. & Williams, J. (2014). Making abstract mathematics concrete in and out of school, *Educational Studies in Mathematics*, *86*, 193-209.

Sweller, J. (1988). Cognitive load during problem solving: Effects on learning, *Cognitive Science*, *12*(2), 257–285.

Tall, D. (1977). *Cognitive conflict and learning of mathematics*. Retrieved 22.09.2010, from http://www.warwick.ac.uk/staff/David.Tall/pdfs/dot1977a-cog-confl-pme.pdf.

Van Dooren, W., Vershcaffel, L. & Onghena, P. (2002). The impact of pre-service teachers' content knowledge on their evaluation of students' strategies for solving arithmetic and algebra word problems, *Journal for Research in Mathematics Education*, *33*, 319–351.

Yoshida, H. & Sawano, K. (2002). Overcoming cognitive obstacles in learning fractions: Equal-partitioning and equal-whole, *Japanese Psychological Research*, *44*(4), 183-195.

◈◇◈