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Cognitive Backgrounds of Problem Solving: A Comparison of Open-ended vs. Closed Mathematics Problems

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Problem solving has been a core theme in education for several decades. Educators and policy makers agree on the importance of the role of problem solving skills for school and real life success. A primary purpose of this study was to investigate the influence of cognitive abilities on mathematical problem solving performance of elementary students. The author investigated this relationship by separating performance in open-ended and closed situations. Findings of the study indicated that the cognitive abilities explained 32.3% (open-ended) and 48.2% (closed) of the variance in mathematical problem solving performance as a whole. Mathematical knowledge and general intelligence were found to be the only variables that contributed significant variance to closed problem solving performance. General creativity and verbal ability were found to be the only variables that contributed significant variance to open-ended problem solving performance.

Keywords: cognitive abilities, mathematical problem solving, open-ended problems, closed problems

INTRODUCTION

As human beings we have been facing with a large amount of problems in daily life. From deciding what to cook for dinner to seeking cure for cancer, different forms of skills and abilities are needed to solve problems. Although we may not be able to solve every problem, we have the capacity to devise strategies and procedures to approach problems (Willats, 1990). According to Butterworth and Hopkins (1988) this capacity appears to be innate. Communicating through cries and grunts, even young babies seek solutions to their comfort and hunger problems (Ellis & Siegler, 1994).

In the field of education, teaching for and of problem solving skills take an important place. In the early 20th century educators who believed that school education should be redesigned around "real-life" situations (Hiebert et al., 1996)

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called often for curricula in which that the knowledge acquired in the classroom could transfer well to the professions, such as medicine, engineering, social work, or education (Boud & Feletti, 1991). Dewey's (1933) ideas about reflective thinking and problem solving provided educators with strong motivation and pathways to redesign school curricula. Dewey believed that reflective thinking was the key to moving beyond the distinction between knowing and doing, thereby providing "a new way of viewing human behavior" (Hiebert et al., 1996). Stemming from Dewey's distinction between knowing and doing, educators and policy makers declared 'problem solving' concept as a core theme in mathematics curricula (NCTM, 1980).

Research on mathematical problem solving has been drawn on, and has evolved from, Polya's (1945) book 'How to Solve It'. In his book, Polya provided the outline of a problem solving framework, a hint of the details necessary to implement it, and a description of steps for solving mathematics problems. Polya's ideas about problem solving influenced the field of mathematics education for decades. The call of the National Council of Teachers of Mathematics (NCTM) in 1980 for problem solving to become "the focus of school mathematics" was widely echoed in the field of mathematics education (NCTM, 1980, p.1). Later, the members of the council endorsed this recommendation with the statement that "problem solving should underlie all aspects of mathematics teaching to give students experience of the power of mathematics in the world around them" (NCTM, 1989). One of the reasons why the council members had emphasized problem solving in its reports was that problem solving "encompasses skills and functions that are an important part of everyday life and furthermore it can help people to adapt to changes and unexpected problems in their careers and other aspects of their lives" (NCTM, 2000, p.4).

State of the literature

- The problem solving concept was referred to by scholars as a high order thinking process that was composed of major intellectual abilities and cognitive processes.
- Problem solving process involves much more than the simple recall of facts or the application of well-learned procedures.
- Cognitive abilities such as intelligence, creativity and originality, spatial ability, verbal ability, working memory, and knowledge have been identified as important factors contributing to problem solving performance.

Contribution of this paper to the literature

- This study proposes that the influences of the cognitive variables (intelligence, creativity, knowledge, memory, reading ability, verbal ability, spatial ability, and quantitative ability) on mathematical problem solving performance differ in open-ended and closed situations.
- In this study mathematical knowledge and general intelligence were found to be the only variables that contributed significant variance to closed problem solving performance.
- Furthermore, general creativity and verbal ability were found to be the only variables that contributed significant variance to open-ended problem solving performance.

In addition to NCTM's emphasis on problem solving, many other researchers also highlighted the importance of problem solving in mathematics education. For example, according to Cockcroft (1982), problem-solving ability lies "at the heart of mathematics" (p.73) because it is the means by which mathematics can be applied to a variety of unfamiliar situations. Carpenter (1989) expressed the view that teaching problem solving is important to encourage students to refine and build onto their own processes over a period of time as they discard some ideas and become aware of further possibilities. In summary, councils and mathematics educators have "elevated a problem solving approach into a position of prominence" (Otten, 2010, p. 14) by locating it at the center of mathematics education.

Problem types

Early classifications of problem types appeared in the field of cognitive science and artificial intelligence. In the studies of Minsky (1961) and Reitman (1964)

problems are categorized into two main types: well defined and ill defined. Minsky (1961) described a well-defined problem as a problem that had an unambiguous solution that could be presented in a systematic manner. Well defined problems have a definite initial state and the goals and operators are known (Dunbar, 1998). Classic examples of well-defined problems include solving an equation (Dunbar) or calculating the perimeter of a circle. On the other hand, ill-defined problems evoke a highly variable set of responses concerning referents of attributes, permissible operations, and their consequences (Reitman, 1964). Unlike well-defined problems, ill-defined problems are ones in which the solver does not know the operators, the goal, or even the current state (Dunbar). Examples of an ill-defined problem might be finding a cure for cancer (Dunbar) or finding a solution for global warming.

Another problem classification was suggested by Getzels and Csikszentmihalyi (1976), who proposed that the structure, method, and solution of a problem could be used for classification purposes. The key finding from Getzels and Csikszentmihalyi's research was that problem solving could be categorized into three types, based on the interaction between the presenter and solver of a problem (Alhusaini & Maker, 2011). The knowledge of both persons about (a) the problem, (b) the method, and (c) the solution made the problem types range from open-ended to closed. Building on Getzels and Csikszentmihalyi's work, Maker and Schiever (1991) proposed the "DISCOVER Problem Continuum" in which six problem types were displayed, along with how much information was known and how much structure was provided for both the problem presenter and the problem solver in each problem type (Table 1).

The Problem Continuum was an expanded version of the model developed by Getzels and Csikszentmihalyi (1976, 1967). Maker and colleagues added Problem Types III and IV to provide a more fluid transition between the Types, based on observations during research (Maker, 1978; 1981; 1993; Whitmore & Maker, 1985). In this context, problems were classified as either closed or open based on the number of alternatives available to the problem solver. For example, a problem was defined as closed if it could be solved in only one way and open if it could be solved in an infinite number of ways. Examples for some problem types in mathematics are presented below:

Type I problem: A Type I problem is a closed problem that is well-defined, and known to both the presenter and the solver. The method and solution also are known to the presenter and to the solver, but the solution is supposed to be derived by the solver (Bahar & Maker, 2011). For example, "Solve this problem: 3 + 4." The problem is well-defined, the method is given by the presenter which is subtraction. The solver needs only to find that "7" is the correct answer.

Type II problem: A Type II problem is a closed problem that is well-defined, and known both to the presenter and the solver. Different from Type I problem, method

Table 1. Problem continuum matrix

	Type	Problem		Method		Solution	
		Presenter	**Solver**	**Presenter**	**Solver**	**Presenter**	**Solver**
Closed	**I**	Specified	Known	Known	Known	Known	Unknown
	II	Specified	Known	Known	Unknown	Known	Unknown
	III	Specified	Known	Range	Unknown	Known	Unknown
Open-Ended	**IV**	Specified	Known	Range	Unknown	Range	Unknown
	V	Specified	Known	Unknown	Unknown	Unknown	Unknown
	VI	Unknown	Unknown	Unknown	Unknown	Unknown	Unknown

Note: Adapted from Maker, J., & Schiever, W. (2010). Curriculum development and teaching strategies for gifted learners (3rd Ed.). Austin, TX: Pro-Ed

to solve the problem is not known to solver. For example, "If a pencil costs $2, how much would two pencils cost?" The solver needs to find that the method is multiplication and that the correct answer was $4.

Type IV problem: A Type IV problem is an open-ended problem which is known to both the presenter and the solver. Different from Type I problem, several correct methods and solutions were possible for a Type IV problem, all of which were known to the presenter but had to be derived by the solver (Bahar & Maker, 2011). For example, "Use these numbers to write as many correct equations as possible (2, 3, 5)." The problem could have four solutions [2+3=5, 3+2=5, 5-3=2, 5-2=3], qualifying this problem as Type IV.

Type V problem: A Type V problem is an open-ended problem that is well-defined and known to the presenter and to the solver. Different from Type V problem, the methods and solutions for a Type V problem were unknown to both. For example, "Write as many problems as possible that have 9 as the answer." Several categories of methods and solutions might be given: [1+8=9, 2+7=9, 10-1=9].

Theoretical framework & statement of goal

The problem solving concept was referred to by scholars (Resnick & Glaser, 1976; Sternberg, 1982) as a high order thinking process that was composed of major intellectual abilities and cognitive processes. Therefore, approaching the problem through a cognitive lens would be more appropriate to answer the research questions of the study.

The theoretical framework of this study is rooted in Newell and Simon's (1972) information-processing (IP) theory of learning. In their theory, Newell and Simon highlighted the similarities between artificial intelligence and human problem solving and they emphasized the role of factors such as working memory capacity and cognitive retrieval of relevant information. They claimed that ability to solve problems successfully depended on a number of factors related to the human information-processing (IP) system. This higher order learning theory has been used to elaborate the cognitive processes of problem solving.

As a complex form of human endeavor, problem solving process involves much more than the simple recall of facts or the application of well-learned procedures (Lester 1994, p. 668). Throughout this complicated process, cognitive abilities such as intelligence (Polya, 1973; Resnick & Glaser, 1976; Sternberg, 1982), creativity and originality (Polya, 1980), spatial ability (Booth & Thomas, 1999), verbal ability (Dodson, 1972), working memory (Swanson 2004), and knowledge (Lester, 1980) have been identified as important factors contributing to problem solving performance. However, prior researchers looked into specific relationships, such as working memory and problem solving performance. In this study, different from the past studies, the author investigated this relationship from a broader perspective and explored how cognitive abilities as a whole (including general intelligence, general creativity, working memory, mathematical knowledge, reading ability, spatial ability, quantitative ability, and verbal ability) predicted problem solving performance.

Another gap in the past studies was that prior researchers did not compare the influence of the cognitive abilities on different types of problems. To fill this gap, the author modeled this relationship by separating performance in open-ended and closed problems. By doing so, the author aimed to investigate how problems with different structures might require different cognitive abilities for reaching successful solutions.

A primary purpose of this study was to investigate the influences of cognitive abilities including intelligence, creativity, memory, knowledge, reading ability,

verbal ability, spatial ability, and quantitative ability on the mathematical problem solving performance. The following research questions guided this study:

1. To what extent do cognitive variables predict mathematical problem solving performance in closed problems? What cognitive variables are the best predictors of the mathematical problem solving performance in closed problems?
2. To what extent do cognitive variables predict mathematical problem solving performance in open-ended problems? What cognitive variables are the best predictors of mathematical problem solving performance in open-ended problems?

METHOD

In this study, for the exploration of potential relationships among these variables, numerical data were analyzed. Therefore, this study was classified as quantitative research. According to Burns et al., (2006), in quantitative research, numerical data are used and statistical analyses are employed to obtain information about the world, giving the opportunity to describe and examine possible relationships among variables.

The research design of this study was a non-experimental and descriptive correlational study. Correlational studies have been used to examine the relationships among two or more variables, and they provide an opportunity to determine the pattern and the strength of the existing relationships and also allow for hypotheses generation. A correlational relationship indicates association between variables in a synchronized manner that does not imply causal relationship. Non-experimental studies are very common in the field of education, because many human characteristics cannot be manipulated experimentally due to natural and ethical reasons. Studies that combine descriptive and correlational characteristics are used to examine variables and to describe relationships among them (Burns et al., 2006; Polit & Beck, 2004; Trochim, 2001).

Participants/subjects

The data were collected from the archives of the DISCOVER projects at the University of Arizona. In the DISCOVER project archives, longitudinal data about students who enrolled in the project's classrooms were available for researchers, including results that were collected using a variety of tests. The participants included 67 students in grade 3. The participants were from four schools in a southwestern state in the U.S. as presented in Table 2. All schools were located in the Diné Nation, and all were in rural, low-income areas. At least 94% of the students at each of these schools came from low income families. Approximately 70% of the children spoke Diné as their home and dominant language, and some were bilingual in English and Diné. Although most of the children spoke some English, many of them would not have been considered proficient in either language when they entered school. Because Diné was many students' first or dominant

Table 2. The number of students at each school

		Gender	
School	**Number**	**Male**	**Female**
School A	20	7	13
School B	13	6	7
School C	17	12	5
School D	17	11	6
Total	67	36	31

language, so early elementary teachers either spoke Diné or worked with an instructional assistant who did. Over 70% of the faculty and staff of all four schools were Diné, and most came from local community.

Measurement of abilities

Measurement of mathematical problem solving performance

The Discovering Intellectual Strengths and Capabilities while Observing Varied Ethnic Responses (DISCOVER) assessment scores were used in this study to measure the participants' problem solving performance in mathematics. The DISCOVER assessment model was developed to identify gifted students from culturally diverse groups by observing the number and the choice of problem solving strategies used by children. The assessment was grounded in the theory of multiple intelligences (Gardner, 1984), the theory of the triarchic mind (Sternberg, 1989), and studies of creativity (Getzels & Csikszentmihalyi, 1976). Problem solving has been a key component of the DISCOVER assessment model. Problem solving was conceptualized in the model based upon the problem classification proposed by Getzels and Csikszentmihalyi (1976). In this context, problems were classified as either closed or open based on the number of alternatives available to the problem solver. For example, a problem was defined as closed if it could be solved in only one way and open if it could be solved in an infinite number of ways.

For the DISCOVER assessment model, problem structure was rated on a scale that ranged from a Type I problem to a Type VI problem (Table 1). A Type I problem would be closed whereas a Type VI problem would be completely open-ended. All conceivable problems could fall somewhere on the continuum between the two extremes.

Previous studies of the DISCOVER assessment showed high inter-rater reliability ranging from 80% to 100% (Sarouphim, 1999; Griffiths, 1996). Sak and Maker (2003) investigated the predictive validity of DISCOVER, and found that it explained 20% of the variance in Stanford 9 Math scores with $p=.007$ and 20% of the variance in AIMS Math scores with $p=.009$. These results provided evidence for the predictive validity of DISCOVER. The results obtained by Sak and Maker (2004) and Maker (2001) showed that moderate correlations existed between the DISCOVER assessment and math achievement ($r= .30$, $p<.01$) and IQ scores ($r=.35$, $p<.01$).2.

The measurement of general intelligence

The Raven's Colored Progressive Matrices (RCPM) assessment scores were used in this study to measure the participants' general intelligence. The RCPM is a norm-referenced standardized test that was designed to measure general intelligence of young children aged 5 through 11. The RCPM is a nonverbal group test in spatial form that can be administered individually or after the age of 8 in a group format. The test consists of 36 items, grouped into three sets (A, Ab, B) of 12 items in each set. Each item in the test had a missing part in a pattern to be completed from the given choices. The bivariate correlations between the item difficulties, for pairs of ethnic groups, ranged from .97 to 1.00 (Jensen, 1998). Also the test-retest reliabilities have ranged from .71 to .92 and concurrent validity (with variety of other instruments) estimates ranged from .55 to .86 (Sattler, 1988).

The measurement of general creativity

The Test for Creative Thinking-Drawing Production (TCT-DP) assessment scores were used in this study to measure the participants' general creativity. The TCT-DP (Urban & Jellen, 1996), was designed as a cross-cultural standardized instrument. The TCT-DP test consisted of six figural fragments that stimulate further drawing in a free and open way: a semi-circle, a point, a large right angle, a curved line, a broken

line, and a small open square outside the large square frame. The drawing product was evaluated and scored by means of 14 evaluation criteria: continuations, completions, new elements, connections made with a line, connections made to produce a theme, boundary breaking that was fragment dependent, boundary breaking that was fragment independent, perspective, humor, affectivity, unconventionality, and speed (Urban & Jellen, 1996). These fourteen scores were then combined into a total score.

The reliability of the TCT-DP was high—from .89 to .97 (Urban & Jellen, 1996). The authors claimed that the validity of the TCT-DP was difficult to evaluate because no comparable instrument existed, and cited studies showing low or no correlations between the TCT-DP and measures of achievement as evidence of its discriminant validity. However, others found correlations ranging from .21 to .41 with the Raven Matrices and the TCT-DP (Urban & Jellen, 1996). The TCT-DP was designed as a nonverbal assessment. The test was field-tested with hundreds of elementary students in 11 countries from diverse backgrounds.

The measurement of mathematical knowledge and reading ability

The Comprehensive Tests of Basic Skills/4 (CTBS/4) assessment scores were used in this study to measure the participants' mathematical knowledge and reading ability. The CTBS/4 was designed to measure achievement in reading, language, spelling, social studies, study skills, mathematics, and science. The mathematics section included subtests of 'concepts and arithmetic', and 'computation'. The CTBS/4 'concepts and arithmetic' score was used as a measure of the participants' mathematical knowledge in this study. The reading section included subtests of vocabulary and comprehension. The CTBS 'reading comprehension' score was used as a measure of the participants' reading ability in this study. Reliability coefficients (KR-20) for the levels used by the sample ranged from .88 to .94 (Shepard, 1985).

The measurement of working memory

The Structure-of-Intellect Learning Abilities (SOI-LA) assessment scores were used in this study to measure the participants' working memory. The SOI-LA is a standardized test (Meeker & Meeker, 1982) that was designed to measure discrete cognitive abilities based on Guilford's structure of intellect (SI) theory, in which intelligence was viewed as being comprised of operations, contents, and products. The SOI test has been available in two alternate forms, A and B. Each form could be individually or group administered to students in grades 2-12. Along with general cognitive assessment, the SOI has been used widely to diagnose learning disabilities, prescribe educational interventions, profile strengths and weaknesses, identify reasons for underachievement, match cognitive style and curriculum material, and screen for gifted students. The test-retest reliabilities of the SOI have ranged from .00 to .74 (Coffman, 1985). For this study, the SOI total memory score was used as a measure of the participants' working memory. The total memory score was calculated as the sum of four subtests: memory of figural units (MFU), memory of symbolic implications (MSI), memory of symbolic systems (MSS), and memory of symbolic units (MSU). The SOI test was described as an effective instrument in identifying gifted students from minority backgrounds (Meeker, 1969). Roid (1985) claimed that the SOI test was an ideal assessment for culturally and linguistically diverse students because of its predominant nonverbal and figural structure.

The measurement of verbal, spatial, and quantitative abilities

The Developing Cognitive Abilities Test (DCAT) scores were used in this study to measure the participants' verbal, spatial, and quantitative abilities. The DCAT (Beggs & Mouw, 1980) is a group administered standardized test that was designed as a measure of learning characteristics and abilities that contribute to academic

performance of students in grades 1-12 (Wick, Beggs, & Mouw, 1989). The test had three categories: verbal, quantitative, and spatial. Each category of the DCAT was comprised of 27 items, for a total of 81 test items. Internal consistency coefficients of the test ranged from .70 to .96, with the majority in the mid .80s (Wick, 1990). The DCAT's verbal, spatial, and quantitative scores were used as measures of verbal, spatial, and quantitative abilities respectively for this study. The DCAT also has been used as a screening measure for identifying potentially gifted students (Wick, 1990). One of the important distinguishing characteristics of the DCAT is the link between specific items and Bloom's (1956) cognitive taxonomy (Canivez, 2000).

Data collection and test scoring

The data related to general intelligence, mathematical knowledge, working memory, verbal ability, spatial ability, quantitative ability, reading ability, and mathematical problem solving were collected at the end of the spring semester of grade 3 in a regular classroom setting. The assessments were administered on different days. Only the data related to general creativity were collected several months later than other data due to the state regulations. Because all of the instruments used in the study were standardized instruments, test administrations and scoring were done in a group format as instructed on the test manuals by trained researchers and graduate assistants.

Data analysis

To answer the research questions, the author employed statistical procedures described below using SPSS. First, preliminary analyses were conducted in the form of a missing data analysis for the data set. These analyses were conducted to explore systematic errors in the data set resulting from missing data or errant unrepresentative results due to observations with excessive influence.

To answer research questions multiple regression analyses were performed to predict students' problem solving performance. To employ regression analyses, intelligence, creativity, memory, knowledge, reading ability, verbal ability, spatial ability, and quantitative ability constituted independent variables. For Research Question 1, the author used closed problem solving performance scores as the dependent variable. For Research Question 2, open-ended problem solving performance scores were used as the dependent variable.

FINDINGS

Single step multiple regression was employed to assess the ability of cognitive variables (intelligence, creativity, memory, knowledge, reading ability, verbal ability, spatial ability, and quantitative ability) to predict mathematical problem solving performance in closed and open-ended problems. Preliminary analyses were conducted to ensure no violation of the assumptions of normality, multicollinearity, and homoscedasticity. The author assessed the normality of independent and dependent variables by examining for skewness (the symmetry of a distribution) and kurtosis (the clustering of scores toward the center of a distribution) values. The skewness and kurtosis values showed an acceptable normality for all variables employed. Descriptive statistics for the cognitive variables and mathematical problem solving performance in closed and open-ended problems have been presented in Table 3.

Multicollinearity occurs when the independent variables are highly correlated. Although multiple regression analysis is a powerful data analysis technique to deal with multicollinearity (Bagozzi and Fornell, 1982), Pallant (2010) recommends checking for multicollinearity before analyzing data. In this study independent

Table 3. Descriptive statistics for variables

Measure	N	Min.	Max.	M	SD
General Creativity	67	9	43	24.75	9.52
General Intelligence	67	16	50	31.85	7.23
Verbal Ability	67	231	561	377.22	69.98
Quantitative Ability	67	213	488	354.58	53.47
Spatial Ability	67	290	552	399.51	53.23
MPSP in Closed Problems	67	1	39	11.75	6.76
MPSP in Open-ended Problems	67	0	35	8.88	8.10
Working Memory	67	9	35	21.36	6.01
Reading Ability	67	486	730	643.58	62.45
Mathematical Knowledge	67	613	744	654.24	30.88

Note. MPSP = mathematical problem solving performance

Table 4. Correlations among independent variables

Variables	1	2	3	4	5	6	7
1. General Creativity	1						
2. General Intelligence	0.435**	1					
3. Verbal Ability	0.289*	0.408**	1				
4. Quantitative Ability	0.358**	0.443**	0.532**	1			
5. Spatial Ability	0.384**	0.485**	0.259*	0.388**	1		
6. Working Memory	-0.054	0.043	0.021	0.01	0.037	1	
7. Reading Ability	0.183	0.047	0.282	0.281	0.167	0.426*	1
8. Mathematical Knowledge	0.393*	0.198	0.333	0.391*	0.472**	0.244	0.45

*** Correlation is significant at the 0.01 level (2-tailed).*
** Correlation is significant at the 0.05 level (2-tailed).*

Table 5. Summary of regression analysis for variables predicting problem solving performance

	Closed Problems			Open-ended Problems		
Model	R	R^2	SEE	R	R^2	SEE
1[a]	0.694	0.482*	3.695	0.568	0.323*	7.264

a Predictors: (Constant), Reading ability, general intelligence, math knowledge, verbal ability, quantitative ability, general creativity, spatial ability, working memory
*** Correlation is significant at the 0.01 level (2-tailed).*
** Correlation is significant at the 0.05 level (2-tailed).*

variables were correlated in a range between -0.054 and 0.532 (Table 4) and a moderate multicollinearity was observed among independent variables.

Research question 1 (predicting closed problem solving performance)

As seen in Table 5, all cognitive abilities (intelligence, creativity, memory, knowledge, reading ability, verbal ability, spatial ability, and quantitative ability) were entered into Model 1 as a block, and the total variance explained by the model as a whole was significant, R2 = 0.482, F (8, 58) = 2.93, p = .02.

The cognitive abilities in the model explained 48.2% of the variance in MPSP in closed problems. As seen in Table 6, students' mathematics knowledge and general intelligence were found to be the only significant predictors of their MPSP in closed problems (Table 6). However, working memory, verbal ability, reading ability, quantitative ability, and general creativity did not explain the variance in MPSP in closed problems at a significant level.

Table 6. Hierarchical analysis of mathematical problem solving performance

Model	Predictor	Closed Problems			Open-ended Problems		
		SE	*β*	t	SE	*β*	t
1	**General Creativity**	0.12	-0.06	-0.99	0.26	0.23	2.45*
	Reading Ability	0.02	0.01	0.02	0.20	-0.08	-1.01
	Math Knowledge	0.04	0.38	4.12**	0.04	0.09	0.82
	General Intelligence	0.18	0.34	3.08*	0.24	0.21	1.83
	Verbal Ability	0.02	0.12	1.14	0.01	0.23	2.21*
	Quantitative Ability	0.03	0.22	1.88	0.01	-0.02	-0.25
	Spatial Ability	0.02	-0.11	-1.69	0.01	0.03	0.48
	Working Memory	0.17	0.21	1.72	0.14	-0.09	-1.44

** Correlation is significant at the 0.05 level (2-tailed).*
*** Correlation is significant at the 0.01 level (2-t ailed).*

Research question 2 (predicting open-ended problem solving performance)

As seen in Table 5, after entry of cognitive variables into Model 1, the cognitive abilities explained 32.3% of the variance in MPSP in open-ended problems, and F change (8, 58) = 2.39, p = 0.044. To explore the best predictors of mathematical problem solving performance in open-ended problems, multiple regression analysis was employed. The model included the following set of predictors: intelligence, creativity, memory, knowledge, reading ability, verbal ability, spatial ability, and quantitative ability. Mathematical problem solving performance (MPSP) in open-ended problems was used as the dependent variable. As seen in in Table 6, students' general creativity ability and verbal ability were found to be significant predictors of their mathematical problem solving performance in open-ended problems. Working memory, spatial ability, quantitative ability, reading ability, and mathematical knowledge did not contribute significantly to the variance in MPSP in open-ended problems.

DISCUSSION

Predicting closed problem solving performance

Using multiple regression analysis, the author found that cognitive abilities were significant predictors of mathematical problem solving performance (MPSP) in closed problems. The cognitive abilities as a whole explained 48.2% of the variance in MPSP in closed problems. In the model, students' mathematics knowledge and general intelligence were the only variables that contributed significant variance to MPSP in closed problems. This finding brings fort an important question to answer: Why are mathematical knowledge and general intelligence the only variables that contributed significant variance to MPSP in closed problems?

To us, the answer is related to the structure of problems. According to information processing (IP) theorists, problem solving is associated with three sets of thinking processes: (a) understanding, (b) searching, and (c) implementing solutions. The influence of cognitive abilities on these thinking processes might vary depending on the type of problem. In this study the problems that were used to measure MPSP in closed problem solving performance were Types I, II, and III problems. As explained in Maker's Problem Continuum model, in Type I, II, and III problems, the problem is known to both the presenter and the problem solver. The presenter also knows the correct solution. As Hong (1998) suggested, in solving these kinds of problems, if problem solvers possess appropriate domain-specific knowledge, including basic concepts, facts, and principles of a particular subject

matter domain, the learners can solve the problem directly without searching for a solution using various searching strategies. When solvers do not have appropriate knowledge to solve the problem, they are required to use specific strategies to search for a solution (Chi, et al., 1982). Fingar (2012) stated that intelligence can help in creating these strategies by reducing uncertainty and providing insight. Therefore, performance in solving these kinds of problems might be associated with domain-specific knowledge or intelligence.

Predicting open-ended problem solving performance

Using regression analysis, the author found that cognitive abilities predicted mathematical performance in open-ended problems significantly. The cognitive abilities as a whole explained 32.3% of the variance in mathematical problem solving performance in open-ended problems. In the model, students' general creativity and verbal ability were found to be the variables that contributed significant variance to MPSP in open-ended problems.

General creativity and verbal ability are the only variables that contributed significant variance to MPSP in open problems. Again, this statistical explanation brings forth the same question: Why are general creativity and verbal ability the only variables that contributed significant variance to MPSP in open-ended problems? In this study, the problems that were used to measure MPSP in open-ended problem solving performance were Type IV and V problems. As explained in Maker's Problem Continuum model, Type IV and V problems are known to the presenter and the solver, but the problem may be solved in more than one way and the presenter knows the range of solutions. These problems can be solved inductively but they have an accepted range of answers. Because of the structure of these problems, the problem solver's fluency is associated with his/her performance in these types of problems. As Mednick (1962) suggested, fluency is an important component of creativity because more responses to a single prompt results in a higher probability that a problem solver will generate an original idea. Therefore, expecting general creativity to be a significant predictor of open-ended problem solving performance is reasonable.

The relationship between performance in solving open-ended problems and verbal ability might seem complicated to explain. One possible explanation for the fact that verbal ability was found to be a significant predictor of MPSP in open-ended problems might be the relation between fluency and verbal ability. Fluency is described as an aspect of verbal ability (Lamar, Zonderman, & Resnick, 2002). Researchers have found that individuals with high verbal ability perform well on measures of fluency (Ayotte, Potter, Williams, Steffens, & Bosworth, 2009). In addition, as declared before, open-ended problems have multiple potential valid solutions and the problem solver's fluency is associated with his/her performance in these types of problems. Therefore, fluency can be described as a moderator between verbal ability and performance in solving open-ended problems, resulting in verbal ability being found to be a significant predictor of MPSP in open-ended problems.

When combining all of these findings we claim that the relationship between cognitive abilities and problem solving performance may vary depending on the structure of a problem. The following problems are nice examples to better understand this distinction.

> Problem A: Solve this problem: 4 + 6 = ___
> Problem B: Write as many problems as possible that have 10 as the answer.

Although these two problems include similar content and concepts, they have different structures. Problem A is defined clearly, has a specific method, and one

right answer (closed, Type I). Problem B is defined clearly (all problems written must have an answer of 10), but the methods are not specified, and in fact, are infinite, an infinite number of solutions can be devised (open-ended, Type V). Therefore, we conclude that closed and open-ended problems require different cognitive abilities for reaching successful solutions. These findings are thought-provoking, and can help educators and researchers understand how structure (type) of a problem might influence the relationship between cognitive abilities and problem solving performance.

This study has several important implications for practitioners and researchers in the field of psychology, education, and other related disciplines. We found that open-ended problems have different processes and components than those of closed problems in mathematics. To develop students' problem-solving skills, mathematics educators and teachers must design proper teaching and learning strategies using methods that correspond to the different characteristics and different nature of problems.

The characteristics of instructional strategies and teaching methods should be in accordance with educational goals. To improve students' mathematical performance in closed problems, educators must focus on enhancing students' mathematical knowledge and they should consider students' intellectual levels. Similarly, to improve students' mathematical performance in open-ended problems, educators must focus on fostering students' creativity and verbal abilities. Underestimating students' cognitive abilities and their interaction with learning processes might possibly result in failures.

Finally, to teachers and other educators, we would say that fostering students' creativity in mathematics classrooms is a major component in learning mathematics. As the findings indicated, creativity contributes to mathematical problem solving performance significantly when the problem has an open-ended structure. Minimal or ineffective use of open-ended problems during instruction has been one common mistake made by teachers in many mathematics classrooms. This common practice does not allow students to use and apply their creativity in mathematics. However, children enjoy creative thinking experiences, and they can learn mathematics while also applying their creative thinking in the use of mathematical principles (Bahar & Maker, 2011). Curriculum designers and educators should produce rich learning settings and materials to address students' creativity. In addition, teachers should create classroom environments in which students can defend their solutions or decisions, and therefore develop their creative thinking.

We believe problem solving should play a key role in education, at all levels, precisely because it is so important in everyday life. Each of us may make hundreds of decisions every day, and the vast majority of these decisions are about how to solve open-ended problems because we are faced with numerous possible variations or alternatives. Deciding what to cook for dinner, for example, typically is an open-ended problem; usually one can see a clear need to do so, yet the method and solution will vary. Likewise is it not imperative for educators to help students develop their problem solving skills, especially to solve complex, open-ended problems? Unfortunately much remains to be done in the standard classroom. Most mathematics problems in school curricula continue to be Problem Types I and II only—drill and practice, find-the-right-answer kinds of approaches. As educators, we cannot expect our students to function in the real world after teaching them twelve-plus years how to solve Problem Types I and II because these Problem Types are virtually non-existent after we graduate.

The substantial influence of cognitive abilities on problem solving performance supports the theoretical framework of the study. However, researchers still need to identify the variables that account for the remaining unexplained variance in mathematical problem solving performance. Therefore, we suggest that future

researchers should consider the fact that abilities and skills develop in a sociocultural context and they should analyze the influence of demographic variables including ethnicity, socioeconomic status, and cultural variables on problem solving performance.

Limitations of the study

This study has few limitations. First of all, this study's sample consisted of only Dine students in grade 3, mostly coming from low income families. From this perspective, the sample of population is unique and homogeneous. Therefore, the results might not be generalized to other populations and all grade levels. Future researchers should have participants from other ethnicities, socio-economic status, and grade levels so that the findings may be generalizable to diverse populations.

Second limitation is the small sample size. Different guidelines concerning the number of cases required for regression analysis were suggested by different authors. For example, Stevens (1996) recommended having at least 10 participants for each predictor used in a study for a reliable analysis. Although the size of the sample in the study met Stevens' criteria, for increased power in statistical analysis, future researchers might consider boosting sample size.

Another limitation of the study is related to the data collection procedures. The data related to general intelligence, mathematical knowledge, working memory, verbal ability, spatial ability, quantitative ability, reading ability, and mathematical problem solving were collected at the end of the spring semester of grade 3. The data related to general creativity were collected several months later than other data due to the state regulations. Although Brocher (1989) found a high pre- and post-test reliability in general creativity scores of elementary students with a coefficient of $r = 0.81$ (after several months), we recommend future researchers to administer testing within a closer time intervals.

Problem solving has been a core theme in education for several decades. Educators and policy makers agree on the importance of problem solving skills for school and real life success and they advocate for locating it at the center of education. We believe, as Otten (2010) pointed out, this emphasis on problem solving will not be self-contained only to the domain of mathematics education but also will transfer into society in positive ways by promoting a knowledgeable citizenry and by creating pathways of advancement for students (Hiebert et al., 1996; Schoenfeld, 2007). Therefore, we hope the next generations will be better equipped to deal with problems, not just in mathematics classrooms, but also in real life.

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