OPEN ACCESS
EURASIA Journal of Mathematics Science and Technology Education
ISSN: 1305-8223 (online) 1305-8215 (print)
2017 13(8):4523-4551
DOI 10.12973/eurasia.2017.00946a

Comparing Grades 10–12 Mathematics Learners' Non-routine Problem Solving

David Mogari
University of South Africa, SOUTH AFRICA
Munyaradzi Chirove
University of South Africa, SOUTH AFRICA

Received 17 October 2016 ▪ Revised 16 January 2017 ▪ Accepted 22 May 2017

ABSTRACT

The study, which is derived from a larger study, compares grades 10 – 12 mathematics learners' non-routine problem solving. An exploratory study was conducted on a convenience sample drawn from three high performing high schools located in Tshwane North District, Gauteng province of South Africa. Learners wrote a non-routine problem solving test. Findings revealed that the 11th grade learners obtained the highest mean score while that of the 10th grade learners was the lowest. High school learners' level of strategy use on solving non-routine problems improved significantly as they progress from grade 10 to higher grades. No significant difference was discovered as learners progress from grade 11 to 12.

Keywords: non-routine problems, non-routine mathematical problem solving, problem solving strategies

INTRODUCTION

Mathematics is hierarchal and gets increasingly complex when progressing through the grades. Conversely, learners need requisite competencies to learn mathematics successfully in each grade. In this regard, the South African (SA) school curriculum sets out to expose learners to mathematical experiences that can develop their essential mathematical skills so that they can identify, investigate and solve problems creatively and critically, and develop mathematical reasoning and creative skills in preparation for more abstract and complex mathematical content (Department of Basic Education [DoBE], 2011, p9 &10). Also, in SA, mathematics is compulsory till grade 9 and generally schools only allow above-average learners to continue with it into senior grades. A learner who achieves poorly in mathematics in either 10th or 11th grade he/she will be made to switch to the less cognitively demanding mathematics literacy. In the senior grades however teaching is still generally examination driven where learners are subjected to drill and rote learning with emphasis on mastery of

Correspondence: David Mogari, *University of South Africa, South Africa.*
mogarld@unisa.ac.za

State of the literature

- Varying results produced in various countries on the level of development of non-routine problem solving ability and amount of (textbook) mathematics learnt.
- Non-routine problem solving continues to pose difficulties to learners across many countries
- Generally, the quality of mathematics teaching remains a challenge in South Africa where emphasis is still on drill and rote learning.

Contribution of this paper to the literature

- Successful routine problem solving does not guarantee any success on solving non-routine problems
- Successful non-routine problem solving seems not to be a function of amount of mathematics content acquired
- Affirmation that conceptual learning under girds successful non-routine problem solving.

algorithms and procedures rather than developing conceptual reasoning strategies and high-order thinking skills (cf. CDE, 2013). Suffice it to state that SA learners'problem solving ability is compromised due to such teaching such that, for example, grade 12 learners could not successfully solve problems that involved high-order thinking and reasoning (AMESA, 2013) and TIMSS 2011 showed that SA learners lagged behind their counterparts in other participating countries with their mathematics competency (HSRC, 2012). Furthermore, Bansilal, Brijlall and Mkhwanazi (2014) measured mathematics teacher's content knowledge and found that on average teachers scored 29% on problem solving questions. Mathematics problem solving is essential in developing thinking and reasoning skills, promotes high-order thinking and facilitates conceptual understanding and meaningful learning (see Mogari & Lupahla, 2013). Webb (2010) notes that engaging learners more on cognitively demanding mathematical tasks requiring critical thinking and reasoning tend to enhance deeper knowledge and understanding which are requisite for developing problem solving competencies. Such competencies, according to OECD (2010), are learners' capacities to engage in cognitive processes in order to understand and resolve tasks where a method to discover the solution to a problem is not obvious. This study is derived from a larger study that sought to establish a link, if any, between learners' non-routine problem solving and their belief systems, and seeks to compare non-routine problem solving strategies of grade 10 – 12 mathematics learners by addressing the questions:

1. Which non-routine problem solving strategies do grades 10 – 12 learners use in mathematics?
2. Is there any significant statistical difference among the scores obtained by grades 10 - 12 learners in non-routine problem solving?

Despite, efforts to promote problem solving learning in schools evidence shows that learners are still not competent enough to solve cognitively demanding problems such as non-routine problems (e.g. Elia, Van den Heuvel, & Kolovou, 2009; AMESA, 2013; DoBE, 2014). CDE (2013) attributes the deficit to poor teaching in lower grades that has left learners with limited understanding of the basics and insubstantial foundational competencies. However, there are schools in SA, such as those in this study, that have displayed a consistent above average performance in grade 12 mathematics. This study therefore seeks to determine whether there is any improvement in the non-routine problem solving capacity of learners of high performing schools as they progress through grades. Van den Heuvel-Panhuizen, Kolovou and Robitzsch (2013) compared the grades 4 – 6 learners' non-routine algebraic problem solving and found an improvement as learners' moved up the grades. Arslan and Altun (2007) investigated the success of grades 7 and 8 learners in using non-routine problem solving strategies and found better success in grade 8; learners displayed pre-knowledge of some non-problem solving strategies which they might have learnt from routine problem solving; and that grade 7 learners learnt some strategies more rapidly than those in grade 8. In TIMSS 2011, SA participated with grade 9 learners but was still outperformed by countries that participated with grade 8 learners (HSRC, 2012). By implication, it does not necessarily mean that learners in senior grades are always more mathematically capacitated than those in lower grades. Moreover, that generally there is emphasis on rigid, recurring and repetitive use of known procedures to solve routine problems in SA mathematics lessons (cf. CDE, 2013).

The South African 2015 and 2016 national senior certificate examinations diagnostic reports (see DoBE, 2015, 2016) highlight a concern on learner poor performance on NRMPs. The reports indicate that learners copied with lower order questions that required application of routine procedures taught in the classrooms. They failed to cope with non-routine problems that required independent or creative thinking. The assumption was that learners were not adequately exposed to the NRMPs. They were not provided with adequate and appropriate classroom exercises that require application of basic knowledge to unfamiliar problem situations. Because of the focus on quantity and quality of mathematics examination results by most teachers as they teach mathematics, and the possible challenge of teaching and learning non-routine problems to both teachers and learners, this study examines high school learners' knowledge of problem solving (PS) strategies and their ability to effectively apply the strategies to PS. Despite effort put on teaching PS, a study by Aslan and Altun (2007) discovered that learners faced difficulties on mastering the non-routine PS skills. In addition, they lacked confidence on selection and application of the strategies to PS. Some challenges on teaching learners PS possibly result from learners failing to effectively learn and apply some heuristics (e.g., 'make a diagram', 'use a simple case') (Schoenfeld, 1992). Instead of teaching general heuristics to PS, Schoenfeld (1992) recommends teachers to teach learners specific strategies that relate or link directly to specific problem categories.

For different reasons, teachers tend to expose non-routine problems only or more often to the most able learners in mathematics; those who finish classwork earlier than others, or simply neglect to teach and adequately cover the problem solving content areas. This practice is against the expectation of CAPS that recommends all learners to be exposed to non-routine problems in order to develop higher thinking and reasoning skills. Exposing learners to non-routine mathematical problems (NRMPs) can improve their ability to search for and apply strategies that are effective in solving problems and appreciate the beauty of mathematics (e.g., there is more than one way to solve a NRMP; mathematics is a product of human creation). Rather than being worried of some possible negative effects (e.g., demotivation), all learners should be encouraged to solve NRMPs to reap the possible benefits attached to it. The need for employees with high proficiency levels in non-routine problem solving by the current work industry (Gilfeather & Regass, 1999) could not be over emphasised.

This study was conducted assuming that all learners are exposed to non-routine problem solving as stipulated in CAPS document (see DoBE, 2011). Therefore, this study was pursued against this backdrop. The study seeks answers to questions: What are the non-routine problem solving strategies the 10th, 11th and 12th graders know? What is the learners' level of using strategies to problem solving?

PREVIOUS RELATED STUDIES

Arslan and Altun (2007) studied the level at which the 7th and 8th grade learners learn and use strategies to solve NRMPs. They discovered that the level of learning and effective application of a strategy to PS was dependent on the age of a learner. For example, as compared to the 8th graders, the 7th graders showed a greater improvement to applying the 'make a list' strategy to PS. On the other hand, as compared to the 7th graders, the 8th graders showed a rapid improvement to learning and application of 'look for a pattern' strategy to PS. A study in Turkey by Yazgan (2015) on the role of each PS strategy on explaining learners' success in solving NRMPs and in discriminating between successful and non-successful 6th graders revealed that success in solving NRMPs depends on knowledge of PS strategies. Specifically, Yazgan (2015) discovered that strategies explained 65% of the learners' success in PS. On this regard, Gilfeather and Regass (1999) contend that learners can be able to discover solutions to NRMPs by being able to use procedures and strategies effectively. The strategies, as ordered in terms of their significant contribution to success in solving NRMPs were 'make a drawing', 'look for a pattern', 'guess and check', 'make a systematic list', 'simplify the problem' and 'work backward'. Yazgan (2015) also discovered that the level of strategy use possibly discriminated between high achievers and low achievers in NRMP solving. For instance, high achievers could use 'look for a pattern' and 'make a drawing' more successful in PS than the low achievers.

This study attempts to fill the following gaps in the literature: Most studies reviewed fail to describe or analyse in full the high school learners' non-routine problem solving abilities. In addition, there is a limited number of studies that analyse, describe and compare high school learners' levels of NRMP solving abilities.

THEORETICAL BACKGROUND

It is argued that in order to explore the learners' non-routine mathematical problem (NRMP) solving capacity it is crucial to get insight into 'What are non-routine problems?'; 'What do NRMP entail?' and 'What does solving NRMP require and involve?'

Non-routine problems

These can be described as

mathematical problems that are more complicated and difficult, do not have straightforward solutions, require productive thinking, are approached in more or less sophisticated ways, are non-standard, involve unexpected and unfamiliar solutions, require an insightful approach and strategic thinking and involve the use of various mathematical concepts (see Mogari & Lupahla, 2013, p95)

Brunning, Schraw and Ronning (1999) view NRMPs as ill-defined problems with several acceptable solutions that can be obtained using several unique strategies. Elia, van den Heuvel-Panhuizen and Kolovou (2013, p607) consider NRMPs 'more complicated and difficult than routine problems' and similarly, according to Mogari and Lupahla (2013, p95), they are 'more cognitively challenging and demanding than routine problems'. NRMPs are tasks based on real life circumstances or models thereof that cannot be solved using familiar methods by replicating known procedures (see Muis, 2004; Arslan & Altun, 2007; Mogari & Lupahla, 2013). To this end, in the view of Arslan and Altun, NRMP reflects the relationship between mathematics and reality. This study hopes to establish whether a link exists between the capacity to solve NRMPs and the amount of mathematics learned when moving up the grades.

Non-routine problem solving

Non-routine problem solving depends considerably on higher-order thinking and reasoning and, according to Carson (2007) and Lester (2013), it requires learners to have capacity to synthesise and coordinate their knowledge and skills and apply them to novel problem situations. Arslan and Altun (2007) consider skills and knowledge consisting, among others, of concepts, formulae and algorithms essential in NRMP solving. Elia et al (2009) report that non-routine problem solving is a non-straightforward process that requires creative thinking and use of heuristic strategies to understand the problem situation as NRMPs are complicated and difficult. Arguably, NRMPs have no definite or rehearsed procedure or pathway to follow when solving them. Seemingly solving NRMPs require thinking, flexibility, creativity, inventiveness and resourcefulness and these are some of the attributes aspired by the SA school curriculum. This study intends to determine whether these attributes develop as learners go up the grades. That is, is there any link between the amount of mathematics learned and the capacity to solve non-routine problems? Lester (2013) contends that for learners to succeed in problem solving, they need adequate and relevant previous experiences in learning how to solve problems, a strong mathematical knowledge base, knowledge of various mathematical models or representations, and have the ability to model or represent mathematical situations and construct or draw patterns of inferences. Kolovou (2011) indicates that the solution process of NRMP requires productive and strategic thinking, unravelling complexity of the problem situation, and insights into how quantities in a problem situation relate to one another. Given that prior instruction plays insignificant role in developing NRMP solving ability (Mabilangan, Limjap & Belecina, 2011), it thus posited that solving many of such problems is essential in this regard and, according to Schoenfeld (1992), success in NRMP solving hinges on cognitive resources, strategy use, beliefs and control process. But, successful NRMP solving depends on proper use of problem solving strategies (Elia et al, 2013). The study therefore seeks to determine whether the degree of use of strategies improves with progression through the grades.

Non-routine mathematical problem solving strategies

Unlike routine problems that require algorithms, solving NRMPs involve the use of strategies (see Polya, 1985; Schoenfeld, 1992; Elia et al, 2009) even though strategies don't guarantee solution to problems. Instead, strategies help to establish procedures or pathways to solutions. We argue that arguably the process is based on learners' thinking and understanding of NRMP. Strategies are defined in Yazgan (2013, p572) as procedures used to explore, analyse and probe aspects of non-routine problems with a view to devising a pathway to the solution. There are two types of strategies, namely, cognitive and meta-cognitive strategies. Cognitive strategies include general strategies, such as trial-and-error; working backwards; finding a pattern; using analogies; considering extreme cases; visual representation; intelligent guessing and testing; systematically accounting for all possibilities; and deductive reasoning (see Muis, 2004; Russell, 2007; Depaepe, 2009; Logsdon, 2007; Malouff, 2011). Learners don't normally use strategies spontaneously when solving NRMPs (Schoenfeld, 1992). They instead glance at NRMP in trying to decide what reckoning to perform and this is a shortcoming on their part (Arslan & Altun, 2007). Also, the cognitive demands of respective strategies vary and this affects the rate of use of each strategy, for example, trial-and-error is set to have minimum cognitive demands and it is commonly use (Elia et al; 2009). Some problems are solved with more than one strategy.

In terms of picking on strategies to use in NRMP solving, learners tend to prefer some strategies to the other albeit with varying outcomes. For example, Mogari and Lupahla (2013) found that Namibian learners most preferred the algebraic strategy to other strategies, even though they successfully solved problems that were illustrated with diagrams. The Filipino learners in Mabilangan, Limjap and Belecina (2011) used the 'making a model or diagram' strategy the most. Learners frequently used trial-and-error strategy with high rate of success in Elia et al. (2009). Hegarty and Kozhevnikov (1999) discovered that Ireland learners performed poorly in problems accompanied with diagrams or pictorial representations, probably, because the diagrams diverted the learners' attention from the main relationships in the problem statements to irrelevant visual details in the diagram or picture. Meta-cognitive strategies include self-regulatory actions such as decomposing the problem, monitoring the solution process, evaluating, and verifying results (Schoenfeld, 1992). This study will explore the cognitive strategies learners use and also determine whether their ability to use the strategies improves as they go up the grades.

In sum, to succeed with NRMP solving, it is argued that it is important for learners to know the features of NRMP so that they can distinguish it from routine problems. As this enables learners realise that NRMP requires different procedures to solve. NRMP solving involves strategy use instead of algorithms, and learners have to understand the potential and efficacy of each strategy. Learners should also know that solution pathways of some NRMP may entail more than one strategy and the solution pathway of each NRMP may not necessarily work for the other NRMP. Also, appropriate strategy use is function of creative thinking. In turn, success in NRMP solving depends on strategy use while ability to think creatively in NRMP solving is informed by one's mathematical prowess.

METHOD

An exploratory study was conducted to determine 10^{th} to 12^{th} grades learners' non-routine problem solving strategies. The first author administered a NRMP solving test to a convenience sample of 395 learners (grades 10; 11 & 12 = 173; 180 & 42[1], respectively) drawn from three high performing high schools located in Tshwane North District, Gauteng province of South Africa. The schools were 9, 7 and 4 kilometres apart, respectively. To ensure that learners in these schools wrote the test in the same way and did not exchange any information, arrangements were made to have the test written on the same day albeit at different times since the first author had to travel between the schools. Learners were made aware of the instructions of the test (see Appendix A). The idea of providing rough work and alternative solutions was informed by the notion that NRMP solving is seemingly a 'haphazard' process and hinges on high-order thinking and reasoning. It was therefore thought it will be worthwhile to note the learners' reasoning and thinking processes invoked in solution pathways hence learners were encouraged to also provide rough work and alternative solutions. Mathematically correct workings were duly credited.

The test had six non-routine problems that did not require aspects of the mathematics school curriculum to solve (see **Table 1**). Problem 1 (P1) was an arithmetic sequence word problem that required derivation of general rule from the pattern and use the rule in the solution process. Problem 2 (P2) was on simple proportion and required logical reasoning. Problem 3 (P3) was an inequality problem that required manipulation of variables. Problem 4 (P4) could be solved by either logic (reasoning) or systematic trial and error. Problem 5 (P5) could be solved by an algebraic method (forming an equation) or by systematic trial and error. Problem 6 (P6) had no clear mathematics referents (i.e. had no numbers in its formulation) but required reasoning and algebraic method to solve. The test was content validated by two heads of mathematics and three experienced mathematics teachers. Cronbach alpha yielded a value of 0.79.

To avoid giving specific grade(s) (especially higher grades) advantage over other grades, the test items were not composed of specific mathematics content taught in the

Table 1. Coding scheme for problem solving strategies

Strategy	Abbreviations	Specifications
Systematic listing	SL	Making an organized list which is composed of at least three values. The steps are of the same size and trials 'move' in one direction.
Modelling	MD	Use of algebra (linear equations, simultaneous equations, linear inequalities), drawing diagrams, or sketches
Trial-and-Error	TE	Making at least two trials of which the last value given is the answer. The steps are not of the same size, and the 'movement' of the trials does not necessarily go in one direction.
Guess, Check and Revise [systematic(sys)/ unsystematic(unsys)]	GCR	Sys:- Making a reasonable guess, checking the guess and revising the guess if necessary Unsys: - Making a guess and lack checking or revision to improve the guess. Making one trial only. Giving the answer only.
Use a formula	F	Selecting a formula to use or substituting values into a formula.
Elimination	E	Eliminating incorrect answers or eliminating possible solutions based on the given information in the problem.
Logical reasoning	LG	Using logical reasoning to justify statements or reach a conclusion. Writing logical statements.
No logical reasoning	NLG	Statements lack logic or does not make sense. Unreasonable (absurd). Naïve, impulsive or unthinking. Not answering the question asked. Unable to detect method used.
Look for patterns	LP	Identifying some common characteristics that can be generalized and used to solve the problem.
Consider a simple case	SC	Includes repeating information from the problem formulation or rewording the problem; dividing the problem into simpler problems; using smaller numbers or working backwards.

different grade levels but were real life problems that were within reach of learners of grades 10, 11 and 12. Learners could apply the mathematics learnt in school to resolve the problems. The expectation was that learners had not previously encountered and solved similar problems set. The problems could be resolved in many different approaches. As such, learners were to search for and apply strategies they think could possibly solve the problems. The problems enabled the researchers to examine the different strategies the learners used to find a solution.

ANALYSIS

To analyse the learners' solutions, a coding scheme (see **Table 1**) derived from Elia et al. (2009) and Mabilangan, Limjap and Belecina (2011) was used. The strategies learners used were identified by matching each learner's solutions against specifications in **Table 1**.

The way a strategy is used in the solution process was categorised into *Thorough/insightful use of strategy*; *Partial use of strategy*; and *Limited use of strategy* as was done in Mabilangan et al. (2011). Details of each category are listed in **Table 2**. The solutions were scored by awarding 5 points for thorough/insightful use, 3 for partial use and 1 point for limited use of strategy. Where the assessment of a solution fell between limited and partial use of strategy 2 points

Table 2. Rubric to classify strategy use

Thorough/Insightful use of strategy	Partial use of strategy	Limited use of strategy
There is some evidence of insightful thinking in problem exploration.	There is some focus but with limited clarity.	There is no central focus and the details are sketchy or not present.
The learner's work is clear and focused.	The learner applies a strategy which is only partially useful.	The procedures are not recorded (i.e., only the solution is present).
The strategies are appropriate and demonstrate some insightful thinking.	The learner starts the problem appropriately, but changes to an incorrect focus.	Strategies are random. The learner does not fully explore the problem and look for concepts, patterns or relationships.
The learner gives possible extensions or generalizations to the solution or the problem.	The learner recognizes the pattern or relationship, but expands it incorrectly.	The learner fails to see alternative solutions that the problem requires.

were awarded. Similarly, 4 points were awarded for between partial and insightful use of strategy (Mabilangan et al., 2011). For blank or wrong answer zero was awarded. The first author scored the learners' solutions and the second author verified the scoring by checking through 50% of randomly selected scored learners' answer sheets. The general practice in SA is to select 20% of the answer sheets for scoring verification.

The respective mean scores on the 6-items problem solving test of the three grades were compared using one-way analysis of variance determined using SPSS. The grades' levels of strategy use in solving NRMPs were compared to determine if the learners' insight in resolving non-routine problems improves as they progress through the grades.

FINDINGS

Learners' strategies

The following are strategies learners used: Systematic Listing (SL); Modelling (MD); Trial-and-error (TE); Use a Formula (F); Systematic Guess, Check and Revise (GCR [sys]); Unsystematic Guess, Check and Revise (GCR [unsys]); Consider a simple case (SC); Logical reasoning (LG); No logical reasoning (NLG); and Look for patterns (LP). However, how strategies were used and their frequency of use differed across the grades and learners.

10th Grade: Table 3 provides percentage numbers of 10th grade learners per strategy used per problem.

In total, 9 strategies (i.e. GCR (unsys); GCR (sys); SL; MD; SC; LG; NLG; F & TE) were used in varying degrees. GCR (unsys); NLG; SC & MD were used in all the problems. Of these strategies, on average, GCR (unsys) appears to be the highly used strategy, followed by NLG, then SC and lastly MD. On the other end of the spectrum, LP was not used in any of the problems, TE; GCR (sys) & SC were used only in one problem. TE & GCR (sys) were used in problem 4 by 1% & 2% of learners, respectively. SC was used in problem 1 by 35% of learners.

Table 3. Frequency of strategy use per problem

Strategy	P1		P2		P3		P4		P5		P6	
	f	%	f	%	f	%	f	%	f	%	f	%
GCR (unsys)	53	31	15	9	47	27	117	68	120	69	117	68
GCR (sys)	0	0	0	0	0	0	4	2	0	0	0	0
SL	61	35	0	0	0	0	0	0	0	0	0	0
LP	0	0	0	0	0	0	0	0	0	0	0	0
MD	3	2	2	1	1	1	5	3	6	3	14	8
SC	6	3	13	8	11	6	18	10	19	11	16	9
LG	0	0	30	17	0	0	0	0	0	0	13	8
NLG	54	31	105	61	77	45	30	17	17	10	10	6
F	7	4	15	9	7	4	2	1	0	0	0	0
TE	0	0	0	0	0	0	1	1	0	0	0	0
Blank sheet	0	0	7	4	34	20	12	7	22	13	17	10

Table 4. Frequency of strategy use per problem by grade 11 learners

	P1		P2		P3		P4		P5		P6	
Strategy	f	%	f	%	f	%	f	%	f	%	f	%
GCR (unsys)	58	32	8	4	53	29	107	59	121	67	104	58
GCR(sys)	7	4	0	0	0	0	0	0	2	1	0	0
SL	79	44	0	0	0	0	0	0	0	0	0	0
LP	3	2	0	0	0	0	0	0	0	0	0	0
MD	19	11	4	2	18	10	34	19	19	11	51	28
SC	9	5	19	11	25	14	36	20	26	14	6	3
LG	1	1	71	39	0	0	2	1	0	0	20	11
NLG	20	11	82	46	60	33	12	7	4	2	0	0
F	2	1	2	1	2	1	1	1	0	0	0	0
TE	0	0	0	0	0	0	1	1	0	0	1	1
Blank sheet	2	1	8	4	34	19	17	9	24	13	21	12

LG was used in problems 2 & 6 by 17 % and 8% of learners, respectively. Strategy F was not used only in problems 5 & 6. All learners attempted problem 1 and the highest number of learners, i.e. 20%, did not attempt problem 3.

Figures 1, 2 and 3 exemplify how a typical learner used strategies to solve 3 different randomly selected problems.

11th grade: Table 4 shows % numbers of 11th grade learners per strategy used per problem.

The group used all the strategies in varying degrees. The most commonly used strategy was GCR (unsys) and was used the highest of all the strategies, i.e. 67%; 59% & 58% of learners, in problems 5; 4 & 6, respectively. In terms of frequency of use in the problems, it was followed,

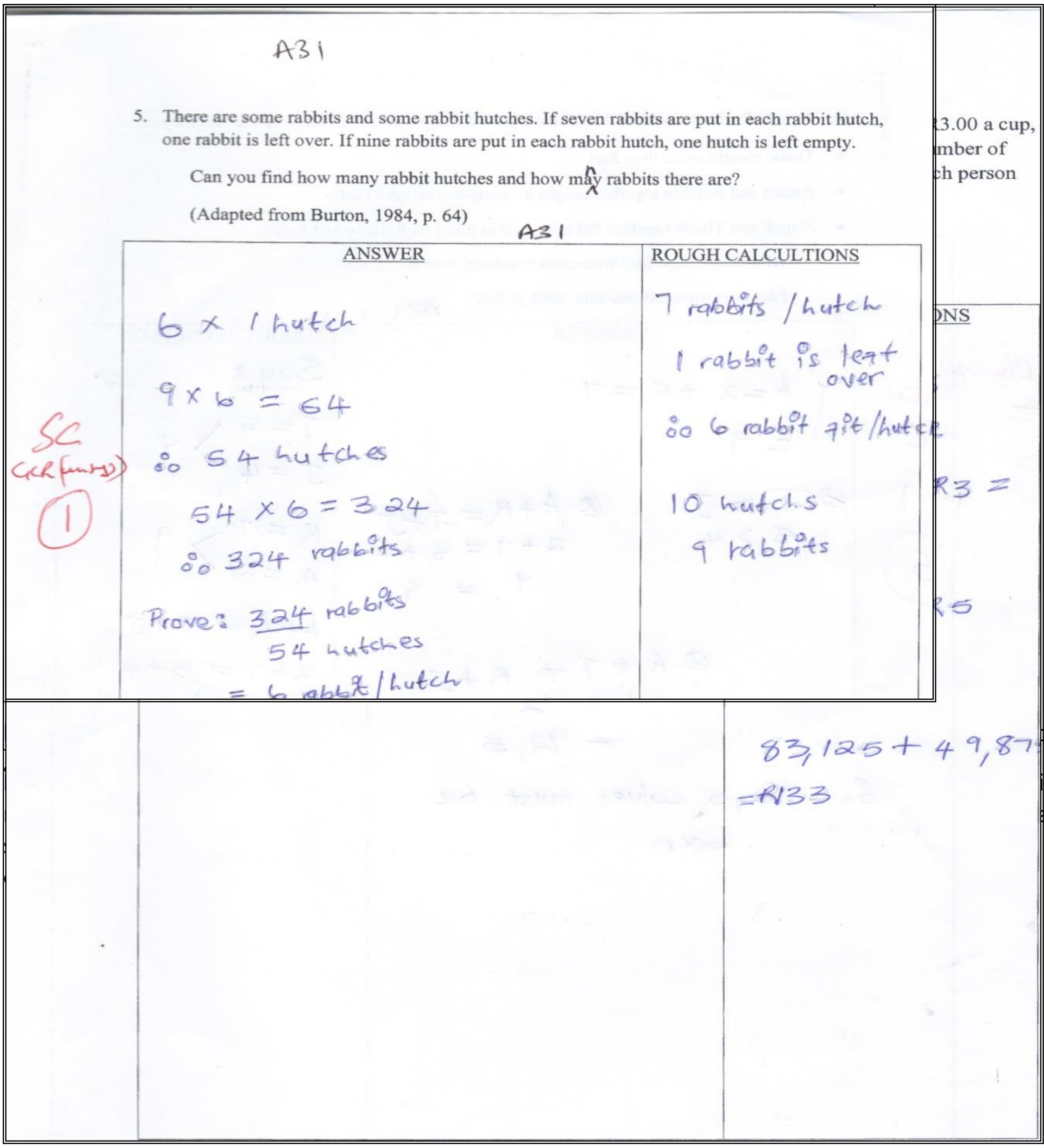


Figure 2. A learner guessed that the bill per person is the sum of the cost cup of tea and cost piece of cake, and then worked out the number of people who had tea by dividing total bill by bill person. The quotient is a fraction and this does not make sense

on average, by MD and then SC. NLG was not used only in problem 6 even though it was used the highest, i.e. 46%, in problem 2. LG was not used in problems 3 & 5, but was the second highest, i.e. 39%, used strategy in problem 2. In addition to not being used in problems 5 & 6, the use of F in other problems did not exceed 1%. GCR (sys) and TE were used only in two

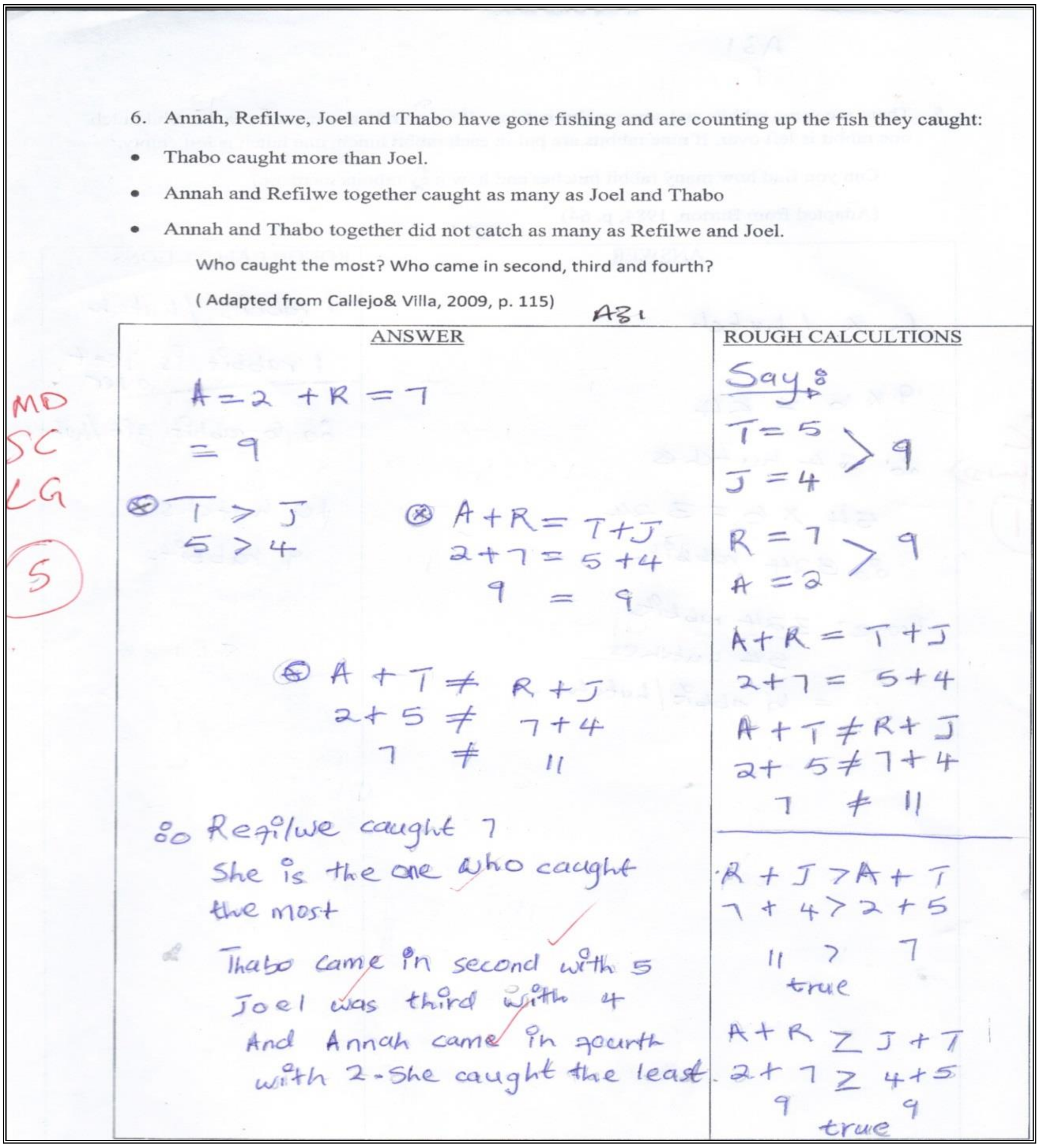


Figure 4. She used SC, MD and LG where she attempted first to express the problem algebraically, then guessed the solution and verified its correctness by substituting numbers in the algebraic statements. The learner continued guessing until a solution that satisfied the algebraic statements was obtained

problems, with GCR (sys) in problems 1 & 5 and TE in problems 4 & 6. TE was the only strategy used in problem 1. SL and LP were only used in problem 1 with SL being the highest, i.e. 44%, of all the strategies used in this problem.

Figures 4, 5 and **6** exemplify a learner's solutions.

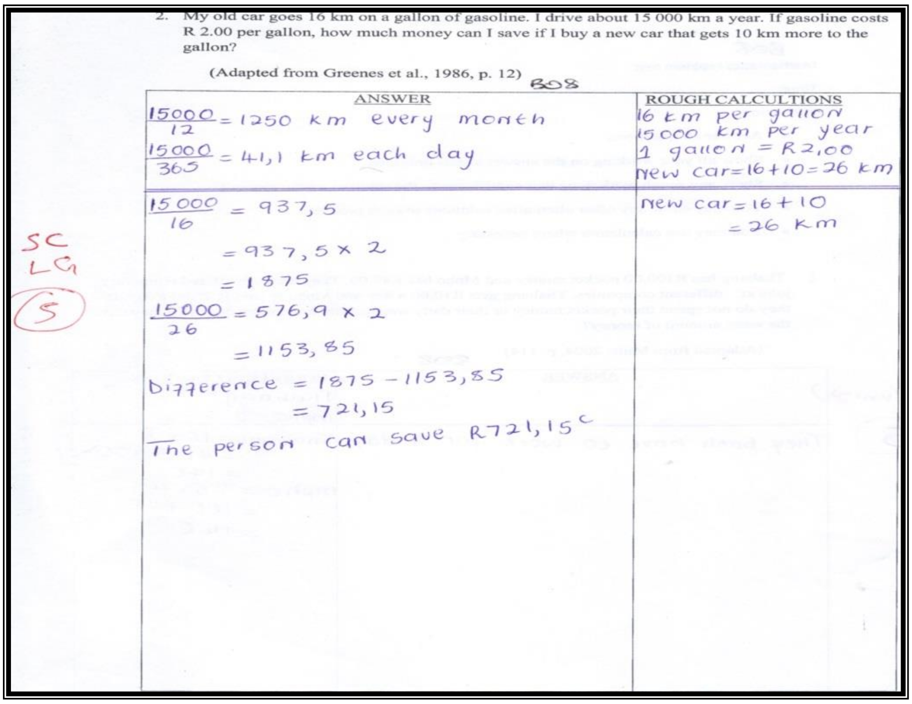
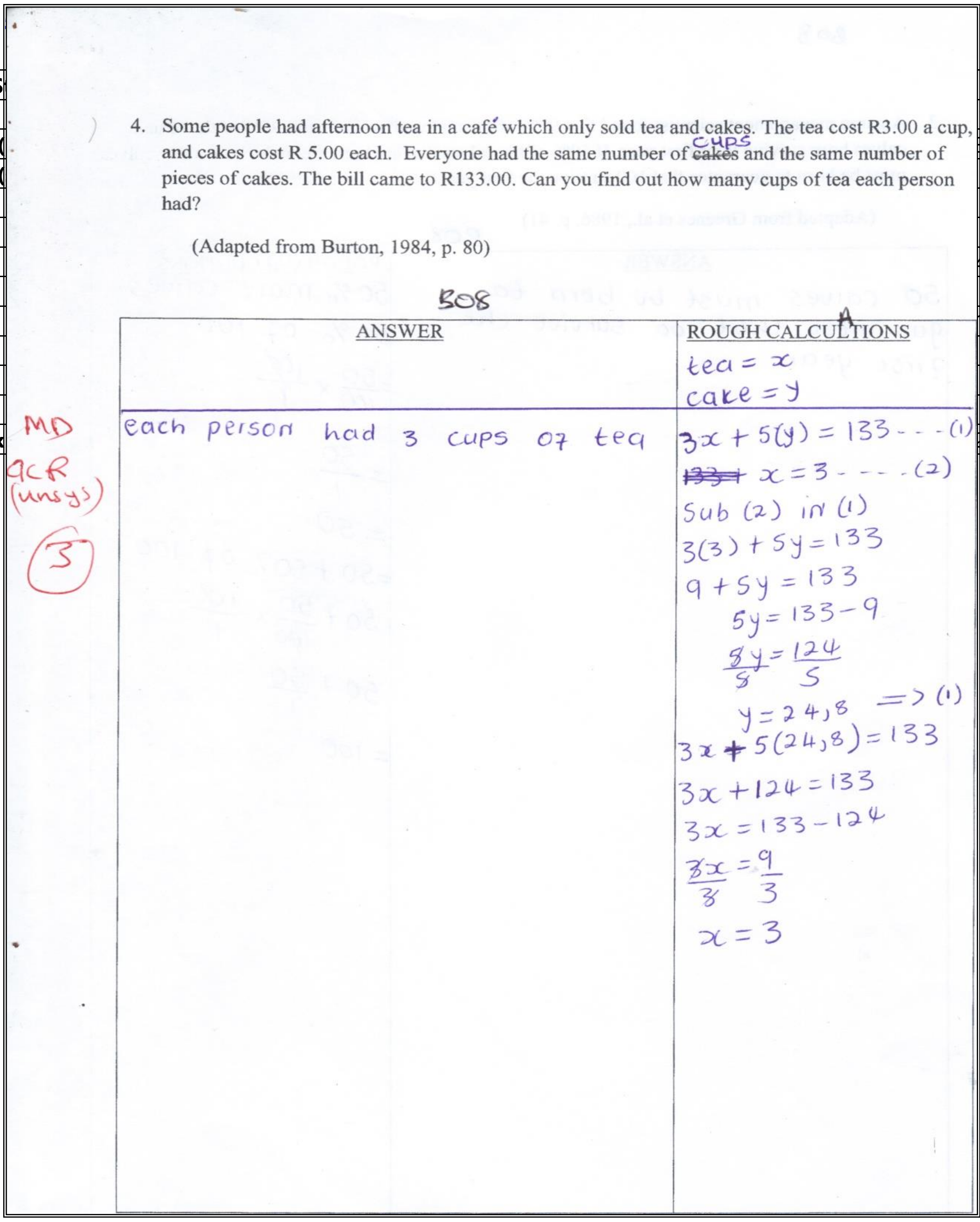


Figure 5. As in Figure 1, the learner first rewrote the information given in the problem and then determined the number of gallons of fuel the car uses per year. Unlike figure 1 where the number of gallons as the cost of using the car per year was considered, the learner successfully converted the number of gallons to the cost of using the car per year. The learner then considered the whole picture of the problem and used all the necessary information in the problem. However, the statements presented were not mathematically correct due to use of a 'piece-wise approach' to problem solving. The learner equated quantities of different values.

12th grade: **Table 5** shows % numbers of 10th grade learners per strategy used per problem.

Table .

S	%
GCR (	36
GCR (	0
SL	0
LP	0
MD	48
SC	5
LG	10
NLG	5
F	0
TE	0
Blank	29


4. Some people had afternoon tea in a café which only sold tea and cakes. The tea cost R3.00 a cup, and cakes cost R 5.00 each. Everyone had the same number of ~~cakes~~ cups and the same number of pieces of cakes. The bill came to R133.00. Can you find out how many cups of tea each person had?

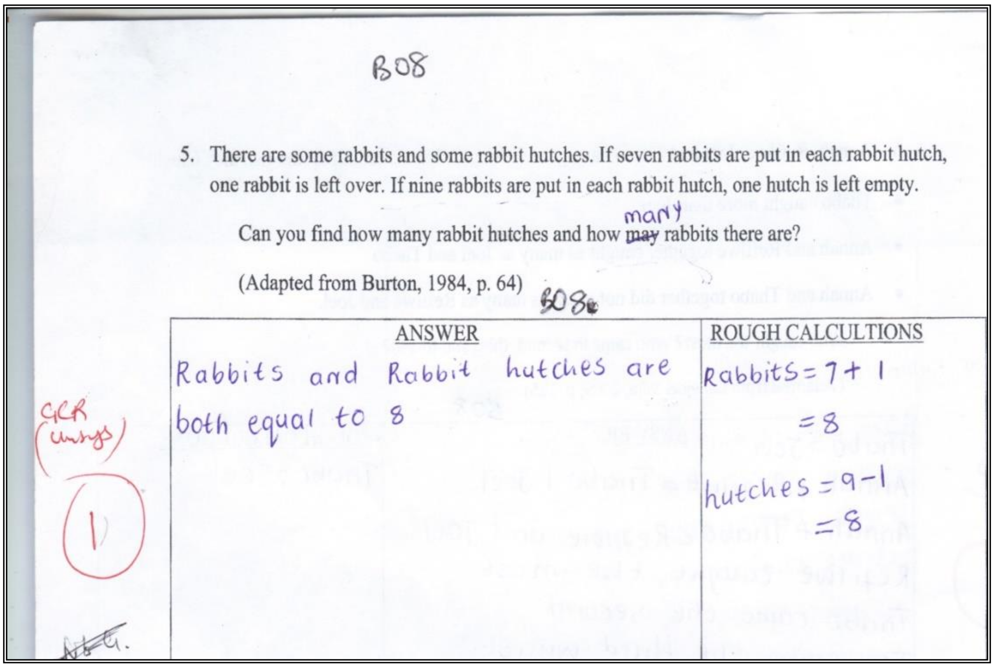
(Adapted from Burton, 1984, p. 80)

B08

ANSWER	ROUGH CALCULTIONS
	tea = x cake = y
each person had 3 cups of tea	3x + 5(y) = 133 ... (1) ~~133 +~~ x = 3 ... (2) Sub (2) in (1) 3(3) + 5y = 133 9 + 5y = 133 5y = 133 − 9 5y/5 = 124/5 y = 24,8 => (1) 3x + 5(24,8) = 133 3x + 124 = 133 3x = 133 − 124 3x/3 = 9/3 x = 3

MD
GCR (unsys)
3

Figure 6. A learner used MD and GCR (unsys) strategies to solve the problem. The learner introduced *x* and *y* variables to represent the number of tea cups and number of pieces of cakes per person, respectively. However, the algebraic expressions and guesses made were incorrect. The learner did not consider the third variable (i.e. the total number of people who took the tea) in the algebraic expressions. The learner overlooked ensuring that the values of *x* and *y* are cardinals



B08

5. There are some rabbits and some rabbit hutches. If seven rabbits are put in each rabbit hutch, one rabbit is left over. If nine rabbits are put in each rabbit hutch, one hutch is left empty.

Can you find how many rabbit hutches and how ~~may~~ many rabbits there are?

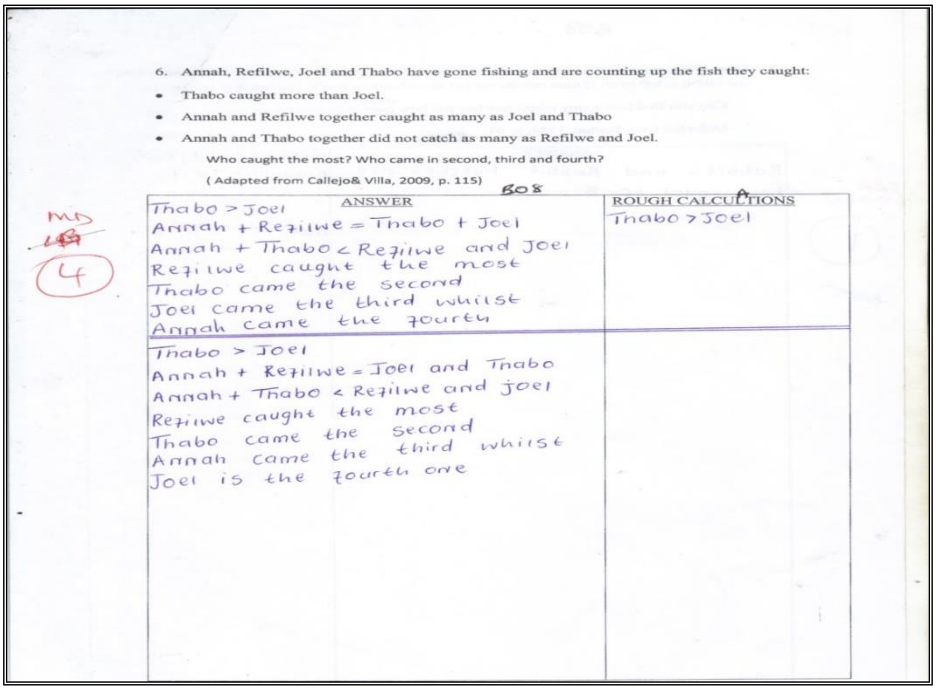
(Adapted from Burton, 1984, p. 64)

ANSWER	ROUGH CALCULTIONS
Rabbits and Rabbit hutches are both equal to 8	Rabbits = 7 + 1 = 8 hutches = 9 − 1 = 8

GCR (unsys)

1

Figure 7. Similar to Figure 3, the learner used GCR (unsys) where she made a single guess of the number of hutches and rabbits and this did not make sense. Rightfully, the number of rabbits should be a multiple of 9 because the rabbits were to be grouped in nines



6. Annah, Refilwe, Joel and Thabo have gone fishing and are counting up the fish they caught:

- Thabo caught more than Joel.
- Annah and Refilwe together caught as many as Joel and Thabo
- Annah and Thabo together did not catch as many as Refilwe and Joel.

Who caught the most? Who came in second, third and fourth?

(Adapted from Callejo& Villa, 2009, p. 115)

B08

ANSWER	ROUGH CALCULTIONS
Thabo > Joel Annah + Refilwe = Thabo + Joel Annah + Thabo < Refilwe and Joel Refilwe caught the most Thabo came the second Joel came the third whilst Annah came the fourth	Thabo > Joel
Thabo > Joel Annah + Refilwe = Joel and Thabo Annah + Thabo < Refilwe and joel Refilwe caught the most Thabo came the second Annah came the third whilst Joel is the fourth one	

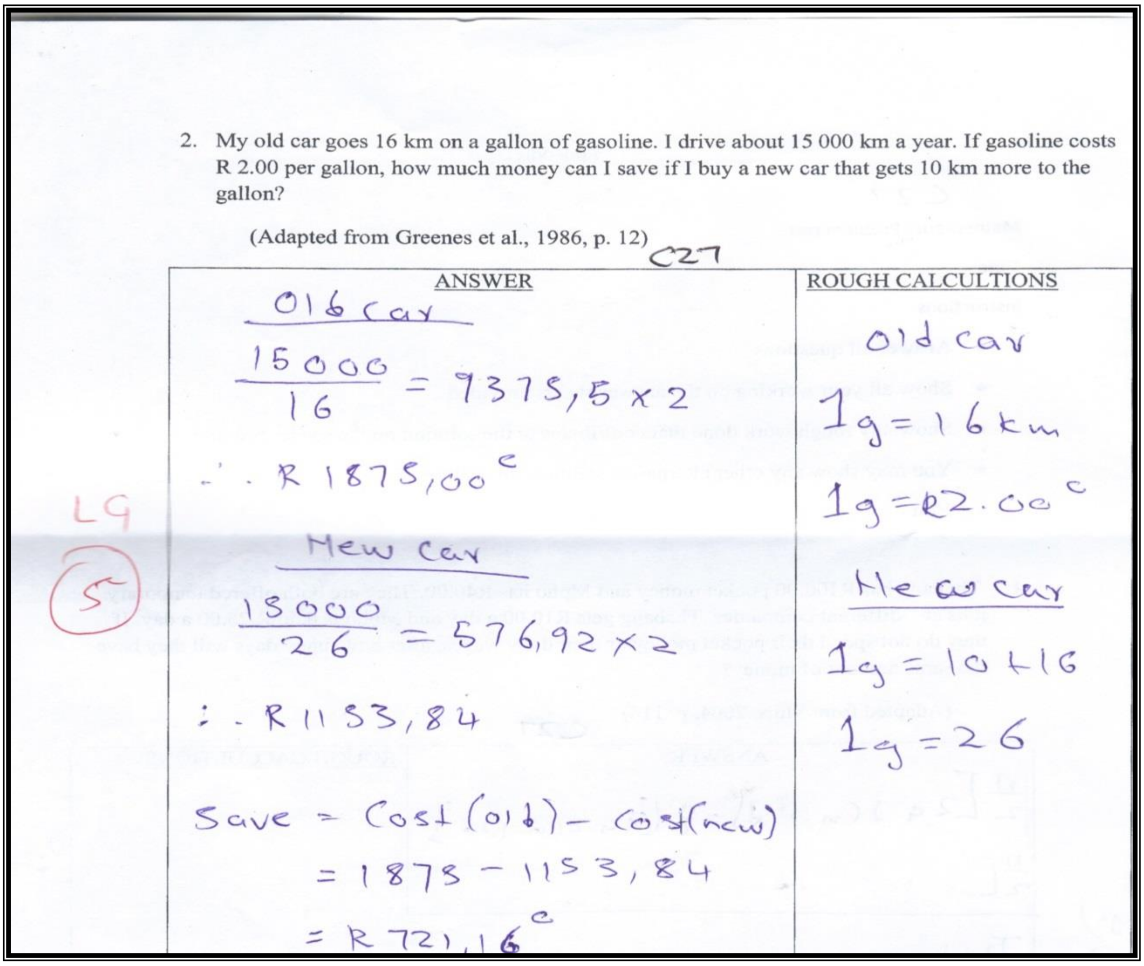
MD

4

Figure 8. Similar to Figure 4, the learner provided algebraic representation of the problem. Unlike in Figure 3, the learner in Figure 6 used names as variables and then provided answers without solution pathways

The learners used 8 strategies where GCR (unsys), MD, SC and NLG were used in all the problems with GCR (unsys) being the highly used and SC the least used among the four strategies. GCR (unsys) was used by 48% of learners in problem 1, by 52% in problem 3 and by 60% in problem 5. The next highest used strategy, on average, was MD, which was used by 48% of learners in problem 6, by 29% in problem 1 and by 17% in problems 4 & 5. Strategy F was not used in problems 5 & 6, LG was only used problems 2 & 6 and LP was only used in problems 1 & 5. SL was only used in problem 1. There were blanks in all the problems with the highest percentage (i.e. 43%) recorded in problem 5.

Figures 7, 8 and **9** are examples of solutions of a typical learner.



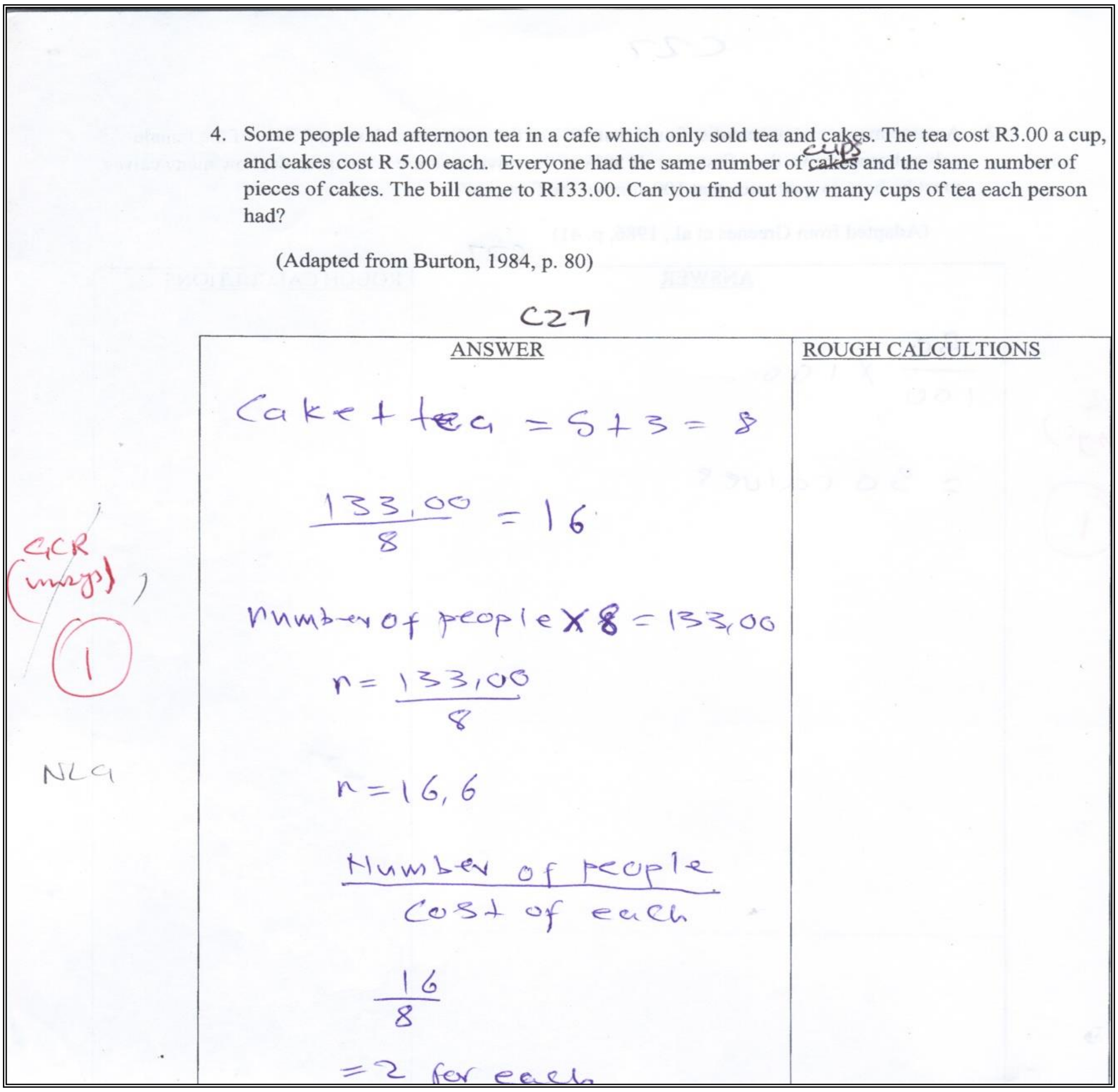
2. My old car goes 16 km on a gallon of gasoline. I drive about 15 000 km a year. If gasoline costs R 2.00 per gallon, how much money can I save if I buy a new car that gets 10 km more to the gallon?

(Adapted from Greenes et al., 1986, p. 12) C27

ANSWER	ROUGH CALCULTIONS
Old car 15000 / 16 = 937,5 x 2 ∴ R 1875,00 New car 15000 / 26 = 576,92 x 2 ∴ R 1153,84 Save = Cost (old) – cost (new) = 1875 – 1153,84 = R 721,16	Old car 1g = 16 km 1g = R2.00 New car 1g = 10 + 16 1g = 26

LG 5

Figure 9. Similar to 10th and 11th grades learners, a 12thgrade learner solved the problem by first rewriting the given information and then determined the cost of using the car per year. He solved the problem the same way as an 11thgrade learner by using logical reasoning and piece-wise approach. Even though the solution was correct, the steps were not mathematically correct because the learner equated quantities of different values



4. Some people had afternoon tea in a cafe which only sold tea and cakes. The tea cost R3.00 a cup, and cakes cost R 5.00 each. Everyone had the same number of ~~cakes~~ cups and the same number of pieces of cakes. The bill came to R133.00. Can you find out how many cups of tea each person had?

(Adapted from Burton, 1984, p. 80)

C27

ANSWER	ROUGH CALCULTIONS
Cake + tea = 5 + 3 = 8 133,00 / 8 = 16 number of people X 8 = 133,00 n = 133,00 / 8 n = 16,6 Number of people / Cost of each 16 / 8 = 2 for each	

GCR (wrongs)

1

NLG

Figure 10. Similar to the 10th grade learners (see **Figure 2**), the 12th grade learner, initially, guessed that each person had one cup of tea and one piece of cake and, thereafter, presented a fractional number of people who took the tea. The learner could not develop a mathematically acceptable solution

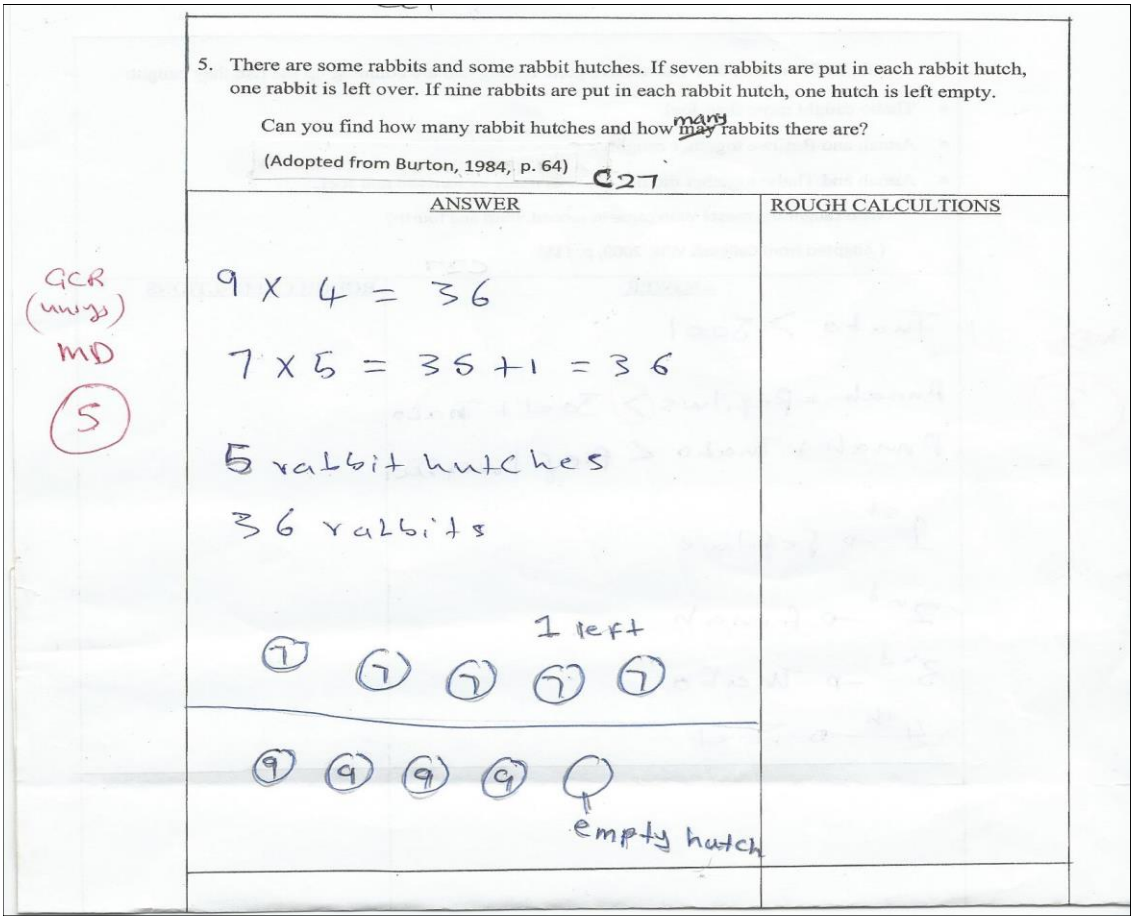
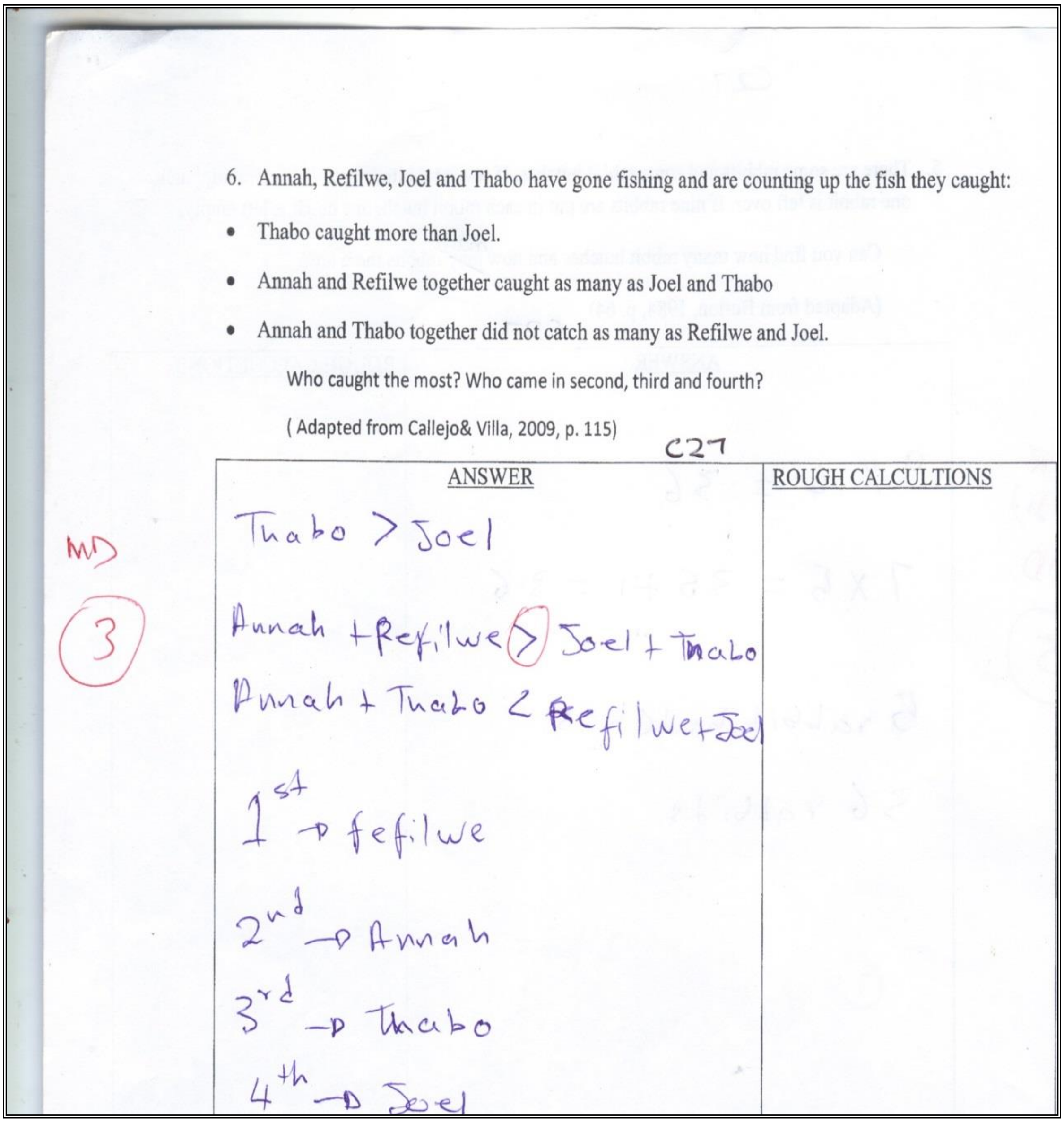


Figure 11. Unlike the 10th and 11th grades, the learner solved the problem using MD and GCR (unsys) strategies. The learner considered all the conditions in the problem and successfully identified the correct number of hutches and rabbits that were there. The learner then verified the correctness of the answers by modelling the situation using diagrams. However, the learner presented on the answer space only one set of guessed values that satisfy the given conditions and no other values that might have been tried in the process of solving the problem were presented


6. Annah, Refilwe, Joel and Thabo have gone fishing and are counting up the fish they caught:

- Thabo caught more than Joel.
- Annah and Refilwe together caught as many as Joel and Thabo
- Annah and Thabo together did not catch as many as Refilwe and Joel.

Who caught the most? Who came in second, third and fourth?

(Adapted from Callejo& Villa, 2009, p. 115)

C27

ANSWER	ROUGH CALCULTIONS
Thabo > Joel Annah + Refilwe > Joel + Thabo Annah + Thabo < Refilwe + Joel 1st -> fefilwe 2nd -> Annah 3rd -> Thabo 4th -> Joel	

MD

3

Figure 12. A learner solved the problem in a way similar to that of the 11th grade (see Figure 6). The learner presented algebraic statements and conclusions without showing how they were deduced from the statements

Reflecting on strategies learners used

The most used strategy by learners in all grades is GCR (unsys) with MD being the second most used strategy among the 11th and 12thgrades. NLG is the second most used strategy by 10thgrades while it is the third most used strategy by 11thgrades. SC is the third most used strategy among the 10th and 12thgrades. TE is the least used strategy by the 10thgrades and is the third least used strategy by the 11thgrades while it was not used by the

12thgrades. LP was not used by the 10thgrades while it was the least used by the 11thgrades and the second least strategy by the 12thgrades. The 11th grades used all strategies with varying degrees to solve all the problems.

All the grades used SC and LG strategies to solve P2. Learners' presentations of incomplete solutions and non-sensible mathematical statements might be due to failure to monitor the solution process. When solving P4, the 10th and 12th grades resorted to guessing and could not present mathematically sensible solutions. An 11th grade learner modelled the problem situation using algebraic expressions. But, the learner did not consider the whole picture of the problem and this led to incorrect models. When solving P5, the 10th and 11th grades failed to consider the constraints described in the problem situation that limited the possible number of hutches and rabbits. They could not look back to check if their answers made sense or satisfy the conditions stated. Though the 12th grade could solve P5, all the mental processes undergone on solving the problem were not put down on the paper. When solving P6, the problem that involved no numbers, the 10th grade did not only model the situation and present a conclusion as was done by 11th and 12th grades, but the learner justified the conclusions by calculations. On the overall, learners tended to use learned and practiced procedures they normally use in solving routine problems when they solved non-routine problems.

How strategies were used?

Analysis of the level of strategy use per question across the grades (see **Table 6**) reveals that 44,5% of the 10th graders showed 'limited use of strategies' on P1. The percentage rate of use of limited strategies on P1 by the 11th and 12th graders were 27,8% and 33,3%, respectively. The 10th graders (20,8%) and the 12th graders (16,7%) had the highest percentage use of 'partial strategies' on P1, while the 11th graders (13,9%) had the lowest percentage use of 'partial strategies' on P1. The 11th graders (58,3%) and the 12th graders (50%) had a highest percentage rate of 'thorough use of strategies' on P1, while the 10th graders (34,7%) had the lowest percentage rate of 'thorough use of strategies' on P1.

The 10th and 11th graders' percentage rate of 'limited use of strategies' on P3 were 99,4% and 97,8%, respectively. The 12th graders (97,6%) had the lowest percentage rate of 'limited use of strategies' on P3. There was no 12th graders who showed 'partial use of strategies' on P3. The 11th graders (1,11%) had a highest percentage rate of 'partial use of strategies' on P3. There was no 10th graders who showed a 'thorough use of strategies' on P3. The percentage rate of 'thorough use of strategies' on P3 for the 11th and 12th graders was 1,11% and 2,38%, respectively. The 10th graders (92,5%) and the 11th graders (75%) showed the highest percentage rate of 'limited use of strategies' on P6. The 12th graders (40,5%) had the highest percentage rate of 'partial use of strategies' on P6. The 11th graders (3,33%) and the 12th graders (2,38%) had the highest percentage rate of 'thorough use of strategies' on P6.

There is evidence of *limited use of strategy* in learners' solutions (see, for example, **Figures 2, 3, 4, 6, 7** and **10**). The percentage rate of this category for the 10th, 11th and 12th grades

Table 6. Frequency of level of strategy use per question

		GRADE 10						GRADE 11						GRADE 12					
		LS		PS		TS		LS		PS		TS		LS		PS		TS	
		f	%	f	%	f	%	f	%	f	%	f	%	f	%	f	%	f	%
STRATEGY USE	P1	77	44.5	36	20.8	60	34.7	50	27.8	25	13.9	105	58.3	14	33.3	7	16.7	21	50
	P2	142	82.1	25	14.5	6	3.47	110	61.1	29	16.1	41	22.8	29	69	7	16.7	6	14.3
	P3	172	99.4	1	0.58	0	0	176	97.8	2	1.11	2	1.11	41	97.6	0	0	1	2.38
	P4	159	91.9	14	8.09	0	0	155	86.1	22	12.2	3	1.67	36	85.7	6	14.3	0	0
	P5	168	97.1	4	2.31	1	0.58	175	97.2	3	1.67	2	1.11	41	97.6	0	0	1	2.38
	P6	160	92.5	12	6.94	1	0.58	135	75	39	21.7	6	3.33	24	57.1	17	40.5	1	2.38
	TOT	878	84.6	92	8.86	68	6.55	801	74.2	120	11.1	159	14.7	185	73.4	37	14.7	30	11.9

Table 7. Breakdown of strategy use

		GRADE							
		10		11		12		TOTAL	
		f	%	f	%	f	%	f	%
STRATEGY USE	LS	878	84.6	800	74.1	185	73.4	1 863	78.6
	PS	92	8.9	120	11.1	37	14.7	249	10.5
	TS	68	6.6	160	14.8	30	11.9	258	10.9
	TOTAL	1038	100	1080	100	252	100	2 370	100

LS = Limited use of strategies (scored 0, 1 or 2 points per question)
PS = Partial use of strategies (scored 3 or 4 points per question)
TS = Thorough/insightful use of strategies (scored 5 points per question)

was 84.6%, 74.1% and 73.4%, respectively. In terms of partial use of strategies, as in **Figures 1, 8** and **12**, the percentage use by 10th, 11th and 12th grades learners was 8.9%, 11.1% and 14.7%, respectively. For thorough/insightful strategy use, as in **Figures 4, 5, 9** and **11**, the percentage use for the 10th, 11th and 12th grades was 6.6%, 14.8% and 11.9%, respectively. **Table 7** provides breakdown of how strategies were used.

Overall, there was considerable 'limited use of strategy' in all grades but % frequency decreased when moving up the grades. There was, however, an increase with 'partial strategy use' when moving up the grades. For 'thorough/insightful strategy use', lowest % frequency occurs in the 10th grade while the 11th grade produced the highest % frequency.

Comparing mean scores across the grades

Table 8 presents the descriptive statistics of the scores obtained by the grades.

The mean scores obtained are all out of 30. One-way analysis of variance (ANOVA) was used to determine whether there was a significant difference between the mean scores at

Table 8. Descriptive statistics

Grade	N	Mean	Std. Deviation	Std. Error	95% Confidence Interval for Mean		Minimum	Maximum
					Lower Bound	Upper Bound		
Grade 10	173	8.27	3.228	.245	7.79	8.76	2	22
Grade 11	180	10.62	4.027	.300	10.02	11.21	2	24
Grade 12	42	9.88	4.133	.638	8.59	11.17	2	19
Total	**395**	**9.51**	**3.865**	**.194**	**9.13**	**9.89**	**2**	**24**

Table 9. ANOVA test

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	491.513	2	245.756	17.856	.000
Within Groups	5395.186	392	13.763		
Total	5886.699	394			

Table 10. Mean equality test

	Statistic[a]	df1	df2	Sig.
Welch	18.686	2	111.194	.000
Brown-Forsythe	16.412	2	141.939	.000

[a] Asymptotically F distributed.

$p < 0.05$. ANOVA F-statistic was highly significant [$F(2,392) = 17.856$; $p < 0.05$] (see **Table 9**). This means that there exists at least one statistically significant difference among the problem solving mean scores.

The results also showed that Levene's test was significant [$F(2,392) = 5.137$; $p = 0.006 < 0.05$]. This confirmed that the variances were significantly different. However, due to the violation of the assumption of homogeneity of variance, the equality of means was tested using more robust procedures, e.g., the Welch and Brown Forsythe F tests. Both the Welch and Brown Forsythe's F statistics were highly significant [$F(2,111) = 18.686$; $p < 0.05$ and $F(2,142) = 16.412$; $p < 0.05$, respectively] (see **Table 10**). This ascertained the existence of at least one significant difference among the means.

Games Howell's post hoc test was conducted to identify mean pair(s) which are most significantly different (see **Table 11**).

Table 11 shows that the mean difference of the 11^{th} and 10^{th} grades was highly statistically significant ($p \approx 0.000 < 0.05$). Similarly, the difference between the problem solving mean scores of 10^{th} and 12^{th} grades was statistically significant ($p = 0.057 \approx 0.05$). The mean difference of the 11^{th} and 12^{th} grades was not statistically significant ($p = 0.552 > 0.05$). A

Table 11. Games-Howell's test

(I) Grade	(J) Grade	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval	
					Lower Bound	Upper Bound
Grade 10	Grade 11	-2.345*	.388	.000	-3.26	-1.43
	Grade 12	-1.609	.683	.057	-3.26	.04
Grade 11	Grade 10	2.345*	.388	.000	1.43	3.26
	Grade 12	.736	.705	.552	-.96	2.43
Grade 12	Grade 10	1.609	.683	.057	-.04	3.26
	Grade 11	-.736	.705	.552	-2.43	.96

* The mean difference is significant at the 0.05 level.

statistically significant mean difference between the 10th and 11th grades and 10th and 12th grades implies that the 11th and 12th grades used more insightful strategies to solve problems than the 10th grades. A statistically insignificant mean difference between the 11th and 12th grades may mean that these classes have comparable insights in the use of problem solving strategies. However, on average, all grades obtained low scores meaning that the learners had difficulties solving the non-routine mathematical problems.

DISCUSSIONS

Data show that learner performance varied across the grades, with 10th grade learners obtaining the least mean score and the mean score of 11th grade learners being the highest. An increase in learner scores from 10th to 11th grade is consistent with the findings by Van den Heuvel-Panhuizen et al (2013) and Arslan and Altun (2007). The decrease of scores from 11th to 12th grade is somehow compatible with the results of TIMSS 2011 (see HRSC, 2012). According to the results, the learners' non-routine problem solving skills, unsurprisingly improved between the 10th and 11th grades and astonishingly deteriorated between the 11th and 12th grades. It may very well be that, in 10th and 11th grades, ostensibly the above-average learners pursuing mathematics will do everything to ensure that they continue to perform well lest they will be made to switch to the career-limiting mathematics literacy if they perform poorly. Learners therefore solve as many problems as possible with the hope of becoming more mathematically competent and thus succeeding. This view is conforming to theory presented by Webb (2010) that one's problem solving capacity improves by engaging in many mathematical tasks as possible. And, according to Arslan and Altun (2007) more routine problem solving tends to have positive effect on NRMP solving. In 12th grade however much time is dedicated to revision and rehearsing by working through previous years' examination question papers where anything that is not examinable is ignored and as implied in CDE (2013) such practice is not beneficial.

All grades used a number of problem solving strategies that include Systematic Listing (SL); Modelling (MD); Trial-and-error (TE); Use a Formula (F); Systematic Guess, Check and Revise (GCR(sys)); Unsystematic Guess, Check and Revise (GCR(unsys)); Consider a simple case (SC); Logical reasoning (LG); No logical reasoning (NLG) and Look for patterns (LP). This is anomalous given that non-routine problem solving tends to receive minimal attention in most SA schools (see AMESA, 2013). The current study fell short to verify AMESAs observation in the 3 participating schools. Nevertheless, the finding accords with that by Mabilangan et al (2011) who discovered that Filipino secondary school learners could apply strategies to solve non-routine problems without any relevant teaching. Albeit learners could apply the strategies to non-routine problem solving, as evident in their solutions, their understanding and effective use of the strategies was problematic. In particular, learners could not plan and effectively execute the strategies. The results also show that GCR (unsys), NLG and SC were frequently and widely used in 10th grade with GCR (unsys) being mostly used. In 11th grade, it was GCR (unsys), MD and SC that were widely used with GCR (unsys) being the most used strategy. For 12th grades, it was GCR (unsys), MD, NLG and SC that were widely used and GCR (unsys) was still the most used strategy. Elia et al. (2009) found TE to be the mostly used strategy and attributed this to the rudimentary state of the 4th grade learners' NRMP solving. In Yazgan (2013, p575), most 12th grade learners benefited from 'use of equation' (or formula as referred to in this study).

The low mean scores indicate that learners battled with NRMPs. Unlike routine problems that require procedures and algorithms to solve, NRMP solving hinges on reasoning and thinking (Kolovou, 2011) coupled with proper strategy use (Elia et al. 2013). Furthermore, success in NRMP solving is a function of mathematical capacity. The low scores thus depict the low level of learners' mathematical capacity and this is consonant with the report by AMESA (see AMESA, 2013).

CONCLUSION

The study shows that it is not a forgone conclusion that learners' NRMP improves progressively as more mathematics is learned or as more routine problems are solved. It seems NRMP depends on the type of teaching learners are exposed to. The teaching that focuses largely on drill and rote learning tends to compromise learners' NRMP solving capacity. It thus concluded that NRMP solving is independent of one's ability to solve routine problems. In terms of strategies learners used, it is evident that learners mostly used strategies that may be considered less sophisticated and are 'haphazard' in nature or follow a 'hit-or-miss' approach, i.e., GCR (unsys), NLD and SC even though MD is the second popular strategy used in grades 11 & 12. The choice of these strategies may be attributed to the learners' under-developed essential mathematical competencies that are crucial in solving NRMP. Another possible thinking is that learners seem to think that what works for routine problems (i.e. algorithms can be replicated on other problems) also works for NRMP. The choice of MD by grades 11 & 12 seems to be influenced by significant amount of word problem solving learners are exposed to. However, pervasive instances of use of ineffective problem solving strategies were noted whereby learners created models without use of all conditions or restrictions in the problem situation. Some created models and, thereafter, abandoned them in favour of strategies set to be 'haphazard' or 'hit-or-miss' oriented, e.g., GCR (unsys) and NLD. This could be, probably, due to learners' low level complexity of their mathematical knowledge that is characterised by lack of creativity, reasoning and high-order thinking. It is thus concluded that prowess use of problem solving strategies tends to be associated with the level of complexity and sophistication of mathematical knowledge and skills.

NOTES

[1] Generally schools are reluctant to avail their grade 12 learners for research studies as attention is devoted to preparing for end-of-examinations. At the time of the study, only 42 grade 12 learners were availed for the study and these were considered the above average cohort.

REFERENCES

AMESA (2013). A report on the 2013 Mathematics & Mathematical Literacy Grade 12 Examination Papers. Retrieved from www.amesa.org.za on 2014/07/31.

Arslan, C. & Altun, M. (2007). Learning to solve non-routine mathematical problems. *Elementary Education Online* 6(1), 50-66.

Bansilal, S., Brijlall, D., & Mkhwanazi, T. (2014). An exploration of common content knowledge of high school mathematics teachers. *Perspectives in Education* 32(1), 30 – 46.

Branca, N. A. (1980). Problem solving as a Goal, Process and Basic Skill. In Krulik, S, & Rey, R. E. *Problem solving in school mathematics, National Council of Teachers of Mathematics, 1980 Yearbook.* Virginia: NCTM.

Brunning, R. H., Schraw, G. J., & Ronning, R. R. (1999). *Cognitive Psychology and instruction* (3rd Ed.). New Jersey: Prentice-Hall Inc.

Carson, J. (2007). A problem with problem solving: Teaching thinking without teaching knowledge. *The Mathematics Educator* 17(2), 7-14.

CDE (2013). Mathematics outcomes in South African schools: What are the facts? What should be done? *CDE Insights*. Johannesburg: Centre for Development and Enterprise

Cherry, K. (2011). What is problem-solving? Retrieved from http://psychology.about.com/od/problemsolving/f/problem-solving-steps.htm on 2011/06/29

Depaepe, F. (2009). The culture and practices in sixth-grade mathematics classrooms: *An attempt to unravel relationships between social and individual aspects in problem-solving lessons.* Retrieved fromhttp://lirias-kuleuven.be/bitstream/.../233766/.../doctoraat+Fien+Depaepe.pdf on 2011/03/29

Department of Basic Education (2011). *Curriculum and Assessment Policy Statement Grades 10-12: Mathematics*. Pretoria: Government Press.

Department of Basic Education (2014). *Report on the annual national assessments of 2014: Grade 1-6 & 9.* Pretoria: Government Press.

Department of Basic Education (2015). National senior certificate examinations 2015. diagnostic report. www.education.gov.za

Department of Basic Education (2016). National senior certificate examinations 2016. diagnostic report. www.education.gov.za

Elia, I., Van den Heuvel-Panhuizen, M., & Kolovou, A. (2009). Exploring strategy use and strategy flexibility in non-routine problem solving by primary school high achievers in mathematics.*ZDM Mathematics Education,* 41: 605-618.

Engel, A. (1998). *Problem-solving strategies.* New York: Springer-verlag.

Ewen, I. (1996). Strategies for problem solving. In Posameties, A. S., & Schulz, W. *The art of problem solving: A resource for the mathematics teacher.* California: Corwin Press, Inc.

Gilfeather, M., & Regass, J. (1999). Routine and non-routine problem solving; Mathematics experience based approach (MEBA); Pentathlon Institute, Inc

Hegarty, M., & Kozhevnikov, M. (1999). Types of visual-spartial representations and mathematical problem solving.*Journal of Educational Psychology,* 91(4), 684-689.

HSRC (2012). Towards equity and excellence.*Highlights from TIMSS 2011: SA Perspective*. Pretoria: HSRC Press.

Kirkley, J. (2003). Principles of teaching problem solving; *Technical Paper* #4.doi:10.1.1.117.8503

Kolovou, A., Van den Heuvel-Panhuizen, M., Bakker, A., & Elia, I. (2011). An ICT environment to assess and support students' mathematical problem-solving performance in non-routine puzzle-like word problems. *ICME 11.*

Lester, F. K. (2013). Thoughts about research on mathematical problem-solving instruction. *The Mathematics Enthusiast*, 10(1/2), 245-278.

Logsdon, A. (2007). How to solve math word problems- learn the SQRQCQ strategy to solve math problems. Retrieved from http://learningdisadisabilities.about.com/od/insttructionalmaterials/ht /sqrqcq.html on 2011/04/01.

Lucangeli, D., & Cabrele, S. (2006). The relationship of metacognitive knowledge, skills and beliefs in children with and without mathematics learning disabilities. In A. Desoete& M. Veenman (eds). *Metacognition in Mathematics Education*. New York: Nova Science Publishers, Inc.

Mabilangan, R. A., Limjap, A. A., &Belecina, R. R. (2011). Problem solving strategies of high school students on non-routine problems: A case study. *Journal of Basic Education,* 5, 23-46.

Malouff, J. (2011). Psychology-Over fifty problem solving solvingstrategies explained. Retrieved from http://www.une.edu.au/bcss/psychology/john.malouff/problem-solving.php on 2011/04/01

Maree, K., Aldous, C., Hattingh, A., Swanepoel, A, & Van der Linde, M. (2006). Predictors of learner performance in mathematics and science according to a large-scale study in Mpumalanga.*South African Journal of Education,* 26(2), 229-252.

Mogari, D., & Lupahla, N. (2013). Mapping a group of Northern Namibian grade 12 learners' algebraic non-routine problem solving skills.*African Journal of Research in Mathematics, Science and Technology Education,* 17(1/2), 94-105.

Muis, K. R. (2004). Epistemic styles and mathematics problem solving: *Examining relations in the context of self –regulated learning.* Retrieved from http://ir.lib.sfu.ca/bitstream/1892/9935/1/b37359599.pdf on 2011/03/30

Muller, R. (2014). Truth about SA's mathematics, science education quality. Retrieved May 18 2016 from http://mybroadband.co.za/news/government/103677-truth-about-sas....

National Council of Teachers of Mathematics (2000). *Principles and Standards for School Mathematics.* Reston, VA: NCTM.

OECD (2009). *Pisa 2009 assessment framework – key competencies in reading, mathematics and science.* Paris: OECD.

Polya, G. (1985). *How to solve it* (2nd ed). Princeton, NJ: Princeton University Press.

Russell, D. (2007). Problem solving strategies. Retrieved April 01 2011 from http://math.about.com/cs/testprep/∞/ps.html

Schoenfeld, A. H. (1992). Learning to think mathematically: Problem solving, metacognition, and sense-making in mathematics. In Grouws, D. A. (Ed.). *Handbook for research on mathematics teaching and learning* (pp. 334-370). New York: Macmillan.

Stephenson, D. (2001). Math problem solving techniques: *Eighty problem solving strategies and practice activities for 9 to 12 year olds.* Greenwood, WA: Ready-Ed Publishers. ISBN: 186397 1106

Van den Heuvel-Panhuizen, M., Kolovou & Robitzsch, A. (2013). Primary school students' strategies in early algebra problem solving supported by an online game. *Educational Studies in Mathematics* 84: 281 – 307.

Webb, N.L. (2010). Content complexity and depth-of-knowledge as applicable to research and practice. In D. Mogari; A. Mji; F. Mundalamo & U.I. Ogbonnaya (eds). *Proceedings of ISTE2010 International Conference: Towards Effective Teaching and Meaningful Learning in MST*, pp. 1 – 19.

Yazgan, Y. (2013). Non-routine mathematical problem-solving at high school level and its relation with success on university entrance exam. *US-China Education Review* 3(8): 571 – 579.

Yazgan, Y. (2015). Sixth graders and non-routine problems: Which strategies are decisive for success? Academic journals. Vol. 10(13), pp. 1807-1816.

Zanzali, N. A. A., & Nam, L. L. (1997). Evaluating the levels of problem solving abilities in mathematics. Retrieved April 01 2011 from http://math.unipa.it/~grim/jzanzalinam.PDF

APPENDICES

Appendix A

Mathematics Problem Solving Test

Time allowed: 1hour

Instructions

- Answer all questions
- Show all your working on the answer sheets provided
- Show any rough work done that contributes to the solution on the spaces provided.
- You may show any other alternative solutions to each problem.
- You may use calculators where necessary

1. Thabang has R100.00 pocket money and Mpho has R40.00. They are both offered temporary jobs at different companies. Thabang gets R10.00 a day and Mpho is paid R 25.00 a day. If they do not spend their pocket money or their daily wages, after how many days will they have the same amount of money?(Adopted from Muis, 2004, p. 114)

2. My old car goes 16 km on a gallon of gasoline. I drive about 15 000 km a year. If gasoline costs R 2.00 per gallon, how much money can I save if I buy a new car that gets 10 km more to the gallon?(Adopted from Greenes et al., 1986, p. 12)
3. There are 18 animals in Thabo's farmyard. Some are chickens and some are cows. Thabo counted 50 legs in all. How many of the animals are chickens and how many are cows?(Adopted from Muis, 2004, p. 114)
4. Some people had afternoon tea in a cafe which only sold tea and cakes. The tea cost R3.00 a cup, and cakes cost R 5.00 each. Everyone had the same number of cups and the same number of pieces of cakes. The bill came to R133.00. Can you find out how many cups of tea each person had? (Adopted from Burton, 1984, p. 80)
5. There are some rabbits and some rabbit hutches. If seven rabbits are put in each rabbit hutch, one rabbit is left over. If nine rabbits are put in each rabbit hutch, one hutch is left empty.

Can you find how many rabbit hutches and how many rabbits there are? (Adopted from Burton, 1984, p. 64)

6. Annah, Refilwe, Joel and Thabo have gone fishing and are counting up the fish they caught:
   - Thabo caught more than Joel.
   - Annah and Refilwe together caught as many as Joel and Thabo
   - Annah and Thabo together did not catch as many as Refilwe and Joel.

Who caught the most? Who came in second, third and fourth?(Adapted from Callejo&Vila, 2009, p. 115)