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Conceptual and Procedural Approaches to Mathematics in the Engineering Curriculum – Comparing Views of Junior and Senior Engineering Students in Two Countries

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ABSTRACT

One challenge for an optimal design of the mathematical components in engineering education curricula is to understand how the procedural and conceptual dimensions of mathematical work can be matched with different demands and contexts from the education and practice of engineers. The focus in this paper is on how engineering students respond to the conceptual-procedural distinction, comparing performance and confidence between second and fourth year groups of students in their answers to a questionnaire comprising conceptually and procedurally focused mathematics problems. We also compare these students' conceptions on the role of conceptual and procedural mathematics problems within and outside their mathematics studies. Our data suggest that when mathematical knowledge is being recontextualised to engineering subjects or engineering design, a conceptual approach to mathematics is more essential than a procedural approach; working within the mathematical domain, however, the procedural aspects of mathematics are as essential as the conceptual aspects.

Keywords: conceptual and procedural knowledge, confidence, engineering education, undergraduate mathematics

INTRODUCTION

In engineering education programmes, mathematics has traditionally been a strong component serving as a necessary basis for a range of applied engineering subjects (Kashefi, Ismail, Yusof, & Mirzaei, 2013). However, particularly in light of the increased use of advanced computing technologies in professional contexts, there is no consensus among educators on what type of mathematical knowledge engineering students need to develop during their

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State of the literature

- Research on the role and place of mathematics in the formal education of engineers has moved away from an earlier main focus on the mathematical content towards a greater interest also on how to study mathematics and what kinds of mathematical skills are valued.
- In light of the development of computational technologies, a need of a broader spectrum of mathematical skills for practicing engineers, including conceptual understanding, rather than a narrow focus on procedural skills has been suggested.
- A concept-based instruction in undergraduate mathematics education can develop conceptual knowledge without losing out on the procedural skills.

Contribution of this paper to the literature

- Engineering students consider procedurally oriented tasks as more common than conceptual tasks in their mathematics studies but the other way round in their engineering studies outside the mathematics curriculum.
- The correlation between performance and confidence on the conceptually oriented tasks increases during their studies.
- The majority of the students consider both the conceptual and procedural tasks as relevant to their general engineering studies.

formal education. For example, Flegg, Mallet and Lupton (2012, p. 718) state that "there seems to be no consistent, research-informed, view of how, what, when and by whom mathematics should be taught to engineering students". Even if a strong mathematical training is seen as necessary for becoming a competent engineer (Kent & Noss, 2003), the kind of mathematical knowledge employers ask for varies (Nguyen, 1998). The employers interviewed by Kent and Noss (2003), for example, put more emphasis on an holistic awareness of the mathematical needs for engineering work than on manipulative skill (p. 9). Referring to Keith Devlin, Flegg et al. (2012) argue that a mainly procedural view on mathematics "may obscure the role that thinking mathematically plays in engineering practices" (p. 718). Indeed, a need of a broader spectrum of mathematical skills for practising engineers has been suggested by several authors (Alpers, 2010; Cardella, 2008; Kent & Noss, 2003). For example, in his review of research on engineering workplace mathematics, Alpers (2010) pointed to a range of aspects that "capture the way mathematical thinking or activities occur during the work on practical problems" (p. 3), such as the contextual embedding of mathematical models, as well as concepts and procedures, and that the use of technical artefacts draws on an understanding of mathematical notations and graphics.

Such a nuanced picture of the role of mathematics in the work of practising engineers contrasts the findings reported by Winkelman (2009) of a mathematical preparation in engineering education often described with reference to intellectual status, gatekeeper, detachment, lack of creativity, and ease of evaluation. When mathematics is studied more to serve as a foundation and pre-requisite, it may lead to more emphasis on procedural fluency than on conceptual understanding. In mathematics education, both as a research discipline and a field of practice, the distinction between these two aspects of doing, teaching and learning mathematics has persisted since the 1970's but has also been criticized. The discussion has highlighted problems inherited in a separation of the ability to apply the computational and algorithmic rules, abundant in mathematics, making up much of its power in solving problems and generating knowledge, and the meaning production in terms of conceptual content and structure.

Research on the role and place of mathematics in the formal education of engineers has thus moved away from an earlier main focus on the mathematical content (e.g. Barry & Steele, 1993; Håstad, 1968) towards a greater interest also on how to study mathematics and what kinds of mathematical skills are valued. While demands from engineering faculties often have been more emphasising computational mathematical skills than mathematical thinking or understanding, in light of the discussion above, analytic and creative knowledge based skills of engineers as a work force are expected (Kashefi et al., 2013; Nguyen, 1998). This would speak for a need of mathematical knowledge oriented towards the conceptual as well as generic skills such as analytical thinking and problem solving often expected to be implicit in mathematical work (cf. Gainsburg, 2015). This is also reflected in more recent policy documents on engineering curricula (Alpers, 2013a, 2013b).

As a contribution to the discussion of the mathematical training of engineers regarding the kind of mathematical knowledge needed for their educational and professional needs, this paper takes its focus on how engineering students respond to the conceptual-procedural distinction. More specifically, we compare performance and confidence between second and fourth year groups of students in their answers to a questionnaire comprising conceptually and procedurally focused mathematics problems. We also compare these students' views on the role of conceptual and procedural mathematics problems inside and outside their mathematics studies.

CONCEPTUAL AND PROCEDURAL APPROACHES TO MATHEMATICS

The distinction between conceptual and procedural aspects of doing, teaching and learning mathematics has since the 1970's been discussed in research literature, mainly within a psychological framework with reference to Piaget and mental schemes, as a critical issue in mathematics education (Hiebert & Lefevre, 1986; Sfard, 1991; Star, 2005). Hiebert and Lefevre (1986), for example, defined conceptual knowledge in mathematics as a network of knowledge in which "the linking relationships are as prominent as the discrete pieces of information" (pp. 3-4). Procedural knowledge was defined by two parts, constituted by step-by-step procedures for solving mathematical tasks, on one hand, and knowledge of the symbolic representations used in such procedures, on the other hand. To be competent in mathematics then involves not only of knowledge of concepts and knowledge of procedures but also of relations between these two types of knowledge. Research suggests that there is a complex interplay between these two constructs regarding their interdependence and development (Schneider & Stern, 2010; Rittle-Johnson & Schneider, 2015). Although some authors see them as independent

(Radu, 2002), others have emphasized dynamic and/or evolutionary interconnectedness (Sfard, 1991; Star, 2005; Baroody, Feil, & Johnson, 2007). Kieran (2013) claims, however, that the distinction is a false dichotomy, with reference to the nature of algebraic symbols. Also in the context of more advanced mathematics, Wu (1999) argues from a number of examples that "in mathematics, skills and understanding are completely intertwined" (p. 2), making the distinction a bogus dichotomy.

In the context of engineering education, studying the relation between conceptual and procedural approaches to solving mathematics tasks, Engelbrecht, Bergsten and Kågesten (2009) found that first year engineering students tended to proceduralise tasks having a conceptual focus. Engelbrecht, Bergsten and Kågesten (2012) designed a questionnaire to investigate junior engineering students' achievement and views on mathematics tasks that were either conceptually or procedurally focused. The students found procedural tasks to be more common than conceptually focused tasks in their mathematics courses, while the latter were more common in engineering subjects; they also reported a higher confidence in their performance on procedural tasks. For their engineering studies, however, both types of tasks were considered relevant by a majority of the students. The same authors also interviewed lecturers in engineering subjects and professional practising engineers; preliminary results indicate that the interviewees found a conceptual approach to mathematics in engineering education as more relevant for engineering courses as well as engineering practice than a procedural approach, though some valued both approaches arguing that conceptual understanding is "underpinned by" procedural fluency (Bergsten, Engelbrecht, & Kågesten, 2015). The engineers expressed "a strong concern for an increased conceptual approach, not only to mathematics but to engineering education as a whole, to better balance the need from workplace engineering" (p. 988)

There is a fundamental difference between modes of teaching taking a procedural or a conceptual approach. To teach for procedural knowledge could mean to present pre-formulated definitions, notations and procedures without first having provided contexts of meaning for the concepts and methods involved. In contrast, teaching for conceptual understanding might start with problems requiring initial reasoning involving meanings of mathematical objects from the students, so that connections to their prior knowledge can be made (Rittle-Johnson & Schneider, 2015). However, not providing a clear guidance for the organisation of teaching in relation to the conceptual-procedural distinction in mathematics, opens up for an influence on curriculum issues from ideologies and traditions rather than from empirical research: "In the traditional curriculum, concept development is viewed as arising from computational proficiency with relevant procedures" (Baker, Czarnocha, & Prabhu, 2004, p. 1). Similarly, Wu (1999) argues that it is necessary to learn procedures first in order to develop conceptual understanding, with reference to meaning making in mathematics. In line with such views, calculus courses tend to begin with definitions and theorems only to quickly move on to work dominated by algebraic computations in a procedural manner (cf. Bergsten et al., 2015).

In a comparative study conducted at undergraduate mathematics level, students that were taught calculus with a conceptual approach performed significantly better on both conceptual and procedural items than students taught with a procedural approach (see Chappell, 2006). This suggests that a concept-based instruction in undergraduate mathematics education can develop conceptual knowledge without losing out on the procedural skills. A qualitative analysis indicated that knowledge acquired in a concept-based learning environment is better extended to novel situations (Chappell, 2006).

PURPOSE, FOCUS AND DESIGN OF THE RESEARCH

This report forms part of a larger collaborative project between two universities in South Africa and Sweden with main objective to investigate the current needs that engineers have of mathematics, with a focus on the conceptual-procedural distinction discussed above. In our previous study (Engelbrecht et al., 2012) we investigated how second year engineering students respond to the conceptual-procedural distinction made in research, as discussed above. In the study presented here, we surveyed senior (4^{th} year) engineering students, comparing the results of these students with those of the junior students. We compare these students' performance and their conceptions on the role of conceptual and procedural mathematics problems within and outside their mathematics studies, as well as about the relevance of these ideas in their engineering studies. As students' confidence about the results of their own mathematical work may interact with their approach being more conceptual or procedural, we also include students' confidence of their answers to the test items in our data. The following research questions guided our study:

- *Do engineering students' performance and confidence of response differ between conceptually and procedurally focused mathematics tasks and between junior and senior students?*
- *How do engineering students view the role and relevance of conceptual and procedural mathematics problems in their education?*

Based on the answers to these questions provided by our data, we will conclude the paper with a discussion of the role of mathematics in engineering education with respect to its conceptual and procedural aspects.

Conceptual and Procedural Approaches to Solving Mathematics Tasks

As the purpose of the present study concerns the balance between a conceptually and a procedurally oriented way of teaching undergraduate mathematics courses for engineering students, there was a need to develop an instrument consisting of tasks that the target group of students generally approach in either a conceptually or a procedurally oriented way. Therefore, descriptions and analyses of conceptual and procedural knowledge in mathematics that are used in the literature need to be operationalized into criteria for what characterizes a solution to a task as mainly conceptually or procedurally oriented. This asks for an external language of description to allow an unambiguous reading of the empirical instances linked to an interpretation of the theoretical descriptions that are employed (Bernstein, 2000). Here, conceptual knowledge is outlined as being concerned primarily with relations between conceptual entities and their meanings, in the sense of seeing a structure through a network of relations among these entities and their representations. As this also involves prior knowledge including extra-mathematical interpretations and use of conceptual entities, a conceptual approach to solving mathematical tasks need to include, among other things, representational versatility involving 'translations' between verbal, visual, numerical and formal/algebraic expressions, and linking relationships including interpretations and applications of concepts (for example with diagrams) to mathematical situations. With an emphasis on how to operate on the mathematical representations to carry out a solution to a task, a procedural approach is characterized by (symbolic and numerical) calculations, employing (given) rules, algorithms, formulae and symbols.

Confidence of Response

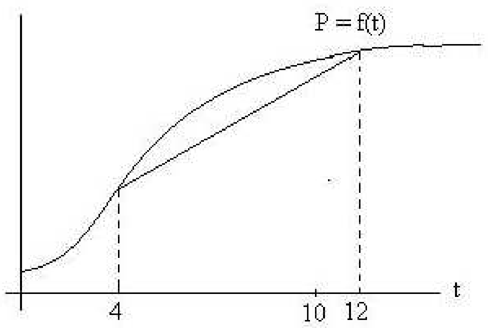
To investigate confidence of response originated in the social sciences, where it is used in surveys. Commonly, a respondent is then requested to indicate the degree of certainty she/he has in her/his ability to select and use well-established knowledge, concepts and rules to arrive at each answer (Potgieter, Malatje, Gaigher, & Venter, 2010). Confidence is most often measured on a scale, in our case 1 - 4, where 1 implies a total guess and 4 a complete confidence. Irrespective of whether the answer is correct or not, low confidence indicates a guess which, in turn, implies a lack of knowledge. However, if the confidence is high and the answer wrong it indicates a misplaced confidence in the person's knowledge on the particular subject matter, either suggesting a misjudgement of her/his own ability or a sign of existing misconceptions. A confidence measure, together with the correctness or not of a response can thus be used to distinguish between a lack of knowledge and either over-confidence or the existence of a misconception (Potgieter et al., 2010).

Development of the Research Instrument

The development and piloting of the questionnaire that was used to address our research questions is described in Engelbrecht et al. (2012). In its final version four pairs of calculus tasks (questions) in a multiple choice format were used. In each pair one question was classified as having a conceptual and the other a procedural focus. In constructing the questions, we used the descriptions of a conceptual and a procedural approach as described above.

No question was explicitly labelled as conceptual or procedural and the order of the conceptual and procedural questions in the four pairs differed. Below one example of a pair consisting of a conceptual item (question A) and a procedural item (question B) is shown, along with follow-up questions about confidence of response and opinion about the questions being common/relevant in their studies. Question A is classified as conceptual because mathematical concepts (function, slope, rate of change, increasing function) and statements must be interpreted in relation to a diagram representing a mathematized situation (population size as a function of time). No calculations need to be performed to answer the question. In contrast, Question B is classified as a procedural item: only standard rules for differentiation must be employed on a mathematical formula followed by a numerical calculation for a specific value of a given variable (symbol). The follow-up questions were the same for all items.

Question A. The function $P = f(t)$ in the sketch gives the population size after t years. The slope of the line is approximately 300. Choose from statements **A - D** which of the statements is NOT true or choose **E**.



A. The average rate of change of population over the interval $4 < t < 12$ *is 300 people per year.*

B. The rate of change of the population at $t = 4$ *is more than 300 people per year.*

C. The rate of change of the population at $t = 10$ *is more than 300 people per year.*

D. The function is increasing.

E. They are all true.

Question B. Find $f'(2)$ if $f(x) = 3xsin\sqrt{x^2 - 3}$

A. $3sin1 - 12cos1$

B. $-3sin1 + 12cos1$

C. $3sin1 + 6cos1$

D. $3sin1 + 12cos1$

E. None of these

Each of these questions was immediately followed by a question about the student's confidence with the answer:

Indicate how certain you are about your answer:

A. Positive B. Almost certain C. Uncertain D. Total guess

After each pair of questions three additional questions were asked:

Regarding the types of questions represented by questions A and B:

1. Which one of the two questions is more common in your studies within your mathematics courses?
2. Which one of the questions is more common in your studies outside your mathematics courses?
3. Which one of the two questions is more relevant for your studies as engineer?

A. Question A B. Question B C. Both D. Uncertain

The four pairs of questions that were used in the final instrument were on four different mathematical topics within one variable calculus: the first pair (as in the example) on differentiation of a single variable function; the second pair on applications (interpretation) of the derivative; the third pair on differential equations; and the last pair on integration.

Research Context and Sample

Two universities took part in the project within which the present study was conducted, one in South Africa and one in Sweden. Both are comprehensive high ranked universities in their respective countries and have strong engineering faculties that have been running engineering education for many years.

In this study we investigate senior engineering students. We selected some major engineering study directions at the two universities and targeted all students enrolled in those directions. The South African group consisted of 236 4^{th} year students, including 78 chemical engineering students and 158 mechanical engineering students. Of the 126 students in the corresponding Swedish group, there were 47 mechanical engineering students from one campus of the Swedish university, 27 electrical engineering students from the same campus, and 52 mixed engineering students from the other campus of the university. The results of the questionnaire from these students will be compared with those of the 2^{nd} year students reported in Engelbrecht et al. (2012). In that study, with a similar distribution of engineering programs, a total of 358 students participated from the South African university and 229 from the Swedish university. The study thus involved altogether 587 2^{nd} year students (junior students) and 362 4^{th} year students (senior students), that is a total of 949 students.

Although the group of 4^{th} year students who were surveyed are not the same students who were surveyed in their 2^{nd} year, there was no major change in curriculum, teaching approach or nature and standard of assessment at any of the two universities during this period. A longitudinal rather than a cross-sectional study would have required the use of different items for the senior students as compared to the junior students.

Students in South Africa enter university after having attended primary and secondary school for 12 years. The engineering program in which the target group students were enrolled can be completed in four years but the majority of students study for five years. The first two years are very similar for all the different fields with mathematics the main component but also some physics, computer science and chemistry. From the third year the program becomes more specialized and practice oriented. The mathematics part of their curriculum consists of courses in single variable calculus and linear algebra in the first year, while in the second year they do courses in multivariable calculus, differential equations, numerical methods and introductory real analysis with applications to differential equations.

In Sweden students enter university after completing nine years of compulsory and three years of upper secondary school. The engineering degree program in which the students in this study were enrolled, is a five years program offering a range of directions. In the first two years there is a strong emphasis on mathematics (30-45% of course volume) and theoretical subjects such as physics, information technology and mechanics. Later on the focus gradually moves to more practice oriented, problem-based projects and students finish their degrees with a master level thesis at a private company or within a research team. The mathematical part of the curriculum has a similar structure as in the South African university.

We targeted all students in a number of specific study directions, as pointed out above. Announced in advance by e-mail, students were asked to volunteer to complete the test immediately after a lecture which was included in the compulsory part of the program. In a short introduction the researcher presented the procedure for completing the forms. Though not all students turned up on the specific day for the lecture, the sample covered a major part of the students enrolled in the specific study direction.

Data Analysis

The multiple choice responses to the questionnaire were processed by an optical reader and the quantitative data were processed and analysed by the researchers. Means and correlations that were computed and compared are presented below in tables and diagrams, along with reports about levels of significance.

The analysis of the data was done twofold:

- *A comparison between the senior and junior students of the two countries (with sample sizes as described above);*
- *A general comparison on the results of the entire group of 362 senior students with those of the 587 junior students.*

The instrument was designed to have items of equal and intermediate levels of difficulty (for the target group) across a variety of types of tasks that are relevant for their studies. To address the inner homogeneity of the test, the correlations between the results (performance) of the individual eight items with the total test results were calculated and are displayed in **Table 1**, along with the performance level of each item. For comparative purposes we include the same results for the second year students.

Table 1. Performance and Pearson correlation coefficients between individual test items and total marks for the test

Item	1A	1B	2A	2B	3A	3B	4A	4B
Conceptual/Procedural	C	P	P	C	P	C	C	P
Performance (%) for 2nd years	60.1	51.4	81.1	56.6	54.9	58.4	38.0	58.4
Performance (%) for 4th years	62.8	40.0	82.5	51.9	47.8	67.8	44.2	53.6
Correlation for 2nd years	0.47	0.42	0.34	0.53	0.51	0.48	0.48	0.49
Correlation for 4th years	0.52	0.43	0.35	0.47	0.54	0.50	0.55	0.48

The level and even distribution of the total marks and the correlation coefficients indicate that with both groups there was a sufficient amount of homogeneity present between the different items in the instrument (all strongly significant). The slightly lower correlations for item 2A, involving differentiation and evaluation of a polynomial, can be explained by the high solution frequency appearing in both groups.

RESULTS

Performance

In **Table 2** the test performance for the two countries is presented.

Table 2. Performance comparison (percentage) in conceptual, procedural and all items

	Conceptual		Procedural		Overall	
	2nd year	4th year	2nd year	4th year	2nd year	4th year
Sweden	58.2	64.5	60.5	64.1	59.3	64.3
South Africa	50.1	52.5	62.1	51.7	56.0	52.1

From **Table 2** it is clear that in Sweden the performance on both the conceptual and the procedural items is higher or slightly higher for senior students than junior students (t-test $p = 0.01$ for conceptual and $p = 0.09$ for procedural), while in South Africa it is slightly higher for senior students than junior students for the conceptual items ($p = 0.06$) but much lower for the procedural items ($p < 0.01$). For the Swedish students, these differences add up to an overall higher performance for the senior students ($p = 0.02$), while for the South African students there is a slightly lower performance for the senior students than for the junior students ($p = 0.07$).

Comparing the two countries, the overall performance of the junior Swedish students - average 59.3% for the whole test - is somewhat higher than that of the South African students - average of 56%. With a t-test p-value of 0.048 this is significant on a 5% level but not on a 1% level. The senior Swedish students also performed significantly ($p < 0.01$) higher than the South African students.

Whereas the Swedish senior students performed higher than their counterparts in South Africa on the procedural questions ($p < 0.01$), for the junior students it was the other way around though not significantly ($p = 0.24$). In the conceptual questions, the Swedish junior as well as senior students performed significantly higher ($p < 0.01$) than the South African groups of students.

Confidence

In **Table 3** we give the average confidence that junior and senior students reflected in answering conceptual and procedural questions as well as the overall averages for the two countries. For measuring confidence, students had to indicate if they are "positive" about their answers (giving a score of 4), "almost certain" would give a score of 3, "uncertain" a score of 2 and "total guess" a score of 1.

Table 3. Confidence comparison (mean scores) in conceptual, procedural and all items

	Conceptual		**Procedural**		**Overall**	
	2nd year	4th year	2nd year	4th year	2nd year	4th year
Sweden	2.91	3.13	3.34	3.36	3.13	3.25
South Africa	3.30	3.00	2.90	3.23	3.10	3.12

All Swedish students showed significantly (t-test p-value < 0.01) more confidence in their responses to the procedural questions than to the conceptual questions, which for the South African students was the case only for the senior students. However, Swedish senior students' confidence in conceptual questions was higher than for the junior students, whereas South African students' conceptual confidence was lower in the 4th year. Procedural and overall confidence was slightly higher in the 4th year than in the 2nd year in both countries. When comparing the overall confidence of students in the two countries, it is somewhat higher for the senior Swedish students than for the junior students, whereas in South Africa there is no significant difference.

In **Figure 1** a comparison of the average confidence of all 362 senior students and the 587 junior students is given. We compare students' confidence in the conceptual items with the procedural items.

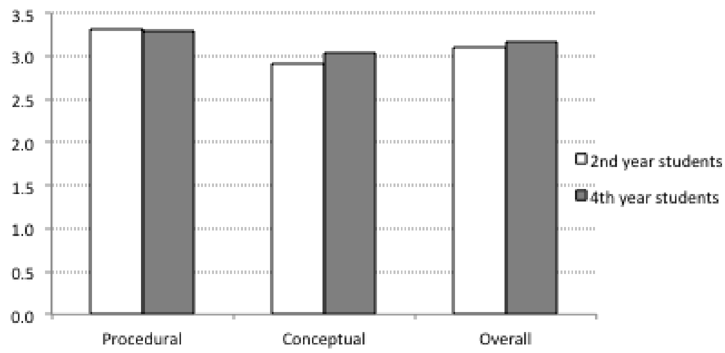


Figure 1. Comparison of the average confidence of the entire group of students

From these graphs it is clear that the junior students were slightly more confident about their answers to the procedural questions than the senior students, while for the conceptual items and overall it was the other way around. Although the senior students did not perform significantly better in the procedural items, they are still more confident about these procedural questions than about the conceptual items (t-test $p < 0.01$).

Correlation Between Performance and Confidence

We calculated the Pearson correlation coefficients between the entire groups of junior and senior students' performance on the conceptual and procedural items, respectively, and the confidence that they have in their responses. This information is reflected in **Figure 2**.

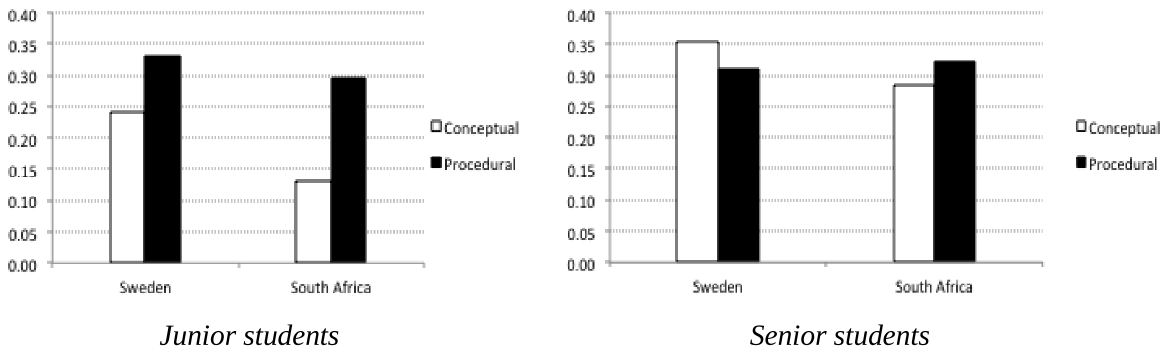


Figure 2. Correlation between performance and confidence

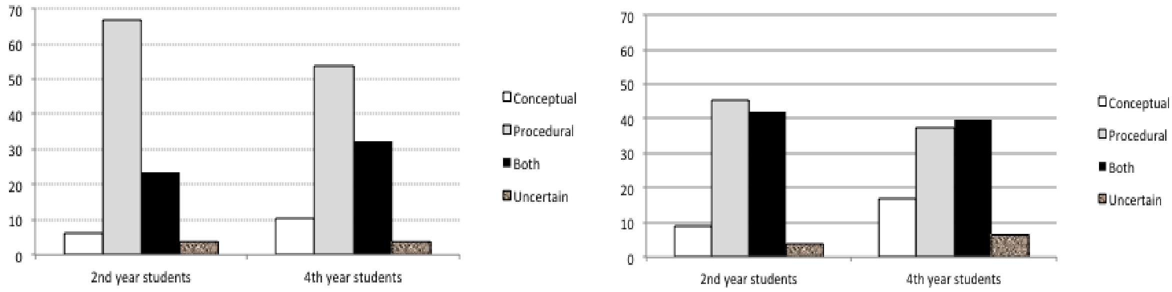
All correlation coefficients indicate a statistically significant correlation on a 1% level. For both Swedish and South African junior students the correlation between confidence and performance for the conceptual items is lower than for procedural items. For the senior students, in Sweden the correlation between confidence and performance for the conceptual items is higher than for procedural items while for South Africa it is lower.

In both countries, the correlation between confidence and performance for the conceptual items is higher for the senior students than for the junior students. There are no major differences between the junior and senior students in confidence and performance for the procedural items.

A calculation of the Pearson correlation coefficient between all students' performance in the conceptual items and their confidence in these items showed that this correlation was higher for the senior students (0.31) than for the junior students (0.17). Between procedural performance and confidence, however, the correlation was almost the same for these groups of students (0.32 and 0.31 respectively). On these large samples of 362 and 587 students these correlations are significant on a 1% level of significance.

Role in the Mathematics Curriculum

After each pair of conceptual and procedural questions the student had to indicate which of the two questions is more common in her/his mathematics curriculum. The percentage distribution of the junior and senior students in the two countries is given in **Figure 3**.



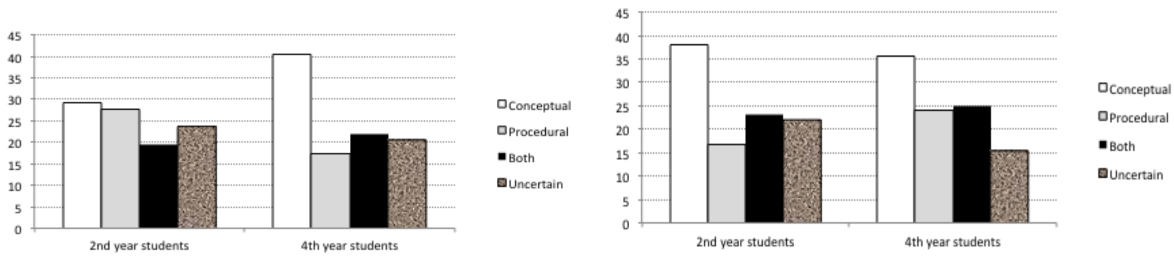
Sweden *South Africa*

Figure 3. Percentage distribution of students' conceptions of items in the mathematics curriculum

The data show that students find the procedural items more common inside mathematics curricula than conceptual items – Swedish students even more so than South African students. This is still true in the 4th year but perhaps not as strongly as in the 2nd year. In both countries there are more senior students that consider conceptual questions as part of the mathematics curriculum than junior students, and a lower number of senior students as compared to junior students that consider procedural questions as part of the mathematics curriculum. It is also clear that a smaller part of the South African than the Swedish students consider both conceptual and procedural items as part of the mathematics curriculum

Role Outside the Mathematics Curriculum

After each pair of conceptual and procedural questions the student had to indicate which of the two questions is more common in her/his studies outside her/his mathematics curriculum. The percentage distribution of the junior and senior students is given in **Figure 4**.



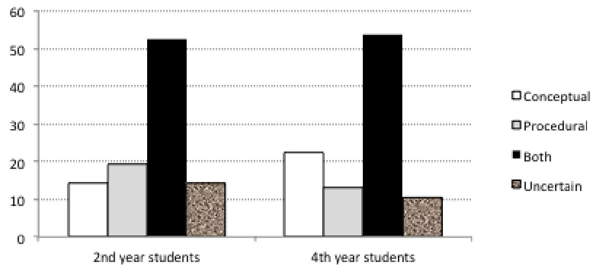
Sweden *South Africa*

Figure 4. Percentage distribution of students' conceptions of items outside the mathematics curriculum

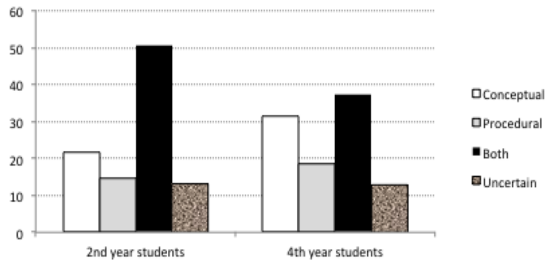
In Sweden the proportion of senior students seeing conceptual questions as important outside the mathematics curriculum is higher than for the junior students, while the proportion of students seeing procedural questions as important outside the mathematics curriculum is lower for the senior students. In South Africa the pattern is the opposite: a slightly lower part of senior students sees conceptual questions as important outside the mathematics curriculum while a larger part of senior students see procedural questions as important outside the mathematics curriculum than for junior students. It seems as if students (both junior and senior students in both countries) to a greater extent see conceptual items as common in their engineering studies outside mathematics. In both countries, a slightly higher proportion of senior students than junior students see both conceptual and procedural questions as important outside their mathematics studies. It also seems as if a lower part of senior students is uncertain about this issue, though still constituting a considerable proportion.

Relevance of Items to Engineering Studies

After each pair of conceptual and procedural questions the student had to indicate which of the two questions is more relevant to her/his engineering studies in general. The percentage distribution of the junior and senior students is given in **Figure 5**.



Sweden



South Africa

Figure 5. Percentage distribution of students' conceptions of items' relevance to engineering studies

For all groups of students, the majority of the students consider both the conceptual and procedural questions as relevant to their general engineering studies. In Sweden the number of students seeing conceptual questions as relevant to their studies is higher in the 4th year than the 2nd, while the number of students seeing procedural questions as relevant to their studies is lower in the 4th year than the 2nd. In South Africa senior students see conceptual as well as procedural questions as more relevant to their studies than junior students.

From these graphs we can conclude that both junior and senior students express the opinion that procedural questions are more common than conceptual questions inside mathematics, and that outside mathematics, conceptual questions are more common than procedural questions. In all cases there was a larger part of senior students seeing conceptual mathematics as an important part of their studies than junior students. However, a majority of both junior and senior students see both, conceptual and procedural questions, as relevant to their studies in engineering in general, though for South African students this proportion was lower for the senior than for the junior students.

DISCUSSION AND CONCLUSIONS

Methodological Concerns

As they had completed a major part of their mathematics studies as well as a fair amount of applied subjects, including some project oriented work, the senior students involved in this study should have an experiential base for having a balanced view of the character of the mathematics courses as well as the use of mathematics for other subjects. The outcomes of the study reported here, with its comparison with the junior students, should therefore be of relevance for discussions on the issues in focus concerning mathematics teaching in engineering education.

An issue of discussion is to what extent quantitative data of the kind presented here are appropriate and useful for providing information on qualitative issues in mathematics education within engineering programs. Knowledge about how large groups of students experience different emphases on procedural or conceptual approaches to mathematical work, from the point of view of their mathematics and engineering studies in general, certainly is useful for engineering curriculum design. Considering the discussion found in research literature, as outlined above, an operationalization of these analytical categories into test items discriminating between them may be questioned. However, based on the pilot testing and qualitative response analysis of the instrument (Engelbrecht et al., 2009), and the outcomes of the study on junior students (Engelbrecht et al., 2012), the results presented here show significant patterns of similarities and differences between junior and senior students' experiences of these constructs.

The outcomes from the comparison of the two countries show more similarities than differences, which are rather subtle and point more to differences in program structure and traditions within engineering education globally than to cultural differences between the countries. Some of these differences have been discussed above but the overall patterns in the data are similar, thus indicating general issues of structure of the education that may also be found elsewhere.

Observed Patterns in the Data

Stevens, O'Connor, Garrison, Jocuns, and Amos (2008) describe three dimensions of an engineering student's route through a tertiary education program: disciplinary knowledge, identification (as an engineer), and navigation (admittance, passing courses, etc. through an educational program). The main interest in this paper, as well as in the project framing it, concerns the role mathematics is given in the education of engineers, with a specific focus on the teaching of mathematics in terms of conceptual and procedural approaches. To this aim, a key question is to what extent students experience mathematical knowledge as "accountable disciplinary knowledge", that is knowledge counted as engineering knowledge (Stevens et al., 2008, pp. 356-357). Though influencing the navigation dimension, standard examination tasks in the introductory mathematics courses, while commonly procedural in character, do not belong to this category. A student might therefore not see mathematics as part of her/his identity as an engineer. The data presented in this paper, comparing the views of junior students with those of more senior students, indicates that this may develop later in the education from the experience of applied courses outside mathematics and work with projects acquired later in the education.

The observation that senior students' views in some of the data differ from those of the junior students is an indication that education matters. The role of mathematics for engineering may not be that clear to the students during the beginning of their studies, more than that it is important in a general unspecified sense. However, after being exposed to more applied and practice oriented subjects and projects later in their studies, a more engineering oriented view on the way mathematical concepts and methods enter into these activities may develop (cf. Gainsburg, 2015; Kashefi et al., 2013). The result that in both countries performance of the conceptually oriented mathematics is higher for the senior students than for the junior students (**Table 2**), as well as the appreciation of its role outside the mathematics studies (**Figure 4**) and for engineering studies in general (**Figure 5**), is an indication of such development.

The outcome that the South African senior students perform lower on the procedural dimension than the junior students, in contrast to the Swedish students (**Table 2**), may be due to differences in the curriculum in the engineering programs. While in South Africa, the program soon takes more focus on the practice of engineering knowledge, and does not include further mathematics studies, in Sweden studies remain more theoretically oriented and, in some of the directions of the program, include more mathematics courses. Indeed, the program as a whole has a strong mathematical component. This fact may also account for the observation that the Swedish students' performance on the conceptual items is higher than the South African students in both groups (**Table 2**). The same fact also provides an explanation to why the senior students perform higher on the procedural items in Sweden in contrast to the junior students where the South African students perform higher (**Table 2**).

These observations are also linked to the students' confidence in the conceptual and procedural items, respectively, which are both higher or the same for the senior than for the junior students in Sweden. In South Africa, however, for the senior students, compared to the junior students, performance is higher but confidence lower on the conceptual items, while for the procedural items the difference is the other way around (**Table 2**, **Table 3**). As the South African junior students have a higher performance than the Swedish junior students, and are not exposed to more procedural work in mathematics, in contrast to the Swedish students, their confidence can be high later in the program, also stronger than for conceptual items. Generally, confidence is higher for the procedural items than for the conceptual items, apart from the junior students in South Africa (**Table 3**). Indeed, when the skill to learn is new, confidence may stay behind the development of the skills, but when later in place and used, confidence may be high even if performance may suffer from not being practised. It is here interesting to note (**Figure 2**) that the correlations between performance and confidence for the procedural items are strong (around 0.30, $p < 0.01$) and similar in both countries for junior and senior students, while for the conceptual items these correlations are much stronger for the senior than for the junior students in both countries and in particular for the South African students (0.28 as compared to 0.14). This points to an increased familiarity with the conceptual aspects of mathematics through its use in applied subjects, and, indeed more so for the South African students but to some extent also for the Swedish students, project oriented engineering work, later in the education.

The observation that all students are of the opinion that procedural questions are more common than conceptual questions within the mathematics curriculum, especially for the Swedish students, while outside mathematics conceptual questions are seen as more common than procedural questions (**Figure 3**, **Figure 4**), indicates that the mathematical training seems to take a different approach than how mathematics is used in the applied subjects (cf. Flegg et al., 2012). This polarisation is somewhat stronger for the Swedish students than for the South African students. A fact that may have had an impact on this result is that in the Swedish programs, different to the case for South Africa, the use of electronic calculators was not allowed in exams, possibly requiring stronger emphasis on calculation skills. There are also some differences between the two countries regarding the opinions on the approaches in the studies outside the mathematics curriculum, an issue where a larger proportion of uncertain students needs to be taken into account, however (not unexpectedly) lower for the senior than for the junior students (**Figure 4**). Interestingly, the procedural approach is to a much lesser extent seen as relevant to their studies in engineering in general, where instead the largest proportion of all the students see a combination of conceptual and procedural questions as relevant (**Figure 5**). This represents a conceptualisation emphasised in the literature reviewed above (e.g. Rittle-Johnson & Schneider, 2015) as a key aspect of being competent in mathematics.

An Interpretation of Observed Outcomes

The observations above point to a possible mismatch between the way mathematics is taught and how it is used in the applied courses in the engineering education at these universities to account for its relevance to engineering. Rather than employing a procedural approach, literature supports conceptually oriented teaching fostering problem-based learning strategies using engineering relevant tasks (Flegg et al., 2012). This view is also in line with the wide approach with a focus on the mathematical thinking required in engineering oriented work, as proposed for example in Cardella (2008). How, more specifically, mathematics courses (commonly provided in the beginning of the engineering curriculum) might be better linked with more applied subject courses and project oriented engineering courses (commonly provided in later years of the engineering curriculum) is however still an open issue (e.g. Flegg, et al., 2012) where literature seems to rely more on opinions than on empirically based research knowledge (e.g. Cardella, 2008). This state of affairs is also mirrored in the changing orientations towards mathematics education for engineers presented in three different curriculum framework documents produced by the SEFI Mathematics Working Group from 1992 (topic oriented), 2002 (learning outcome oriented), and 2013 (mathematical competence oriented); see Alpers (2013a).

The data analysed in this paper concern the conceptual-procedural dimensions of mathematics in relation to three fundamental demands placed on the mathematical studies in engineering education – two internal educational demands, one stemming from the mathematical studies and one from the applied subjects, and one external professional demand from the practical and theoretical work performed by engineers (cf. Alpers, 2010; Flegg et al., 2012). The educational demand from mathematics defines the knowledge and skills of the basic courses needed to pursue the more advanced mathematics courses as well as mathematical courses of more applied character such as optimization, statistics and numerical methods. The educational demands related to the applied subjects are qualitatively different – here mathematical methods are used as tools to deal with something else than mathematics. This means that the mathematical knowledge must be recontextualised from the perspective of the principles of the non-mathematical domain into which it is "moved" (on the notion of recontextualisation, see Bernstein, 2000; Dowling, 2014). Also the professional demand (for example as outlined in Alpers, 2010) requires a recontextualisation of the mathematical knowledge acquired during the education for the purpose of, for example, modelling, engineering design, problem solving and computing/programming in relation to different contexts.

One challenge for an optimal design of the mathematical components in curriculum development in engineering education is to understand how the procedural and conceptual dimensions of mathematical work can be matched with these different demands and contexts. Our data suggest that engineering students, during their education, increasingly emphasise both the presence and the importance of a conceptual approach to mathematics, especially in combination with the procedural aspect, for the applied courses and engineering studies in general. A combination or integration of the two approaches focused on in this study is also pointed out in the literature as essential for the development of mathematical knowledge per se, though some authors emphasise their distinct characters. One interpretation of our data is that when mathematical knowledge is being recontextualised (to applied subjects or engineering design, for example), a conceptual approach to mathematics is more essential than a procedural approach. For the professional practice of engineering, where mathematics is being recontextualised for example in modelling activities (cf. Gainsburg, 2007), the technical aspects of mathematical work are largely performed by computing technologies. Working within the mathematical domain, however, the procedural aspects of mathematics are as essential as the conceptual aspects, having a key role in technical parts of proofs and problem solving. This interpretation of the data is also supported by the views of practicing engineers and lecturers in engineering subjects expressed in the interview study by Bergsten et al. (2015), where the issue of the conceptual-procedural aspects of mathematics education in the engineering curriculum was discussed.

The outcomes from the comparison of the two countries show more similarities than differences, which are rather subtle and point more to differences in program structure and traditions within engineering education globally than to cultural differences between the countries. Some of these differences have been discussed above but the overall patterns in the data are similar, thus indicating general issues of structure of the education that may also be found elsewhere.

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