Eurasia Journal of Mathematics, Science & Technology Education, 2008, **4**(4), 381-386

www.ejmste.com

Constructivism and Peer Collaboration in Elementary Mathematics Education: The Connection to Epistemology

Anoop Gupta
University of Windsor, Windsor, CANADA

Received 06 November 2007; accepted 08 August 2008

In this paper, an attempt is made to determine if peer collaboration increases student achievement in teaching elementary mathematics. Empirical evidence and philosophical problems with constructivist epistemology are considered. Two things are argued: first, it is reasonable to think, for elementary mathematics, peers collaboration is useful (especially in heterogeneous groups). Peer collaboration is an appendage to instruction, not a replacement for the didactics of an expert, or individual problem solving (which occurs both at its inception, when mathematics is discovered as well as advanced levels). There is reciprocity between individual and social settings in learning mathematics. Second, for the teaching of mathematics an adequate epistemology will guide, to some extent, a successful pedagogy.

Keywords: Constructivism, Peer Collaboration, Mathematics Education

INTRODUCTION

Pedagogical constructivism entails three principles: *encouraging collaboration*, primitive activity and exploration, respecting multiple points of view and emphasizing authentic problem solving (Solomon, 2000, p. 328). Pedagogical constructivism (henceforth "constructivism") is also sometimes taken to be a full blown philosophical position about the nature of knowledge; namely, we make knowledge up like the rules of chess. I argue that the value of peer collaboration is contingent upon the context and limited by our epistemological stand in specific ways that is little noticed by constructivists.

I proceed by first considering the conditions under which peer collaboration in mathematics is appropriate. Second, I consider the claim that in order make constructivism generally plausible, we must separate epistemological and pedagogical variants. Finally, I argue however, that employing peer collaboration in mathematics must be determined in relation to the student, teacher, nature of the subject matter, and is likely to be guided by our epistemological stance.

Peer collaboration could be studied independently of constructivism. Considering peer collaboration and constructivism together is justified: to jettison peer collaboration requires revising constructivism. It is reasonable to think that the debate over peer collaboration in mathematics must be resolved by empirical studies, however (Fawcett & Gourton, 2005). I am not conducting an empirical study, and, rather, offer a philosophical comment on the debate over constructivism and peer collaboration. Further, I shall use examples from science and advanced mathematics because the sources I use do so. Finally, when I discern the relationship of our epistemology to our pedagogy (Figure 1), some possible adherents of several views related to them are inferred for the purpose of illustration alone. Scholars of individual thinkers referred to may attempt to amend their place in the picture I sketch, which would yield debates that will transcend the purpose of this paper.

Correspondence to: Anoop Gupta, PhD in Mathematics Education, Faculty of Education, University of Windsor, Windsor ON N9B 3P4, CANADA
E-mail: akgupta03@yahoo.ca

E-ISSN: 1305-8223

Peer Collaboration

According to Fosnot, Professor of Education, Director of Mathematics in the City, New York, and Dolk, researcher at the a the Freudenthal Institute in the Netherlands, mathematics is either about transmitting knowledge (didactic learning) or constructing meaning, but not both (Fosnot & Dolk, 2005). Fosnot, also editor of *Constructivism, Theory, Perspectives, and Practice,* offers the following definition:

Constructivism is a theory about *knowledge and learning.* It describes knowledge not as truths to be transmitted or discovered, but as emergent, developmental, nonobjective, viable constructed explanations by humans engaged in meaning making in cultural and social communities of discourse. Learning from this perspective is viewed as a self-regulatory process of struggling with the conflict between existing personal models of the world and discrepant new insights constructing new representations and models of reality as a human meaning-making venture…([Emphaisis mine]. Fosnot, 2005, p. ix)

Fosnot concluded, "Major restructuring is needed in the schools if we are to take constructivism seriously" (Fosnot, 2005, p. xi). Philips, editor of a volume published by the National Society for the Study of Education, dedicated to the theory, remarked, "Constructivism is currently a fashionable magic word in the Western intellectual firmament… (Philips, 2000a, p. 1). Ernest von Glaserfeld, the first social constructivist, puts it this way:

The key idea that sets constructivism apart from other theories of cognition was launched about 60 years ago by Jean Piaget. It was the idea that what we call knowledge does not and cannot have the purpose of producing representations of an independent reality…(Glaserfeld, 2005, p. 3)

Putting aside the constructivist appropriation to Piaget, it is at least clear many have tried to develop it in relation to Vygotsky in order to emphasize peer collaboration. Pichat and Ricco (2001), psychologists at the University Paris 8, noted that there are three poles in the classroom: the student, teacher and knowledge. For Vygotsky, upon whom they rely, cognitive mediation (contractual expectations) is the main factor in understanding (Piachat & Ricco, 2001). Mastery of mathematics, according to Pichat and Graciel, is more than knowledge of procedures, but knowing when to employ them, which requires the guidance of the teacher in the Vygotskian zone of proximal development (ZPD).

In "Small-group Searches for Mathematical Proofs and Individual Reconstructions of Mathematical Concepts"*,* Vidakovick and Martin agreed that constructivist theory provides the basis for co-operative and collaborative learning (Vidakovic & Martin, 2004). Discussion, they claimed, leads to deeper understanding.

According to Vidakovick and Martin, we internalize culture and externalize it by passing it on. By missing an opportunity for externalization, we limit internalization; that is to say, if we do not have a chance to explain our thought to someone else we fail to solidify learning (Vidakovic & Martin, 2004). They emphasized that in the mid-20th century two theories have dominated mathematics education research, Piaget's information processing model and Vygosky's social-constructivism. Vidakovic and Martin advocated co-constructivism that reconciles both the individual and social aspects of Piaget and Vygotsky (Vidakovic & Martin, 2004). Viadaok and Martin concluded that mathematics learning can be enhanced by peer-collaboration in small groups, provided there are some common understandings of what counts as a proof.

Lillian M. Fawcett and Alison F. Gourton, in "The Effects of Peer Collaboration on Children's Problem-Solving Ability", pointed out that group work, according to constructivists, enhances learning through participation, makes transition to the wider community easier, and maximizes use of limited resources (Fawcett & Gourton, 2005). For Vygotsky cognitive change is linked to collaborative interaction. For Piaget, learning results from peer interaction, which provides conflict: cognitive development depends on a conflict between what is known and not, creating disequilibrium (Fawcett & Gourton, 2005). For Vygotsky, the notion of a community of learners supports the idea of group work (Fawcett & Gourton, 2005).

According to Fawcett and Gourton, peer collaboration increases student achievement, though depends on complex factors like age, ability level, partners, motivation, confidence, gender and task. Further, there are more cognitive benefits when participants listened and reflected on logical consistency and precision (Fawcett & Gourton, 2005).

There must be exposure to a higher level of reasoning, active participation (active reasoning and the exchange of ideas), and communication (Fawcett & Gourton, 2005; Vidakovic & Martin, 2004). Different skill levels lead to the conflict necessary for conflict (in ZPD and for Piaget). Active participation and verbal interaction are necessary for internal reorganization, as well as cognitive change.

Philosophical Quandaries

Philosophical problems with constructivism clarify what would make peer collaboration desirable, and we can begin with the critics. Sriraman, in a recent article in *Mathematical Behavior*, has pointed out that deduction or induction from particular cases (i.e., generalizing activity) requires working over an extended period of time (Sriraman, 2004). Slezak (2000), director of the program in cognitive science at New South Wales, worried, "On these [constructivist] views education becomes indoctrination, pedagogy is propaganda, and ideas are merely conventional conformity to social consensus" (p. 93). Constructivism leads to relativism, which is at its "heart" (Slezak, 2000, p. 93).

Matthews (2000), in "Appraising Constructivism in Science and Mathematics Education", agreed with Fosnot, for instance, that constructivism is not just a theory about learning but also "our culture's greatest and most enduring achievement, namely science" (p. 162). Constructivism, as Matthews pointed out, could also be a theory of cognition, learning, teaching, education, personal knowledge, scientific knowledge, educational ethics, politics, and a worldview (Matthews, 2000, p. 163). According to him, the semantic and epistemological domains are often confused.

Matthews disagreed that constructivism must entail idealism (Matthews, 2000, p. 163). Social constructivism, as held by Glaserfled, leads to paradoxes, like that of self-refutation (i.e., the theory itself is constructed) (Matthews, 2000, p. 167). Matthews separated educational, philosophical, and sociological constructivism.

Matthews wrote, "Language, especially scientific and mathematical language needs to be mastered and, at the end of the day, transmitted" (Matthews, 2000, p. 171). Definitions need to be taught, and are not always made up by learners:

One might reasonably ask, at this point, whether learning theory

or ideology, is simply getting in the way of good teaching. Why must learners construct for themselves ideas of potential energy, mutation, linear inertia, photosynthesis, valiancy and so on? (Matthews, 2000, p. 180)

Several commentators suggested separating epistemological issues from pedagogical ones (Matthews, 2000; Burbules, 2000). Burbules (2000), in "Bridging the Impasse", wrote, "Focus on trying to understand the practices and procedures by which constructions come to be created, adjudicated and commonly shared" (Burbules, 2000, p. 326).

Burbules concluded that teachers need different tools, and that constructivism may be one of them (Burbules, 2000). Constructivism, after all, has the virtue of attempting to produce the kinds of conditions that drive scientific [and mathematic] exploration in the first place (Burbules, 2000; Ball & Bass, 2000). If peer collaboration in mathematics has value, it will be because pedagogy requires and accepts it.

The Reciprocity between Practice and Epistemology

When longitudinal studies are wanting, ethnographic ones intimate a solution. James W. Stigler and Harold W. Stevenson, who have conducted ethnographic studies of mathematic education, attempted to explain the "startling" higher achievement of Asian students in mathematics, compared to their American counterparts (Stigler & Stevenson 1999, 66). Stigler and Stevenson claim that the Asian class is "constructivist", yet also involves *less peer collaboration and more instructional time with the teacher* (Stigler & Stevenson, 1999, pp. 69, 71). Stigler and Stevenson contend that we need to question if individualized or group learning is better than whole-class instruction (Stigler & Stevenson, 1999).

The value of peer-collaboration can only be determined perhaps for a specific subject, class, and lesson. To be sure, the fruitfulness of peer collaboration will also depend on the teacher and culture of the students.

Looking at matters from a neurological perspective, Kong and associates, publishing in *Cognitive Brain Research*, have showed that the parts of the brain used to carry out addition operations are also used for subtraction, which is useful in breaking the stranglehold between pro- and anti-constructivists. Kong and associates conjectured:

Children usually start learning arithmetic with simple addition, then subtraction. They later learn the more complicated aspects of addition and subtraction like carrying. This developing order may be reflected in the neural circuitry of mental calculation and may explain why the neural network of simple addition is the basis of other calculation types. (Kong et al., 2005, p. 407)

In mathematics we move from simplicity to complexity, reflecting the nature of the subject matter.

Furthermore, the factory model of education is the setting in which constructivists implement peer collaboration. Long before populations were committed to mass education, we learned in a master-disciple relationship. The apprenticeship system was universal: the blacksmith, carpenter, musician and mathematician, trained the apprentice. In the apprenticeship system, collaboration is between someone who has vast experience with solving problems in the given field.

An important point is revealed about peer collaboration from the apprenticeship system: it is useful when one of the participants is knowledgeable enough to guide others. Also, it is still reasonable to think that peer collaboration is generally useful.

Confirming previous work, Schliemann and Carraher (2002), in "The Evolution of Mathematical Reasoning: Everyday versus Idealized Understandings" noted that mathematics involves personal discovery, as well as conventional symbols and contexts (Schiemann & Carraher, 2002, p. 242).

Mathematics relies upon specific representations and tools, which play a role in the structure and role of mathematical thinking (Schiemann & Carraher, 2002, p. 244). Constructivists, they emphasized, must realize that some notions are more useful in the long run (even if at odds with individual ways of doing things). "17 – 6 = 11" is more useful that working with a fish bowl (Schiemann & Carraher, 2002).

We can distinguish between the common and deep contexts. The *common context* is grade 5, mathematics students, at Coronation Public School, in Windsor, Ontario, Canada. There is also the *deep context* (or culture) which includes previous experiences and assumptions of the class. We return to complexity: in the relationship between the student, teacher, and subject matter, there is a balance to be had. To resolve the debate, I propose we consider both philosophical foundations of peer collaboration and the implications for pedagogical practice.

Epistemology and Group Work

Philosophers since at least Frege (1884/1953) have scrutinized the relationship between what has come to be known as the *context of discovery* from that of the *context of justification.* We may wish to recall that Frege separated how we discover something from how we justify it. Yet considering group work in elementary mathematics prompts us to add the *context of learning.* We can distinguish, in different ways that have been held by various scholars, between how knowledge is discovered (e.g., when it first was discovered), justified (e.g., proved), and learned (i.e., how we teach *accepted* knowledge). The following chart (table 1) depicts the relations between the three contexts of discovery, justification, and learning.

Table 1. Some relations of the contexts of discovery, justification, and learning (explained in detail below).

View	Context of Discovery	Context of Justification	Context of Learning
Metaphysical realist-1	+	-	+
Metaphysical realist-2	+	-	-
Naturalist-1	+	+	+
Naturalist-2	+	+	-
Skeptic	-	-	-
Radical Constructivist-1	-	+	+
Radical Constructivist-2	-	+	-

For metaphysical realists the contexts of discover and justification must be separated, in principle, yet there is disagreement about the consequences for learning. For the metaphysical realist-1, there is knowledge to be discovered that we may never reach and learning is modeled on practices of inquiry in the relevant field. For the metaphysical realist-2, like Frege, there is knowledge to be discovered that we may never obtain and learning need not be modeled on current practices in the relevant field.

As is well known, naturalists blur Frege's distinction: how knowledge is acquired is how it must be justified. Yet, like metaphysical realists, may disagree with the consequences for pedagogical practice. The naturalist-1 holds that their epistemology provides the ground of a pedagogical practice. Conversely, the naturalist-2 agrees with Frege only in this: our epistemology need not reflect our pedagogy.

The global skeptic suspends judgment about the possibility of knowledge, its justification, and it is reasonable to think, must make learning an arbitrary matter: there cannot be any science of teaching anymore than anything else. At best, we can obtain solidarity.

Philosophical constructivists attempted to avoid skepticism by rejecting the recognition transcendence of truth and by inextricably tying it to our methods of justification. The radical constructivist-1, like von Glasserfeld, denied that knowledge is mind-independent: all truth is constructed within modes of justification. The radical constructivist-1 holds that knowledge should be taught the way we justify it. The radical constructivist-2, like it is reasonable to conjecture, Hilbert formalist thought of the 1920s, are not wedded to a pedagogy modeled on the way knowledge is produced.

It is apparent from the two species of metaphysical realism, naturalism, and radical constructivism discussed that whatever view we have of knowledge does not entail a pedagogical program. At the same time, it is reasonable to think that the first species of metaphysical realism, naturalism, and the radical constructivist, where there is some connection between the contexts of discovery or justification and learning could be the basis of compelling arguments in that direction. That is, if the naturalist-1 is right we would have one reason to teach in a way that models how we actually discover and justify knowledge, as much as is feasible. If the radical constructivist-1 is right, we would have a reason to emphasize mathematics as a social game where we attempt to master the rules of symbol manipulation.

Without straying too much further into epistemological debates, suffice it to say that the realist has an edge: mathematical truth is eternal and unchanging, guiding even constructivist pedagogy. We are directed in terms of content: there is one and only one mathematics. Conventional notation and methods, further, are guided by both biology and mathematics. "159 – 7 = 152" is easier to solve than "CLix – vii = CLii", which is why in fact we use Arabic numerals not Roman ones. Some cultures do not count numbers greater than identified body parts. A number system, like our Arabic-Indian one (the ten base number system with a "0"), is necessary for calculations involving high cardinalities, since we first need to conceive of those numbers (or have a procedure for constructing them). In mathematics, social constructions are circumscribed.

Practice and Group Work

We can consider the implications for practice by reflecting on the suggestions of constructivists emphasis upon group work, which is consistent with their epistemological assumptions about the social nature of knowledge. Nelson (1996), a biology professor at Indiana University who has won awards for excellence in teaching, outlined what he dubbed "the myths of rigor" in traditional pedagogy (e.g., tough courses, thwarting grade inflation, lecturing, focusing on content, emphasizing student responsibility, and handing in work on time). He argued, however, that didactic pedagogy favors the upper-middle class and supports discrimination against non-traditional students.

Nelson, reviewing the relevant literature, noted that those from upper middle class backgrounds automatically formed collaborative groups to get through calculus, increasing their "status" for helping others, whereas underprivileged children think that "only weak students study together" (1996, p. 166). He observed from studies and his experience that modifying traditional pedagogy with active learning, discussion, and peer collaboration, benefits all students and the weakest segments of the population the most.

Nelson concluded that traditional didactic ways of teaching are comparatively ineffective and bias. He contended that the reason faculty members continue with ineffective teaching practices is self-serving, relies upon erroneous attribution schemes (blame the victim), and dysfunctional illusions of rigor.

It is altogether reasonable to think that we learn best when we have to interact and communicate with others. Building upon the social dimensions of learning will increase student achievement, by boosting interrelated factors, such as meta-cognition, memory retention, motivation, and the understanding that comes with having to explain what we think to others. It is a platitude but worth reciting: we are social creatures.

One reason group work is sometimes effective is because it increases motivation. We are more motivated to excel in a discipline when it is considered a value, culturally or in our interpersonal groups, which we internalize. Group work can change the value of mathematics for disadvantaged groups (where mathematics has little value), for both constructivist and naturalist theorists (e.g., behaviorists).

Prospects for Group Work

Group work, in fact, functions as part of a carefully considered pedagogical strategy, which though not entailed, may be at the very least consistent with our overall philosophical view of knowledge. I argued two things: first, for mathematics, it is reasonable to think, peer collaboration is an appendage to instruction, not a replacement for the didactics of an expert or individual problem solving. I can only conjecture that peer collaboration is useful at the elementary as well as some advanced settings (in heterogeneous groups that facilitate instruction). There is a knowledge we need to transmit. In addition, if we are to follow the system of apprenticeship, mathematics is done individually or in dyads (insofar as we spend time practicing), both at its inception, when discoveries are made, and at advanced levels. A great deal of practice is required to develop the skills and the appropriate neural networks, to excel in any discipline. At least some of the practice must be done alone, which is consistent with what we know of those who have excelled, across disciplines. Though pedagogy cannot, it is reasonable to think, cannot fully mimic the way knowledge is discovered, it is desirable to move in that direction because it is more likely to produce an authentic context for learning.

In fact second, for the teaching of mathematics an adequate epistemology can usefully reflect a successful pedagogy, its principles, which take into account the student, teacher, and subject matter. Pedagogy, at the very least, must reflect the fact that epistemic discoveries are made by individuals that rely upon a social store of previously accumulated knowledge. There is reciprocity between individual and social settings in learning mathematics. The pedagogue must keep the entire repertoire of heuristics at her disposal, both what follows her assumed epistemology and what departs from it.

Some constructivists, however, not only reject all dialectic methods but do not realize when they rely upon rote learning, a reward system, and independent study. My aim has not been to argue that peer collaboration has no place, but rather to critically reflect on how we adjudge its worth. Our epistemology is one landmark in guiding our choice of heuristic. It is important to keep in mind, however, that both naturalist and constructivist epistemologists can embrace the same pedagogy of active learning where group work is prominent. Group work has a place and does not entail a knockdown argument against naturalists. On the contrary, naturalists need to detail the implications of their epistemological stand is for scholarship of teaching and learning.

REFERENCES

Ball, D. L., & Bass, H. (2000). Making believe: The collective construction of public mathematical knowledge in the elementary classroom. In D. C. Philips (Ed.) *Constructivism in education: Opinions and second opinions on controversial issues* (ninety-ninth yearbook of the National

Society for the Study of Education) (pp. 193-224). Chicago: University of Chicago Press.
Brooks, J. G., & Brooks, M. G. (1999). *In search of understandings: The case for constructivist classrooms.* Virginia: ASCD.
Burblules, N. C. (2000). Moving beyond the impasse. In D. C. Philips (Ed.) *Constructivism in education: Opinions and second opinions on controversial issues* (ninety-ninth yearbook of the National Society for the Study of Education) (pp. 308-330). Chicago: University of Chicago.
Fawcett, L. M., & Gourton A.F. (2005). The effects of peer collaboration on children's problem-solving ability, *British Journal of Educational Psychology, 77*, 157-169.
Fosnot, C. T. (Ed.) (2005). Constructivism: Theory, perspectives and practice (2nd ed.) NY: Teacher's College Press.
Fosnot, C. T., & Dolk, M. (2005). "Mathematics" or "mathematizing"? In C. T. Fosnot (Ed.) Constructivism: Theory, perspectives and practice (2nd ed.) (pp. 175-191). NY: Teacher's College Press.
Frege, G. (1953). *Grundlagen der arithmetik* [*The foundations of arithmetic: A logico-mathematical enquiry into the concept of number*]. (J. L. Austin, Trans.). Oxford: Blackwell. (Original work published in 1884)
Glaserfeld, E.v. (2005). Introduction: Aspects of constructivism. In C. T. Fosnot (Ed.) Constructivism: Theory, perspectives and practice (2nd ed.) (pp. 3-7). NY: Teacher's College Press.
Kong, J., Wang, C., Kwong, K., Vangel, M., Chua, E., & Gollub, R. (2005). The neural substrate of arithmetic operations and procedural complexity, *Cognitive Brain Research, 22*(3), 397-405.
Nelson, G. E. (1996). Student diversity requires different approaches to college teaching, even in math and science. *American Behavioral Scientist, 40*(2), 165-175.
Philips, D. C. (Ed.) (2000). *Constructivism in education: Opinions and second opinions on controversial issues* (ninety-ninth yearbook of the National Society for the Study of Education). Chicago: University of Chicago Press.
Philips, D. C. (2000a). An opinionated account of the constructivist landscape. In D. C. Philips (Ed.) *Constructivism in education: Opinions and second opinions on controversial issues* (ninety-ninth yearbook of the National Society for the Study of Education) (pp. 1-16). Chicago: University of Chicago.
Piaget, J. (1932). *The moral judgments of the child.* London: Routledge and Kegan Paul.
Piaget, J. (1959). *The language and thought of the child* (3rd ed.). London: Routledge and Kegan Paul.
Pichat, M., & Ricco, G. (2001). Mathematical problem solving in didactic institutions as a complex system: The case of elementary calculus, *Journal of Mathematical Behavior*, *20*, 43-53.
Schliemann, A. D., & Carraher, D. W. (2002). The evolution of mathematical reasoning: Everyday versus idealized understandings, *Developmental Review, 22*, 242-266.
Slezak, P. (2000). A critique of radical social constructivism. In D. C. Philips (Ed.) *Constructivism in education: Opinions and second opinions on controversial issues* (ninety-ninth yearbook of the National Society for the Study of Education) (pp. 91-126). Chicago: University of Chicago.
Solmon, J. (2000). The changing perspectives of constructivism: Science wars and children's creativity. In D. C. Philips (Ed.) *Constructivism in education: Opinions and second opinions on controversial issues* (ninety-ninth yearbook of the National Society for the Study of Education) (pp. 283-307). Chicago: University of Chicago.
Sriraman, B. (2004). Reflective abstraction uniframes and the formalization of generalizations, *The Journal of Mathematical Behavior, 23*(2), 205-222.
Stigler, J. W., & Stevenson, H. W. (1999-2000). How Asian teachers polish each lesson to perfection. In *Child Growth and Development* (6th ed.) (pp. 66-77). Connecticut: McGraw-Hill.
Vidakovic, D., & Martin, W.O. (2004). Small group searches for mathematical proofs and individual reconstructions of mathematical concepts, *Journal of Mathematical Behavior, 23*(4), 465-492.
Vygotsky L. S. (1978). *Mind in society.* Cambridge Mass.: Harvard University Press.

◈◇◈