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Definite Integral Automatic Analysis Mechanism Research and Development Using the "Find the Area by Integration" Unit as an Example

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ABSTRACT
Using the capabilities of expert knowledge structures, the researcher prepared test questions on the university calculus topic of "finding the area by integration." The quiz is divided into two types of multiple choice items (one out of four and one out of many). After the calculus course was taught and tested, the results revealed that the Q matrix of the second type of multiple choice items (one out of many) is closer to that of the experts compared with that of the first type of multiple choice items (one out of four). The statistical information was then analyzed using Bayesian probability analysis methods to establish the "find the area by integration" unit model as an example of a Bayesian network adaptive diagnostic test system and to establish how this structure could be used as the foundation for developing remedial teaching approaches.
Keywords: Bayesian network, DINA model, error type, expert knowledge structure, Q matrix

INTRODUCTION

Regardless of the teaching method used, a general dissatisfaction with calculus courses has emerged in various countries in the last decade (Tall, 1993). In the UK, a recent report from the London Mathematical Society acknowledges the difficulty of university mathematics and the need to reduce the content and reorganize the course (London Mathematical Society, 1992). Students face difficulties in learning the computational procedures without understanding the concepts (Serdina Parrot & Eu, 2014; Artigue, Batanero, & Kent, 2007). For the overwhelming majority of students, calculus is not a body of knowledge, but a repertoire of imitative behavior patterns that are abstract, complex and difficult to understand (Tapare, 2013; Moise, 1948). The focus of this study was on methods for helping students discover their own errors and revise their own concepts, separating bothersome theory from practical methods. This approach was particularly applied to solve problems with the definite integral in university-level calculus courses.

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State of the literature

- This study has no relation to the price, there is no criticism of the earlier article, and there is no plagiarism. This study wanted to be a lively network of learning calculus to attract students to enhance students' confidence and thus stimulate interest.

Contribution of this paper to the literature

- Based on the literature showing that computerized tests are generally used for more basic primary and secondary school mathematics, the present study of university calculus indicates that "Find the Area by Integration" constitutes an attempt
- The present study discusses "error types" on the "Find the Area by Integration" unit
- The present study develops items related to "Find the Area by Integration" and builds a computerized test interface to assist Bayesian network diagnostic analysis.

LITERATURE

Related Calculus Literature

Mathematics education is based on problem solving, which has been widely recognized as a framework for analyzing the learning of mathematical processes (Fernando & Aarón, 2013; Marchis, 2013). As students explore mathematical problem solving processes, they must understand the foundations of mathematics. However, their existing mental structures are not fully consistent with the new structures of mathematics they are learning (Herscovics, 1989; Hiebert & Carpenter, 1992). In new contexts, old experiences can cause serious conflicts (Tall, 2004). The students then have cognitive obstacles, such as the symbolic representation of integrals, the integral sign itself, and the cognitive concept of the 'area under the curve' interpretation (Mallet, 2011). However, without a complementary conceptual understanding, the students will be less able to generalize their skills to other contexts and thereby lose much of the problem-solving power of calculus (Zerr, 2010).

It does seem to be a fact of life that few prosper in mathematics. Therefore, a good begin to struggle and need appropriate help to be able to pursue mathematics further (Tarmizi, 2010). As instructors, we, the 'experts,' need to recognize the difficulties placed on learners by our reliance on common symbols for new concepts (Mallet, 2011). Thus, to make progress in the actual teaching and learning of mathematics, one needs to deepen and further one's understanding of the theoretical ideas that frame teaching and learning (Larson et al., 2010).

Mistakes in mathematics are as important and as significant as correct answers. In some cases, they are more significant (Schwarzenberger, 1984). Errors help us understand the students' thoughts and why the errors occur and can thereby be a diagnostic tool that helps us to understand the context of mathematical errors and support the development of mathematics instruction.

Computer-based Tests

Traditional tests have been criticized for having only limited effectiveness because they cannot accurately assess potential changes in students (Caffrey, Fuchs, & Fuchs, 2008). Additionally, observing a particular student's situation in a large class is difficult (Chatzopoulou & Economides, 2010). Therefore, increasingly more instructors use software to assist in the process of teaching and learning in higher education (Schroeder, Minocha, & Schneider, 2010). The development of information technology over the last two decades has made computer- based testing feasible in both educational research and practice (Bunderson, Inouye, & Olsen, 1988).

Furthermore, today's e-learning technology enables organizations to start adopting online instructions as well as online testing (Tao, Wu, & Chang, 2008). These test systems can effectively improve students' learning (Chatzopoulou etc, 2010; Wauters, Desmet, & Noortgate, 2010). Given this success, computer technology is gradually being introduced to the testing field. Moreover, computer-based tests save time and have a better implementation of remedial teaching.

Q matrix

This study is aimed at different types of items designed to obtain different Q matrices, using the DINA estimated property subject cognitive status, and to identify the rate of assessment indicators. To clearly explain the relationship between the items and skills, a cognitive diagnosis model is primarily used as a skills matrix Q to influence the item table (Tatsuoka, 1985). If there is a test of J × K items and skills, the Q matrix with J × K matrix elements is defined as follows:

$$q_{jk} = \begin{cases} 1, \text{to answer the j} - \text{th item needs k} - \text{th skill.} \\ 0, \text{otherwise.} \end{cases}$$

where $j = 1,2,, \dots, J. k = 1,2, \dots, K.$

For example, if a test has two items and three skills, then $\mathrm{Q}_{23} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$, to answer the first item, you need skills 1 and 3, and to correctly answer item 2, you need skills 1 and 2.

A Q-matrix can be viewed as a cognitive design matrix that explicitly identifies the cognitive specification for each item (Torre, 2009). Using the Q matrix to determine the required skills for each test item, subject experts must also decide on the degree of mastery of each skill and how it affects the chances of correctly answering the question. Teachers can use the displayed students' Q matrix to estimate the cognitive properties of the students is master or not, and the students show different learning state.

Table 1. The recognition rate

		Estimated value	
		True	False
True value	True	n_{11}	n_{10}
	False	n_{01}	n_{00}

Note: n_{ij} = 1, if the estimate value is consistent with the true value; otherwise, n_{ij} = 0.

Recognition Rate

For all items of cognitive diagnostic modes, the recognition rate is the consistency of the results of students' tests, where experts determine the attribute states. In this study, the recognition rate between the true value (e.g., expert judgment) and the estimated value (e.g., where the DINA mode determines the results) is 1 when the two results agree and 0 when they are inconsistent, as shown in **Table 1**.

This study describes the DINA mode as follows.

DINA Mode (Deterministic Inputs, Noisy and Gate Model)

Junker and Sijtsma (2001) first used the DINA mode, which is the basis of various cognitive diagnostic assessments (Doignon & Falmagne, 1999; Tatsuoka, 1995). This mode is considered to have all cognitive attributes of the item required to correctly answer this item, but the answer probability of this item will be affected by two parameters: carelessness and guessing.

$$P\left(Y_{ij}=1|\eta_{ij}=1\right)=(1-s_j)^{\eta_{ij}}{g_j}^{(1-\eta_{ij})} \tag{1}$$

$$\eta_{ij}=\prod_{k=1}^{K}\alpha_{ik}^{Q_{jk}}$$

$$s_j=P\left(Y_{ij}=0\,|\eta_{ij}=1\right),$$

$$g_j=P\left(Y_{ij}=1\,|\eta_{ij}=0\right)$$

where Y_{ij}: the reaction of the i-th student to the j-th item is as follows:

s_j: the student has the required cognitive attributes for the j-th item, but, because of carelessness, has a probability of giving the wrong answer.

g_j: the student does not have the required cognitive attributes for the j-th item, but, because of guessing, has a probability of giving the correct answer.

α_{ik}: the i-th student either has the k-th cognitive attribute or not; if he has the property, then he is given the label 1; otherwise, he is given a label of 0.

This model assumes that the student, who needs to have all required cognitive properties, can answer correctly questions, but the probability of the answer questions, will be subject to the impact of two parameters - slip and guessing. DINA model divides students into two categories, one answers correctly the question - to master the all required cognitive properties for the test item or to solve the item by means of the probability of guessing, the other answers incorrectly the item - the lack of one or some of the necessary cognitive properties to reduce the chances of their correct answers, or careless. While the DINA model only considers two parameters, i.e., careless s_j and guess g_j. But it has a good model fit (de la Torre & Douglas, 2008) and is a simple and easily interpretable model (de la Torre & Douglas, 2004). It is often used in tests. Based on this, this study will use this model as a tool to assess the cognitive properties of students.

Bayesian Network

In recent years, the Bayesian network has become an important modeling method for decision making problems in real-world applications (Irad, 2007). The Bayesian network (BN) is also known as a belief network, probability network or causal network. A graphical representation mode, a reasoning tool for knowledge representation, was based on Bayes' theorem to explain the degree of probability of the interaction between several variables, which is a probabilistic expert system. The content is formulated as follows: Suppose $U = \{S_1, S_2, \ldots, S_n\}, S_i$ which is finite (for i =1,2,…,n). In the event E, the probability of occurrence of S_i is

$$P(S_i|E) = \frac{P(E|S_i)P(S_i)}{P(E)} = \frac{P(E|S_i)P(S_i)}{\sum_i P(E|S_i)P(S_i)} \quad (2)$$

where $P(S_i)$ prior probability, $P(E|S_i)$ sample probability, $P(S_i|E)$ posterior probability.

Bayes' theorem uses the prior probability and sample probability to infer the posterior probability. These conditional dependencies in the graph are often estimated by using known statistical and computational methods. The directed acyclic graphical Bayesian network is based on Bayes' theorem to constitute a series of conceptual nodes and arrow symbols between nodes. In the knowledge structure figure, each node represents an event, or a random variable in the network. A connecting link indicates a causal relationship between events, an association or a causal relationship between two variables in a network. The presence or absence of links and the influence degree are represented by the strength of the conditional probability. Thus, this directed graph is a joint probability distribution representation of several variables. These graphical structures are used to represent knowledge about an uncertain domain. Students' errors could be caused by random and other uncertainty factors (Friedman, Goldszmidt, Heckerman, & Russell, 1997). Yet the Bayesian network can still be used with incomplete or missing information.

Some of the main advantages of Bayesian networks are their ease of understandability, simplicity in the acquisition of prior knowledge, potential for causal interpretation, and

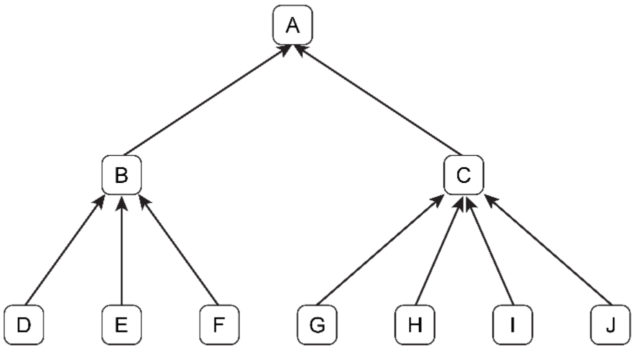

Figure 1. The knowledge structure

effective handling of missing data (Wu, Kuo, & Yang, 2012). In this study, the students' responses were input into the Bayesian network models, and the joint probabilities were calculated by using the MATLAB analysis software. According to the updated the posterior probability distributions of the sub skills or error types, a standard (critical ratio = 0.3) was used to make the decision to deduce the recognition rates of Bayesian network to estimate whether the students have mastered the sub-skills, or whether exhibit error types, allowing us to understand the abilities and errors of students.

RESEARCH

Assessment of Content Analysis

Calculus includes functions, limits, derivative functions, integrals, etc., and interesting applications to real life. A closed area gives the size of a two-dimensional region, and the degree of coverage represents a particular area, which refers to the size of the surface to be covered. In elementary school, one learns about the areas of rule graphs, such as squares, rectangles, triangles, circles, and trapezoids. However, how will we find the areas of irregular shapes? To do so, this study employs the "find the area by integration" issue.

Expert Knowledge Structure

After much discussion, the domain experts decide the sequence of the concept development and relationships among these concepts to depict the expert knowledge structure for each unit in a tree diagram (Wu et al., 2012).

In this study, first, five university teachers are urged to analyze our expert knowledge structure for the "find the area by integration" unit of the one-year compulsory calculus textbook and to understand the teaching experience and knowledge that the students should theoretically have. There are many relationships between concepts, which can be divided into vertical and horizontal relationships (Rosch, 1975). The vertical relationships between the concepts were discussed in this study. In this knowledge structure diagram, each concept is a node, and concept B and C should be mastered before concept A can be attempted. The

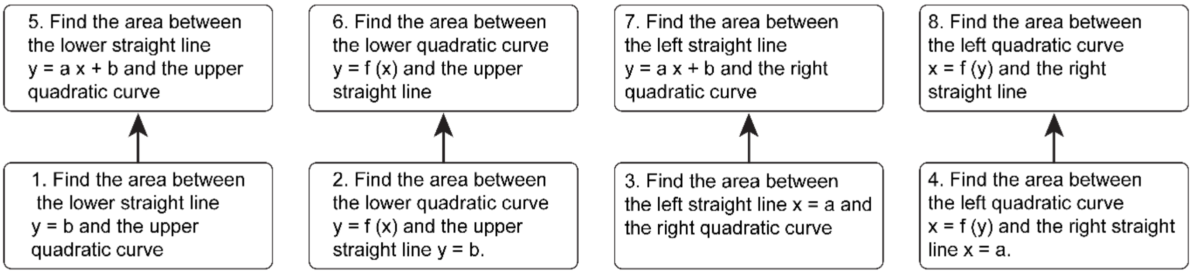

Figure 2. Expert knowledge structure

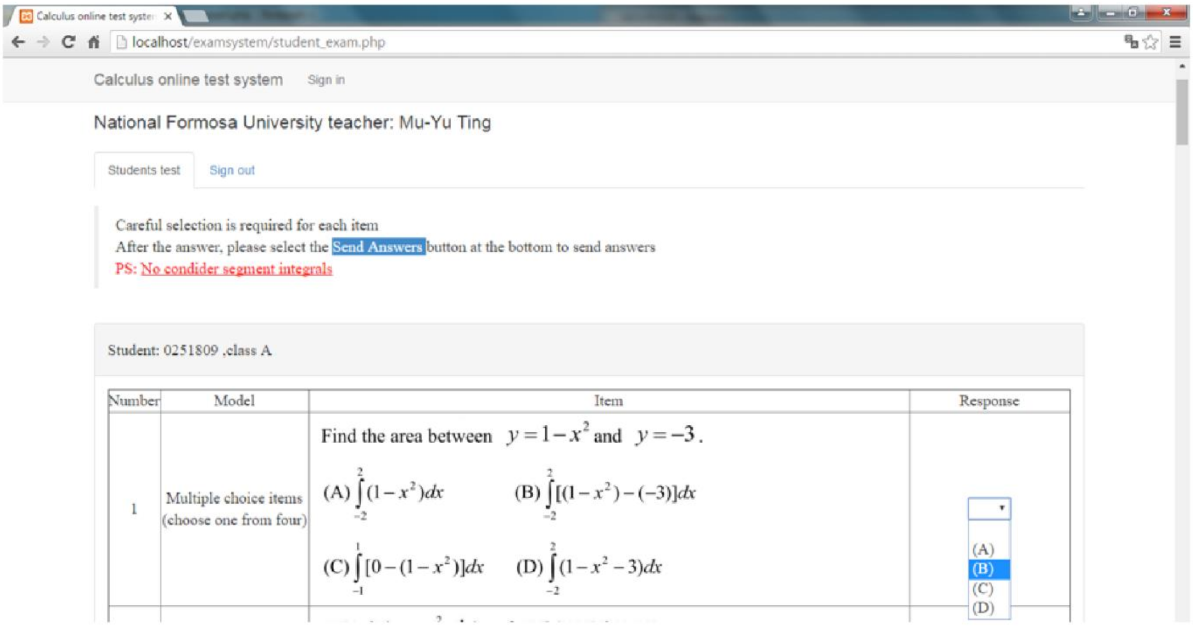

Figure 3. A survey of the first type of multiple choice items (one out of four)

concepts D~ F should be mastered before concept B can be attempted. The concepts G~J should be mastered before concept C can be attempted. The knowledge structure is shown in **Figure 1**.

This study organizes the items according to expert knowledge structures. The expert knowledge structure, refers to Ting & Kuo (2016) is shown in **Figure 2**.

A survey of the first type of multiple choice items (one out of four) is given as the **Figure 3**.

A survey of the second type of multiple choice items (one out of many) is given as the **Figure 4**.

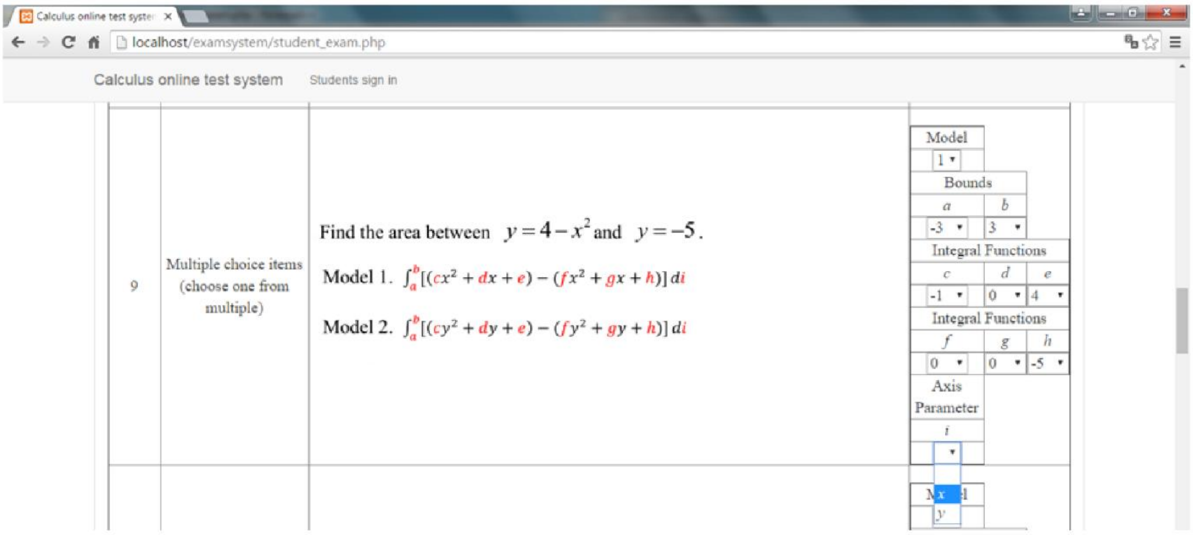

Figure 4. A survey of the second type of multiple choice items (one out of many)

Table 2. Sub-skills of "finding the area by integration"

Node	Sub-skill
S1	Find the left and right bounds
S2	Determine the upper and lower integrating functions
S3	Find the upper and lower bounds
S4	Determine the left and right integrating functions
S5	Find the axis parameter

Sub-skills of "Finding the Area by Integration"

To find the area by integration, this study uses the following sub-skills as Table 2:

Participators

In this study, a paper and pencil test was administered at one university, and then, the items were altered to obtain more suitable questions. A second test was then held at the same university. The effective sample size is 157 freshmen. A third academic year was held a formal test, 155 valid samples.

Error Type Analysis

Misconceptions may come from students' self-learning and from traditional mechanical teaching. These misconceptions result in fuzzy concepts and produce different error types. Teachers who want to help students must first determine the cause of the students' error types. The researchers determine the teaching materials necessary to construct a quantitative analysis of error types as the smallest unit of teaching so that teachers can directly estimate students' error types from their responses as Table 3.

Table 3. Student error types

Node	Error type
E1	Ignore the inevitability of the lower or the left integrated function.
E2	Mispick or un-select the lower or the left integrated function.
E3	Mispick or un-select the upper or the right integrated function.
E4	Misplace the axis parameter.
E5	Misplace or un-select the lower boundary.
E6	Misplace or un-select the upper boundary.

Table 4. Expert-defined Q matrix

Attribute Item	S1	S2	S3	S4	S5
I1	1	1	0	0	1
I2	1	1	0	0	1
I3	0	0	1	1	1
I4	0	0	1	1	1
I5	1	1	0	0	1
I6	1	1	0	0	1
I7	0	0	1	1	1
I8	0	0	1	1	1

Research Tools

As shown in **Figure 2**, which lists eight nodes of the expert knowledge structure, the researcher designed problem sets were divided into two sets of eight multiple choice items (one out of four and one out of many). Each item set represents only a concept of the knowledge structure, which is represented in the diagram as a node. For the multiple choice items with four possible answers, only one option is correct, and the remaining answers represent the most common errors that students make on the pre-test. Based on the student's answers, the researcher can determine the student's error type. Multiple choice items with many possible answers must be listed with a reaction formula, so that the student's thinking process can be clearly seen. In this study, when the true value is consistent with the estimated value, the recognition rate is considered 1; otherwise, it is 0.

RESULTS AND DISCUSSION

When students complete online quizzes, their response procedures can be obtained from a database. Using experts' interpretations of the wrong answers, the researcher used MATLAB to write programs that can automatically determine the wrong types of problem solving processes for future students. According to the students' responses, this study calculates the joint probability using a Bayesian network to estimate whether students have the necessary sub-skills or whether they exhibit error types; this approach also allows one to understand the ability of students given the errors they make.

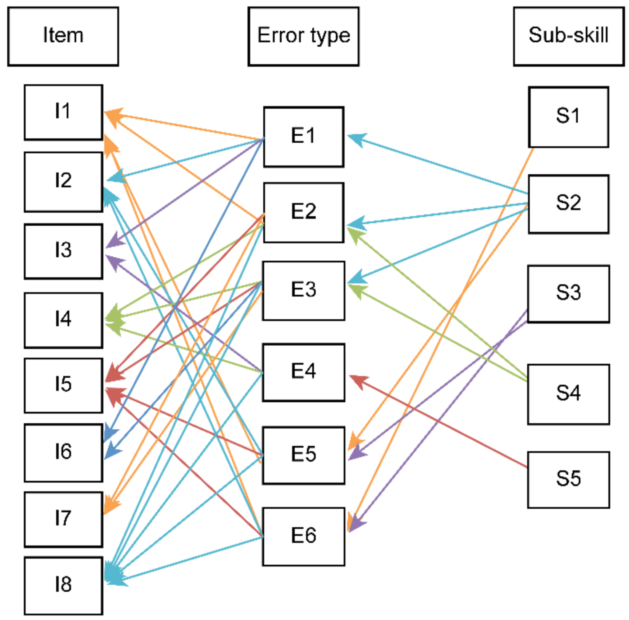

Figure 5. Mode 1 Bayesian network

Given a quiz to assess specific items of cognitive skill or the concepts of a single desired set of cognitive attributes, a Q matrix is obtained. Given J × K items and a test of cognitive attributes, a corresponding Q matrix of size J × K will be defined by experts as shown in **Table 4.**

The recognition rate for the Q matrices for the first type of multiple choice items (one out of four) and the experts is 0.7635. The recognition rate for the Q matrices for the second type of multiple choice items (one out of many) and the experts is 0.8888. The Q matrix for the second type of multiple choice items (one out of many) is closer to that of the experts.

The Bayesian network diagram is divided into three layers from left to right: the first layer is the items; the second layer is the error types; and the third layer is the sub-skills. As shown in **Figures 5** and **6**, the single solid-line arrow represents the association between both layers. In the definite integral Bayesian network diagram, the link inputs in the MATLAB 7.0 codes "0" or "1" between nodes. This research explores the following two modes:

Mode 1: This mode includes eight of the first type of multiple choice items (one out of four), with I1 ~ I8, error types E1 ~ E6 and sub-skills S1 ~ S5.

Mode 2: In addition to Mode 1, eight of the first type of multiple choice items (one out of four) (I1 ~ I8) were then incorporated with eight of the second type of multiple choice items (one out of many) (M1 ~ M8) to evaluate the corresponding error types as **Figure 5**.

The multiple choice items (one out of many) correspond to additional error types, as **Figure 6**:

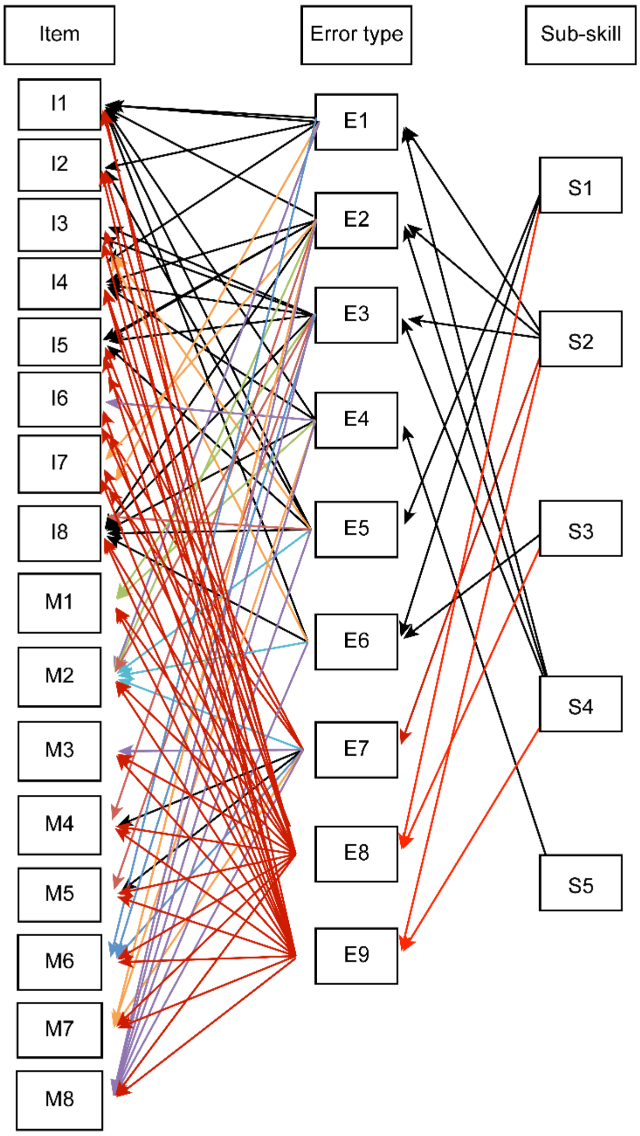

Figure 6. Mode 2 Bayesian network. The black lines represent the first connection mode, and the colored lines represent increased connections

E7: The two functions of the integral are inverted.

E8: There is an error in solving simultaneous equations.

E9: There is a cognitive error in size or height.

The average recognition rates of skills, from Models 1 and 2, are 0.7642 and 0.8023, respectively. The average recognition rates of errors, from Models 1 and 2, are 0.8106 and 0.8215, respectively. The average recognition rates of Models 1 and 2 are 0.7874 and 0.8119, respectively. The results show that the overall recognition rate of the BN (Model 2) is better than that of Model 1, which has only the multiple choice items.

In the future, teachers may consider using the type of multiple choice items (one out of many) to replace the traditional the type of multiple choice items (one out of four)in order to achieve Tarmizi's viewpoint (2010), i.e., to be able to immediately understand the learners learning situation, to help them continue to learn; and to conform to the opinion of Schroeder et al. (2010), that is to say, more and more people use software to help teaching and learning in the process of teaching and learning in higher education, so as to achieve the goal of improving the quality of teaching and learning by means of the characteristics of computer multimedia. This research focuses on definite integral diagnostic tests and automated analysis mechanisms. Future studies may focus on remedial teaching.

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