MODESTUM
OPEN ACCESS

EURASIA Journal of Mathematics, Science and Technology Education, 2019, 15(3), em1678
ISSN:1305-8223 (online)
Research Paper https://doi.org/10.29333/ejmste/103031

Description Language of Educational Content Structure: Possibilities of Modern Mathematics

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Received 31 July 2018 ▪ Revised 25 November 2018 ▪ Accepted 15 December 2018

ABSTRACT

In modern conditions of intensive growth and differentiation of scientific knowledge, continuing reforms of educational systems, the pedagogical society is constantly facing the problem of justification of the content of education and its structuring. One of the tasks referring to this problem is a task of development of the description language of the educational content structure. Research objective consists of justification of the possibility of use of fractal geometry statements for the description of the educational content structure. As a main research method we have chosen a theoretic methodological analysis of scientific works concerning a development of an educational content theory and describing empirically stated defining peculiarities of its structure. Comparison of the description language of the educational content structure accepted in pedagogy and the description methods of structure of objects researched by the fractal geometry. The hypothesis about fractal nature of the educational content structure has been formed and proved immediately by means of the empiric material: the content structure of education and its "through" branches has a mosaic nature consisting of elements with different qualities (for example, mathematic, scientific and humanitarian disciplines or: basic, vocational and polytechnic education. The mosaic elements create a multistage system and are characterized by essentially divergent proportions. With the increasing of density or weight (with respect to "size") of elements of bearers of some properties we can state that the element of a larger proportion consisting of smaller elements expresses mainly the given property (so it plays a key role). The general picture looks like a mosaic board consisting of elements made themselves as mosaic pictures. This procedure occurs again on several levels. This description presents properly an idea of a multifractal formed by means of overlapping fractals, because the iteration procedure of creation of geometrical fractals looks on every step as a mosaic picture. The article may be useful for educationists researching the questions concerning the educational content and its structure and also for practitioners selecting the training documentation while developing different levels of educational programs.

Keywords: content of education, educational content structure, implicit and apical components, through lines of educational content, fractal

INTRODUCTION

In modern social pedagogical conditions, the quantity of information accepted during the education process increases essentially (Thibaut et al., 2018). The existent changes of the content of education, not only quantitative but also qualitative, are defined by a continuing intensive growth and differentiation of scientific knowledge - one of its determinants (Lednev, 1991a, 1991b; van Driel, Slot & Bakker, 2018). In such conditions the pedagogic society

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Contribution of this paper to the literature

- Difficulties arising in theoretical pedagogics during analysis of educational content structure are revealed by the authors.
- It is proven that the root cause of these difficulties is usage of classical geometry terms like 'vicinity', 'boundary', 'intersection of areas', et al. during analysis of educational content structure.
- It is shown that adequate description of educational content structure requires the usage of fractal geometry terms introduced in 1950-es.
- A necessity to study the possible usage of inexact sets theory and fuzzy logic terms as a language of educational content elements description are established. A goal of the study is a formation of optimal structure of natural education, and education at all. It will allow to eliminate a problem of selection between discrete (Romano-Germanic) and continuous (English) styles of narration of natural sciences.

is constantly facing the problem of justification of the content of education and its structuring. One of the tasks referring to this problem is a task of development of the description language of the educational content structure.

The questions of the educational content structure are the most completely and successively examined in investigations by Lednev (1969, 1971, 1973, 1980, 1988, 1989, 1991a, 1991b) and his followers (Kubrushko, 2001; Lednev & Kubrushko, 2001; Lednev, Kuznetsov & Sova, 1977; Lednev, Ryzhakov & Shishov, 1994). That is why for solving the above stated task we paid our attention to the educational content structure presented in works by Lednev (1969, 1971, 1973, 1980, 1988, 1989, 1991a, 1991b) and predominantly to the terminology used for description of this structure.

The terminology developed by Lednev (1969, 1971, 1973, 1980, 1988, 1989, 1991a, 1991b) for description of the educational content structure is not a traditional method of a structure description for mathematics. But its relative correspondence to the properties of the researched objects makes its use efficient as long as the more rigorous mathematically formed instrument is absent. In this regard there appears a necessity to examine in details the causes leading to the conclusion about a fractal nature of the objects of the educational content investigated by V.S. Lednev. In the present article the generalized results of the analysis of this conceptual construct on the ground of the statements of fractal geometry are presented (Gapontseva, Fedorov & Gapontsev, 2009a, 2009b, 2010).

RESEARCH METHODOLOGY

In the article we have investigated huge empiric material integrated in the works by Lednev (1969, 1971, 1973, 1980, 1988, 1989, 1991a, 1991b).

Methodology and research methods include a personal activity approach developed by Lednev (1991a) for the analysis of the educational content structure, theoretic methodological analysis of scientific works concerning a development of an educational content theory and describing empirically stated defining peculiarities of its structure. Comparison of the description language of the educational structure content accepted in pedagogy and the description methods of structure of objects researched by the fractal geometry.

RESULTS

The necessity to address back to the analysis of the educational content structure is connected with the intensive growth of scientific knowledge and educational crisis caused by this growth. As the fullest examination of the used conceptual construct it seems advisable to select a monograph (Lednev, 1989), where the principles of "double including" and "functional fullness" are formed summarizing the large empiric material.

But, herewith, it is important to note that in the seventy years of the last century the new possibilities appeared which were supplied by the fractal geometry as a branch of the modern mathematics. Its active development is connected with the name of Benoît B. Mandelbrot. The first success of the new geometry is aligned with the research of the coastline of islands (Mandelbrot, 1967). Then these ideas found application in description of molecular structure of a surface of a condensed phase and percolating clusters (Aharony et al., 1985; Avnir, Farin & Pfeifer, 1984; Avnir & Pfeifer, 1983; Mandelbrot, Passoja & Paullay, 1984), and also in description of diffusion and in physics of condensed mediums (Mandelbrot, 1983; Olemskoi & Flat, 1993; Pietronero & Tosatti, 1986; Zeldovitch & Sokolov, 1985). The questions of the fractal geometry as a specific branch of mathematics are analyzed in the following works (Bozhokin & Parshin, 2001; Falconer, 1990; Feder, 1988). The interest to the fractal geometry is preeminently connected with the possibility of its use in different spheres from the description of tree crowns to the investigation of behavior of complicated dynamic systems including market behaviors (Gimaliev et al., 2018; Isaeva et al, 2004; Mandelbrot, 1978, 2009; Mandelbrot & Hadson, 2006; Mavrikidi, 2008; Mutavchi et al., 2018; Peters, 2004; Ryabchenko et al., 2018). The demonstrativeness of fractals and their esthetic expressiveness are of a great

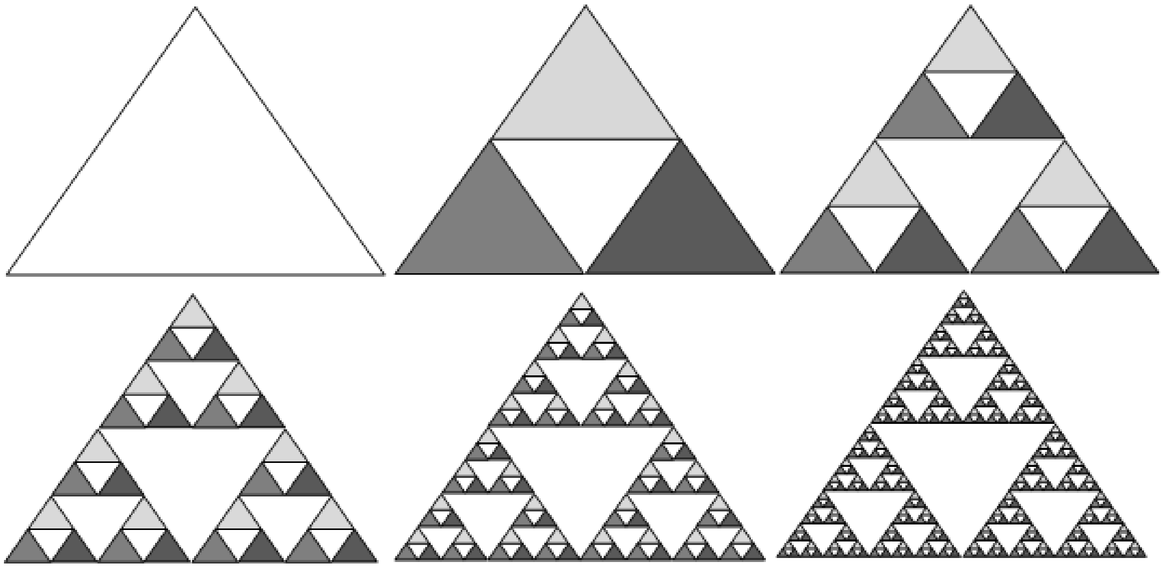

Figure 1. Building of a triangular Sierpinski gasket from elements with different gradations of a grey color

importance including our work too (Peitgen & Richter, 1986). As it will be stated below, namely the demonstrativeness of the geometrical fractals allows to use the ideas of the fractal geometry for interpretation of qualities of a so complicated object as the educational content. In addition to it, the natural prerequisite for an attempt to explain the educational content structure using the methods of the fractal geometry is their omnitude attested in the names of works (Barnsley, 1988; Mandelbrot, 1982).

The main qualities of fractals differing them from the usual geometrical objects are outlined as follows: the usual objects consist of elements of a set (of dots) located elbow to elbow, i.e. constantly, so that between any two elements it is possible to place one more element. Namely this continuity property of a set allows to use the idea about the boundary between different parts of this set and the idea about the neighborhood of any selected element of this set. The example of the continuous set is a geometrical basis - bearer of a fractal (its “backing board”), and the fractal itself, that fills the backing board, is a set consisting of isolated elements. Geometrical basis - bearer of a fractal (its “backing board”) is a continuous set with integer-valued dimension, that is a topological dimensionality, which value equals the number of measurements. Many isolated elements are located on this backing board, so this is a fractal itself. The method of their location is often defined by an iteration procedure of a fractal building. As an example we name **Figure 1**. In this picture the stages of building a triangular Sierpinski gasket are presented. The first triangle (the white one) in the upper row is a continuous set with the topological dimensionality value equaling 2. Then the parts are removed from it and are replaced by triangles with different gradations of a grey color. During the endless continuation of the iteration procedure on the white backing board there appears a fractal consisting of isolated elements having three different gradations of a grey color.

Individuality of a fractal as a unified whole is connected with mutual arrangement of its elements on the backing board. This mutual arrangement is characterized by a self-similarity property: in any neighborhood of every fractal element the nature of the mutual arrangement of other fractal elements fully repeats itself without distinction of the size of its neighborhood. Particularly, it is seen in **Figure 1**, that at any stage of building of this fractal the mutual arrangement of contacting elements with different gradations of a grey color is the same, i.e. similar to the mutual arrangement that was set on the first step of the building and self-similar for the whole fractal and its parts.

In **Figure 1** the starting stages of building the “Sierpinski gasket” on the ground of three initial elements with different gradations of a grey color is introduced. The triangles with different gradations of a grey color present the elements with different qualities, for examples, mathematics, physics, chemistry. Their joint presence in any area of the “backing board” (geometrical basis - bearer of a fractal) is evident, because at any stage of building the elements with different gradations of a grey color contact to each other. During the endless continuation of the process of iterations the initial backing board (geometrical basis - bearer) turns out to be filled by isolated elements of three types, whose mutual arrangement has the same nature as at any stage of a fractal building.

The structure of fractals differs essentially from the structure of usual geometrical objects. Firstly, it is impossible to define it pointing out the ending list of constituent parts of their mutual arrangement. Secondly, in the case of fractals it is not allowed to use an idea of a boundary between its parts. Thirdly, it is not correct to state that in a spontaneously selected area of a fractal there are elements of only one definite content. In any part of a fractal there are elements of all types defined during the building the fractal, as it is seen in **Figure 1**.

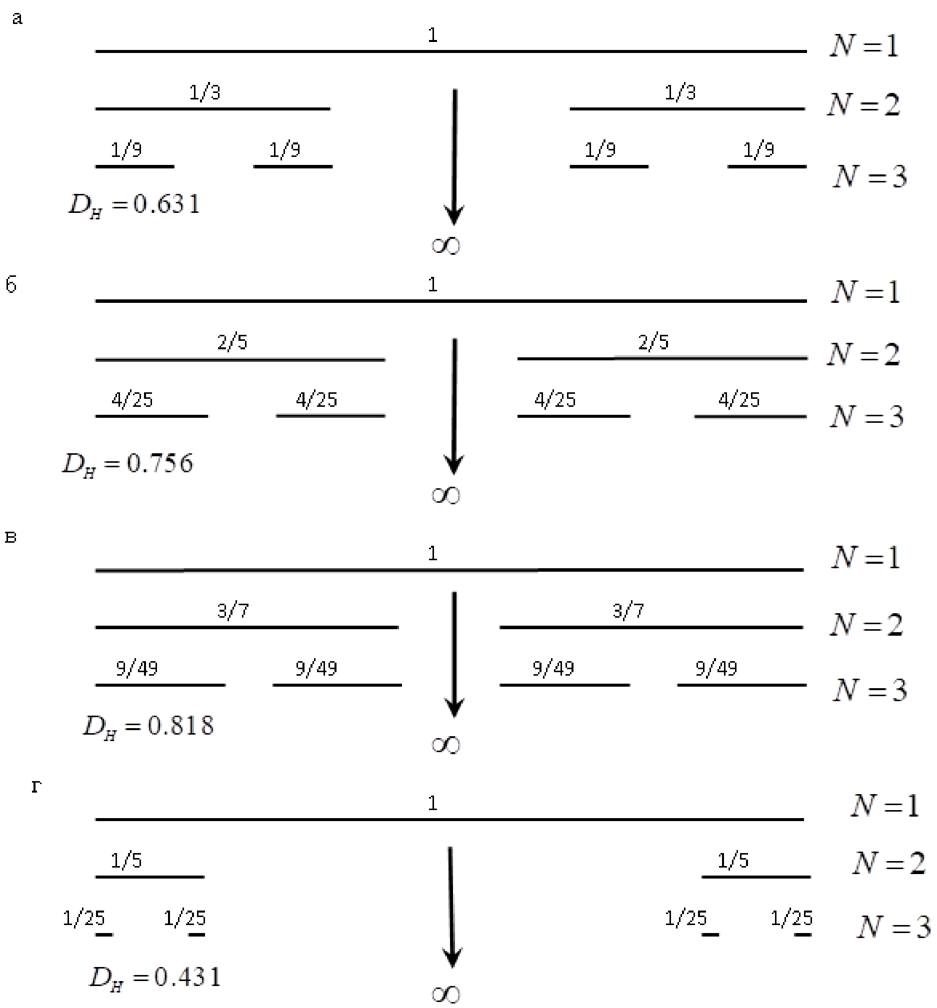

Figure 2. Cantor sets building: we cut from the middle parts a) one third, b) one fifth, c) one seventh and d) three fifth, D_H – fractal dimension of the "Cantor dust"

The topological dimensionality of continuous geometrical objects equals the number of dimensions and its value is an integral number. So, for a bidimensional geometrical object the value of the topological dimensionality equals 2 (see, for example, the first triangle in **Figure 1**, that is a geometrical basis - bearer of the fractal "Sierpinski gasket"). For a monodimensional object the value of the topological dimension equals 1 (see, for example, the initial segments when N=1 in **Figure 2**, which are a geometrical basis - bearer of a fractal «Cantor dust»).

The quantitative parameter of a fractal is fractal dimension (it is also called Hausdorff dimensionality D_H). It has a fractional value as distinct from a topological dimensionality of continuous geometrical objects. The value of the Hausdorff dimensionality can change from the value of the topological dimensionality of the "backing board" D_T (a geometrical basis - bearer of a fractal) up to the value less by 1. Its exact value depends on average density of filling the "backing board" by elements of the fractal. The higher is the average density, the more is the value of the Hausdorff dimensionality and the nearer is it to the value of the topological dimensionality of the geometrical basis-bearer of the fractal (see **Figure 2**).

The fractals, whose building is shown in **Figure 2**, have symmetrical location of elements with regard to the middle of the geometrical basis-bearer. This occasional property of these fractals is connected with the specific peculiarity of its building, i.e. symmetrical location of removed segments. But if the cut part is displaced from the middle of the initial segment, the final arrangement of the fractal elements turns to be unsymmetrical. It has vital importance when a multifractal is being built. A multifractal is formed by means of a combination of several different fractals on one and the same backing board. It is possible, because fractals consist of isolated elements and, if these elements are located on the backing board in non-coincident positions, any number of different fractals can be placed on the same backing board. Together they form a multifractal. This multifractal is shown in **Figure 3.** The peculiarity of this multifractal is its inhomogeneity. It is expressed in the fact, that its elements fill the backing

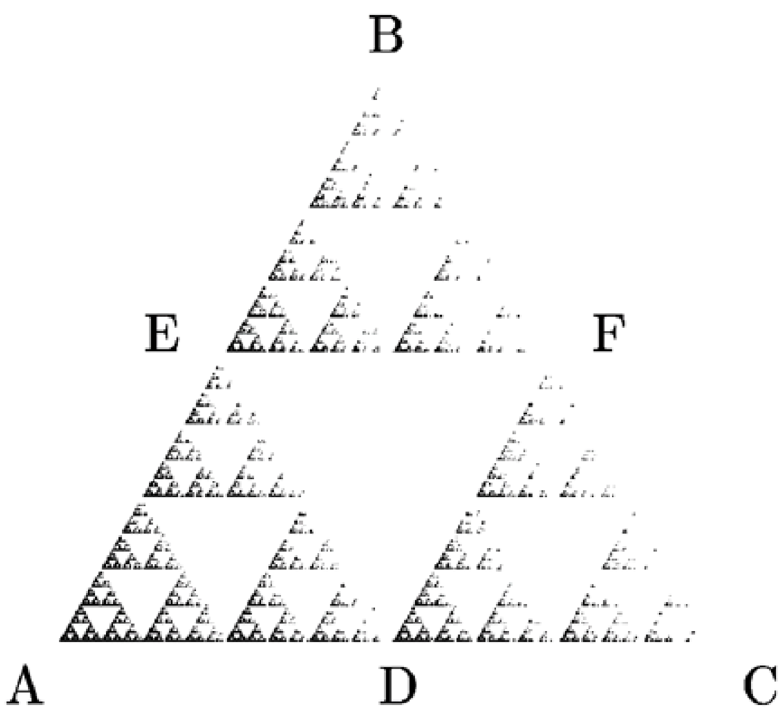

Figure 3. Inhomogeneous fractal (multifractal), built with the help of a method of random generation of locations of dots while forming a triangular Sierpinski gasket (Bozhokin & Parshin, 2001))

board irregularly. Their density reduces with the moving from vertex A of the triangle to side BC, correspondingly, the value of a local Hausdorff dimensionality D_H reduces from 2 to 1.

The concept of a multifractal has found its application in, for example, description of turbulence (Benzi et al, 1984; Meneveau & Sreenivasan, 1987).

The concept of a multifractal is important for us, because it allows to describe the specific peculiarities of the educational content structure, empirically stated by Lednev (1969, 1971, 1973, 1980, 1988, 1989, 1991a, 1991b) and called by him implicit and apical elements of the structure, combinations of which form "through lines", elements of the next level. The "through" components or branches of a personal education are its consequently developed in the course of time branches which, figuratively speaking, penetrate all consequential types and stages of education" (Lednev, 1989) p. 59. This concept was introduced by Lednev for the first time in a lecture course that he delivered for students and listeners of the engineering pedagogic department of Moscow Institute of Agricultural Engineering named after Goriachkin in 1986/87 academic year. According to Lednev, "through lines" are elements of the educational content structure formed as an alternation of apical and implicit components of the same content, for example, a through line "mathematics" consists of separate courses " arithmetic", "geometry", "algebra" etc. and an implicit including of different elements of mathematics into the content of courses of "physics", "biology", "literature", "history" and so on. According to Lednev (1989, 1991a, 1991b), it is an element which can be selected from a structure as a unified whole without changing its functional properties. The example can be a course "Planimetry" in a "through line Mathematics". According to Lednev, in this case the elements with the observed property cannot be selected without changing their functional qualities. As an example you can observe the use in the course "Logics" such geometrical objects as closed plane areas defining the volumes of inductive logic concepts. Different mutual arrangement of these areas shows the including of one concept into another, their independency and other forms of correlation of the concept volumes see (Chelpanov, 1994).

"Apical" and "Implicit" Elements of the Educational Content Structure

Lednev (1991a, 1991b) limited reasonably his analysis of the educational content structure within two relations: he did not concern philosophical and mathematical aspects of the content of the concept "structure". The primary task of his analysis was description of the **real structure** of the educational content from the point of view of huge empiric material, accumulated by pedagogy and its adjustment on positions accepted in theoretical pedagogy.

Such approach is proved, because, before the possibility to present a mathematical description of an object appears, it is necessary to describe it in the terms of an exact sphere, in which it was firstly stated. That is why so indefinite within the boundaries of mathematics terms as "apical" and "implicit" elements are used for characterizing the structure of the pedagogical objects, personal structure, personal activity structure (Lednev, 1989). The implicit elements are characterized by the fact that they cannot be selected (isolated) from a set, while the apical elements are able to exist by themselves. These are limiting characteristics of the elements, but in the reality there are *intermediate states* of all kinds.

The content of education (according to Lednev) looks at the first stage as a complicated multilevel hierarchy system of implicit and apical elements. For its adjustment a concept of basic components of the educational content structure was introduced. Defining these components Lednev (1988, 1989, 1991a, 1991b) uses a stated by him principle of double including. According to this principle, a basic component is implicitly included into all apical components of the educational content structure, but it is also included into the structure as a strongly pronounced apical component. As a result, the basic component is present in all elements of the educational content structure and at all stages of the deployment of the educational content. So, the basic components form a new type of an element of the structure "through line".

At the next step of description of the educational content structure its most important through lines are pointed out. Lednev (1991b) uses for this purpose an activity approach, according to which the educational content structure has two main determinants: activity object structure (subject matter) and activity subject structure (an object to which the education is directed).

During the period of Lednev's (1969, 1971, 1973, 1980, 1988, 1989, 1991a, 1991b) active work on the theory of the educational content structure the concept of fractal objects was known only by an inner circle of specialists – mathematicians and physicians. The concept of a fractal itself was introduced by B.B. Mandelbrot in the sixty years of the last century, but it became widely known only at the end of the 20th century. It is the main reason, why Lednev's research of the educational content structure draws upon his own authoring language and not upon the language of mathematics.

Because of inaccuracy of mathematical characteristics of the elements of the educational content structure it is possible to understand the difficulties, which Lednev faced while describing this structure. In our opinion, these difficulties are a main reason, because of which the theoretical pedagogy has not overcome the phase of an empiric description of a researched object yet.

The limitation of the use of mathematics, initially introduced by V.S. Lednev while analyzing the educational content structure, led expectedly to a definite inaccuracy in the problem solving. But the analysis of the huge empiric material worked out by him (Lednev, 1971, 1991a, 1991b), allowed us to make a conclusion about fractal nature of the objects he analyzed (Gapontseva, Fedorov & Gapontsev, 2009a, 2009b, 2010).

For a multifractal built on the ground of fractals consisting of elements with different qualities (see the example of building of such a fractal in **Figure 1**), proportions of elements of different qualities in an area of a "backing board" depend on location of this area on the "backing board". This situation was stated empirically by V.S. Lednev (1988, 1989, 1991a,b) with regard to the objects researched by him: personality, personal activity, educational content, its determinants and elements. And, though the description of the researched objects, presented by him, has qualitative nature, it points exactly their topological properties out.

Two Types of the Structure of Pedagogical Objects According to V.S. Lednev

In the monograph (Lednev, 1989) the structures "selected according to two criteria" are observed. The first method of such selection is based on one of these criteria, stated exactly with pointing to two extreme types of the structure. Firstly, these are systems consisting of autonomous structural elements combined in one system, but having its independent integrity, so that they can be transferred into other systems. Secondly, there are implicit structures, which can be seen by an observer of the system, but cannot be separated from it. The implicit components of the structure are characterized as follows: they are "… a kind of a structural projection of a system, or its cut-outs. …they reflect objectively the system from some standpoint, but at the same time they are abstractions" (Lednev, 1989).

To the first ones such element refers as, for example, a part of general education system, which is a subject taught in upper secondary school and in a technical secondary school with the use of one and the same textbook. To the second ones such element refers as, for example, development – a component of a triune process of personality formation: education, upbringing and development, which is not pointed clearly out in the organizational form as distinct from education. According to Lednev's (1989, 1991a, 1991b) opinion, all other structures selected on grounds of separateness of the system elements are situated between these two extreme types of a structure.

Here we can see an idea about existing of a range of similar structures selected according to one sign and located between two extreme ones. So, it should be considered, that though an implicit structure is an extreme member of this range, but it consists of some parts as its other members. It is also understood, that the separate parts of an implicit structure cannot be selected without changing their qualitative properties, but herewith they keep unchanged a part of their individual description, that allows to identify them.

The second method of description of the structure of pedagogical objects is not supported by a clear definition of a criterion of selection of structural elements. It is defined by a name of specific peculiarities of a structure and is illustrated in a range of examples (Lednev, 1989): "The other approach to the selection of structures… is connected with pointing out two interrelated structures of one and the same system. They are internal and external structures. Herewith, the internal structures tend to be basic in relation to external ones, though it is not still definitely clear, whether it is always so or not".

As an example, we can observe a structure of a lesson. Its external structure consists of separate, clearly selected, consecutive components: "…organizational aspect, home task checking, presentment of a new topic material, revision of the studied material, and, in the end, the final part of the lesson… But in the structure of a lesson there is one more "layer" or a sub-structure, elements of which… create its own peculiar integrity and mostly hidden from an observer. It is a constant control of the activity of students, motivation of their activity etc". This last sub-structure, as distinct from the first one, whose elements are clearly pointed out, is called an internal one. It is stated, that "A combination of internal and external structural components can be viewed as sub-structures of one and the same system. The structures of a similar type are specific for pedagogical systems" (Lednev, 1989).

We should pay our attention to the fact, that in the observed example the sign of system components, creating its internal structure, coincides with the sign of implicit components. Really, it is said the following about the elements of an internal structure: "a sub-structure, which elements… are in a great measure hidden from an observer". At the same time while describing the elements of an external structure the following is underlined: "…the consecutive components of a lesson, well-known to any teacher, are observed: an organizational aspect…" So, the elements of an external structure can be pointed out. In other words, they are relatively autonomous (these are apical elements), otherwise it is senseless to say that they are clearly selected. Herewith, the elements of an internal structure have a small degree of separateness, they are "dissolved" in the system, "intertissued" into it, i.e. they are implicit according to an initial meaning of this term.

So, the following question appears: is there any distinction between the discussed methods of the structure introduction or not? Or, maybe, the next question will be more correct: is this distinction essential from the point of view of those theoretical consolidations, that allowed Lednev (1991a, 1991b) to adjust effectively the huge empiric material of the pedagogical practice?

Two principles refer predominantly to those generalizations, serving as important tools of the theory of the educational content: *a principle of double including of basic components into the system and a principle of functional fullness of education and of functional fullness of its content components.* If in the course of an analysis of these principles it is possible to replace, without sense damage, the term "an autonomous component (or, the equivalent ones "strongly pronounced", or "apical") by the term "a component (an element) of an external structure", we should accept, that from the point of view of basic principles of the theory of the educational content the concept "autonomous component" is equal to the concept "external structure component".

The problem with the concept "an implicit component" (as equivalent the following terms are used: "not selected clearly", "a component observed in some projection, cut-out, layer…", "an abstract quality of a system appearing on looking at it through some lens"), is solved in the same way: its name, without sense damage, can be replaced by a concept "a component of an internal structure", so the concept "an implicit component" is equal to the concept " a component of an internal structure" (Gapontseva, Fedorov & Gapontsev, 2010).

Principles of "Double Including" and "Functional Fullness"

Because of significance of these principles we present them in a proprietary formulation. The principle of double including says: "every basic component of any sub-system of the educational content is included into its general structure in two ways: firstly, as a "through line" in relation to apical structural components, secondly, it is performed as one of apical, clearly expressed components" (Lednev, 1989). This principle was formulated and used by Lednev (1988, 1989, 1991a, 1991b) for the first time for analysis of the structure of scientific knowledge (Lednev, 1971). The principle of functional fullness says: "any system, including a pedagogical one, cannot operate effectively or operate in total, if a set of its essentially significant sub-systems (elements of a system) is not functionally full" (Lednev, 1989).

It is necessary to point out two things. The first one: while formulating these principles the concepts "a basic component" and "an essentially significant sub-system" are not preliminary defined. The analysis of practice of using these concepts showed, that they, as a rule, coincide, so, from now forth, we will use one of them, namely, the concept "a basic component". The second one: a concept of a "through line" is introduced here. This through line is formed by apical, clearly expressed components of the educational content together with the components, opposed to them, i.e., to all seeming, not clearly expressed.

The definition of the principle of "double including" do not contain a single-valued properties of the components of the educational content, opposed to apical components. With regard to it, it is necessary to turn to authorial examples illustrating this principle for specification. “The first example we’ll get specially from a neighboring area – from a theory of a lesson structure. It is well-known, that during the whole lesson a teacher, providing management, controls regularly students’ activity. In other words, not observing the structure of the whole lesson, we can definitely note, that the control of students’ activity is a “through” component of a lesson, i.e. it is present in one form or another at any stage of the lesson, from its very beginning up to the end. This is one of lines of including the observed component into the whole system of the lesson activity.

But we should draw our attention to another fact: such control of students’ activity as a home task checking is one of autonomous consecutive elements of a lesson. This is a second type of performing one and the same element – control of students’ activity - in the whole system of a lesson activity.” (Lednev, 1989). So, in the second part of this example, while characterizing the second way of including one and the same element into the system structure it is estimated from the point of view of separateness.

On the other side, a term “apical” was introduced by Lednev (1991a, 1991b) and systematically used by him for characterizing an external structure of a system. It means, that from the point of view of a principle of double including and according to application practice the concepts an “apical”, “autonomous” and “clearly selected” component and a component of “an external structure” are identical. Within the boundaries of a stated identity we will observe these concepts further on.

The made analysis shows, that a concept “an autonomous component” and a concept equivalent to it “a component of an external structure” and also a concept “an implicit component” and a concept of equal value “a component of an internal structure” comply in the same way with the principle of double including. The difference between these two systems of concepts and the two ways of characterizing the structure of objects of pedagogical research by Lednev (1991a) (according to a degree of separateness of the elements and, from the other point of view, in terms “external” and “internal” structures) is not apparently connected with the content of these concepts but seems to be aligned with their volume, i.e. with the sphere of their application.

While introducing the concepts “a component of an external structure” and “a component of an internal structure” it is strongly pronounced that it is a case of two ways of including one and the same component (element), i.e. an element having one nature regardless of a way of including into a system. For example, a control function can be executed with the help of a home task checking and distributed control of the situation at any stage of a lesson (i.e. a continuous fixation of a current situation in a teacher’s mind and almost unintentional controlling reactions, not disturbing the logics of the given stage of a lesson: he can waggle his finger, shake his hand, say “Be quiet” etc.). it means, that when the terms “autonomous elements” and “implicit elements” of a system are used in relation to the components of a system being of one nature, they become fully identical (equivalent according to the content and to the volume), as inductive logical concepts, to the terms “an element of an external structure” and “an element of an internal structure” correspondingly.

The definition of a principle of functional fullness, cited before, does not contain a direct pointing to the characteristics of a system structure (external – internal, autonomous – implicit element). It says about “essential sub-systems” (elements or components – this word is used in the same meaning as an “element” while characterizing an object in the same situations) of a system. Essential elements are meant to be those elements, without which a system cannot exist or operate effectively.

As simple examples we can present systems containing autonomous (in the accepted terminology) components, removing of which leads to full impossibility of further system operation: “Simply said, a car without an engine is not a car, a bird without wings cannot fly, and a room without a door cannot be inhabited. In the same way, an education system cannot be effective, if, for example, mathematic or esthetic education is absent in it” (Lednev, 1989). But then more complicated cases are observed, when removing of an autonomous element does not lead to a full stop of its functioning. The difficulty is connected here with the fact, that “the more complicated is the system, the more difficult is to define a concept of its normal functioning. …a concept of effectiveness of a complicated system operation is always multifunctional, because this system is connected with the surroundings by means of a huge number of bonds” (Lednev, 1989).

For characterizing a role of autonomous elements of a system from the point of view of sufficiency of its functioning it is offered to introduce an index of functional significance, falling within the range of between 1 (an engine of a car) and 0 (a mascot on a car’s hood).

The connection of the principle of functional fullness with the nature of a system structure and a principle of double including of the components is seen while observing a compensation property peculiar for complicated systems. This property appears when “a component is lost, whose index of functional significance is less than 1 and whose value does not achieve some critical position”.

According to Lednev (1989), compensation in complicated systems after removing of any autonomous component appears as a result of a partly transferring of its functions to other autonomous elements of a system. As far as definite functions can be associated with some quality (property, behavior aspect etc.) peculiar for the nature of a given element of a system, it is hard to escape a conclusion, that this element, having been removed as an autonomous one, remains (and strengthens) as an implicit component of a structure of other autonomous elements (elements of other nature, bearing other qualities). It follows from what has just been described above, that the compensation property appears when there is a "through line" connected with the given element (component), i.e. when it is included not only into an external, but also into an internal structure of a system.

Lednev's (1989) monograph does not contain this conclusion directly, but it can be extracted from it by means of its analyzing. Really, as an example of a compensation mechanism we can introduce a situation existing in the USSR in the first half of the last century concerning the polytechnic education. (Lednev, 1989): "So, in the thirty years, even removing such an important component, as the polytechnic education, from the curriculum of the general education school did not lead, as it is known, to the destruction of the whole general education school, though did a lot of damage to the public education. The compensation function appeared in that case in the following way. Firstly, the upbringing of common labor qualities, in which the polytechnic education played the main role, was transferred to the remained elements of the education, secondly, the general-purpose technical and technological (polytechnic) information was included into neighboring subjects (physics, chemistry, biology, geography, mathematics etc.)"

So, on the ground of the fulfilled analysis it is possible to form a hypothesis about fractal nature of the educational content structure in the following way. The general description of the educational content structure has nature formed as "mosaic in mosaic" on several levels of the dimension. The basic components of the educational content are components, included into its structure, at least on two proportion levels: i.e. as implicit ones on one proportion level and as apical ones on the other proportion level. The alternation of the areas of the educational content, in which the given component performs either implicit or apical nature, leads to formation of "through lines" in the educational content providing the presence of the given element in all structure elements. It creates a mechanism of realization of the principle of functional fullness of a system during the variation of external and internal conditions of its existing by means of redistribution of the content of the given component between different proportion levels of its including.

The sign showing the fact, that the given component is basic, can be formed as follows: it is basic, if its removing on two neighboring proportion levels leads to impossibility for a system to operate. At the same time, the principle of double including of this component stops its functioning, because its whole "through line" is excluded.

The same idea may be expressed in another way: a basic component of a system selected as a whole one (on all the levels) has a close to 1 index of functional significance. Really, it is impossible to imagine any lesson without any form of the control and a home task checking and without any tracking of students' behavior, as well as a car is impossible without an engine and as education is impossible without development or upbringing and, at least, as a personality is impossible without temperament or memory.

The analysis made before allows to accept as an admissible hypothesis a statement concerning the fact, that *the educational content structure has fractal nature.* This conclusion is supported by an analysis of empiric regularities, stated by V.S. Lednev, in particular, the principles of "double including" and "functional fullness". The further proving of this hypothesis requires an immediate appealing to empiric description of the educational content.

DISCUSSIONS

While analyzing the description of the educational content structure some contradictions were defined. We start with a quotation of a text, that is a basis of our investigation: "Education… has a complicated hierarchical structure which is characterized by **mutually intersecting** components, in particular: experience digestion (in a form of knowledge and skills), upbringing of behavior qualities, physical and mental development …performs a triune process… and this triunity is peculiar: the process of education is immediately directed to experience learning by students. Upbringing and development are executed **indirectly"** (Lednev, 1989). We should note, that the use of a term "indirectly" in the last sentence cannot have any sense except the following: within the boundaries of education upbringing and development are executed only in the process of education and cannot exist separately from it.

On the other hand, we see, that **Figure 4** (that repeats the corresponding picture in the monograph (Lednev, 1989)) reproduces exactly an idea of **mutual intersection** of the components expressed in the cited abstract. The contradiction of the description language (including its graphical aspect) and the nature of the studied objects appears here visually for the first time. The schemes of representation of structures, identical to the given one, are then persistently repeated and they all are analogous to an attempt to introduce a "solution" as "intersection" of its components and not as their "mixture", in which every component is distributed throughout the mixture. We could not draw your attention to this inaccuracy of the description of the observed objects, but it contains a source of essential logic contradictions connected with the fact, that a real sense forming logics of discussion is given to

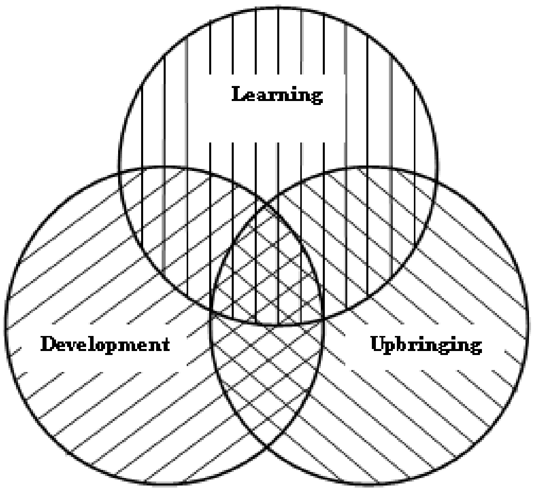

Figure 4. Structure of education (personality aspect) (2. **Fig. 2.1**)

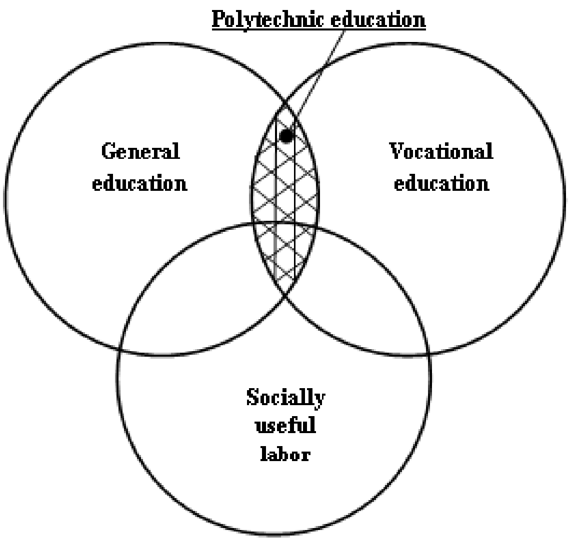

Figure 5. Correlation of general and vocational education

different areas of sets and, in particular, to the areas of their intersection in **Figure 4**. This logics is, of course, incorrect, because in the reality these areas occur together.

The further logics, which leads inevitably to reproducing of inaccuracy, contained in the schemes in **Figure 4**, **5**, is the following: content of education is education but without taking into account its technology. So, the structure of educational content repeats in its basis the structure of education. That is why there are no doubts, when the scheme described in **Figure 5** is selected as the basis of definition of one of the three "through" branches of education, namely polytechnic education.

For description of "through" branches of education we present a scheme (**Figure 6**), identical to the scheme in **Fig. 2.3.1** in (Lednev, 1989). We can see, that this scheme reflects incorrectly the nature itself of the observed objects. It contradicts to the main conclusion of the monograph, in which it is stated, i.e. to the principle of double including and to the principle of functional fullness and also to the empiric material contained in the monograph.

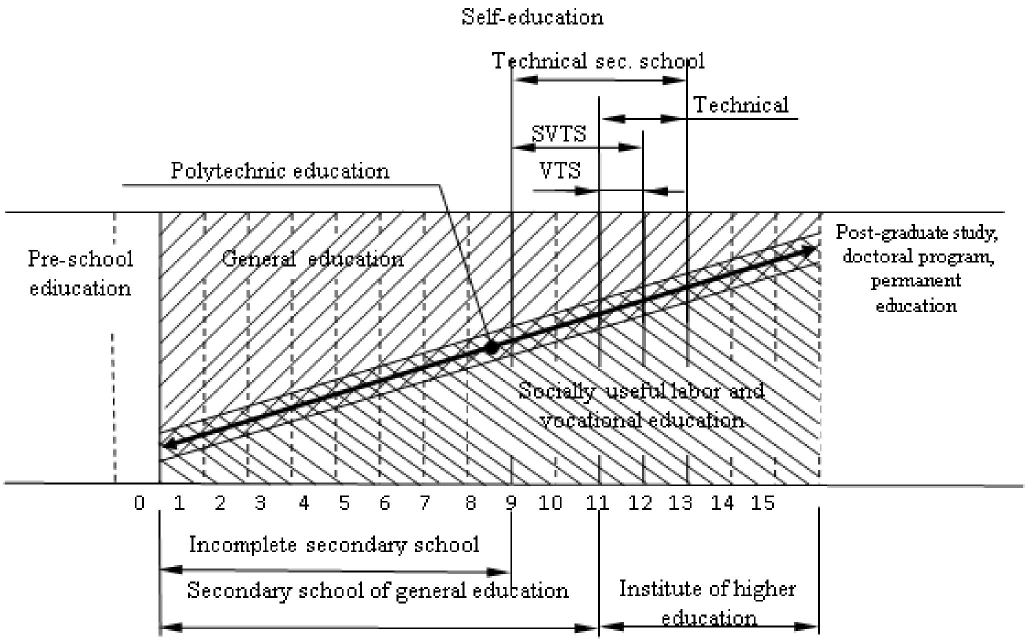

Figure 6. Main structural parallels ("through" branches) and consecutive stages (steps) of education (2. **Fig. 2.3.1**)

Let's pass to the analysis of this material. "As the main "through" branches of education the basic and the vocational education are performed, and also the area of their intersection – the polytechnic education… it is not difficult to see, that every "through" branch begins really even in the pre-school childhood, then they go as "through" lines through a secondary school of general knowledge and a professional institution finding their further development in the process of permanent education" (Lednev, 1989).

The following statement proves this idea: "the pre-school upbringing usually precedes the systematic education. The term "upbringing" in this case underlines the fact that at this stage there is no organized education. … As a whole the pre-school upbringing is real education, because it has all three main components: upbringing, development, teaching. At this stage of education we can trace practically all "through" components, and some of them are presented naturally in propaedeutical form" (Lednev, 1989). We can introduce many examples, but it is clear without them, that in pre-school education the elements of all three through lines of education are connected very closely, i. e. basic, polytechnic and vocational educational, and they are practically dissolved in each other. When observed attentively, it turns out to be obvious, that the elements of all three main types of education (basic, polytechnic, vocational) are in one or another form interwoven and pierce in this form through all consecutive stages of education. The last statement coincides exactly with the description given by V. S. Lednev: "This quite clear idea needs, perhaps, only one explanation: how should we understand vocational education in a school of general knowledge? …".

In other words, school of general education is called so not because it has not got special training, but because the basic training is a leading line of education in this type of educational institution.

In a similar way, a vocational school, that always has got a cycle of basic subjects, is called vocational according to its major task and purpose" (Lednev, 1989).

It seems to be impossible to make on the ground of the presented quotation a direct conclusion, that the presence of elements of special training in basic school, and also presence of basic subjects in a vocational school means including the elements of vocational education into basic one and vice versa. V.S. Lednev draws our attention to this fact: «…we should note, that in the stated idea of a concept "general education" and education executed in a basic secondary school do not coincide, like the concepts of vocational education and education executed in vocational education institutions».

Let's examine this idea on the example of education. As it is seen in the scheme (**Figure 6**), general education is a really "through" branch. The training of general knowledge begins in the pre-school childhood and reaches its climax at the Secondary General School, which main purpose, as it follows from its name, is basic general training of young people. But the general education is not confined by the general school, it continues in vocational education institutions of all types, because there the general education is executed alongside with the vocational training. It continues further, after finishing the systematic education in a permanent form. That is why we called the basic general education a "through" branch. In a similar way, as we will see later, both vocational and polytechnic education can be observed" (Lednev, 1989).

This attempt to distinguish the concepts of "general education" and "school of basic knowledge" does not allow to make a conclusion, that the general education can be singled out from the education in a whole and can be separated from the vocational education, and does not allow in such a way to substantiate simultaneously the "through" nature of basic, vocational and polytechnic education and the possibility to depict them graphically as continuous and persistent. Here there is a confusion of a selection of structural elements itself and ways of their selection.

The selection of elements, characterized according to their primary quality, for example, the general education, can be made taking into account a sign of a territorial localization, i. e. in one building (an exact school), in hard and soft copies in the plans of development of school education accepted by a ministry, i.e. taking into account organizational opportunities of the society. According to the description of the educational content structure an exact mechanism of selection of elements and their grouping is not so important as their qualitative characterizing (identification of their individuality), typical proportions, mutual arrangement (interaction), and the dimensions of the area, where they are grouped.

From this point of view, while describing the structure, the difference between the general education and the basic school is not essential. The general school is only one of the selected (according to Lednev's own terminology) elements of the "through" branch "general education". As a result, describing the educational content structure we should point out, that one of its elements "secondary general school" covers a definite area of the content of education (a list of subjects), some of them have predominantly a basic nature (a list of subjects), the period of study at school (a definite study period), average number of students (territorial characteristics), distribution of subjects according to the period of education and, perhaps, some other details.

Let's introduce a list of structural elements with essentially different levels of proportions (it was taken from (Lednev, 1989)):

The first level is a content of education in a whole.

The second level is a content of education according to main education stages (general school, technical and vocational education, advanced education, higher education)…

The third level of organization of the content of education is cycles of training courses (subjects)…

The fourth level is the training courses (subjects) themselves, i.e. mathematics, physics, chemistry, some language etc…

The fifth level: the training courses, in their turn, have a complicated structure and are divided into separate training disciplines…

The sixth level: the components of the fifth hierarchical level of organization, i.e. disciplines, also have a complicated structure and are divided, as a rule, into chapters, topics, lessons. In other words, at least three hierarchical levels of organization of the content of education can be pointed out.

It is clear, that if we examine the elements of the educational content structure of two remote levels, between which there exist (but are not included into the current picture) the elements of several intermediate levels (maybe, we do not distinguish them because of technical or organizational reasons, or according to the current tradition of describing), we will occur in a situation, where on a larger scale of examination the elements of a smaller scale are indiscernible, but their presence will be observed as a degree of some quality. It explains the phenomena of "implicit" presence of a component and of quasi-persistence (continuity) of a selected element of a larger proportion. The last statement gives rise to the existence of "through lines" (components, branches) as the elements of a structure.

CONCLUSION

From the above stated we can make a conclusion, that the formed before hypothesis concerning the nature of the educational content structure is proved immediately by the empiric material. So, the structure of the educational content and its "through" branches has a mosaic nature made from elements with different qualities (general, vocational and polytechnic education), the mosaic elements create a hierarchical system, i.e. are characterized by essentially different proportions. When the quantity (or "size") of elements of bearers of an exact quality raises, we can state, that an element of a larger scale, consisting of smaller elements, expresses predominantly the given quality (i.e. it plays the main role). The common view has a form of a mosaic picture made of the elements, which are themselves created in a form of mosaic. This procedure repeats on several proportion levels. The description is adequately expressed by means of an iteration procedure of building a geometrical fractal, which on every step of building has a form of a mosaic picture, for example, a picture made of elements with different degrees of a grey color in **Figure 1**.

The description language of the content of education, accepted in practice, is supported by a system of traditional concepts, such as a boundary, internal part of a set, persistence, which allow to use graphical illustrations, but do not correspond to the real nature of an object. The disturbance of the correspondence of the description language of the educational content structure to the nature of the described object damages essentially the theoretical investigations in pedagogy and leads to great organizational and material losses.

The achieved results allow to give a new content to two main principles of the theory of the educational content, stated by Lednev: the principle of double including of basic elements into a system and the principle of functional fullness of a system. In other words, the following hypothesis can be considered to be proved: *the basic components of the content of education are the components, which are included into its structure on at least two proportion levels: as implicit and as apical ones. The common picture of a structure has a nature of "mosaic in mosaic" and is possible on several proportion levels. It leads to appearing of "through lines" in the content of education, providing implicit presence of a given element in all apical elements of the structure, and creates a mechanism of realization of functional fullness of the system during the variation of external and internal conditions of its existence by means of re-distribution of the content of the given component between different proportion levels of its including*.

So, we can point out the following stated qualitative properties of the educational content structure:

- fused presence of structural components;
- absence of definite boundaries between the structural elements;
- possibility to rarefy and to thicken the components up to the full predominance of one of them in a structural element;
- presence of self-similarity property in the characteristics of the educational content structure.

The pointed qualitative peculiarities define more or less unambiguously an object of modern geometry, which topological nature corresponds to them. *This object is a fractal or a multifractal.* These are geometrical objects, which started to be learned not so long ago within the limits of applied topology. There typical peculiarity is a fractional fractal dimensionality (Hausdorff dimensionality - D_H) in the case of fractals and a whole spectrum of fractional dimensionalities for a multifractal (Bozhokin & Parshin, 2001; Feder, 1988; Mandelbrot, 1978).

The usual to us geometrical objects have always got an entire Hausdorff dimensionality, which equals to a topological dimensionality (a number of measurements). Their structure is characterized by an ultimate or countable set of parts divided by clear boundaries. This logics of structure description is usually oriented to these traditional objects. But in the case of the content of education this logics turns out to be non-adequate to the nature of objects. For adequate description of their structure (personality, activity, scientific knowledge and its parts etc.) it is necessary to appeal to the language of fractal geometry.

Callous conception of organization of pedagogical objects, including the content of education may be presented in a form of a mosaic picture consisting of elements of several different colors introducing the elements of different qualitative content, for example mathematic elements correspond to the red color, elements of scientific disciplines correspond to the blue color, elements of humanitarian disciplines – to the yellow one. Herewith, the mosaic elements are mosaic pictures themselves, but they consist of smaller elements of the same main colors. So, the elements of a large scale contain the elements of the other colors, but the elements of some given color occur predominantly. The elements of a smaller scale are, in their turn, made in a form of mosaic from smaller elements of the same main colors.

From the point of view of fractal geometry this picture can be interpreted in the following way: a set of elements of a given quality are an inhomogeneous multifractal similar to that presented in **Figure 3.** The superposition of multifractals from elements of different colors (different quality) creates an integral object, which itself is an inhomogeneous multifractal. In it there are areas of thickening of elements with the given quality, they are, according to Lednev's terminology, apical components of the structure, but also there are areas with thickening of elements of other qualities, i.e. other apical components, in which the elements typical for the rest of apical components occur implicitly. In **Figure 7** we can see a scheme describing a case of an inhomogeneous multifractal built from elements of three different colors (qualities). The intensity of the given color in some area of the backing board is proportional to the density of elements with this quality or to the local Hausdorff dimensionality of the inhomogeneous multifractal made from elements of the corresponding "color". If different colors correspond to such properties as "learning", "development" and "upbringing", **Figure 7** reflects more precisely the sense of the concept "structure of education", than the scheme introduced in **Figure 4** ((2. **Fig. 2.1**)). If the different colors correspond to such elements as "mathematic", "scientific" and "humanitarian" disciplines, we get a sectional drawing of a through line "general education". So, in any part of the section of this through line there are elements of all three types, belonging not only to mathematic, but also to scientific and humanitarian disciplines in different relations to each other. This circumstance has much in common with the idea by A. Einstein, that the problems of modern physics are impossible to be solved adequately without taking into consideration the data of psychology and semantics (Einstein, 1935).

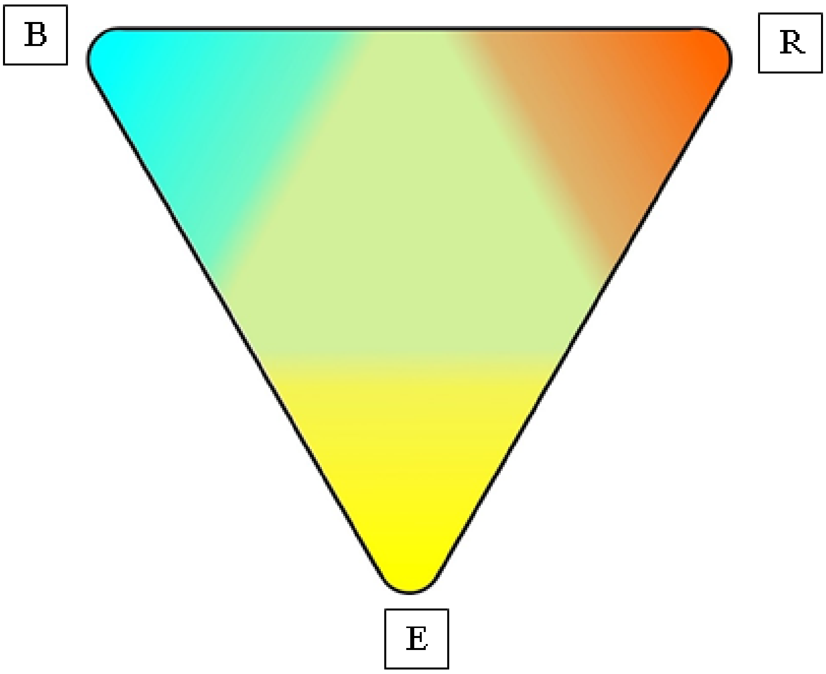

Figure 7. The distribution of the density of elements (or local values of Hausdorff dimensionalities) of a multifractal consisting of three inhomogeneous multifractals, whose elements have qualitative differences. In the corners there are apical components made of elements with the given quality, but the same elements are implicitly present in the rest of the backing board area. Here the colors may correspond to the following qualities: the blue one – to general education, the red one – to vocational education, the yellow one – to socially useful labor. In the other variant: the blue one corresponds to elements of mathematic disciplines, the red one – to elements of scientific disciplines and the yellow - to elements of humanitarian disciplines etc. in the second case it is a schematic representation of a section of general education

It is clear, that a direct attempt to characterize qualitatively and quantitatively the above described structure of the educational content faces great difficulties, for example, because its realization requires a systematical processing and analysis of the empiric material referring to different levels. These levels differ by spatiotemporal proportions. So, the typical proportions of an engineering cycle of disciplines (period of exposition and contingent) are essentially larger than the corresponding typical proportions of one of the disciplines of this course, and definitely larger than the proportions of a separate exact topic. However, the typical proportions themselves are fixed in traditional technologies of education, including the level of normative materials. But it is necessary to have in mind, that nowadays a segment of education supported by computer technologies develops rapidly, and it contains the new levels and new spatiotemporal proportions.

As a result, we should note, that the direct use of methods of fractal geometry for description of the content of education and its structure will be possible only in the long term and at the present moment seems to be premature. But the made analysis allows to point out one more perspective opportunity. It is connected with the fact, that the description of inhomogeneous multifractals uses, in fact, the language of the theory of probability (see the comment to **Figure 3**) for describing the presence or the absence of an element of a multifractal in the given area of the "backing board". When it is said about a multifractal consisting of elements with different qualities (elements of different colors), as in the case illustrated in **Figure 7**, the probabilistic description should take into account the possibility of a variety of a "color". In both cases the probability to find out an element can be explained as a function of belonging of the element to an inexact set (Kofman, 1982). This leads naturally to the opportunity to describe the content of education on the ground of use a fuzzy logics (Novak, Perfilieva & Mochkrozh, 2006). Its main difference from the classical formal logics is connected with the absence of a principle of excluded middle in it (Nazarov & Konysheva, 2018), that allows to broaden the application of methods of logics to the description of situations characterized by a fuzzy nature of the presented data (Zak, 2016), including the most of situations observed in the pedagogical science. The fuzzy-set theory and the fuzzy logics have a developed mathematical apparatus, but, at the same time, they have already found a wide application in car, airspace and transport industry, analysis and making management decisions and in other spheres. In business and economics the fuzzy logics won the recognition after the expert system was the only to foresee a stock market crash on the ground of fuzzy rules. It gives the basis to consider the fuzzy logics as a perspective tool for analyzing and planning of the educational content.

ACKNOWLEDGEMENT

The work is performed according to the Russian Government Program of Competitive Growth of Kazan Federal University.

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