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Do Trading Volume and Downside Trading Volume Help Forecast the Downside Risk?

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ABSTRACT

This paper uses the downside realized semi variance to measure the downside risk and then the HAR-DR, HAR-DR-V and HAR-DR-DV models on the basis of the HAR-RV model are built. Finally, by comparing the three models' prediction ability for downside risk in the stock spot market and futures market, we test whether the trading volume and downside trading volume of the two markets can be used to predict the downside risk. And we also study the differences under different samples and different models. The results indicate that trading volume and downside trading volume have different prediction effects for the downside risk in different periods. The trading volume and downside trading volume exhibit much forecasting power in the futures market. However, they show little forecasting power in the spot market.

Keywords: downside realized semi variance; stock spot market, futures market, risk periods, forecasting power

INTRODUCTION

Risk management is one of the most pressing questions in finance (Bolton et al, 2011; Dai and Wen, 2014; Rampini et al, 2014; Wen et al, 2014 a, 2014b, 2016, 2017; Gong and Lin, 2017), and is closely related to the assets pricing (Han 2013; Liu et al. 2014). The traditional research has generally used the variance and beta to measure financial risk. Two classic examples of published literature are Markowitz (1952) and Sharpe (1964). Markowitz (1952) built portfolios by analyzing the mean-variance relationship. Sharpe (1964) used beta to measure financial risk in the CAPM. In addition, Engle (1982) developed the ARCH model which can depict financial assets' volatility. Volatility has become a common indicator for measurement of risk according to the Engle (1982)'s study. Taking variance, beta and volatility to measure financial risk implies the uncertainty or variation of asset price. This risk not only includes the uncertainty caused by downward movement of asset prices but also includes the uncertainty caused by any upward movement. However, most investors incur losses only when asset prices move downward and upward movement benefits investors. Therefore, the risk of asset prices moving downward is the real risk for investors. It is more important to study the downside risks of asset prices.

Although the downside risk is very important, it is not been studied much. On the one hand, previous studies have mainly focused on the measures of downside risk. For example, Markowitz (1959), Price et al. (1982), Barndorff-Nielsen and Shephard (2008) used the downside variance and deviation, downside beta, and downside realised semivariance to measure the downside risk, respectively. On the other hand, many works have examined the downside risk-return relationship. Post and Vliet (2006) found downside risk can explain the high average returns of small/value/winner stocks and it plays an important role in asset pricing; Bali et al. (2009) examined the intertemporal relationship between downside risk and expected stock returns; Huang et al. (2012) studied the

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Contribution of this paper to the literature

- The proposed HAR type models are creatively applied to predict the downside risk and find that the proposed HAR models are a very promising tool in forecasting the downside risk.
- The forecast effect of trading volume and downside trading volume on the downside risk are examined and it shows that the trading volume and downside trading volume can effectively predict the volatility of futures market.
- We find that the models of HAR-DR, HAR-DR-V and HAR-DR-DV have a strong predictive effect on the weekly and monthly downside risk, while a weak predictive effect on the daily downside risk. Therefore, it is necessary to consider the different time period for the downside risk predicting by HAR models.

relationship between extreme downside risk and expected stock returns in American stock market. Sévi (2013) analyzed the downside risk-return trade-off at daily frequency.

Extant literatures about the measure of downside risk and downside risk-return trade-off play an important role in studying the downside risk but the prediction of downside risk is a very important thing. However, the mass of literature about the prediction of downside risk is minimal. Rubia and Sanchis-Marco (2013) argued accurate downside risk forecasting is very elusive. They found downside risk forecasting may be improved considerably by the use of volume-related variables. What's more, the mixture distribution hypothesis (MDH) introduced by Clark (1973) provided theoretical support for the fact that the trading volume predicts the risk in financial assets. According to the MDH, the variance of daily price changes and trading volume are both driven by the information in the market. The arrival of unexpected "good news" results in a price increase whereas "bad news" causes a price decrease. When the trading volume and downside trading volume increase, prices of financial assets change more and financial downside risk increases. According to Rubia and Sanchis-Marco (2013) and the mixture distribution hypothesis, the trading volume should be able to predict the downside risk. And our research attempts to examine whether the view is founded.

The rest of this paper is organized as follows. The next section is the methodology. Section 3 presents the descriptive statistics. In Section 4, whether trading volume exhibits the in-sample forecasting power for the downside risk in the stock spot market and futures market is analyzed. In Section 5, we show the out-of-sample trading volume's forecasting power for downside risks. Section 6 provides the conclusions.

METHODOLOGY

Downside Risk Proxy

In this paper, we choose the downside realised semivariance which is developed in Barndorff-Nielsen and Shephard (2008) based on the realized volatility (Andersen and Bollerslev 1998) to measure the downside risk. We suppose a trading day t, divide the total day's trading into N parts and $P_{t,i}$ is the i^th^ (i=1, $\cdots$, N) closing price of the trading day t. And $r_{t,i}$ is the return of the i^th^ on trading day t, $r_{t,i} = 100(\ln P_{t,i} - \ln P_{t,i-1})$. The RV on trading day t (RV_t^0) can be written as

$$RV_t^0 = \sum_{i=1}^{N} r_{t,i}^2 \tag{1}$$

Andersen and Bollerslev (1998) studied the realized volatility in the currency exchange market. But unlike the currency exchange market, trading isn't continuous in stock market. Therefore the realized volatility calculated with expression (1) can reflect the market volatility only during the trading periods and market volatility caused by overnight information cannot be reflected. Therefore, according to Huang et al. (2013) and Gong et al. (2014, 2017), considering the overnight return variance, we adjust the realized volatility as

$$RV_t = RV_t^0 + r_{t,n}^2 = \sum_{j=1}^{M} r_{t,j}^2 \tag{2}$$

where $r_{t,n}$ are the overnight return, $r_{t,1} = r_{t,n} = 100(\ln P_{t,o} - \ln P_{t-1,c})$.

Then, referring to Barndorff-Nielsen and Shephard (2008), on the basis of Formula (2), we get the expression of downside realised semivariance (RS^-):

$$RS_t^- = \sum_{j=1}^{M} r_{t,j}^2 I(r_{t,j} \leq 0) \tag{3}$$

where $I(\cdot)$ is the indicator function taking the value 1 if the argument I is true. In this paper, we use the RS^- to measure the downside risk. So the downside risk of phase t can be expressed as

$$DR_t^d = RS_t^- = \sum_{j=1}^{M} r_{t,j}^2 I(r_{t,j} \leq 0) \tag{4}$$

Econometric Model

The general forecasting model (HAR-RV model) can be expressed as

$$RV_{t+H}^d = c + \alpha_d RV_t^d + \alpha_w RV_t^w + \alpha_m RV_t^m + \varepsilon_{t+H} \tag{5}$$

where RV_{t+H}^d represents the realized volatility in the future H days, $RV_{t+H}^d = (RV_{t+1}^d + RV_{t+2}^d + \cdots + RV_{t+H}^d)/H$. RV_t^d is the daily realized volatility in phase t. $RV_t^w = (RV_t^d + RV_{t-1}^d + \cdots + RV_{t-4}^d)/5$ means the weekly realized volatility. $RV_t^m = (RV_t^d + RV_{t-1}^d + \cdots + RV_{t-21}^d)/22$ shows the monthly realized volatility. We use the downside risk to replace the realized volatility of Model (5) and get the HAR-DR model.

$$DR_{t+H}^d = c + \beta_d DR_t^d + \beta_w DR_t^w + \beta_m DR_t^m + \varepsilon_{t+H} \tag{6}$$

where DR_{t+H}^d represents the downside risk in the future H days, $DR_{t+H}^d = (DR_{t+1}^d + DR_{t+2}^d + \cdots + DR_{t+H}^d)/H$. DR_t^d is the daily downside risk. $DR_t^w = (DR_t^d + DR_{t-1}^d + \cdots + DR_{t-4}^d)/5$ means the weekly downside risk. $DR_t^m = (DR_t^d + DR_{t-1}^d + \cdots + DR_{t-21}^d)/22$ shows the monthly downside risk.

To analyze whether the trading volume can predict downside risk, we add the daily, weekly and monthly trading volumes and downside trading volumes to Model (6), and get HAR-DR-V and HAR-DR-DV models.

$$DR_{t+H}^d = c + \phi_d DR_t^d + \phi_w DR_t^w + \phi_m DR_t^m + \varphi_d V_t^d + \varphi_w V_t^w + \varphi_m V_t^m + \varepsilon_{t+H} \tag{7}$$

$$DR_{t+H}^d = c + \gamma_d DR_t^d + \gamma_w DR_t^w + \gamma_m DR_t^m + \lambda_d DV_t^d + \lambda_w DV_t^w + \lambda_m DV_t^m + \varepsilon_{t+H} \tag{8}$$

where V_t^d in Model (7) is the daily trading volume. V_t^w is the weekly trading volume,$V_t^w = (V_t^d + V_{t-1}^d + \cdots + V_{t-4}^d)/5$. V_t^m is the monthly trading volume, $V_t^m = (V_t^d + V_{t-1}^d + \cdots + V_{t-21}^d)/22$. And DV_t^d in Model (8) is the daily downside trading volume, $DV_t^d = \sum_{\mathrm{j=1}}^{\mathrm{M}} v_{t,j} I(r_{t,j} \leq 0)$, where $v_{t,j}$ is the j[st] trading volume. DV_t^w is the weekly downside trading volume, $DV_t^w = (DV_t^d + DV_{t-1}^d + \cdots + DV_{t-4}^d)/5$. DV_t^m is the monthly downside trading volume, that is $DV_t^m = (DV_t^d + DV_{t-1}^d + \cdots + DV_{t-21}^d)/22$C

Andersen et al. (2001) found that the log realized volatility is more approximate to normal distribution than the realized volatility. Compared with the original model, they also found that prediction accuracy of the logarithmic model is higher. The downside risk in this paper is measured by the downside realised semivariance which is a part of the realized volatility. Thus, in order to improve the robustness and prediction accuracy of the models, we also translate Models (6), (7) and (8) into their respective logarithmic models.

$$\ln(DR_{t+H}^d) = c + \beta_d ln(DR_t^d) + \beta_w ln(DR_t^w) + \beta_m ln(DR_t^m) + \varepsilon_{t+H} \tag{9}$$

$$\begin{aligned} \ln(DR_{t+H}^d) = c &+ \phi_d ln(DR_t^d) + \phi_w ln(DR_t^w) + \phi_m ln(DR_t^m) \\ &+ \varphi_d ln(V_t^d) + \varphi_w ln(V_t^w) + \varphi_m ln(V_t^m) + \varepsilon_{t+H} \end{aligned} \tag{10}$$

$$\begin{aligned} \ln(DR_{t+H}^d) = c &+ \gamma_d \ln(DR_t^d) + \gamma_w \ln(DR_t^w) + \gamma_m \ln(DR_t^m) \\ &+ \lambda_d \ln(DV_t^d) + \lambda_w \ln(DV_t^w) + \lambda_m \ln(DV_t^m) + \varepsilon_{t+H} \end{aligned} \tag{11}$$

DATA DESCRIPTION

In this paper, we use 5-minute high-frequency data from April 16, 2010 to March 5, 2014 (939 days) for the CSI 300 (stock spot market) and CSI 300 futures (stock futures market).

Table 1 lists the descriptive statistics of all variables in HAR-DR, HAR-DR-V and HAR-DR-DV models. Mean values of the downside risk, trading volume and downside trading volume in spot market and futures market (**Table 1**) show that mean values of all variables in the spot market are greater than the futures market, which shows the spot market is more active than the futures market. Skewness and kurtosis of all variables in the spot market show that daily, weekly and monthly downside risks are "right skewed" and "fat tail". And the daily, weekly and monthly trading volumes and downside trading volumes are "right skewed" and "thin tail". In the stock futures market, the downside risk is "right skewed" and "fat tail", and the daily, weekly and monthly trading volumes and downside trading volumes are "right skewed" and "thin tail". There are large differences between skewness and kurtosis of daily, weekly and monthly trading volumes and downside trading volumes.

Table 1. Descriptive statistics of all variables

		$\ln(DR_t^d)$	$\ln(DR_t^w)$	$\ln(DR_t^m)$	$\ln(V_t^d)$	$\ln(V_t^w)$	$\ln(V_t^m)$	$\ln(DV_t^d)$	$\ln(DV_t^w)$	$\ln(DV_t^m)$
Spot market	Mean	-2.270	-2.107	-2.032	17.84	17.85	17.86	17.12	17.15	17.16
	SD	0.743	0.535	0.451	0.393	0.355	0.316	0.396	0.334	0.297
	Skew	0.561	0.805	0.755	0.190	0.157	0.161	0.267	0.285	0.272
	Kurt	3.742	4.030	3.010	2.797	2.873	2.578	2.935	2.928	2.601
	JB	69.22***	139.8***	87.33***	7.082**	4.406	10.79***	11.08***	12.59***	17.40***
Futures market	Mean	-2.317	-2.176	-2.112	12.50	12.62	12.65	11.78	11.91	11.94
	SD	0.767	0.561	0.472	0.958	0.648	0.567	0.963	0.645	0.564
	Skew	0.238	0.557	0.640	-1.332	-0.207	0.175	-1.327	-0.195	0.181
	Kurt	3.192	3.613	3.065	5.261	2.587	1.666	5.245	2.544	1.682
	JB	10.07***	61.94***	62.75***	467.1***	13.10***	72.71***	462.4***	13.77***	71.49***

Note: Asterisks indicate statistical significance at the 1% (***), 5% (**) or 10% (*) level

Table 2. Estimation results of HAR-DR model under full sample

	Spot market			Futures market		
	1-day	1-week	1-month	1-day	1-week	1-month
c	-0.833***	-0.781***	-1.077***	-0.551***	-0.609***	-0.890***
	(-6.771)	(-6.504)	(-9.119)	(-5.375)	(-5.051)	(-20.65)
$\ln(DR_t^d)$	0.108***	0.065**	0.041**	0.080*	0.071**	0.042*
	(2.683)	(2.423)	(2.098)	(1.721)	(2.547)	(1.742)
$\ln(DR_t^w)$	0.206***	0.210***	0.129*	0.403***	0.287***	0.155***
	(2.759)	(2.640)	(1.891)	(5.016)	(3.800)	(3.350)
$\ln(DR_t^m)$	0.374***	0.366***	0.307***	0.335***	0.371***	0.385***
	(4.702)	(3.609)	(3.594)	(3.967)	(3.839)	(9.326)
$Adj.R^2$	0.175	0.301	0.257	0.280	0.407	0.368

RESULTS ANALYSES

Results on Full Sample

To test whether trading volume and downside trading volume can be used in-sample to predict downside risk in the spot market and futures market, we estimate the parameters of the HAR-DR, HAR-DR-V and HAR-DR-DV models. The downside risk, trading volume and downside trading volume in these models have overlapping variables, so we use the OLS with Newey-West to estimate the parameters of the models. And we choose the samples in the period from April 16, 2010 to March 5, 2014.

Table 2 lists estimation results of the HAR-DR model under full sample. In the spot market and futures market, the coefficients of $\ln(DR_t^d)$, $\ln(DR_t^w)$ and $\ln(DR_t^m)$ are significantly positive when the HAR-DR model predicts the 1-day, 1-week and 1-month downside risks, respectively. It shows that the downside risk in the spot market and futures market has strong sustainability or long memory and historical downside risk contains forecast information for the downside risk. Meanwhile, it also shows that the downside risk is affected by the different components of downside risk in the past. Different downside risk components are attributable to investor behaviors with different holding terms (short-term, medium-term and long-term). Thus, this result also proves the existence of heterogeneous investors in the spot market and futures market, which accords with the heterogeneous market hypothesis. In addition, comparing the $Adj.R^2$ of different prediction periods in the same market, and the same prediction period in different markets, we find that the HAR-DR model exhibits more in-sample forecasting power for 1-week downside risk than 1-day and 1-month downside risks, and the model shows more in-sample forecasting power for the stock futures market than the spot market.

Table 3 lists estimation results of the HAR-DR-V model under full sample. The HAR-DR-V model analyzes whether the trading volume can predict the downside risk in the spot market and futures market. In the spot market, the coefficients of $\ln(V_t^d)$, $\ln(V_t^w)$ and $\ln(V_t^m)$ are not significant when the HAR-DR-V model predicts 1-day, 1-week and 1-month downside risks in the future, which means the daily, weekly and monthly trading volumes cannot predict downside risk in the spot market. In the stock futures market, the coefficients of $\ln(V_t^d)$, $\ln(V_t^w)$ and $\ln(V_t^m)$ are significant when the HAR-DR-V model predicts 1-day downside risk in the future, and the coefficients of $\ln(V_t^d)$ and $\ln(V_t^w)$ are significant when the HAR-DR model predicts 1-week downside risk, but the coefficients of $\ln(V_t^d)$, $\ln(V_t^w)$ and $\ln(V_t^m)$ are not significant when the HAR-DR-V model predicts 1-month downside risk. This shows that the trading volume exhibits much in-sample forecasting power for the 1-day and 1-week downside risks but little in-sample forecasting power for 1-month downside risk in the stock futures

Table 3. Estimation results of HAR-DR-V model under full sample

	Spot market			Futures market		
	1-day	**1-week**	**1-month**	**1-day**	**1-week**	**1-month**
c	-4.321*** (-2.991)	-5.730*** (-3.426)	-7.203*** (-4.410)	-1.034** (-2.002)	-1.110* (-1.697)	-1.667*** (-2.272)
$\ln(DR_t^d)$	0.091** (2.308)	0.048** (1.943)	0.025 (1.370)	0.057 (1.194)	0.083*** (2.712)	0.044* (1.839)
$\ln(DR_t^w)$	0.186** (2.361)	0.196** (2.430)	0.104 (1.591)	0.450*** (5.582)	0.309*** (4.013)	0.159* (1.817)
$\ln(DR_t^m)$	0.379*** (4.707)	0.341*** (3.260)	0.270*** (3.059)	0.302*** (3.514)	0.330*** (3.275)	0.361*** (3.789)
$\ln(V_t^d)$	0.042 (0.349)	0.117 (1.241)	0.056 (0.737)	0.062* (1.726)	-0.043* (-1.695)	-0.012 (-0.598)
$\ln(V_t^w)$	0.313 (1.505)	0.090 (0.505)	0.190 (1.229)	-0.253*** (-2.712)	-0.131 (-1.618)	-0.025 (-0.344)
$\ln(V_t^m)$	-0.163 (-1.048)	0.064 (0.358)	0.088 (0.589)	0.227** (2.375)	0.212** (2.055)	0.095 (1.035)
$Adj.R^2$	0.185	0.328	0.320	0.284	0.418	0.371

Table 4. Estimation results of HAR-DR-DV model under full sample

	Spot market			Futures market		
	1-day	**1-week**	**1-month**	**1-day**	**1-week**	**1-month**
c	-4.492*** (-3.044)	-5.781*** (-3.359)	-6.997*** (-4.247)	-1.010** (-2.042)	-1.050* (-1.671)	-1.575** (-2.214)
$\ln(DR_t^d)$	0.074* (1.763)	0.028 (1.015)	0.015 (0.669)	0.049 (1.004)	0.083*** (2.715)	0.044* (1.844)
$\ln(DR_t^w)$	0.175* (1.959)	0.206** (2.361)	0.081 (1.156)	0.479*** (5.807)	0.327*** (4.190)	0.162* (1.876)
$\ln(DR_t^m)$	0.404*** (4.624)	0.345*** (3.051)	0.308*** (3.380)	0.281*** (3.202)	0.312*** (3.044)	0.359*** (3.743)
$\ln(DV_t^d)$	0.095 (0.776)	0.157 ** (2.031)	0.045 (0.758)	0.067* (1.864)	-0.039 (-1.569)	-0.011 (-0.589)
$\ln(DV_t^w)$	0.311 (1.337)	0.022 (0.111)	0.334** (2.026)	-0.300*** (-3.283)	-0.179** (-2.158)	-0.034 (-0.433)
$\ln(DV_t^m)$	-0.197 (-1.161)	0.105 (0.525)	-0.042 (-0.275)	0.270*** (2.851)	0.254** (2.438)	0.099 (1.070)
$Adj.R^2$	0.186	0.327	0.322	0.286	0.422	0.371

market. In addition, the significance and symbols of coefficients of $\ln(DR_t^d)$, $\ln(DR_t^w)$ and $\ln(DR_t^m)$ in the HAR-DR-V model are the similar as the HAR-DR model, which further verifies that the downside risk of the spot market and futures market has strong sustainability or long memory.

Table 4 lists the estimation results of HAR-DR-DV model under full sample. The HAR-DR-DV model is used to analyze whether the downside trading volume can predict the downside risk in the spot market and futures market. In the spot market, the coefficient of $\ln(DV_t^d)$ is statistically significant when the HAR-DR-DV model predicts 1-week downside risk in the future, and the coefficient of $\ln(DV_t^w)$ is statistically significant when the HAR-DR-DV model predicts 1-month downside risk in the future, but the other coefficients of $\ln(V_t^d)$, $\ln(V_t^w)$ and $\ln(V_t^m)$ are not significant. This shows the downside trading volume exhibits weak in-sample forecasting power for 1-week and 1-month downside risks, and the downside trading volume shows no in-sample forecasting power for 1-month downside risk in the spot market. In the futures market, coefficients of $\ln(DV_t^d)$, $\ln(DV_t^w)$ and $\ln(DV_t^m)$ are significant when the HAR-DR model predicts 1-day downside risk and coefficients of $\ln(DV_t^d)$ and $\ln(DV_t^m)$ are significant when the HAR-DR-DV model predicts 1-week downside risk but coefficients of $\ln(DV_t^d)$, $\ln(DV_t^w)$ and $\ln(DV_t^m)$ are not significant when the HAR-DR model predicts 1-month downside risk. This shows the downside trading volume exhibits much in-sample forecasting power for 1-day and 1-week downside risks but the downside trading volume shows little in-sample forecasting power for 1-month downside risk in the futures market. In addition, the significance and symbols of coefficients of $\ln(DR_t^d)$, $\ln(DR_t^w)$ and $\ln(DR_t^m)$ in the HAR-DR-DV model are similar to the HAR-DR model, which verifies that the downside risk has strong sustainability or long memory in the spot market as well as the futures market.

Table 5. Estimation results of HAR-DR model under the sub-samples

		Spot market			Futures market		
		1-day	1-week	1-month	1-day	1-week	1-month
Sub-sample 1	c	-0.901***	-0.909***	-1.269***	-0.625***	-0.677***	-0.965***
		(-5.119)	(-5.123)	(-6.877)	(-3.811)	(-3.270)	(-14.62)
	$\ln(DR_t^d)$	0.122**	0.089**	0.043	0.072	0.083**	0.036
		(2.318)	(2.285)	(1.541)	(1.119)	(2.106)	(1.203)
	$\ln(DR_t^w)$	0.179	0.136	0.077	0.398***	0.165	0.102
		(1.525)	(1.369)	(0.889)	(3.539)	(1.499)	(1.573)
	$\ln(DR_t^m)$	0.338***	0.329**	0.223*	0.313**	0.453***	0.410***
		(2.772)	(2.105)	(1.834)	(2.371)	(2.902)	(7.064)
	$Adj.R^2$	0.133	0.219	0.147	0.237	0.344	0.350
Sub-sample 2	c	-0.825***	-0.761***	-1.061***	-0.486***	-0.549***	-0.827***
		(-4.644)	(-4.291)	(-7.387)	(-3.724)	(-4.060)	(-14.66)
	$\ln(DR_t^d)$	0.089	0.035	0.036	0.086	0.059	0.047
		(1.416)	(0.955)	(1.426)	(1.280)	(1.472)	(1.261)
	$\ln(DR_t^w)$	0.236***	0.294***	0.181*	0.407***	0.414***	0.212***
		(2.656)	(2.432)	(1.741)	(3.669)	(4.291)	(3.296)
	$\ln(DR_t^m)$	0.377***	0.338***	0.295***	0.351***	0.280**	0.350***
		(3.403)	(2.540)	(2.627)	(3.322)	(2.532)	(6.100)
	$Adj.R^2$	0.169	0.303	0.261	0.311	0.460	0.376

Results on Sub-Samples

The full sample is divided into two sub-samples, Sub-sample 1 with 470 items of data from April 16, 2010 to March 22, 2012 and Sub-sample 2 with 469 data from March 23, 2012 to March 5, 2014. The HAR-DR, HAR-DR-V and HAR-DR-DV models are estimated by using the two sub-samples.

Table 5 lists estimation results of HAR-DR model under two sub-samples. Most results in **Table 5** are similar to those under the full sample. For example, the HAR-DR model exhibits more in-sample forecasting power for 1-week downside risk than 1-day and 1-month downside risks. However, a few results are different from the full sample. For example, most coefficients of $\ln(DR_t^w)$ are not significant under Sub-sample 1 and all coefficients of $\ln(DR_t^d)$ are not significant under Sub-sample 2, but all coefficients of $\ln(DR_t^d)$ and $\ln(DR_t^w)$ are statistically significant under the full sample. These results show the downside risks of the spot market and futures market under the two different periods and both exhibit different long memories. In addition, comparing the $Adj.R^2$ of the HAR-DR model under Sub-sample 1 and Sub-sample 2, we find the $Adj.R^2$ of the HAR-DR model under Sub-sample 2 is higher than under Sub-sample 1. This shows the HAR-DR model exhibits more in-sample forecasting power under Sub-sample 2 than in-sample forecasting power under Sub-sample 1.

Table 6 lists the estimation results of HAR-DR-V model under Sub-sample 1 and Sub-sample 2. In Sub-sample 1, coefficients of $\ln(V_t^d)$, $\ln(V_t^w)$ and $\ln(V_t^m)$ in the spot market and futures market are not significant when the HAR-DR-V model predicts 1-day, 1-week and 1-month downside risks, which shows the daily, weekly and monthly trading volumes cannot predict the downside risk for the period from April 16, 2010 to March 22, 2012. In Sub-sample 2, coefficients of $\ln(V_t^d)$, $\ln(V_t^w)$ and $\ln(V_t^m)$ are not significant when the HAR-DR-V model predicts 1 day downside risk in the spot market, which also shows the daily, weekly and monthly trading volumes cannot predict the downside risk of the spot market during the period from March 23, 2012 to March 5, 2014. However, the coefficient of $\ln(V_t^d)$ is significant when the HAR-DR-V model predicts 1-week downside risk, the coefficient of $\ln(V_t^w)$ is significant when the HAR-DR-V model predicts 1-day downside risk and the coefficient of $\ln(V_t^m)$ is significant when the HAR-DR model predicts 1-day, 1-week and 1-month downside risks. These results show the trading volume exhibits in-sample forecasting power for downside risk under Sub-sample 2. In addition, comparing the $Adj.R^2$ of the HAR-DR model under Sub-sample 1 and Sub-sample 2, we find that the HAR-DR-V model exhibits more in-sample forecasting power in Sub-sample 2 than that in Sub-sample 1.

Table 6. Estimation results of HAR-DR-V model under the sub-samples

		Spot market			Futures market		
		1-day	1-week	1-month	1-day	1-week	1-month
Sub-sample 1	c	-3.718*	-4.367*	-5.881***	-1.929	-1.262	-1.851
		(-1.722)	(-1.792)	(-2.832)	(-1.172)	(-0.636)	(-1.321)
	$\ln(DR_t^d)$	0.102**	0.073**	0.034	0.063	0.086**	0.038
		(2.004)	(2.175)	(1.328)	(0.930)	(2.096)	(1.146)
	$\ln(DR_t^w)$	0.166	0.133	0.058	0.419***	0.179*	0.101
		(1.374)	(1.311)	(0.766)	(3.755)	(1.669)	(0.834)
	$\ln(DR_t^m)$	0.366***	0.329**	0.220*	0.279**	0.422***	0.393***
		(2.981)	(2.029)	(1.765)	(2.074)	(2.676)	(3.469)
	$\ln(V_t^d)$	0.145	0.187	0.032	0.033	-0.018	-0.011
		(0.880)	(1.296)	(0.287)	(0.808)	(-0.553)	(-0.482)
	$\ln(V_t^w)$	0.205	-0.046	0.110	-0.137	-0.086	0.024
		(0.704)	(-0.191)	(0.564)	(-1.218)	(-0.860)	(0.297)
	$\ln(V_t^m)$	-0.193	0.051	0.112	0.208	0.149	0.057
		(-0.859)	(0.201)	(0.561)	(1.167)	(0.753)	(0.411)
	$Adj.R^2$	0.140	0.234	0.191	0.235	0.345	0.348
Sub-sample 2	c	-5.956***	-8.827***	-10.83***	-2.563	-3.835**	-7.872***
		(-3.069)	(-3.892)	(-4.232)	(-1.509)	(-2.081)	(-3.772)
	$\ln(DR_t^d)$	0.078	0.016	0.012	0.036	0.077*	0.044
		(1.248)	(0.422)	(0.473)	(0.527)	(1.755)	(1.352)
	$\ln(DR_t^w)$	0.198*	0.265**	0.156	0.520***	0.463***	0.248**
		(1.959)	(2.114)	(1.411)	(4.302)	(4.320)	(2.046)
	$\ln(DR_t^m)$	0.323***	0.218	0.123	0.216*	0.100	0.041
		(2.762)	(1.426)	(0.931)	(1.731)	(0.670)	(0.256)
	$\ln(V_t^d)$	-0.069	0.052	0.090	0.103	-0.073*	-0.015
		(-0.393)	(0.478)	(0.971)	(1.553)	(-1.792)	(-0.472)
	$\ln(V_t^w)$	0.410	0.206	0.246	-0.449**	-0.234	-0.096
		(1.445)	(0.851)	(1.054)	(-2.548)	(-1.596)	(-0.650)
	$\ln(V_t^m)$	-0.066	0.172	0.184	0.493**	0.539**	0.603***
		(-0.343)	(0.736)	(0.893)	(2.495)	(2.586)	(3.205)
	$Adj.R^2$	0.181	0.354	0.377	0.322	0.491	0.430

Table 7 lists results of the HAR-DR-DV model under Sub-sample 1 and Sub-sample 2. In Sub-sample 1, the coefficient of $\ln(DV_t^w)$ in the futures market is significant when the HAR-DR-DV model predicts 1-day downside risk but all other coefficients of $\ln(DR_t^d)$, $\ln(DR_t^w)$ and $\ln(DR_t^m)$ are not significant when the HAR-DR-DV model predicts 1-day, 1-week and 1-month downside risks of the spot market and futures market. It shows the downside trading volume exhibits a little in-sample forecasting power for downside risk of the futures market, but has no in-sample forecasting power for downside risk of the spot market from April 16, 2010 to March 22, 2012. In Sub-sample 2, the coefficient of $\ln(DV_t^d)$ is significant when the HAR-DR-DV model predicts 1-day downside risk but all other coefficients of $\ln(DR_t^d)$, $\ln(DR_t^w)$ and $\ln(DR_t^m)$ are not significant when the HAR-DR-DV model predicts the 1-day, 1-week and 1-month downside risks of the spot market. The results show that the trading volume exhibits a little in-sample forecasting power for the spot market for the period from March 23, 2012 to March 5, 2014. However, coefficients of $\ln(DV_t^d)$ and $\ln(DR_t^w)$ are not significant when the HAR-DR-DV model predicts 1-month downside risk and all other coefficients of $\ln(DR_t^d)$, $\ln(DR_t^w)$ and $\ln(DR_t^m)$ are significant when the HAR-DR-DV model predicts 1-day, 1-week and 1-month downside risks of the futures market. These results show the downside trading volume exhibits a lot of in-sample forecasting power for the futures market under Sub-sample 2. Besides, we also find that the HAR-DR-DV model shows more in-sample forecasting power for downside risk under Sub-sample 2 than in-sample forecasting power under Sub-sample 1.

Table 7. Estimation results of HAR-DR-DV model under the sub-samples

		Spot market			Futures market		
		1-day	1-week	1-month	1-day	1-week	1-month
Sub-sample 1	c	-3.861*	-4.524*	-6.019***	-1.936	-1.180	-1.882
		(-1.744)	(-1.777)	(-2.844)	(-1.188)	(-0.600)	(-1.313)
	$\ln(DR_t^d)$	0.093*	0.060	0.028	0.059	0.085**	0.039
		(1.659)	(1.578)	(0.938)	(0.857)	(2.031)	(1.169)
	$\ln(DR_t^w)$	0.152	0.142	0.039	0.439***	0.197*	0.103
		(1.134)	(1.319)	(0.506)	(3.881)	(1.847)	(0.875)
	$\ln(DR_t^m)$	0.387***	0.323*	0.243*	0.259*	0.406**	0.388***
		(2.877)	(1.823)	(1.892)	(1.907)	(2.569)	(3.491)
	$\ln(DV_t^d)$	0.082	0.142	0.011	0.032	-0.010	-0.011
		(0.479)	(1.285)	(0.132)	(0.773)	(-0.311)	(-0.514)
	$\ln(DV_t^w)$	0.308	-0.050	0.241	-0.175*	-0.146	0.005
		(0.898)	(-0.175)	(1.204)	(-1.658)	(-1.483)	(0.064)
	$\ln(DV_t^m)$	-0.218	0.116	0.021	0.253	0.196	0.082
		(-0.891)	(0.400)	(0.103)	(1.402)	(0.953)	(0.541)
	$Adj.R^2$	0.139	0.232	0.196	0.236	0.349	0.348
Sub-sample 2	c	-6.223***	-8.844***	-10.29***	-2.476	-3.466*	-7.334***
		(-3.118)	(-3.741)	(-3.988)	(-1.456)	(-1.828)	(-3.384)
	$\ln(DR_t^d)$	0.047	-0.016	-0.006	0.022	0.081*	0.044
		(0.739)	(-0.406)	(-0.220)	(0.324)	(1.837)	(1.321)
	$\ln(DR_t^w)$	0.204*	0.284**	0.131	0.560***	0.481***	0.245**
		(1.749)	(2.070)	(1.055)	(4.454)	(4.279)	(1.979)
	$\ln(DR_t^m)$	0.344***	0.225	0.180	0.189	0.087	0.051
		(2.771)	(1.401)	(1.299)	(1.426)	(0.552)	(0.304)
	$\ln(DV_t^d)$	0.120	0.193*	0.104	0.111*	-0.075*	-0.017
		(0.696)	(1.914)	(1.443)	(1.737)	(-1.906)	(-0.511)
	$\ln(DV_t^w)$	0.281	0.046	0.366	-0.512***	-0.268*	-0.085
		(0.965)	(0.172)	(1.398)	(-2.804)	(-1.720)	(-0.525)
	$\ln(DV_t^m)$	-0.101	0.210	0.042	0.549***	0.560**	0.580***
		(-0.469)	(0.820)	(0.195)	(2.632)	(2.485)	(3.091)
	$Adj.R^2$	0.182	0.352	0.373	0.325	0.493	0.422

Results on Other Models

Sections 4.1 and 4.2 describe whether the trading volume and downside trading volume forecast the downside risk by using the HAR-DR, HAR-DR-V and HAR-DR-DV models. In this section, we further study whether the trading volume and downside trading volume forecast the downside risk in the spot market and futures market by using the LHAR-DR-J, LHAR-DR-J-V and LHAR-DR-J-DV models. The LHAR-DR-J, LHAR-DR-J-V and LHAR-DR-J-DV models can be represented as follows:

$$\ln(DR_{t+H}^d) = c + \beta_d \ln(DR_t^d) + \beta_w \ln(DR_t^w) + \beta_m \ln(DR_t^m) + \chi_d R_t^{d-} + \chi_w R_t^{w-} + \chi_m R_t^{m-} + \delta_d \ln(1 + J_t^d) + \varepsilon_{t+H} \quad (12)$$

$$\ln(DR_{t+H}^d) = c + \phi_d \ln(DR_t^d) + \phi_w \ln(DR_t^w) + \phi_m \ln(DR_t^m) + \eta_d R_t^{d-} + \eta_w R_t^{w-} + \eta_m R_t^{m-} + \kappa_d \ln(1 + J_t^d) + \varphi_d \ln(V_t^d) + \varphi_w \ln(V_t^w) + \varphi_m \ln(V_t^m) + \varepsilon_{t+H} \quad (13)$$

$$\ln(DR_{t+H}^d) = c + \gamma_d \ln(DR_t^d) + \gamma_w \ln(DR_t^w) + \gamma_m \ln(DR_t^m) + \nu_d R_t^{d-} + \nu_w R_t^{w-} + \nu_m R_t^{m-} + \theta_d \ln(1 + J_t^d) + \lambda_d \ln(DV_t^d) + \lambda_w \ln(DV_t^w) + \lambda_m \ln(DV_t^m) + \varepsilon_{t+H} \quad (14)$$

where R_t^{d-}, R_t^{w-} and R_t^{m-} are daily, weekly and monthly leverages, respectively, namely, $R_t^{d-} = R_t * I\{R_t < 0\}$, $R_t^{w-} = (R_t + R_{t-1} + \cdots + R_{t-4}) * I\{(R_t + R_{t-1} + \cdots + R_{t-4}) < 0\}/5$, $R_t^{m-} = (R_t + R_{t-1} + \cdots + R_{t-21}) * I\{(R_t + R_{t-1} + \cdots + R_{t-21}) < 0\}/22$, R_t is return at period t, $I\{\cdot\}$ is an indicator function; J_t^d represents the jump obtained by the method proposed by Barndorff-Nielsen and Shephard (2004, 2006).

In this section, we also use the OLS with Newey-West to estimate the LHAR-DR-J, LHAR-DR-J-V and LHAR-DR-J-DV models under the full sample. The results of the LHAR-DR-J model under full sample are shown in Table 8. In this table, almost all coefficients of $\ln(DR_t^d)$, $\ln(DR_t^w)$ and $\ln(DR_t^m)$ are significantly positive when the LHAR-DR-J model predicts the 1-day, 1-week and 1-month downside risks, which further proves that the downside risk of the spot market and futures market has strong sustainability or long memory. Then, the coefficient of R_t^{m-} is significant when the LHAR-DR-J model predicts 1-month downside risk in the spot market and the coefficient of

Table 8. Estimation results of LHAR-DR-J model under the full sample

	Spot market			Futures market		
	1-day	**1-week**	**1-month**	**1-day**	**1-week**	**1-month**
c	-0.758***	-0.698***	-0.910***	-0.601***	-0.597***	-0.875***
	(-4.819)	(-4.292)	(-5.864)	(-5.595)	(-4.717)	(-6.931)
$\ln(DR_t^d)$	0.155***	0.071*	0.056*	0.088*	0.084***	0.047**
	(3.093)	(1.740)	(1.789)	(1.876)	(2.990)	(2.064)
$\ln(DR_t^w)$	0.171*	0.224***	0.146**	0.349***	0.249***	0.136
	(1.917)	(2.771)	(2.080)	(3.851)	(3.276)	(1.466)
$\ln(DR_t^m)$	0.387***	0.375***	0.331***	0.371***	0.406***	0.406***
	(4.253)	(3.498)	(4.193)	(4.179)	(4.309)	(4.413)
R_t^{d-}	3.584	-0.520	0.921	-0.087	-0.098***	-0.033
	(0.835)	(-0.193)	(0.493)	(-1.486)	(-2.605)	(-1.218)
R_t^{w-}	-8.576	-1.182	-3.895	-0.201	-0.181	-0.142
	(-1.070)	(-0.193)	(-0.523)	(-1.314)	(-0.996)	(-0.716)
R_t^{m-}	-1.002	15.48	36.86**	0.169	0.448	0.337
	(-0.064)	(0.920)	(2.370)	(0.600)	(1.278)	(0.935)
$\ln(1+J_t^d)$	-0.734**	-0.245	-0.138	-0.083	-0.408	-0.111
	(-2.170)	(-1.057)	(-0.939)	(-0.212)	(-1.611)	(-0.472)
$Adj.R^2$	0.179	0.302	0.274	0.281	0.414	0.369

Table 9. Estimation results of LHAR-DR-J-V model under the full sample

	Spot market			Futures market		
	1-day	**1-week**	**1-month**	**1-day**	**1-week**	**1-month**
c	-6.533***	-6.997***	-7.480***	-1.147**	-1.020	-2.062***
	(-3.845)	(-3.530)	(-3.990)	(-1.991)	(-1.428)	(-2.929)
$\ln(DR_t^d)$	0.099**	0.030	0.017	0.065	0.094***	0.055**
	(2.110)	(0.921)	(0.641)	(1.336)	(3.058)	(2.435)
$\ln(DR_t^w)$	0.077	0.156*	0.073	0.405***	0.278***	0.145*
	(0.823)	(1.714)	(1.013)	(4.393)	(3.594)	(1.712)
$\ln(DR_t^m)$	0.408***	0.351***	0.296***	0.326***	0.358***	0.347***
	(4.457)	(3.048)	(3.478)	(3.531)	(3.628)	(4.008)
R_t^{d-}	2.575	-1.048	0.244	-0.093	-0.093**	-0.038
	(0.624)	(-0.425)	(0.144)	(-1.611)	(-2.509)	(-1.512)
R_t^{w-}	-17.70**	-6.005	-7.848	-0.087	-0.088	-0.134
	(-2.300)	(-0.731)	(-1.087)	(-0.587)	(-0.485)	(-0.735)
R_t^{m-}	-38.09**	-19.80	0.453	-0.011	0.291	0.076
	(-2.197)	(-1.140)	(0.030)	(-0.035)	(0.804)	(0.214)
$\ln(1+J_t^d)$	-0.555*	-0.106	0.019	-0.124	-0.413	-0.192
	(-1.688)	(-0.492)	(0.137)	(-0.306)	(-1.581)	(-0.839)
$\ln(V_t^d)$	0.112	0.139	0.071	0.063*	-0.042	-0.016
	(0.958)	(1.407)	(0.905)	(1.759)	(-1.597)	(-0.867)
$\ln(V_t^w)$	0.449**	0.177	0.234	-0.228**	-0.105	0.026
	(2.092)	(0.933)	(1.390)	(-2.509)	(-1.304)	(0.374)
$\ln(V_t^m)$	-0.258	0.018	0.041	0.206**	0.179*	0.060
	(-1.625)	(0.091)	(0.283)	(2.145)	(1.790)	(0.721)
$Adj.R^2$	0.197	0.330	0.320	0.284	0.422	0.414

R_t^{d-} is significant when the LHAR-DR-J model predicts 1-week downside risk in the futures market but all other coefficients of R_t^{d-}, R_t^{w-} and R_t^{m-} are not significant. The results show the "leverage effect" of downside risk in the spot market and futures market is weak. In addition, analyzing the significance of coefficient of $\ln(1+J_t^d)$, we find the jump shows little in-sample forecasting power for the future downside risk in the spot market and futures market.

Estimation results of the LHAR-DR-J-V model under full sample are shown in **Table 9**. The significance of coefficients of $\ln(V_t^d)$, $\ln(V_t^w)$ and $\ln(V_t^m)$ in the LHAR-DR-J-V model is in accord with the HAR-DR-V model. In the spot market, the daily, weekly and monthly trading volumes show little power for in-sample forecasting of the downside risk. In the stock futures market, the trading volume exhibits much power for in-sample forecasting of the 1-day and 1-week downside risk but not for 1-month.

Table 10. Estimation results of LHAR-DR-J-DV model under the full sample

	Spot market			Futures market		
	1-day	**1-week**	**1-month**	**1-day**	**1-week**	**1-month**
c	-5.982***	-6.316***	-7.001***	-1.106**	-0.961	-1.515**
	(-3.658)	(-3.305)	(-3.896)	(-2.058)	(-1.437)	(-1.978)
$\ln(DR_t^d)$	0.100**	0.034	0.018	0.055	0.093***	0.049*
	(2.039)	(0.978)	(0.645)	(1.128)	(3.047)	(1.944)
$\ln(DR_t^w)$	0.093	0.181*	0.068	0.432***	0.301***	0.140
	(0.949)	(1.943)	(0.916)	(4.630)	(3.812)	(1.477)
$\ln(DR_t^m)$	0.414***	0.347***	0.318***	0.309***	0.338	0.383***
	(4.393)	(2.927)	(3.708)	(3.291)	(3.367)	(3.998)
R_t^{d-}	5.518	1.468	1.327	-0.100*	-0.095*	-0.036
	(1.369)	(0.481)	(0.632)	(-1.723)	(-2.548)	(-1.317)
R_t^{w-}	-11.63	-2.955	-3.749	-0.101	-0.076	-0.118
	(-1.510)	(-0.402)	(-0.541)	(-0.676)	(-0.422)	(-0.606)
R_t^{m-}	-38.06**	-12.78	1.684	0.005	0.304	0.268
	(-2.229)	(-0.742)	(0.111)	(0.017)	(0.857)	(0.729)
$\ln(1+J_t^d)$	-0.576*	-0.133	0.023	-0.127	-0.436*	-0.068
	(-1.816)	(-0.635)	(0.172)	(-0.311)	(-1.658)	(-0.279)
$\ln(DV_t^d)$	0.195*	0.184**	0.066	0.072**	-0.035	-0.010
	(1.669)	(2.000)	(0.939)	(2.049)	(-1.383)	(-0.530)
$\ln(DV_t^w)$	0.352	0.045	0.317*	-0.287***	-0.168**	-0.015
	(1.521)	(0.218)	(1.770)	(-3.260)	(-2.041)	(-0.183)
$\ln(DV_t^m)$	-0.259	0.084	-0.047	0.255***	0.232**	0.075
	(-1.513)	(0.413)	(-0.305)	(2.723)	(2.308)	(0.818)
$Adj.R^2$	0.195	0.326	0.320	0.286	0.425	0.371

Estimation results of the LHAR-DR-J-DV model under full sample are shown in **Table 10**. The significance of coefficients of $\ln(V_t^d)$, $\ln(V_t^w)$ and $\ln(V_t^m)$ in the LHAR-DR-J-DV model are similar to the HAR-DR-DV model. The results show the downside trading volume exhibits a little in-sample forecasting power for downside risk in the spot market, and has much in-sample forecasting power for the stock futures market.

OUT-OF-SAMPLE FORECAST

Rolling Window Forecast

We choose the rolling window prediction method to analyze whether trading volume and downside trading volume exhibit out-of-sample prediction ability for the downside risk in the spot market and futures market. The rolling window is 500, forecast sample is 439 data from May 11, 2012 to March 5, 2014. Applying the the rolling window prediction method, we can get 439 predicted values of 1-day, 1-week and 1-month downside risks.

SPA Test

SPA test was used to verify whether the out-of-sample predictive power of HAR-DR-V and HAR-DR-DV models is better than the HAR-DR model. If the out-of-sample predictive power of the HAR-DR-V model for downside risk is better than the HAR-DR model, the implication is that trading volume can be used to predict downside risk of the spot market and futures market. And if the out-of-sample predictive power of the HAR-DR-DV model is better than the HAR-DR model, then downside trading volume contains some out-of-sample predictive information of downside risk in the spot market as well as the futures market.

The SPA test is for a kind of superior predictive ability with Bootstrap properties. It was developed by Hansen (2005) on the basis of loss functions. Hansen (2005) found that the ability of the SPA test to discriminate models is better than that of the RC test developed by White (2000) and the SPA test has better robustness.

Results of SPA Test

We get p-values of the SPA test when we choose the HAR-DR, HAR-DR-V or HAR-DR-DV model as the benchmark model and choose two other models as the compared models. **Tables 11a**, **11b** and **11c** list the results of the SPA test when the HAR-DR, HAR-DR-V and HAR-DR-DV models predict the 1-day, 1-week and 1-month downside risks in the spot market and futures market. In these three tables, the second row lists the benchmark models and the second column lists the compared models. If the p-value of Model X as a benchmark model and

Table 11a. SPA test results of forecasting 1-day downside risk (500)

		Spot market			Futures market		
		HAR-DR	HAR-DR-V	HAR-DR-DV	HAR-DR	HAR-DR-V	HAR-DR-DV
MAE	HAR-DR	——	0.253	0.230	——	0.205	0.130
	HAR-DR-V	0.747	——	0.249	0.801	——	0.087
	HAR-DR-DV	0.782	0.727	——	0.536	0.304	——
HMAE	HAR-DR	——	0.557	0.038	——	0.112	0.061
	HAR-DR-V	0.484	——	0.113	0.594	——	0.089
	HAR-DR-DV	0.533	0.511	——	0.519	0.340	——
MSE	HAR-DR	——	0.090	0.037	——	0.224	0.154
	HAR-DR-V	0.678	——	0.108	0.788	——	0.115
	HAR-DR-DV	0.674	0.648	——	0.493	0.262	——
HMSE	HAR-DR	——	0.485	0.119	——	0.475	0.502
	HAR-DR-V	0.211	——	0.125	0.261	——	0.111
	HAR-DR-DV	0.479	0.470	——	0.294	0.446	——

Table 11b. SPA test results of forecasting 1-week downside risk (500)

		Spot market			Futures market		
		HAR-DR	HAR-DR-V	HAR-DR-DV	HAR-DR	HAR-DR-V	HAR-DR-DV
MAE	HAR-DR	——	0.046	0.069	——	0.165	0.281
	HAR-DR-V	0.630	——	0.551	0.829	——	0.759
	HAR-DR-DV	0.626	0.185	——	0.758	0.220	——
HMAE	HAR-DR	——	0.007	0.013	——	0.033	0.072
	HAR-DR-V	0.652	——	0.597	0.529	——	0.335
	HAR-DR-DV	0.576	0.422	——	0.491	0.629	——
MSE	HAR-DR	——	0.001	0.004	——	0.015	0.046
	HAR-DR-V	0.689	——	0.595	0.577	——	0.509
	HAR-DR-DV	0.662	0.397	——	0.522	0.534	——
HMSE	HAR-DR	——	0.074	0.092	——	0.109	0.103
	HAR-DR-V	0.620	——	0.215	0.549	——	0.196
	HAR-DR-DV	0.640	0.461	——	0.510	0.456	——

Table 11c. SPA test results of forecasting 1-month downside risk (500)

		Spot market			Futures market		
		HAR-DR	HAR-DR-V	HAR-DR-DV	HAR-DR	HAR-DR-V	HAR-DR-DV
MAE	HAR-DR	——	0.467	0.449	——	0.395	0.402
	HAR-DR-V	0.496	——	0.321	0.607	——	0.505
	HAR-DR-DV	0.538	0.652	——	0.610	0.490	——
HMAE	HAR-DR	——	0.393	0.341	——	0.264	0.284
	HAR-DR-V	0.647	——	0.262	0.734	——	0.373
	HAR-DR-DV	0.633	0.720	——	0.739	0.635	——
MSE	HAR-DR	——	0.243	0.230	——	0.191	0.199
	HAR-DR-V	0.498	——	0.377	0.492	——	0.464
	HAR-DR-DV	0.803	0.630	——	0.464	0.516	——
HMSE	HAR-DR	——	0.174	0.183	——	0.141	0.132
	HAR-DR-V	0.491	——	0.234	0.484	——	0.233
	HAR-DR-DV	0.478	0.437	——	0.490	0.474	——

Model Y as a compared model is greater than that of Model Y as a benchmark model and Model X as a compared model, the out-of-sample prediction performance of Model X for the downside risk is better than that of Model Y.

In **Table 11a**, SPA test results under different loss functions are different. The p-value of the HAR-DR-V model as a benchmark model and the HAR-DR model as the compared model is greater than that of the HAR-DR model as a benchmark model and HAR-DR-V model as the compared model in the spot market under the MAE and MSE, but the result is the opposite in HMAE and HMSE. This shows that the out-of-sample prediction performance of HAR-DR-V model for 1-day downside risk isn't better than the HAR-DR model and the trading volume doesn't exhibit out-of-sample forecasting power for 1-day downside risk in the spot market. **Tables 11b** and **11c** lead to similar conclusions for the spot market and futures market. The p-value of the HAR-DR-V or HAR-DR-DV model as a benchmark model and the HAR-DR model as the compared model is greater than that of the HAR-DR model as a benchmark model and HAR-DR-V or HAR-DR-DV model as the compared model under all loss functions,

Table 12a. SPA test results of forecasting 1-day downside risk (300)

		Spot market			Futures market		
		HAR-DR	HAR-DR-V	HAR-DR-DV	HAR-DR	HAR-DR-V	HAR-DR-DV
MAE	HAR-DR	——	0.439	0.443	——	0.743	0.694
	HAR-DR-V	0.026	——	0.464	0.268	——	0.361
	HAR-DR-DV	0.044	0.519	——	0.273	0.649	——
HMAE	HAR-DR	——	0.507	0.599	——	0.559	0.456
	HAR-DR-V	0.097	——	0.661	0.442	——	0.047
	HAR-DR-DV	0.093	0.342	——	0.574	0.385	——
MSE	HAR-DR	——	0.587	0.552	——	0.399	0.446
	HAR-DR-V	0.009	——	0.460	0.162	——	0.207
	HAR-DR-DV	0.005	0.160	——	0.234	0.428	——
HMSE	HAR-DR	——	0.562	0.564	——	0.479	0.483
	HAR-DR-V	0.117	——	0.536	0.218	——	0.049
	HAR-DR-DV	0.114	0.186	——	0.231	0.482	——

Table 12b. SPA test results of forecasting 1-week downside risk (300)

		Spot market			Stock futures market		
		HAR-DR	HAR-DR-V	HAR-DR-DV	HAR-DR	HAR-DR-V	HAR-DR-DV
MAE	HAR-DR	——	0.459	0.490	——	0.367	0.339
	HAR-DR-V	0.137	——	0.480	0.011	——	0.051
	HAR-DR-DV	0.116	0.211	——	0.039	0.574	——
HMAE	HAR-DR	——	0.646	0.630	——	0.270	0.282
	HAR-DR-V	0.035	——	0.625	0.698	——	0.104
	HAR-DR-DV	0.032	0.336	——	0.574	0.546	——
MSE	HAR-DR	——	0.617	0.581	——	0.715	0.559
	HAR-DR-V	0.047	——	0.711	0.299	——	0.140
	HAR-DR-DV	0.051	0.275	——	0.398	0.572	——
HMSE	HAR-DR	——	0.762	0.746	——	0.173	0.152
	HAR-DR-V	0.034	——	0.627	0.540	——	0.144
	HAR-DR-DV	0.071	0.363	——	0.563	0.543	——

which shows the out-of-sample prediction performance of HAR-DR-V and HAR-DR-DV models is better than that of the HAR-DR model and the trading volume and downside trading volume exhibit out-of-sample forecasting power for 1-week and 1-month downside risks in the spot market and futures market.

According to **Tables 11a**, **11b** and **11c**, when we choose 500 as the rolling window, we find the trading volume and downside trading volume exhibit certain out-of-sample forecasting power for downside risk in the spot market, which is stronger in the futures market.

Results on Other Rolling Windows

In Section 5.1, we choose 500 as the time window and predict the downside risk in the period from May 11, 2012 to March 5, 2014. In order to further analyze whether the trading volume and downside trading volume can predict the downside risk in other periods, we choose 300 as the time window to predict the downside risk of the period from July 12, 2011 to March 5, 2014.

Tables 12a, **12b** and **12c** list the results of SPA test when we choose 300 as the rolling window to predict the downside risk in the spot market and futures market. In these tables, the second row lists the benchmark models and the second column lists the compared models. Analyzing the results in these three tables, we find most p-values of the HAR-DR model as a benchmark model and the HAR-DR-V or HAR-DR-DV model as the compared model are greater than those of the HAR-DR-V or HAR-DR-DV model as a benchmark model and HAR-DR model as the compared model. This shows the out-of-sample prediction performance of HAR-DR model for the downside risk in the spot market and futures market is better than that of the HAR-DR-V and HAR-DR-DV models, and the trading volume and downside trading volume don't exhibit out-of-sample forecasting power for downside risk in the period from July 12, 2011 to March 5, 2014.

Table 12c. SPA test results of forecasting 1-month downside risk (300)

		Spot market			Futures market		
		HAR-DR	**HAR-DR-V**	**HAR-DR-DV**	**HAR-DR**	**HAR-DR-V**	**HAR-DR-DV**
MAE	HAR-DR	——	0.532	0.539	——	0.457	0.472
	HAR-DR-V	0.123	——	0.464	0.049	——	0.290
	HAR-DR-DV	0.131	0.546	——	0.046	0.673	——
HMAE	HAR-DR	——	0.551	0.773	——	0.507	0.491
	HAR-DR-V	0.146	——	0.245	0.113	——	0.241
	HAR-DR-DV	0.187	0.748	——	0.169	0.756	——
MSE	HAR-DR	——	0.749	0.689	——	0.676	0.633
	HAR-DR-V	0.264	——	0.446	0.322	——	0.221
	HAR-DR-DV	0.279	0.558	——	0.370	0.761	——
HMSE	HAR-DR	——	0.443	0.390	——	0.219	0.218
	HAR-DR-V	0.541	——	0.081	0.504	——	0.263
	HAR-DR-DV	0.624	0.641	——	0.544	0.742	——

Table 13a. SPA test results of forecasting 1-day downside risk (500)

		Spot market			Futures market		
		LHAR-DR-J	**LHAR-DR-V-J**	**LHAR-DR-DV-J**	**LHAR-DR-J**	**LHAR-DR-V-J**	**LHAR-DR-DV-J**
MAE	LHAR-DR-J	——	0.601	0.519	——	0.644	0.369
	LHAR-DR-V-J	0.421	——	0.173	0.368	——	0.036
	LHAR-DR-DV-J	0.473	0.453	——	0.613	0.415	——
HMAE	LHAR-DR-J	——	0.719	0.490	——	0.265	0.087
	LHAR-DR-V-J	0.269	——	0.496	0.719	——	0.094
	LHAR-DR-DV-J	0.281	0.227	——	0.433	0.420	——
MSE	LHAR-DR-J	——	0.085	0.072	——	0.355	0.181
	LHAR-DR-V-J	0.607	——	0.525	0.667	——	0.102
	LHAR-DR-DV-J	0.613	0.484	——	0.500	0.352	——
HMSE	LHAR-DR-J	——	0.503	0.505	——	0.491	0.709
	LHAR-DR-V-J	0.195	——	0.507	0.308	——	0.154
	LHAR-DR-DV-J	0.267	0.242	——	0.290	0.493	——

Results on Other Models

In this section, we further study whether the trading volume and downside trading volume forecast the downside risk in the period from May 11, 2012 to March 5, 2014 by using the LHAR-DR-J, LHAR-DR-J-V and LHAR-DR-J-DV models.

Tables 13a, **13b** and **13c** list the results of SPA test when we choose 500 as the rolling window to predict the downside risk in the spot market and futures market by using the LHAR-DR-J, LHAR-DR-J-V and LHAR-DR-J-DV models. Analyzing p-values from these tables, we find the results are similar to Section 5.1. The out-of-sample prediction performances of LHAR-DR-J-V and LHAR-DR-J-DV models for downside risk in the spot market and futures market are mostly better than the LHAR-DR-J model. The trading volume and downside trading volume exhibit out-of-sample forecasting power for downside risk in the spot market and futures market and it is stronger in futures market.

Table 13b. SPA test results of forecasting 1-week downside risk (500)

		Spot market			Futures market		
		LHAR-DR-J	LHAR-DR-V-J	LHAR-DR-DV-J	LHAR-DR-J	LHAR-DR-V-J	LHAR-DR-DV-J
MAE	LHAR-DR-J	——	0.189	0.187	——	0.314	0.321
	LHAR-DR-V-J	0.528	——	0.758	0.711	——	0.726
	LHAR-DR-DV-J	0.826	0.228	——	0.696	0.272	——
HMAE	LHAR-DR-J	——	0.034	0.036	——	0.100	0.102
	LHAR-DR-V-J	0.522	——	0.532	0.485	——	0.755
	LHAR-DR-DV-J	0.650	0.128	——	0.503	0.240	——
MSE	LHAR-DR-J	——	0.019	0.030	——	0.108	0.130
	LHAR-DR-V-J	0.526	——	0.535	0.533	——	0.754
	LHAR-DR-DV-J	0.571	0.098	——	0.517	0.239	——
HMSE	LHAR-DR-J	——	0.103	0.102	——	0.113	0.094
	LHAR-DR-V-J	0.548	——	0.549	0.547	——	0.283
	LHAR-DR-DV-J	0.534	0.117	——	0.538	0.691	——

Table 13c. SPA test results of forecasting 1-month downside risk (500)

		Spot market			Futures market		
		LHAR-DR-J	LHAR-DR-V-J	LHAR-DR-DV-J	LHAR-DR-J	LHAR-DR-V-J	LHAR-DR-DV-J
MAE	LHAR-DR-J	——	0.585	0.508	——	0.569	0.526
	LHAR-DR-V-J	0.450	——	0.088	0.438	——	0.128
	LHAR-DR-DV-J	0.531	0.636	——	0.449	0.630	——
HMAE	LHAR-DR-J	——	0.435	0.372	——	0.343	0.381
	LHAR-DR-V-J	0.607	——	0.147	0.652	——	0.207
	LHAR-DR-DV-J	0.661	0.810	——	0.639	0.756	——
MSE	LHAR-DR-J	——	0.398	0.380	——	0.399	0.375
	LHAR-DR-V-J	0.618	——	0.318	0.622	——	0.237
	LHAR-DR-DV-J	0.606	0.679	——	0.637	0.769	——
HMSE	LHAR-DR-J	——	0.207	0.193	——	0.109	0.188
	LHAR-DR-V-J	0.456	——	0.689	0.488	——	0.519
	LHAR-DR-DV-J	0.442	0.322	——	0.450	0.487	——

DISCUSSION

Sections 4.1, 4.2 and 4.3 indicate that the trading volume and downside trading volume exhibit much in-sample forecasting power in futures market but little power in the spot market. Then, the trading volume and downside trading volume play different roles in predicting the downside risk in different periods. They exhibit more in-sample forecasting power for downside risk under the period from March 23, 2012 to March 5, 2014 than under the period from April 16, 2010 to March 22, 2012. In addition, we get approximately the same results under the different models, which shows our results are robust.

Firstly, the SPA test need set several loss functions. In this paper, we choose the four most common loss functions, the mean absolute error (MAE), heteroskedastic adjusted mean absolute error (HMAE), mean squared error (MSE) and heteroskedastic adjusted mean squared error (HMSE). Then, according to the method of Hansen (2005), we get the p-value of the "Bootstrap" method. When we compare the predictive powers of different models, the greater p value means that compared with other models, the benchmark model exhibits better out-of-sample predictive power.

However, p-value of the HAR-DR-DV model as a benchmark model and the HAR-DR model as the compared model is greater than that of the HAR-DR model as a benchmark model and HAR-DR-V model as the compared model under MAE, HMAE, MSE and HMSE, which shows the out-of-sample prediction performance of HAR-DR-DV model is better than that of the HAR-DR model, and the downside trading volume exhibits good out-of-sample forecasting power for 1-day downside risk in the spot market. In stock futures market, p-value of the HAR-DR-V or HAR-DR-DV model as a benchmark model and the HAR-DR model as the compared model is greater than that of the HAR-DR model as a benchmark model and HAR-DR-V or HAR-DR-DV as the compared model under all loss functions except the HMSE, which shows the out-of-sample prediction performance of HAR-DR-V and HAR-DR-DV models is better than the HAR-DR model and the trading volume and downside trading volume exhibit a little out-of-sample forecasting power for 1-day downside risk in the stock futures market.

According to Sections 5.3, 5.4 and 5.5, we find the results are different when we predict the downside risks in different periods. The trading volume and downside trading volume exhibit some out-of-sample forecasting power for downside risks in the period from May 11, 2012 to March 5, 2014, but they show little out-of-sample forecasting power from July 12, 2011 to March 5, 2014. In addition, the trading volume and downside trading volume exhibit more out-of-sample forecasting power in the futures market than in the spot market.

CONCLUSION

In this paper, we test whether the trading volume and downside trading volume can be used to predict the downside risk in the stock spot market and futures market. In the in-sample analysis, the trading volume and downside trading volume exhibit much in-sample forecasting power in the futures market but little in-sample forecasting power in the spot market. What's more, the results are different in different periods and are similar under different models. In the out-of-sample analysis, the results are different when we predict downside risk of different periods. The trading volume and downside trading volume exhibit more out-of-sample forecasting power in futures market than in the spot market.

According to the results from the in-sample and the out-of-sample analysis, we find that trading volume and downside trading volume show different predictive powers in different periods and they exhibit more forecasting power in futures market but little forecasting power in the spot market. Furthermore, we can get similar results under different models, which shows the results are robust.

RECOMMENDATIONS

The arrival of unexpected "good news" results in a price increase whereas "bad news" causes a price decrease therefore predicting the risks are of high importance. The investors, economists as well as policy makers should consider the results revealed in this study to be able to keep the stability of market.

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