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Effect of using Desmos on high school students' understanding and learning of functions

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Abstract

This study examines the effect of using Desmos on students' performance in learning functions. An embedded mixed methods design was applied and involved 98 students from an upper secondary school in Sweden. Students' assessments (pre- and post-test) and opinion polls were the two main data collection instruments. The results show that both groups (experimental and control) experienced a significant improvement in their post-test scores. However, the experimental group had a statistically significant improvement in comparison with that of the control group across the five constructs used in this study. The qualitative data revealed that the majority of the students ascribed a positive effect of the use of Desmos on their general understanding of function concepts, their ability to analyze functions and check their answers through visualization, which are difficult when working using paper and pencil.

Keywords: Desmos, digital tool, functions, learning experience, understanding

INTRODUCTION

Mathematics education in the Swedish curriculum, like that of most other countries, aims to develop students' ability to understand mathematical concepts and methods, develop different strategies for solving mathematical problems, and use mathematics in society. Research has shown a general perception of mathematics as a demanding subject, and the concept of functions has long been a challenging area for many students (Sierpinska, 1992). Graham and Thomas (2000) identified the perception of variables, process, object duplication, the relationship between different representations of functions, and the interpretation of graphs as some of the challenges. It is thus not surprising that researchers like Cheung (2013) and Fabian et al. (2016) have argued that the integration of technology and digital tools in the teaching of such concepts can improve students' understanding and development of conceptual knowledge. This is consistent with the Swedish schools inspectorate's report (ref 2019) and Skolverket's (2023) report, that teaching and learning of mathematics and other school subjects should include different forms of methods and that students should have the opportunity to develop the ability to use digital tools to promote the development of their knowledge, deepen their mathematical learning and broaden the areas in which mathematical knowledge can be used and solve mathematical problems.

The use of digital tools in mathematics classrooms has become popular amongst innovative pedagogical practices to design and execute lessons that are challenging, active, learner-centered, and motivating (Hoyles, 2018). The use of computers or digital tools in a thoughtful way can improve students' performance and train students to develop a conceptual understanding of mathematical problems (Wallin et al., 2017). Different digital tools, such as graphing calculators and GeoGebra, have been implemented to enhance students' understanding and learning experience of various mathematical concepts (Akcay, 2017). Desmos is one such tool that has not been widely integrated into mathematics classrooms within Sweden, despite being considered one of the more user-friendly tools for learning mathematics (Chorney, 2021). This study examines how the use of Desmos affects students' learning experiences and performance when learning about functions in a Swedish upper secondary school among the students who attend the humanities and social sciences programs.

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Contribution to the literature

- The findings from this study show that the use of Desmos as a technological tool for learning functions improves students' performance and helps them develop conceptual understanding through visualization and by being a user-friendly tool.
- This study also adds to the literature on the effect of Desmos on Swedish students' learning experiences and the perceptions of how helpful Desmos is and the challenges associated with its use.
- An embedded mixed methods design enabling examination of learning from different perspectives, providing a holistic picture of the implementation of Desmos.

This study addresses the following research questions:

1. How does the use of Desmos affect upper secondary school students' cognitive performance in relation to mathematical functions?
2. In what ways can Desmos contribute to students' knowledge acquisition when they solve tasks on functions?
3. How do students perceive the effects of using Desmos on their understanding of functions?

REVIEW OF RELATED LITERATURE

Functions in Swedish Secondary School Mathematics Curriculum

The concept of functions is one of the main concepts in mathematics taught from primary to advanced level and appears in all curricula (Burnett-Bradshaw & Camille, 2012; Carlson & Oehrtman, 2005; Sebsibe & Dorra, 2019). In Sweden, it is mandatory for upper secondary school students from humanities and social sciences programs to take the course Ma1B, where they should develop a conceptual understanding of the concept of function (the linear, the exponential, and the power function). Students taking the course Ma2B (in their second year of secondary school) should learn and understand the concept of a linear system of equations, the methods for solving systems of linear equations, and the concept and properties of functions. More specifically students learn how to find the domain and range of functions, solve functions graphically, calculate the maximum and minimum points in graphs, how graphs can be shifted in a coordinate system, how to create a function representing a graph, and vice versa, and how to apply their knowledge in problem-solving regarding real-life situations.

After taking the courses Ma1B and Ma2B, students should show a deeper understanding of functions and be able to apply this knowledge when solving mathematical and other related problems. However, several studies, including Henrekson and Jävervall (2016) and Henrekson and Wennström (2022), have established a decline in the performance of Swedish students in national and international tests in areas that require the application of knowledge on functions. Several reasons have been proposed to explain this decline, and many are consistent with earlier research like Tall and Vinner (1981), who argued that most students who learn functions do not develop the concept image needed to gain a deep understanding of the concept. Tall and Vinner (1981) defined the concept image as the cognitive structure of the concept, which may include mental pictures associated with the properties of the concept. They defined concept definition as the mathematical definition of a concept gained from external sources.

Good concept images may be sufficient to handle tasks, but when the student comes in contact with the concept in a broader context, difficulties can emerge due to a mismatch between the concept image and the concept definition (Tall & Vinner, 1981). Dubinsky (2001) and Sajka (2003) reinforce Tall and Vinner's (1981) description of students' concept image and concept definition by indicating that students have a limited understanding of function definition. For instance, the property that for each x-value there is only one definite y-value and how this can be tested with a vertical line test seems to be difficult for students. Another challenge is the ability to recognize that a function is represented infinitely by many discrete points in a coordinate system, but that the function need not have these constraints. Students also express that constant functions are not functions, which demonstrates the limitations they have regarding the concept of function (Dubinsky, 2001).

The call for more innovative approaches in teaching mathematics in general, and more specifically the concept of functions, has broadened the discussion about student-centered approaches to help students develop concept images and concept definitions, which are needed to gain conceptual understanding. Thus, one can argue that the creation of knowledge is not as simple as one may think but involves the integration of many different concepts, tools, and factors to help the learner develop new knowledge, and digital tools play a crucial role in this process (Duffy & Jonassen, 2013).

Benefits of Integrating Technology into Mathematics Teaching

It is documented in the literature that there is importance in integrating technological tools in teaching. Murphy (2016) notes that the use of computer technology:

> can increase student engagement, increase motivation to learning, allow for better teacher-student interaction, support student collaboration, assist in the accuracy of mathematical computation, and help students not only feel more comfortable with learning mathematics but also allow for a deeper understanding of the mathematical concepts. (p. 298)

This was also described by Davidenko (1997). Swanepoel and Gebrekal (2010) posit that the use of computers has a positive impact on learners' achievements, problem-solving skills, and the investigation of mathematical ideas within functions. Similarly, a study by Astafaieva et al. (2020) shows that the use of digital tools in mathematical disciplines improves conceptual understanding of mathematics, supports intuition about predicting a possible result, and facilitates the search for a method (idea) of formal proof, replaces technically sophisticated calculations, and allows to check results obtained analytically. All this can contribute positively to the building of mathematical competencies among students. When Chorney (2021) examined the practical advantages of using Desmos, he observed that digital technology in practice can enable unique ways to engage with mathematics. He further argued that Desmos provides the opportunity to conduct, observe, and analyze mathematical experiments. For example, the ability to zoom in high resolution can clarify mathematical processes. The main advantage of using digital tools can be their ability to visualize mathematical concepts and their application, which is valuable when learning about functions, as it allows the user to draw graphs and explore their characteristics. Enabling learners to explore and visualize the concepts easily, increases the accessibility to understanding the concepts. Students can then integrate symbolic, numeric, tabular, graphical, and visual modes to create their mathematical knowledge. The use of such digital tools can thus provide teachers with different options to be adapted or adopted to tailor the teaching to the needs of individual students (Beckmann et al., 1999; Birgin & Acar, 2022; Murphy, 2016; Nicaise & Barnes, 1996; Waits & Demana, 1999).

Digital Tools for Teaching and Learning Mathematics

As discussed above, research has shown the significant impact that can be achieved using technology in mathematics education in terms of being better able to promote student learning and understanding (Cheung & Slavin, 2013; Fabian et al., 2016). The most common forms of digital tools used in teaching and learning mathematics include, but are not limited to, graphical calculators, GeoGebra, and Desmos. Studies (e.g., Cheung & Slavin, 2013) have shown positive outcomes when using digital tools in teaching. They all have similar disadvantages, with minor differences, making it difficult to decide which tool is better for teaching and learning a particular concept. One significant difference between GeoGebra and Desmos is that GeoGebra can be used for geometry and other mathematical topics, while Desmos targets function learning. While other tools, such as Mathematica and MATLAB, can provide access to the visualization of functions and dynamic interaction with graphical representations of functions, this requires substantial programming experience and is beyond the scope of the pupils considered in this study. Hence, the choice of Desmos for this study has been informed by its user-friendliness and high visualization resolution to help students study and understand the different components of a graph and see how the shape of a graph changes.

Graphing Calculators

Graphing calculators are the most widely used digital tool. Graphing calculators have a range of uses in mathematics. They are used for creating graphs, computing descriptive statistics, and operating on functions, matrices, vectors, and complex numbers (McCulloch et al., 2018). There are several scientific articles about graphing calculators in mathematics teaching. For example, Bos (2009) showed that the use of Texas Instruments (TI) interactive calculators has mathematical, pedagogical, and cognitive impact on the performance of students in their second year of high school who study second-order functions. The results from this study further demonstrated that students' mathematical performance was better when using TI interactive instruments compared to students using a traditional teaching method. However, Ocak (2008) examined how mathematics students at university used calculators when solving challenging problems in the function domain and showed that although the tool was used as an aid for visualization, correcting handwritten graphs, and comparing different graphs, this did not lead to better understanding and improved performance. The results also established that students' previous experiences with graphing calculators influenced their ability to use the tool beneficially but did not necessarily increase their performance. Students with less experience with graphing calculators were disadvantaged and drew graphs without identifying critical points in the curves. Only experienced students with a positive attitude benefited in terms of understanding when introduced to the more complex elements of the functional domain. Godwin and Sutherland (2004) tested how two different digital tools helped students' understanding and the impact of teacher interaction on this implementation. Omnigraf (software for drawing graphs) and graphing calculators were used in the study, and the results showed that both digital tools enabled exploration and testing of different sub-types of functions e.g., $y = x + a$, $y = bc$, $y = bx + a$ in a short time and thus promoted students' understanding. However, the authors pointed to the importance that teachers engage the individuals to experiment and explore mathematical concepts, but that the teacher must know how to create a collective understanding and not run the risk of having a class, where all individuals have understood the concept in different ways.

GeoGebra

GeoGebra is a free software used for learning mathematics. Its several functions are helpful in different areas, such as functions, geometry, and algebra. Researchers have examined the use of this tool in the teaching and learning of mathematics from different perspectives. Zulnaidi and Zakaria (2012) argued that GeoGebra has a positive effect on understanding as they observed a significantly higher conceptual knowledge in their experimental group compared to their control group. They further posed that this improvement was due to the graphical representations of functions accessed through GeoGebra, as the students found it much easier to understand the function domain through visualization and the ability to manipulate a function to examine its changes.

Similarly, Zulnaidi et al. (2020) also reported that the use of GeoGebra in the learning of functions had a positive effect on students' achievement in comparison to the use of traditional teaching methods. The study indicates that the use of GeoGebra as a tool in mathematics classrooms results in the reduction of misconceptions about various mathematical topics.

Abadi and Fradah (2018) analyzed and described students' understanding of the rules of function-shifting with the use of GeoGebra. The use of GeoGebra in the classroom supported the students in understanding function-shifting by allowing them to test out and make mistakes until they found a hypothesis about the general form of function-shifting. Students were then able to answer the questions and explain their answers with examples that supported their understanding and reasoning.

Bakar et al. (2015) presented results that show an improvement in students' performances regarding second-order functions when using GeoGebra. However, there was no statistically significant evidence when the students' results were correlated with their spatial visualization abilities. Thus, it can be argued that although the tool can help improve students' performance, it may not have the same effect on visualization skills. Celen (2020) also stated that despite the numerous benefits associated with the use of GeoGebra, most students did not find it user-friendly, especially students who lack sufficient computer knowledge. Thus, there could be a need for a user-friendly technological tool in the teaching and learning of mathematics (Godwin & Sutherland, 2004).

Desmos

Desmos is a free online graphing calculator that runs as a browser application and as a mobile app. Like other research on the advantages of graphical calculators, McCulloch et al. (2018) show a positive response from teachers who used Desmos in teaching. They reported that using Desmos was easy, fun, and similar to the graphing calculator, producing graphs quickly and correctly. They also saw that it supported students in developing their understanding of mathematics. King (2017) tested Desmos in mathematics education and observed a positive result on students' achievement and deeper conceptual understanding when learning about functions. The implementation of Desmos in the classroom gave the students a chance to investigate, explore and test functions and their graphs as parameters change and how this affects the appearance of the graph, and gave them a better understanding of the domain and range of graphs. According to King (2017), students appreciated the use of the tool and the opportunity to work on their own, at their own pace.

Based on this literature review, one can conclude that prior studies demonstrate a positive impact of the use of digital tools in mathematics education. These learning improvements can meet many of the difficulties learners demonstrate when working to understand function representations. All the tools presented utilize the same concepts and are similar to graphing calculators. Desmos is a digital tool that, according to previous research, improves students' understanding and allows for clear visualization. The observed effects of using Desmos are similar to the effects of the other digital tools presented here. Based on this, Desmos is a representative digital tool that has ease of use and features (e.g., zooming with high resolution) that can help students understand functions through better visualization (Chorney, 2021).

METHODOLOGY

Research Design

An embedded mixed methods design is used in this study, consisting of a quantitative phase (Quasi-experimental approach) and a qualitative phase. A visual representation of the research design guiding the current study is depicted in **Figure 1**. The experimental group and control group did a pre-test at the beginning of the study. Later both groups started to learn non-linear functions (quadratic functions, power functions, and exponential functions) as a part of their mathematical course. The experimental group used Desmos in the classroom during the five weeks of the intervention, and they also did a semi-structured

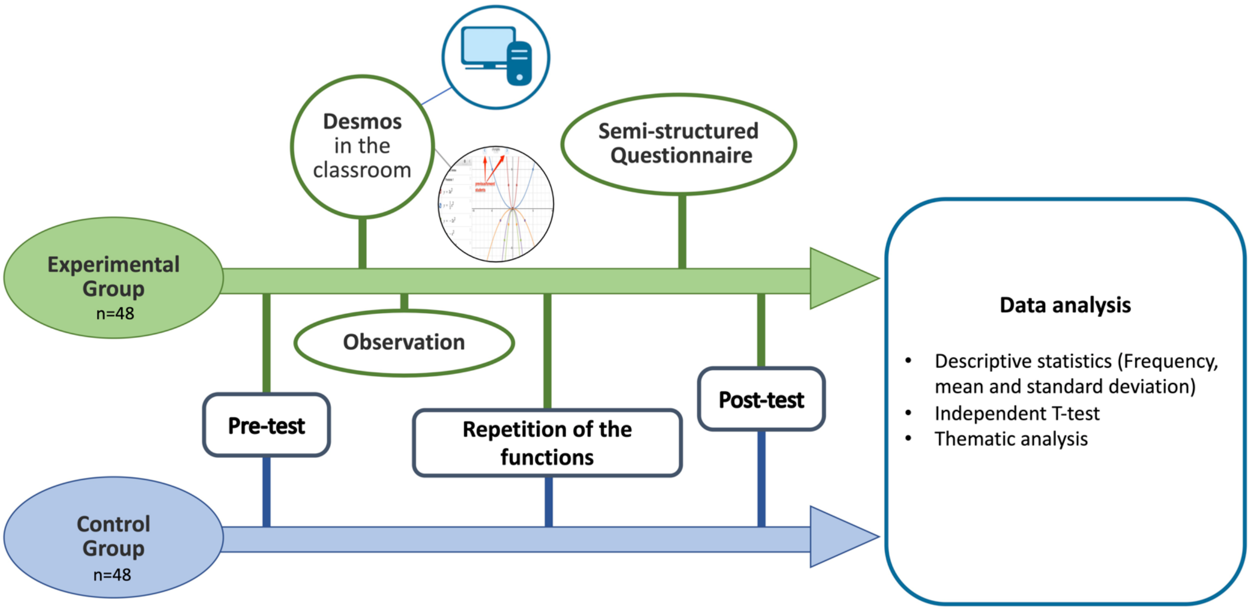

Figure 1. Research design of the study (Source: Authors' own elaboration)

questionnaire toward the end about how they perceived their use of the digital tool. In the middle of the period, the students finished their learning about non-linear functions and had time to revise and repeat the whole chapter about functions (linear and non-linear functions). The study ended with a post-test, the same as pre-test to evaluate the students' knowledge after the experiment period.

The pre-test was conducted for both groups simultaneously during the time the students were learning about the concept of linear functions. Desmos was introduced to the experimental group when the students began to learn the concept of non-linear functions, which lasted for five weeks. The same teaching trajectory was used during all lessons for both groups and consisted of different phases. Firstly, the teacher introduced the theme of the day and explained it on the board while asking students questions along the way (around 20 minutes). Next, there was reserved time for the students to solve questions individually in the book (30 minutes). Both teachers in the study (one for each group) thus used the same teaching method, tasks, and textbooks, but students in the control group did not have access to any digital or visualization tool. They could draw on board, paper, and had regular calculators without the possibility to draw functions in them. The students in the experimental group, on the other hand, started using Desmos during their first non-linear functions lesson. During this lesson they were asked to draw the graphs $y = x^2$and $y = -x^2$ on paper by creating an $x - y$ value table. Students were then introduced to Desmos and asked to draw the same graphs with the tool. Then they compared their graphs (the ones drawn by hand and drawn with Desmos) and discussed what happened to the graph when the coefficient of x^2 changed.

Figure 2 shows what drawing graphs can look like in Desmos. After the non-linear functions lesson, both groups revised the section of functions in the book (the linear and the non-linear functions) and the experimental group used Desmos during this period by being encouraged to draw functions and solve mathematical problems with the tool, and the teacher also used the tool during the whole-class teaching every lesson. The students also drew graphs and examined how the shapes of the graphs change as a result of the changes in the components of the functions. In addition, the students also used the tool to examine the maximum and minimum turning points of curve, the intersections with the x-axis and y-axis, as well as the domain and range of the functions. The post-test was carried out for both groups to observe the development of the student's knowledge. During the post-test, the students from the experimental group were not allowed to use Desmos as this would have given them an advantage over their peers from the control group.

Participants

The target population for the study was 146 second-year upper secondary school students from five classes in a school in Stockholm, who were studying the "mathematics 2B course". The control group consisted of 61 students from two classes. There were 55 students from two classes in the experimental group. A fifth class of 30 students was used for a pilot study, and their results are not a part of the presented results. However, only 98 students (from both groups) took the pre-test while 102 students completed the post-test. Only results from students who did both the pre and post-test from both groups were used to measure the effectiveness of the intervention. Hence, the results include outcomes from 96 students (48 students from the control group and 48 students from the experimental group). The

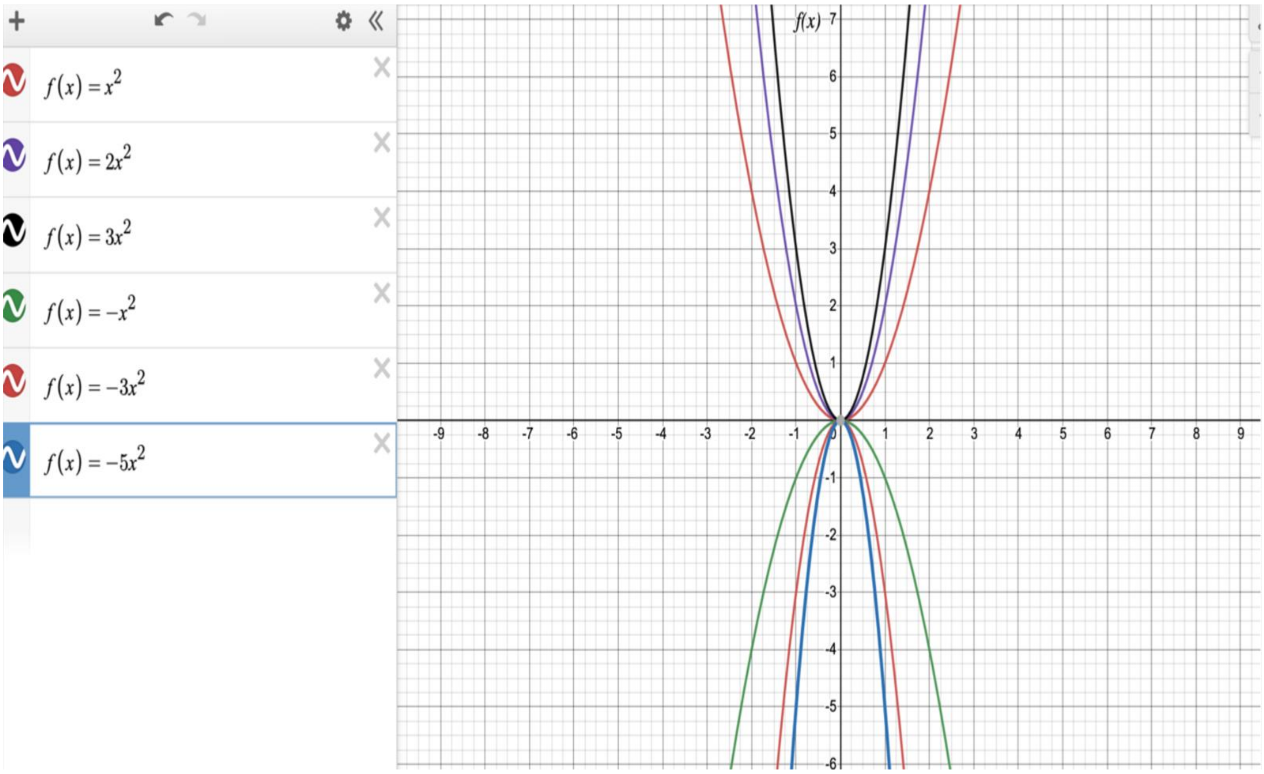

Figure 2. Using Desmos in the lesson (Source: Authors' own elaboration, screenshot when using Desmos.com)

experimental group had the first author of this study as their teacher, while the control group had another teacher that works in the same school.

Informed consent, voluntary participation, and confidentiality are the three main ethical principles that have been adhered to during this study. Before the start of the study, we explained the purpose of the study to the students and their role in this research. The students were informed that their participation was voluntary and that they could quit the study at any time. No traceable data or information was collected during the data collection process, and the students were provided with unique identification codes when handling their data. No names are used when presenting results from the study, and there is also no information about how to identify any student.

Instrumentation and Data Collection

Two data collection tools were used, the students' assessments from the pre and post-test and an opinion poll. The student assessment protocol (used as a pre- and post-test in the study) was designed to test students factual, conceptual, procedural, and metacognitive knowledge according to the six levels of Bloom's Taxonomy (i.e., remembering, understanding, applying, analysis, evaluation, and creating) (Huitt., 2011). This resulted in 20 questions that spread across the six levels, which also were consistent with the content of the mathematics 2B textbook. The questions in **Appendix A** were distributed as follows; knowledge and understanding (questions 4, 5, 8, 9, 10, 11, and 12), application (questions 1, 2, 3, and 13), analysis (questions 6, 7, 18, and 19), evaluate (questions 14, 15, 16, and 17) and create (question 20).

The instruments were validated by the three authors who read through the items and discussed the appropriateness of the items for the Swedish context. Also, since most of the questions were taken from study.com and were written in English, and thus had to be translated into Swedish, it was important to ensure that the appropriate Swedish terminologies were used so that the questions had the same meanings. Before starting the study with the student assessment protocol, a pilot was performed with 30 students from a different class, and an alpha Cronbach reliability of 0.730 was achieved. Based on the results from the pilot, we modified the final tool, and most of the changes were language corrections.

In addition to the student assessment, the experimental group completed an opinion poll (a semi-structured questionnaire) after the post-test. The purpose of the opinion poll was to gather qualitative data from the participants to understand their views regarding the effect of Desmos in their learning of the concept of functions. The opinion poll had two sections, sections A and B. Section A had three questions and elicited the participant's background information: their age, program and gender. Section B had three open-ended questions to collect information on their experiences regarding using Desmos during the five weeks of the intervention. The first question asked the respondents to describe what they found particularly useful about Desmos; the second question elicited

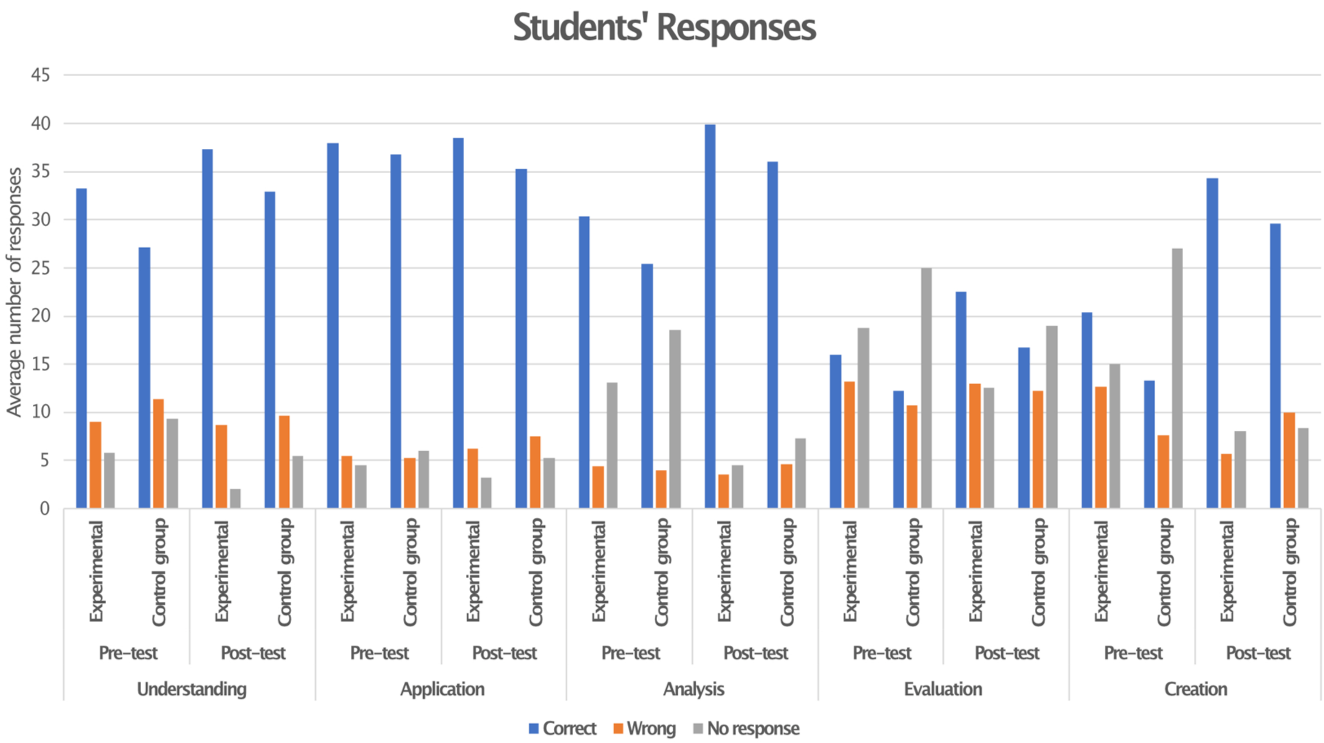

Figure 3. Responses (correct/wrong/no response) for each question per category (Source: Authors' own elaboration)

participants' information about the challenges they encountered when using Desmos. The last question asked the respondents to describe how they would use Desmos to learn better**.** In total, 46 out of the 48 students in the experimental group completed the opinion poll.

Data Analysis

The student's assessment sheets were first marked with the arbitrary codes 1 (correct response), 2 (incorrect response), or 3 (no response) for each question. The codes were then transferred into SPSS (version 28) for both descriptive and inferential statistical analysis. The number of students who did or did not respond to each question is depicted in **Figure 3**. Descriptive (mean and standard deviations) and inferential (independent t-test) statistics were used to generate quantitative results to answer the research questions. For analysis purposes, "remembering" and "understanding" questions were merged into one group, as these two are low-level questions. As discussed above, like pilot study, an alpha Cronbach reliability was run and resulted in a reliability coefficient of 0.894, which shows that scales are reliable. To ascertain inferential statics test (i.e., parametric or non-parametric), a normality test was done to make sure that all individual items were normally distributed before independent t-test was done.

The last research question is about students' experiences of using Desmos in the learning process. The data that answers this question in our research design is from the students' answers to open-ended questions in the opinion poll about how they experienced the use of Desmos. When analyzing this data, we used the thematic analysis procedure to understand the students' experiences, thoughts, and behaviors (Braun & Clarke, 2012). According to Kiger and Varpio (2020, p. 2), thematic analysis has a "reflexibility to be used in a wide range of epistemological frameworks and can be applied to a wide range of questions, designs, and sample sizes". The analysis of the open-ended questions was done using Braun and Clarke's (2006) six-level thematic analysis, which consists of the following steps: familiarizing yourself with the data, generating initial codes, searching for themes, reviewing themes, defining, naming the themes, and producing the report.

RESULTS

Students' Responses

In the pre- and post-test, a majority of the students responded to all the questions, but the number of students who did not answer some questions was higher in the control group, both in the pre- and post-test. Most of those unanswered questions were testing analysis, evaluation, and creating knowledge.

RQ1: How does the use of Desmos affect upper secondary school students' cognitive performance in relation to mathematical functions?

To answer the first research question, the pre- and post-test scores are presented and compared to show if there is any significant difference between the performances of students from both groups. The descriptive and independent t-test results are depicted in **Table 1** and **Table 2**. The higher the mean values are,

Table 1. Pre-test results

Cognitive domain	Group	n	Mean	SD	df	t	p
Remembering & understanding	Experimental	50	1.69	0.61	96	0.3	**0.470**
	Control	48	1.73	0.69			
Application	Experimental	50	1.45	0.36	96	0.6	**0.550**
	Control	48	1.46	0.41			
Analysis	Experimental	50	1.54	0.76	96	1.1	**0.390**
	Control	48	1.69	0.78			
Evaluation	Experimental	50	2.07	0.65	96	1.5	**0.160**
	Control	48	2.26	0.66			
Creation	Experimental	50	1.91	0.85	96	2.2	**0.080**
	Control	48	2.28	0.85			

Note. *$p<0.05$

Table 2. Post-test results

Cognitive domain	Group	n	Mean	SD	df	t	p
Remembering & understanding	Experimental	48	1.24	0.41	94	3.0	**0.002**
	Control	48	1.43	0.61			
Application	Experimental	48	1.27	0.35	94	1.8	**0.035**
	Control	48	1.38	0.47			
Analysis	Experimental	48	1.26	0.57	94	1.9	**0.032**
	Control	48	1.40	0.69			
Evaluation	Experimental	48	1.79	0.68	94	2.8	**0.003**
	Control	48	2.04	0.58			
Creation	Experimental	48	1.45	0.77	94	1.7	**0.043**
	Control	48	1.55	0.73			

Note. *$p<0.05$

the more students are answering wrong and/or not answering the questions, implicating that they cannot solve the questions. The lower the mean values are the more students are answering the questions correctly.

The pre-test scores show that there is no significant difference between the performances of the two groups. However, it is worth noting that the mean values of the experimental group are smaller than that of the control group at all the different levels, thus suggesting that students in the experimental group perform slightly better than their colleagues in the control group. One would have expected that the students would perform better in the remembering and understanding questions as these questions are low-level questions, but it was in the questions about the application that both groups performed better. The questions testing evaluation and creation abilities are the ones that a majority of the students found challenging. But differences in student performances in both groups are small and there are no statistically significant differences, which legitimizes the need for further comparison between these groups after the intervention and from the opinion poll.

The post-test shows an improvement in performance by both groups for all the cognitive domains. Similar to results from the pre-test, the experimental group performs better than the control group at all levels, but now with a statistically significant difference between the two groups. The most significant change in performance was on the items measuring students' creation skills. Here there are 0.46 and 0.73 increases for experimental and control groups respectively, and there is a significantly better performance by the experimental group compared with the control group within this category, with a p-value of 0.043. The second most significant difference was observed in questions testing students remembering and understanding, with a 0.45 and 0.18 increase for experimental and control groups respectively. This domain also reported a significant difference between the groups in the post-test, with a p-value of 0.002. This suggest that all students' understanding of the concept of functions increased significantly after the sessions and the revision practice.

RQ2: In what ways can Desmos contribute to students' knowledge acquisition when they solve tasks of functions?

The purpose of this research question is to examine students' views on how the use of Desmos may have contributed to their understanding, knowledge, and general learning experiences. To answer this, the students from the experimental groups were asked to respond to 16 questions (seven questions about how the use of Desmos has influenced their knowledge and understanding and nine questions on how it has affected their general learning experiences) using a 5-point Likert scale (0=do not know, 1=strongly disagree, 2=partially disagree, 3=partially agree, 4=strongly agree). The results are presented in **Figure 4** and **Figure 5**.

The results in **Figure 4** display that students answered positively to almost all the statements implying that using the digital tool has contributed to their knowledge gains. That is consistent with the results from the post-test assessment, which showed improved performance on all tested parts of the function subject.

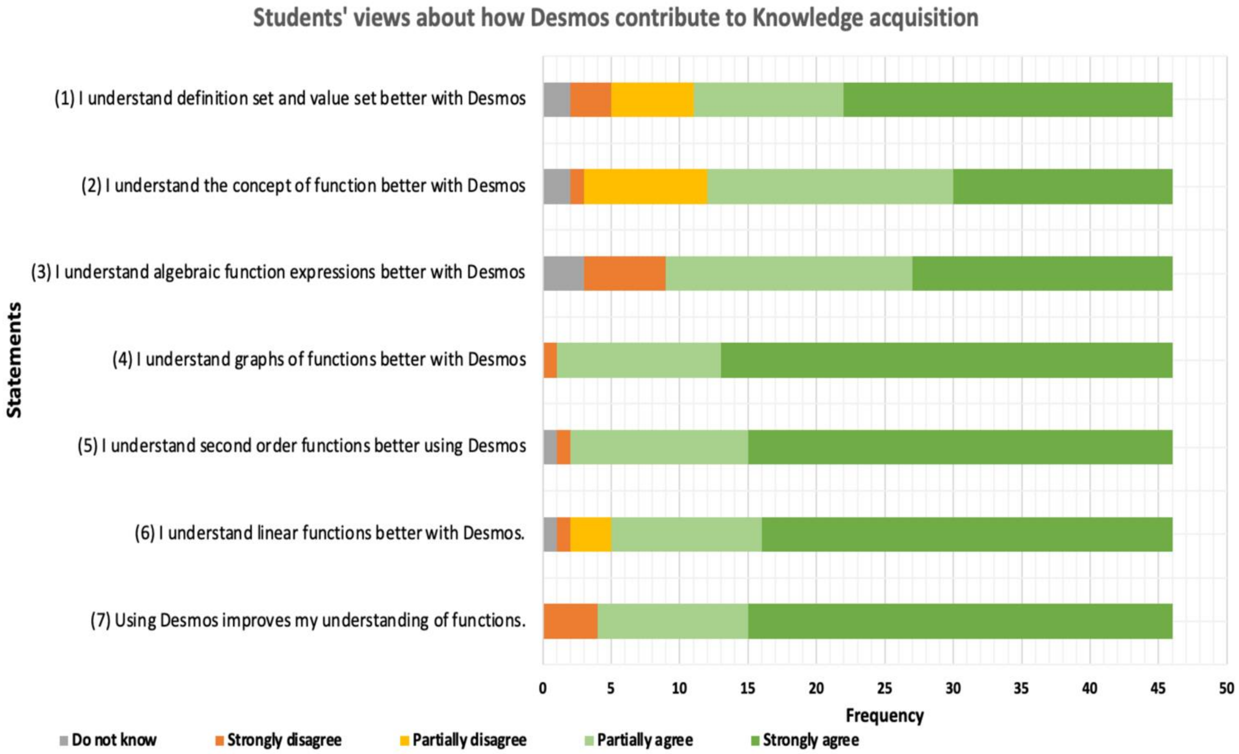

Figure 4. Students' views on how Desmos contribute to knowledge acquisition (Source: Authors' own elaboration)

More specifically, there are very high perceived knowledge gains on every question related to graphical representation or multiple representations, which is reasonable when using such a digital tool (**Figure 4**). The perceived knowledge gains were low in questions related directly to algebraic representations and concept of functions in general. For example, concerning the question "I understand algebraic function expressions better with Desmos", the majority (80.4%) of the students ascribed positively to the statement, but nine students (19.6%) were either not sure or disagreed strongly with the statement. Similarly, 12 students (26.1%) were either not sure or disagreed with the statement "I understand concept of function better with Desmos".

Figure 5 presents the students' views regarding how the use of Desmos has contributed to their learning

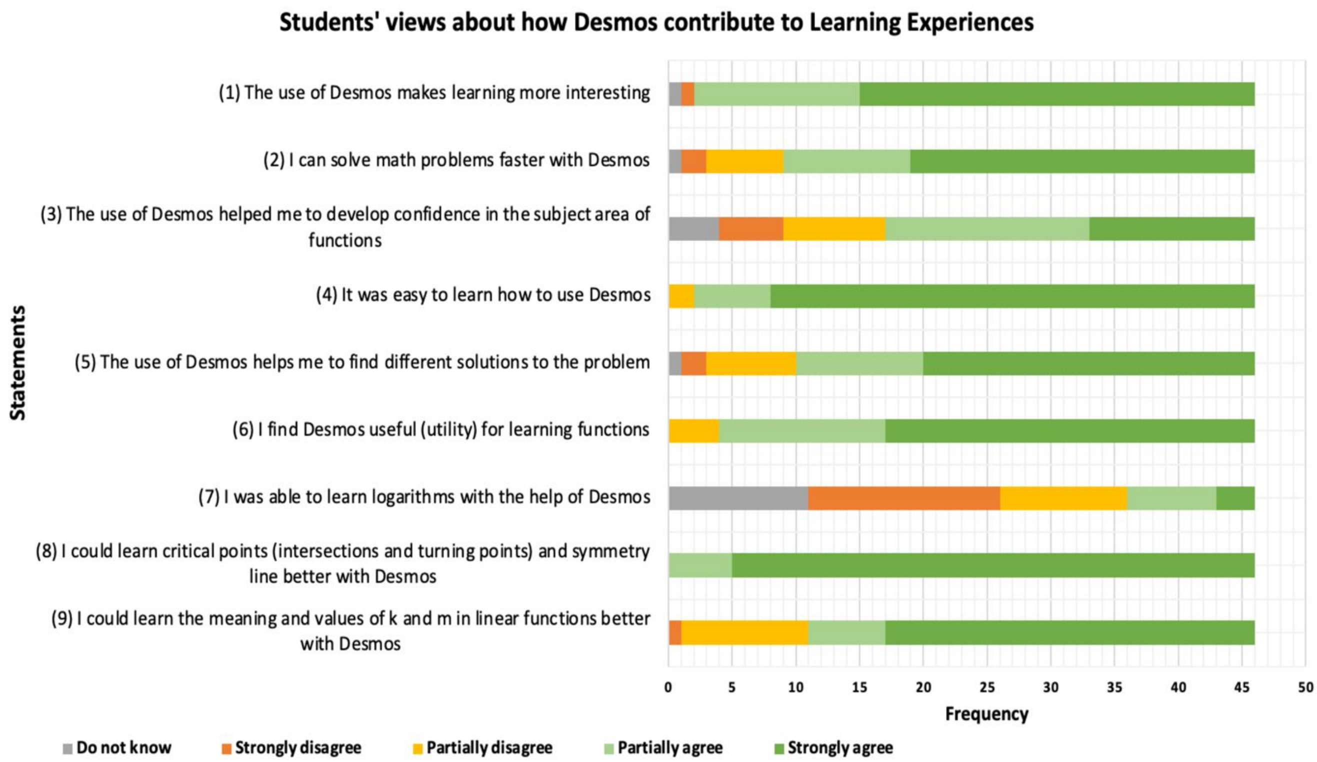

Figure 5. Students' views about how Desmos contribute to learning experiences (Source: Authors' own elaboration)

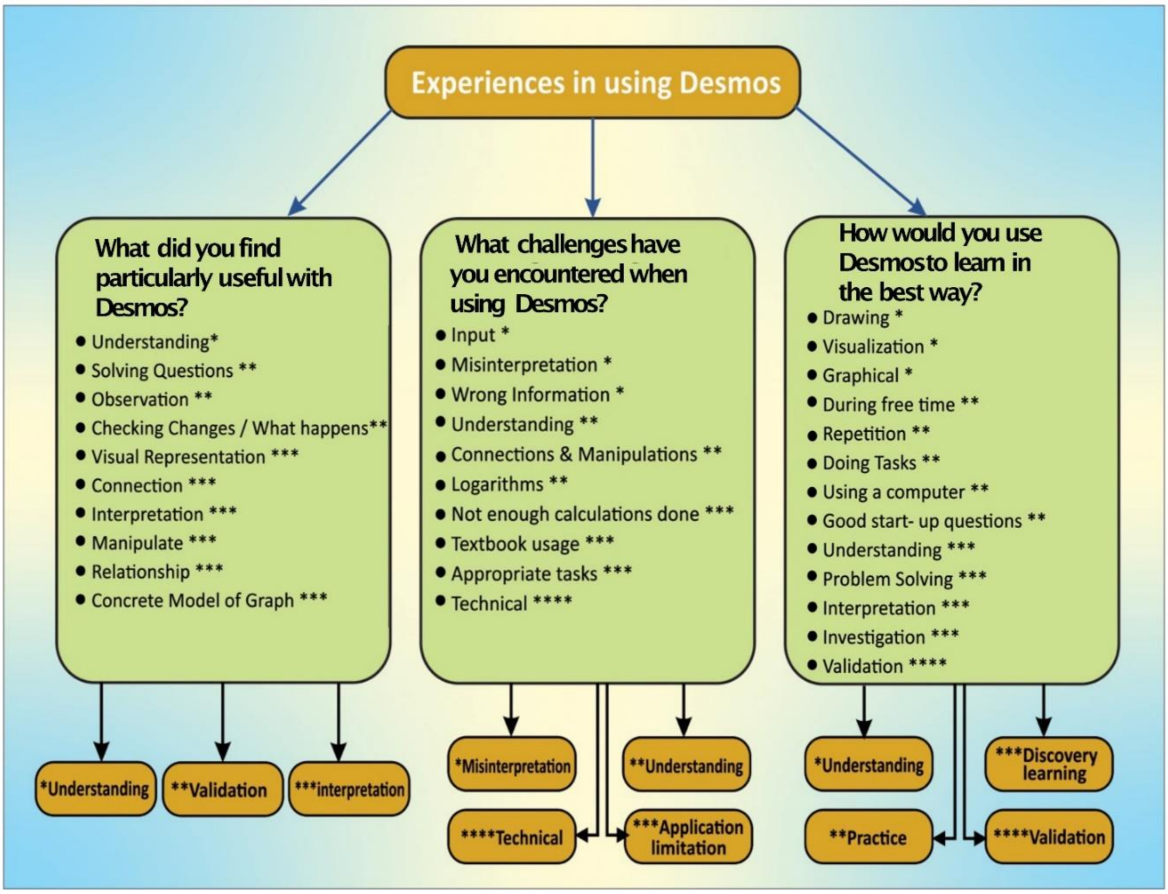

Figure 6. Visual presentation of codes & themes/categories (Source: Authors' own elaboration)

experiences. The answers from the majority of the students show that their learning experiences with Desmos have been generally positive. That is evident in their responses regarding how easy they found the use of the tool, how they could use the tool for multiple representations, and be able to solve questions much faster. However, the use of Desmos does not seem to have increased the students´ self-confidence prominently in terms of knowledge of functions, but a majority of the students still answered positively to the statement (statement number 3 in **Figure 5**). A majority of the students (78.3%) ascribed negatively to the question "I was able to learn logarithms with the help of Desmos" and 11 students (23.9%) were either not sure or disagreed to the statement "I could learn the meaning and values of k and m in linear functions better with Desmos". On the other hand, every student agreed to some point to the statement about learning critical points better (statement number 8).

RQ3: How do students perceive the effects of using Desmos on their understanding of functions?

The last research question sought to examine students' perceived effect of using Desmos on their learning of functions. The answers were collected from three open-ended questions: usefulness of Desmos, challenges in using Desmos, and how to best use Desmos. We followed Braun and Clarke's (2006) six-level thematic analysis process when analyzing this qualitative data. For each question, we read through all the students' answers and found words/expressions and meanings that we gave a specific "code" (presented in the green boxes in **Figure 6**). The codes were then put into categories/themes (in orange boxes in **Figure 6**), and the major themes found in the students' answers to each question are presented in **Figure 6**.

After reading through the responses from the students, we identified 10 different reasons (as shown in the first box in **Figure 6**) from the answers to the question about what they found useful with Desmos. We also identified 10 challenges from their responses as shown in the second box in **Figure 6**, and 13 reasons were identified for the question about how they will use Desmos to learn in the best way. These codes were then put into three, four, and four different categories respectively, as shown beneath the boxes in **Figure 6**.

The symbols *, **, and *** were used to differentiate the codes that were used to generate the different categories. For example, in the first box, which presents students responses about what they find useful with Desmos, *, represents understanding, and ** is for validation and *** for interpretation categories.

The reflections from the participants regarding the usefulness of Desmos mainly concerned the use of the tool for understanding, validation, and interpretation. According to them, understanding was the main usefulness of the digital tool, as they indicated that it helped them to understand the concept of functions better. For example, one student reported that:

> To understand tasks that could not be solved on my own. The answer is only given in the answer sheet, but with Desmos it is possible to understand why the answer turned out the way it did, even if you could not solve the problem on your own. Value and function sets were also very easy to understand with Desmos. It was also particularly useful to examine the *a*, *b*, and *c* values of the function $y=ax^2+bx+c$ with Desmos.

Also, another student described how use of Desmos had helped him/her understand how graphs are drawn and the meaning of the components of the graph.

> Desmos helps you to understand a function graphically. Just looking at an equation can be confusing, but when the function is plotted in front of you and changes as you change the graph, it's easier to understand what each component means in the function.

Almost all the respondents indicated that they could use Desmos to check their results and understand why they got those results, as they could manipulate the variables in the equations to get a clearer picture of the question and the answer. For example, one student indicated that:

> you could quickly check your answers and sometimes solve problems graphically when you would otherwise have had to do the quadratic formula with many decimals, for example. It was also possible to understand how the different variables in a function affected the graph.

Four main challenges were identified from the students' responses: lack of understanding, misinterpretation, application limitation, and technical issues. Some students had challenges with understanding how to use the tool. For example, one student reported that:

> it was a bit hard to understand how to use Desmos at first, but after a while, you got the hang of it. Maybe some people just use it as a recipe, instead of experimenting and thinking, which I think is best.

This re-echoes the previous view that there is a need to allow students to gain a complete understanding of the different ways that Desmos could be used, not just for solving graphical problems in functions. This was expressed by another student who indicated that it is

> hard to understand how to use it for logarithms.

Misinterpretation, which includes the input of wrong information and not understanding how to read and interpret results, was also evident in the students' responses. When describing some of the challenges they had, one student indicated:

> sometimes I input wrong information and Desmos misinterprets it.

Similarly, another student also reported:

> sometimes I would accidentally type in the wrong number, and it would be chaos.

If students keep receiving such wrong answers from the tool, this could be a possible deterrent for students and their learning. The students' ability to be independent and take responsibility for their learning can be the start of developing critical thinking skills. Thus, students may need to have some level of willingness and ability to use the tool, so they can take benefit from its use. Hence the students were asked to indicate how they think they could better make use of Desmos, and four main themes were evident in their responses: understanding (ability to classify relations and compare functions), practice (ability to use the tool to draw graphs of functions), discovery learning (problem-solving and investigations), and validation (checking of results). Discovery learning through investigation and problem-solving is one of the ways to utilize Desmos, and some of the responses from the students were consistent with that:

> to investigate why the answer turned out the way it did when you did not understand the task/could not answer it. Also, to investigate a, b, c value in second-order functions and k and m value in linear functions.

Learning through verification and validation of results could be an added benefit as it provides alternative solutions that could be tested. Apart from using Desmos only to check for correct answers, one student suggested using it to get an in-depth understanding of the concept.

> I would first solve a problem and then put the equation/function into Desmos to check my answer and see the equation in front of me for deeper understanding. If I did not understand a task, I would put the equation/function into Desmos to get an overview and to be able to see in front of me what was in the numbers.

Similarly, another student was of the view that he/she would not use the tool only to check the answers but to have a clearer picture of the situation under consideration:

> To check my answers but also to use the Desmos to draw different graphs so you can learn how things are related to each other and how the graph changes depending on what values you put in. Through Desmos you can easily get a clearer picture of the graph unlike if you were to draw it by hand.

Triangulation of the Study Results

The present study adopted methodological and data triangulation approaches to help understand the issues under consideration. As highlighted above, the use of embedded mixed methods design helped the researchers examine the issue from different methodological perspectives. Apart from methodological triangulation, data triangulation is one of the essential techniques in qualitative research, as highlighted by Creswell (2013). First of all, the integration of the results from the quasi-experimental design and the interviews shows the usefulness of Desmos in learning functions. The quantitative data shows a significant improvement in students' performance after they were introduced to the use of the tool, and this was reflected in the students' responses in the opinion poll, as a more significant majority of the respondents indicated that the use of the tool has helped them to develop conceptual understanding of the concept. Apart from the triangulation of quantitative and qualitative data, the authors examined the different responses received under the sub-headings, how Desmos contribute to knowledge acquisition (**Figure 4**), how Desmos has contributed to students' learning experiences (**Figure 5**), and the usefulness of Desmos (**Figure 6**). A critical analysis of the data from these responses shows that using the tool to understand the concept of functions was paramount. As a result of the tool, the students were able to know how the graph of a function changes when the components of a function are manipulated, which students may find difficult to understand when using pencil and paper.

DISCUSSION

The purpose of this study was to examine the effect of the use of Desmos on students understanding and learning of functions and their perceptions of the use of Desmos. Like the findings of other research, for example, Zulnaidi and Zakaria (2012), this study showed that students who used Desmos during lessons had a significant increase in their performance as compared with their colleagues in the control group. In addition to this, the majority of the students were of the view that the use of Desmos has not only given them an understanding of the concept but has provided a different perspective of examining the behavior of different functions something that they could not do using paper and pencil.

Thomas (2016) conducted a study also showing the positive effects of using Desmos in mathematics education, and Birgin and Acar (2022) and Shahriari (2019) have shown that the use of different digital tools also improves students' learning of functions, all supporting and agreeing with our findings.

Despite the numerous advantages indicated by the students, students' understanding and ability to apply the digital tool in learning other mathematical concepts is an issue of concern, though this was not mainly investigated in our study. Similar to the findings by Godwin and Sutherland (2004) the results show that some of the students were not confident when it comes to how to use Desmos for learning mathematical concepts, such as function (statement 3 in **Figure 6**). This could be explained by that the students did not have time to use the tool for all the different types of functions in a class, we focused on the non-linear during the period of the study even though the pre- and post-test covered both linear and non-linear functions. Another factor that could affect the development of confidence in functions could be that when students started using Desmos they opened a new world, where they only got to discover a small part of it, which left them with feeling that there was a lot left they had not explored and learned.

There is a plethora of research and discussions about the integration of technology in teaching and learning and more specifically in mathematics, however, this could be influenced by the way we see mathematics as evident in the work of Olive and Makar (2010) who argued that:

> if one considers mathematics to be a fixed body of knowledge to be learned, then the role of technology in this process would be primarily that of an efficiency tool, i.e., helping the learner to do mathematics more efficiently. However, if we consider the technological tools as providing access to new understandings of relations, processes, and purposes, then the role of technology relates to a conceptual construction kit (p. 138).

Despite the complex nature of integrating technology as teachers are required to have some key competencies, its positive effect on students learning and achievement cannot be underestimated (Thomas & Chinnappan, 2008). For example, Damick (2015) in her analysis of implementing technology in an algebra classroom argued that:

> in mathematics, it is important to understand the difficulty students have with understanding abstract concepts. Technology is a strategy that can make abstract concepts concrete. Once a student has understood the concept, technology can help students achieve fluency in mathematical facts (p. 14).

This study shows a positive effect of the use of Desmos in the teaching of functions. Other mathematical concepts can be taught with this tool that we did not focus on, such as logarithms (exponential functions). The students' answer to the statement about this area shows that only touching the concepts briefly is not enough for the students to learn it with the tool. There needs to be plans and time for students to learn how to use the tool to better understand concepts. Just handing them a tool is not enough. This is a call for teachers to be innovative in providing students with the opportunity to learn how to use the tool with other concepts or make students aware of the limitations of the tool and provide other alternative tools that could be used for learning the other mathematical concept. However, as noted by Jankvist et al. (2019) technology and for that matter, digital tools could not be used in every situation and can also foster misconceptions. Based on students' views about how best to use Desmos, students indicated that the use of digital learning approaches could be considered a great asset in our quest for training students in problem-solving, discovery, and experiential learning approaches to construct new knowledge.

Limitations

The current study is limited in scope as it is only observes one upper secondary school in the Stockholm Region of Sweden. Hence the results cannot be generalized to the larger population of all upper secondary schools in Sweden. Expanding this to other schools in Sweden would have added some more value to the results to get a holistic picture of the situation under discussion. Also, one factor that needs to be discussed is the role of one of the authors as the teacher for the experimental group. Her role as a teacher and part of the research team could have influenced the experimental group's results in one way or another. Students in the experimental group could have benefited when answering the tasks and using Desmos to solve the questions since they may be familiar with the teacher's teachings, practices, and expectations. But at the same time, it was important to do the experiment with her students because she wanted to know how to test the implementation of the digital tool and the limitations/issues associated with it. It could be argued that using other teachers and students would have been appropriate as this could help reduce the Hawthorne effect. However, the student groups participating in this study have received instruction from the same teachers throughout an 18 month period prior to the intervention. The pre-test results indicate that the groups had comparable abilities regarding concepts and operations regarding functions. While it is conceivable that the positive outcome is due to some variant of the Hawthorne effect, we consider this extremely unlikely. Using Desmos in one class with an expert instructor cannot be considered to bias the results. In addition, this study employed opinion polls, where the participants were asked to provide in-depth descriptions of their experiences; using interviews would have been helpful, and the researchers would have had the opportunity to ask some follow-up questions.

CONCLUSIONS

This study examined the effect of Desmos on students' learning experiences and performance. The findings from this study show that despite the significant increase in students' performances from both the experimental and control group when comparing their pre- and post-test scores, students from the experimental group made a significantly greater improvement than the control group. The use of Desmos seemed to affect the students' understanding and remembering, and creation skills the most, but also all the other domains of questions we examined in this study. Through our survey, we also display that the students found the use of Desmos positively influencing their understanding of the concept of functions, and they could use it to check and explore their answers.

It is worth noting that introducing the digital tool was not without challenges. For example, one of the challenges had to do with the availability of laptops or computers during the lessons, as not all students brought their laptops, so they either had to find one or share with their peers. Time was another challenge. Since this was the first time the students were introduced to using this tool, it took a long time before they got ready and set for the lessons, as most students were not sure that they had to do when using the tool. Individual students' preparedness was another challenge as they were not sure what this new way of teaching and learning was going to be like and what kind of adjustments in their studying routines they had to make. However, the students who used the tool were eager to use innovative approaches such as problem-solving and discovery learning. For example, a student indicated that the best way to learn by using Desmos is

> to investigate why the answer turned out the way it did when you did not understand the task/could not answer it. Also, to investigate a, b, c value in second-order functions.

Some students also had challenges using the tool for learning functions and tried solving some questions using paper and pen, suggesting that some things are easier to learn than others with Desmos. Thus, there could be a need to look into how students can be supported to use the tool better in other mathematical concepts to create a better understanding of functions when using the tool. It can be argued that students should explore all the different ways that they can use the digital tool, to learn many general mathematical concepts and experience the full benefits of using the tool.

Conclusively, one can argue that the use of such technologies is likely to increase students' mathematical fluency and appreciation, which in turn will change their attitudes toward the learning of the subject. One of the main limitations of the current study has to do with the scope and sample size. Since this study was conducted in only one school, we reason that to continue advancing this research field, it is important to consider expanding the scope of the study to various student groups, different mathematical classes, and different school types (well and less-resourced schools). Evidence from this investigation suggests the existence of challenges that the students can face and a lack of confidence in using the tool among the students. We thus point to a need for further studies to examine the use of Desmos, or other digital tools in the learning of mathematical concepts and to investigate the implementations of these tools. But we can conclude that the use of Desmos when learning about non-linear functions resulted in good knowledge amongst students, better visualization, and potentially a better grip on this mathematical concept.

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Ethical statement: Authors stated that informed consent, voluntary participation, and confidentiality were adhered to during this study. The study's purpose and role in this research were explained to the students. A written informed consent was signed by all participants. The students were informed that their participation was voluntary and that they could quit the study anytime through a written form. No traceable data or information was collected during the data collection process, and the students were provided with unique identification codes when handling their data. No names are used when presenting results from the study, and there is also no information about how to identify any student. Since the participants were over 16 years old no official ethical permit was needed..

Declaration of interest: No conflict of interest is declared by authors.

Data sharing statement: The data for this research (which is in Swedish) could be made available for use when needed.

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APPENDIX A: TASKS

Pre-Test and Post-Test

This study aims to collect data to understand students' understanding of the concept of functions.

All answers will be kept in strict confidentiality. To maintain your privacy, your name and any private information that identifies you will not be collected in this study.

Your answers will only be used for research purposes and will not affect your course grade.

Thank you for your help

_____ Yes, I will participate in the study by allowing the researcher access to my answers.

_____ No, I do not want to participate in the study by allowing the researcher access to my answers.

If you are participating in the study, please provide the following information.

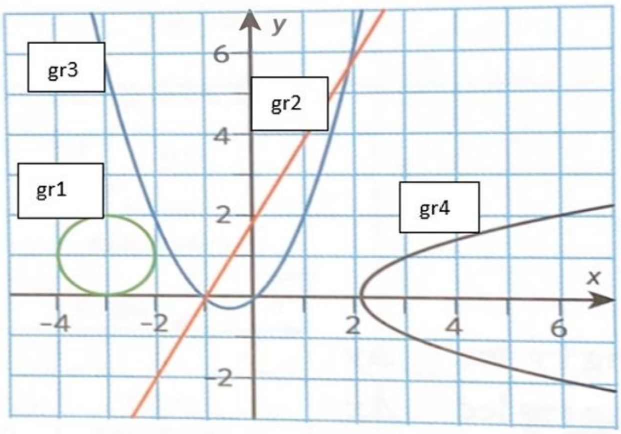
Class: __________________

Age:___________________

Gender: Female / Male / Other

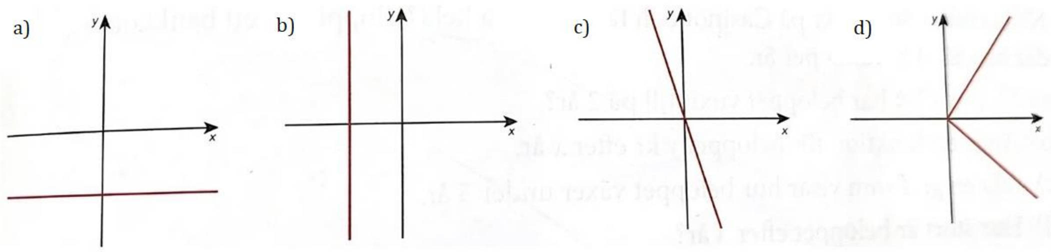
Answer the following 20 questions. Please read each of the questions carefully and check your answers. The written answers are very important, so try to give a full explanation/account where asked.

1. For the function $f(x) = 2x - 1$, calculate $f(5)$.
   $f(5) =$ --
2. For the function $f(x) = x^2 + 4$, calculate $f(2)$
   $f(2) =$ --
3. For function $f(x) = -3x^2,\ calculate\ f(-1)$
   $f(-1) =$ --
4. Which of the graphs show a function?



   a) Graph 1: ---------------
   b) Graph 2: ---------------
   c) Graph 3: ---------------
   d) Graph 4: ---------------
   e) I don’t know: ---------------

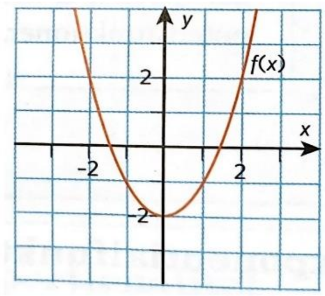
5. Which of the following graphs describe functions?



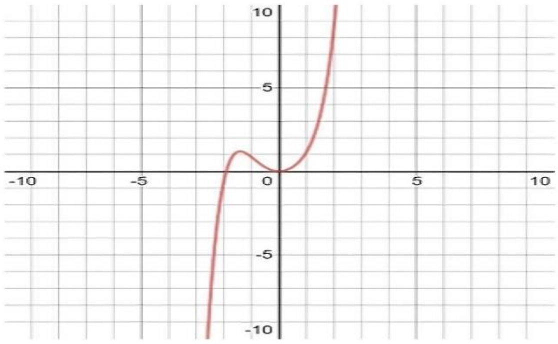
   a) Graph a: ---------------
   b) Graph b: ---------------
   c) Graph c: ---------------
   d) Graph d: ---------------
   e) I don't know: ---------------

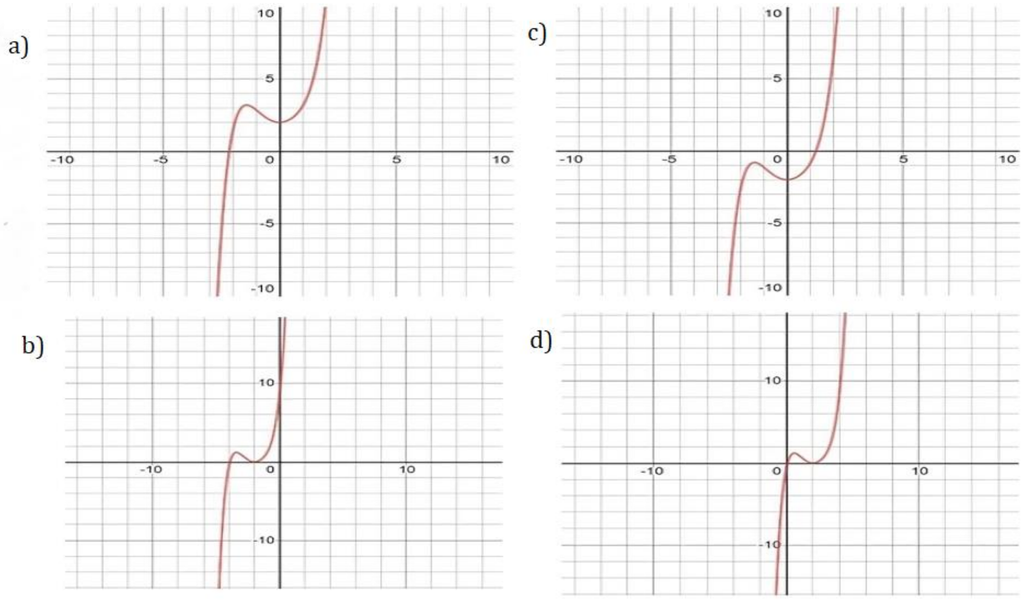
Explanation: ---
----

6. Solve the following problems using the graph
   a) $f(2)=$ ---------------
   b) $f(0)=$ ---------------
   c) $f(x) = 2, x =$ ---------------
   d) $f(x) = -1, x =$ ---------------



7. If the following is the graph of $f(x)$, which of the graphs a, b, c or d represents the graph of $f(x) - 2$? Mark the answer.





8. Decide whether $y = x^2 + 4$ is a function or not.

--------------- a) yes

--------------- b) no

--------------- c) don't know

Explanation: --

----

9. Determine whether or not $x = y^2 - 3$ is a function or not

--------------- a) yes

--------------- b) no

--------------- c) don't know

Explanation: --

----

10. Determine whether $y = x^3 - 7$ is a function or not.

--------------- a) yes

--------------- b) no

--------------- c) don't know

Explanation: --

----

11. Is the following relation a function $\{(-4,3),(6,-3),(-2,3),(-1,-3)\}$

--------------- a) yes

--------------- b) no

--------------- c) don't know

Explanation: --

----

12. Is the following relation a function?

x	y
-2	2
3	-2
-2	3
-3	-1

--------------- a) yes

--------------- b) no

--------------- c) don't know

Explanation: --

----

13. Is the following relation a function?

$\{(-1,3),(4,1),(-1,2),(-1,-3)\}$

--------------- a) yes

--------------- b) no

--------------- c) don't know

Explanation: --

----

14. What is the domain of the function $y = \frac{1}{x+2}$?

Answer: --

Explanation: --

----

15. What is the domain of the function $g(x) = \sqrt{x}$

Answer: --

Explanation: --

----

16. Determine x such that $f(x) = 30$ since $f(x) = 2(x+5)$

Answer: --

Explanation: --

----

17. The function $f(x) = x + 2$ is defined for $0 \leq x \leq 6$. What is the value set of the function?

Answer: --

Explanation: --

----

18. The functions $f(x) = x^2 + 6x - 3$ and $g(x) = x^2 + 4x - 5$ are given. Solve the equation $f(x) = g(x)$

Answer: --

Explanation: --

----

19. Bo examines two linear relations $f(x) = 3 + 2x$ and $g(x) = 4 + 1{,}5x$.

   a. Which relationship grows fastest?

Answer: --

Explanation: --

----

   b. Which relationship has the greatest value when $x = -2$

Answer: --

Explanation: --

----

20. The volume of water in a pool that is emptied can be described with the formula $V(t) = 600 - 2t$ m³, where t is the number of minutes the pool is emptied.
    a) Calculate and explain with words what $V(60)$ means.
    b) How much time does it take to empty the whole pool?
    c) Identify the domain and range of the function.

Answer:

a) --
----

b) --
----

c) Domain: --
Range: --