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Elementary Mathematics Teachers' Perceptions and Lived Experiences on Mathematical Communication

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Mathematical thinking skills and meaningful mathematical understanding are among the goals of current mathematics education. There is a wide consensus among scholars about the purpose of developing mathematical understanding and higher order thinking skills in students. However, how to develop those skills in classroom settings is an area that still needs attention. Fostering mathematical communication in classroom settings is a strategy to develop students thinking skills by mathematical talk, discussion and activities. The purpose of the study is to explore teachers' perceptions and lived experiences about using mathematical communication in their classrooms. The study is qualitative and employed a phenomenological approach to gain in-depth insight about the nature of mathematical communication in the classroom setting. Participants were experienced elementary school mathematics teachers. They discussed about the concepts associated with, strategies to enhance, purposes and benefits and application of mathematical communication as well as impact of mathematical communication on thinking skills, during the interviews. The findings are expected to provide a basis for discussion about using mathematical communication to foster students' higher order thinking skills and mathematical understanding.

Keywords: elementary, mathematical communication, mathematical understanding, mathematical thinking skills, phenomenology

INTRODUCTION

Current national elementary school mathematics education curriculum in Turkey strongly emphasizes students' higher order mathematical thinking skills. Students are expected to solve and construct mathematical problems that require the utilization of higher order thinking skills such as

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critical thinking and reasoning (Ministry of National Education (MoNE), 2009). In addition, international assessment institutions emphasize the need to formulate, make use of and interpret mathematics in a variety of contexts (OECD, 2010). To be able to master those tasks students have to employ skills like mathematical reasoning and problem solving. Higher order thinking skills are also mentioned as essential for coping with the demands of 21st century (Silva, 2009).

Although there is a wide consensus about the importance of developing mathematical thinking skills, it is also worthwhile to discuss how to foster those skills in educational settings. A number of studies have been conducted to find effective ways to develop mathematical understanding and thinking skills. A considerable amount of studies have focused on the concept of mathematical communication as an important classroom activity to foster mathematical thinking skills (Brendefur & Frykholm, 2000; Franke et. al, 2009; Pape, Bell & Yetkin, 2003). Moreover, communication is recognized as one of the essential tools for developing students' conceptual understanding, thinking, problem-solving skills, and reasoning in mathematics (Jung & Reifel, 2011). In Turkey, mathematical communication is also emphasized as an important classroom activity in national elementary school mathematics curriculum (MoNE, 2009).

Despite the wide emphasis on mathematical thinking skills in elementary mathematics curriculum, the overall mathematics achievement of students in Turkey is far from being exemplary. The evidence for this underachievement can be found both in international and national examinations. For example, the results of international assessment programs such as PISA (2006, 2009, 2012) and Trends in International Mathematics and Science Study (TIMSS) (1999, 2007), pointed out that Turkey is one of the lowest performing countries. As a matter of fact, students from Turkey are not successful in engaging tasks involving reasoning and problem solving skills (International Association for the Evaluation of Educational Achievement, 2008; OECD, 2010).

Considering students' performance in mathematical thinking which is described above, it can be inferred that teachers need to employ classroom activities to be more effective in developing their students' mathematical understanding and thinking skills. In this context, the present study focused on mathematical communication as an important way to foster students' mathematical thinking. The study is carried out among experienced teachers working in elementary schools, thus it is important to understand their views and experiences on mathematical communication as a tool for developing mathematical thinking skills. Understanding teachers' views and experiences will provide a basis to discuss about using mathematical communication to support students' mathematical thinking and understanding processes.

Regarding this context, the purpose of this study is to explore experienced elementary mathematics teachers' perceptions and lived experiences of using mathematical communication to foster students' mathematical thinking skills.

In this vein, this study addresses following research question: What are the perceptions and lived-experiences of elementary mathematics teachers on using mathematical communication in the classroom setting?

State of the literature

- Developing higher order mathematical skills such as problem solving, reasoning, conceptual understanding and critical thinking are amongst the prominent purposes of mathematical education.
- There is a considerable number of studies conducted on effective development of mathematical skills and understanding in classroom setting. Enhancing mathematical communication is one of those strategies, which focuses on students' mathematical thinking and understanding processes.
- Mathematical communication is defined as planned interaction in classroom setting, which includes strategies such as questioning, discussions and group activities. The purpose of mathematical communication is to encourage students to express, share and reflect on their ideas.

Contribution of this paper to the literature

- This study employed a qualitative design to gain an in-depth understanding about the nature of mathematical communication in classroom settings. Data was collected through in-depth interviews with experienced mathematics teachers.
- There is a limited number of studies using qualitative methods to explore teachers' perceptions of mathematical communication. Results of the study provide in-depth information about mathematical communication as a strategy to develop mathematical understanding and thinking skills.
- The study is expected to raise issues about strategies to develop mathematical thinking skills, using mathematical communication in classroom settings and teachers' role to facilitate students' thinking and understanding processes.

Literature Review

Several studies which have been investigating on mathematics education reveal that it is not adequate for students to learn and use only procedural and declarative knowledge (Kostos & Shin, 2010; Lynch & Bolyard, 2012). Recent research focuses on improving higher order thinking skills instead of merely attaining and using knowledge. For instance, several researchers, including Kilpatrick, Swafford and Findell (2001), indicated that mathematics education refers to conceptual understanding, strategic competence, adaptive reasoning, productive dispositions and procedural fluency. In other words, learning mathematics needs to address problem solving, showing and expressing ideas, recognizing patterns and transferring learned skills to original situations (Trafton & Claus, 1994). Romberg and Kaput (1999), on the other hand, emphasized mathematical expression, reasoning and generalization as the objectives of mathematical education. From these points on, it can be inferred that in mathematics education, there is a shift from doing mere calculations and applying procedural knowledge, towards developing students' higher order thinking skills.

Although the goals of mathematics education are widely accepted by scholars and educators, the issue of creating classroom environments to reach those goals still needs an effort to elaborate on (Hiebert et al. , 2005; Pape, Bell & Yetkin, 2003). Teachers have an essential role in creating an atmosphere which fosters mathematical thinking and understanding of their students. There are a number of studies on how to design such learning environments. Some of these studies focus on mathematical communication as a way of developing students' mathematical thinking (Brendefur & Frykholm, 2000; Cooke & Buchholz, 2005, Franke et. al, 2009; Pape, Bell & Yetkin, 2003; Sfard, 2001).

According to socio-cultural perspective on learning knowledge, skills and dispositions are developed through social interactions with more skilled others (Bandura, 1969). Gupta (2008), indicates that when students interact and communicate with each other skills such as understanding, meta-cognition and motivation are most likely to develop. Mathematics inquiry is also described as an apprenticeship model where mathematical thinking skills are developed within reflective classroom communication (Cobb, Boufi, McClain & Whitenack, 1997). Various forms of communication are considered central to explore and deepen students' understanding of mathematical ideas, and make connections between other concepts of mathematics and other fields of knowledge (Hiebert, 1992). Facilitating student talk on mathematical problems, concepts and procedures enhance students' understanding so that they can make deeper and clearer connections (Chapin, O'Connor & Anderson, 2003). Furthermore, Sfard (2001) pointed out the importance of mathematical communication by describing thinking as a case of communication. In other words, thinking is a dialogical effort, where one asks questions, investigate possible solutions and reflect upon them.

In a classroom environment where mathematical communication is employed, students are expected to listen, comment and reflect on their friends' mathematical thinking (Pape, Bell & Yetkin, 2003). In such an environment teachers wisely use questioning techniques to understand their students' mathematical thinking. Posing questions also creates an opportunity for students to reflect upon and develop their mathematical thinking. In a study focusing on informal strategies to support communication, Cooke & Buchholz (2005), described strategies to enhance mathematical thinking of the students as follows:

Providing opportunities for self expression
Serving as a facilitator
Providing opportunities for students to connect new understandings to prior knowledge
Connecting administrative task/classroom routines to mathematics
Asking a variety of questions
Encouraging the use of appropriate math terms (pp. 366-368)

Chapin, O'Connor and Anderson (2003) defined effective mathematical communication as "a respectful but engaged conversation in which students can clarify their own thinking and learn from others through talk" (p.5). Using appropriate guidelines (Bruce, 2007), working with students on prerequisite skills (Mooney, Briggs, Fletcher, Hansen & McCullouch, 2009) and creating a respectful environment (Chapin, O'Connor & Anderson, 2003) are among the suggestions to enhance an effective mathematical communication in the classroom.Studies show that mathematical communication is not employed sufficiently in classroom settings. For instance in the study of Yang (2013), students pointed out that activities fostering classroom communication such as cooperating with peers and participating in-class discussions were employed less by the teachers.

Reviewing the literature, it can be inferred that to achieve new goals of mathematics education, creating learning environments which foster mathematical communication is essential. Moreover, teachers have a key role in creating classroom environments where mathematical communication is an indispensable dimension of mathematics instruction. Understanding teachers' perceptions and experiences on using mathematical communication classroom setting is expected to lead to a discussion

about developing students' higher order thinking skills and mathematical understanding through talking, sharing and asking questions in the classroom.

METHOD

Research Design

This study was employed a qualitative research design with a phenomenological approach to describe mathematic teachers' perceptions and lived experiences on mathematical communication in classroom setting. Qualitative methods provided the means to "grasp and sense the lived experience" of participants on the nature of mathematical communication (Berg, 2001; Creswell, 2012; Streubert & Carpenter, 1999). Based on the review of literature and the lack of research examining perceptions about mathematical communications in classroom, the research question was refined to an open-ended guide. This qualitative inquiry employed a phenomenological approach, which is used to investigate how an individual makes sense of an experience and transforms this experience into consciousness (Hart & Swars, 2009, p.162). In a phenomenological study, the researcher gains insight of the phenomenon of interest through interviewing knowledgeable participants (Flower & Schraw, 2000). Specifically, this study was sought to explore lived experiences of elementary mathematics teachers, to understand the nature and usage of mathematical communication in classroom setting.

Participants

A purposive, convenience sample was recruited from a population of participants who had taught mathematics at least four years at an elementary school. Participants were selected based on their lived experiences with the phenomenon and the ability to share that knowledge on mathematical communication. This type of sampling was used to obtain information-rich cases for in-depth analysis (Denzin & Lincoln, 2005). Participants of this study were nine (n=9) mathematics teachers working in elementary schools in the largest metropolitan city of Turkey. All participants have bachelor's degree and four of them hold also master's degrees. According to school types, both public (5 teachers) and private schools (4 teachers) are represented. Years of experience ranged from 4 to 17 years. 2 participants were male, and 7 were female. Participants aged between 27 and 34 (m=31.9)

Data Collection

The data was collected via unstructured interviews with the participants. The individual interviews consisted of open-ended and in-depth questions about their perceptions and lived-experiences on description, usage and benefits of mathematical communication. An interview guide was prepared by the researcher and later revised by five experts on the field of curriculum and instruction. The questions in the interview guide were used as a starting point for further discussion about teachers' perceptions and experiences related with mathematical communication during the interviews. For the reliability and validity of the interview questions, first of all, five experts' opinions were taken and the questions were revised accordingly. These questions were applied to two participants as a pilot study and questions were finalized before the actual interviews took place.

The interviews lasted approximately 40-60 minutes. Each interview was audio-taped. Interviewing took place in different locations such as school, café and home according to participants' convenience. The participants were informed about the research process and assured that the information they give will be kept confidential. To assure voluntary participation, teachers signed a consent form and were informed that they can resign from the interview and research process whenever they prefer.

Data Analysis

After finishing the interviews the audio recordings were transcribed. The transcriptions were read many times and then classified accordingly to identify the themes. The analysis of data is done by hand. For the analysis, steps of (1) exploring the general sense of data, (2) coding the data and (3) specifying the themes were followed (Creswell, 2012). Transcription of the interviews was also interpreted by two different colleagues in the education field and themes were revised accordingly.

Rigor refers to establishing the credibility and trustworthiness of the data and in the present study it was demonstrated through attention to, and confirmation of, information discovery (Denzin & Lincoln, 2005). Lincoln and Guba (1985) have outlined the current gold standard criteria for qualitative researchers by which to establish the trustworthiness of qualitative data. These criteria include credibility, dependability, conformability, and transferability. Several operational techniques were used

to increase the likelihood that credible findings would be produced (Streubert & Carpenter, 1999). For example the transcripts were examined by different experts and this multiple examinations contributed to the trustworthiness of the analysis. After the themes were emerged, two of the participants were presented with the themes and asked for further interpretation and confirmation. According to the results of these interviews the themes were confirmed and found appropriate by the participants.

RESULTS

Thematic Analysis

Nine participant teachers responded the researchers' questions. Responses were categorized around five broad thematic topics: (1) Concepts associated with mathematical communication, (2) Strategies to enhance mathematical communication, (3) Purposes and benefits of mathematical communication, (4) Impact of mathematical communication on mathematical thinking skills, (5) Application of mathematical communication in the classroom. Results of this study were presented under these themes.

Theme One-Concepts Associated With Mathematical Communication

Participants associated different concepts with mathematical communication by their definitions. Three sub- themes emerged from the concepts that teachers used most frequently when defining mathematical communication: (a) understanding mathematics, (b) usage of mathematics in real life contexts and (c) usage of mathematical language.

Most of the participants emphasized the concept of understanding mathematics when they define mathematical communication. For example, one teacher's definition of mathematical communication underlined the importance of mutual understanding between teachers and students. He said that, *"mathematical communication is a process that my students can understand what I'm trying to explain to them and I can understand the answers they give to my questions."* This definition points out the multi-directional aspect of mathematical communication. In other words, the communication is not only from the teacher to the students, it can also occur from students to teachers (Mooney, Briggs, Hansen & McCullouch, 2009). Moreover, the emphasis is on understanding each other, which is the main purpose of communication (Sfard, 2001). Another two participants' definitions also emphasized mutual understanding. One of them said:

I can define mathematical communication as students' understanding of the topic…It (mathematical communication) encompasses the questions I ask students and the ones students ask to me.

The other teacher asserted, "*mathematical communication is the relationship between students and me when I try to enhance their understanding of the topic*"

Usage of mathematics in real life contexts is another concept used for the definition of mathematical communication. One teacher said, *" (mathematical communication) is that students can use their mathematical skills and transfer them into real life"*, the other one stated, *"…students application of knowledge in real life problems" and another teacher defined mathematical communication as "using mathematics in their own life"*. "It (mathematical communication) is how students use and transfer their mathematical knowledge in real life settings" was another teacher's definition emphasizing transfer of mathematical knowledge and skills in real life contexts. Most of the participants related mathematical understanding with the ability to apply knowledge in real life problems. This approach is consistent with the literature where application of knowledge is regarded as a form of understanding (Wiggins & McTighe, 2005). From another perspective, ability to use mathematical knowledge in real life settings is also seen as a form of communication of mathematical understanding.

Mathematical language is the third sub-theme that emerged from teachers' responses. One teacher said "*a mutual understanding of mathematical terms"*, similarly another teacher stated, *"students' usage of mathematical language". The definition of one teacher described a translation process: " (mathematical communication is) translating a verbally described problem into mathematical language or vice versa."* Using mathematical language is a prerequisite for effective mathematical communication. In order to discuss and talk about mathematical problems both students and teachers have to use mathematical language properly (Chapin, O'Connor & Anderson, 2003).

Theme Two-Strategies to enhance mathematical communication

Strategies identified by the participant teachers to enhance mathematical communication are (a) giving real life examples, (b) question-answer technique, (c) peer learning techniques, (d) activities and games

Teachers use real life examples to help students talk about a mathematical concept. They give and receive examples from the real life to enhance understanding and as a starting point for a discussion. One teacher explained "*I use real life examples most frequently. They are current for the children and they can understand more*". Teachers use real life examples to help their students grasp the mathematical language in real life settings. For example one teacher stated:

They (students) need to see the relationship between real life and mathematical language. What does x+4=5 mean (in real life)? It is very beneficial to talk about that. They need to understand what the numbers represent. Therefore, you need to ask how they interpret problems mathematically in real life.

Teachers use question and answer technique also very frequently. Almost all of the participant teachers employ this strategy, because it is an efficient way to help students express their ideas. Teachers underlined the importance of posing quality questions to create a fruitful mathematical communication environment. Some teachers' assertions were related with posing 'why' and 'what if' questions to enhance meaningful mathematical communication. For instance one teacher said, *"for example I ask why we slide one digit by multiplication (of two multi digit numbers). They (the students) answer 'because it is so', but they have to think about why this happens."* Another teacher underlined the importance of posing questions to understand the nature of mathematical operations by stating, "*I have to ask what does addition and subtraction mean"*. One teacher focused on questions to help students see the relationships between concepts, she said *"I ask them (students) questions to help them see the causal relationships in mathematics."* Another teacher uses questions to trigger students' thinking process, she said that, "*by explaining the topic I ask what happens if...questions to help them think about the subject to be learned."*

Chapin, O'Connor and Anderson (2003) also emphasized asking higher order questions. They pointed out that students can deepen their understanding of the operation and practice their computational skills by talking what they are doing and why.

Another common strategy used by the teachers is several forms of peer learning. This strategy is different from others regarding the direction of the communication. Strategies, such as real life examples and question-answer mostly suggest a dialogue between teacher and students. On the other hand, peer learning activities provide communication between students. It is surprising to get responses emphasizing peer learning strategies, since most of the teachers locate themselves in a major position even though they appreciate and employ different communication strategies. For example one teacher explained how and why she uses peer learning activities:

I use peer learning strategies. One student explains the problem to the other. Students do not see it as learning. When the teaching is not done by the teacher they are more open and listening to their friends more carefully. They can be resistant to the teacher, but they are not resistant to their friends.

Some teachers underlined the fact that students like and benefit from these activities. One of them said:

I let them (students) explain the subject to each other. They make good explanations to their friends. Sometimes they explain better than me. The children become so happy that they sometimes ask if their friends can explain the topic to them.

Two teachers emphasized the special communication that occurs between students, for example one of them said:

I don't understand what they (students) say, but they understand each other." The other teacher stated "because they (students) are at the same age they explain the topic different than me. If I can reach 10 students the student explaining the problem can help for example 13 students to understand the subject.

Some teachers, on the other hand, approach peer to peer communication with caution because of the risk of distraction. This point is very clear in one of the teacher statements, she said, *"I don't prefer student to student communication, because they can be easily distracted."*

Although participants use them very rarely, teachers appreciate employing hands-on activities and games to increase students' participation and to provide opportunities to help them express their ideas. One teacher focused on student participation when playing games by stating: *"when they are playing games, I only watch them talking about mathematics and sharing ideas"*. Participants also considered activities as important tools to set a framework for expression and discussion of students' ideas. For example one teacher said, "hands-on activities can also enhance mathematical communication". Another teacher gave an example for an effective activity, she said, *"in a 'data analysis' activity we talked and made lots of interpretations. Students discussed about the topic a lot and I felt very happy."*

Two unexpected responses emerged regarding strategies for mathematical communication. One of them is one teacher's usage of false examples to make students talk about what went wrong. She stated that, *"they all want to talk about the mistake; there is a noise in the classroom, but all students talk about the problem. I watch them with a smile on my face."* Another teacher uses technology to broaden the communication outside of the classroom. He stated that, *"we have a facebook group. I can communicate with my students' parents and give feedback to the students. They feel my presence also outside the classroom"*.

Teachers use a number of strategies to enhance mathematical communication in the classroom setting. They use real life examples, questions, peer learning, hands-on activities and games to create a discussion environment for mathematical ideas. One important point to make about these strategies is that most of them create a communication from the teachers to students. In other words, although teachers defined mathematical communication as multi-directional, they use techniques fostering mostly a one way communication in the classroom setting.

Theme Three-Purposes and benefits of mathematical communication

Participant teachers use mathematical communication strategies for many purposes. Three sub-themes emerged according to responses from the interviews: (a) Monitoring the learning process, (b) creating a positive learning environment, (c) facilitating the understanding of abstract topics.

Teachers most frequently use mathematical communication to monitor their students' learning process. In their assertions, teachers indicated both the importance of students' understanding of their mistakes and teacher's ability to assess their students' understanding. In other words, while students can see their strengths and weaknesses by talking about their thinking processes, teachers also find an opportunity to evaluate their students' learning and improve their instructions accordingly. Some teachers interpret students' effective mathematical communication as an evidence of understanding. One of them said, *"if a student gives his own example about a subject, it means he understood the subject." Another teacher stated, "students who use mathematical communication effectively are the students who understood the subject."* Teachers used students' mathematical expression for evaluation of their understanding. For example, one of them said, "(by enhancing mathematical communication) you can understand where he got the wrong idea and help him to understand the subject correctly." Another teacher stated, *"we learn about their (students') misconceptions of the subject"*. Some teachers gave clues about metacognitive processes in their statements. For example one teacher said, *"they think and discuss about what they and their friends think."* The other one stated, "one can never reach the speed of thinking, but when a student expresses himself, he has a chance to evaluate his thinking." Another teacher expressed, *"sharing the ideas is very important in mathematics. We have to help children talk (about mathematics) in order to make them see their mistakes"*

Using mathematical communication to create a positive learning environment is another important aspect especially for Turkey. As a nation not having the highest achievement levels at mathematics, students can have negative attitudes, fear and prejudice towards mathematics. When teachers use mathematical communication to embrace all of their students, even students with negative attitudes and fears towards mathematics can participate and have something to share. One teacher said, *"by communication with students we make them feel valuable and this increases their motivation to learning."* Another teacher emphasized the importance of helping less capable students to participate in class discussions by stating, *"when we ask an easy question to a less capable student and appreciate his answer, his motivation becomes higher"*. Some teachers believe mathematical communication facilitates students' positive approach towards mathematics as a science. For example one of them said *"their (students') self expression skills develop. When they begin to communicate with the teacher confidently they also communicate with mathematics."* Another teacher emphasized an important concept by stating, *"mathematical communication helps to develop a mathematics culture."*

Teachers repeatedly pointed out that communication is more useful to facilitate understanding of abstract topics. For example one teacher said, "*I prefer to use it in abstract topics. When you teach an abstract topic, it is important to help students communicate with their friends and me.*" Another teacher stated, *"they want to talk mostly about the abstract topics."* The trend of using mathematical communication by explaining abstract topics originates from students' need to talk about the abstract topics to make connections with previous learning, other concepts or real life examples (Michaels, O'Connor & Resnick, 2008; Van de Walle, Karp & Bay-Williams, 2010).

Theme Four-Impact of mathematical communication on mathematical thinking skills

Participants believe that mathematical communication improves mathematical thinking skills. It was common in their statements that they use mathematical communication as a tool to develop their students' mathematical thinking skills. Teachers mostly underline the impact of mathematical communication on conceptual understanding, problem solving, reasoning and critical thinking skills. In line with the participants' association of communication with understanding they also think communication develops conceptual understanding of their students. One teacher said *"when there is an interaction in the classroom, students engage with mathematics. When they engage, they try to understand the concept and apply the knowledge in their lives"*. Another teacher asserted, *"with mathematical communication they are doing an inquiry and develop connections"*. It is known that in conceptual understanding it is important to make interrelations between the concepts (Rittle-Johnson, Siegler & Alibali, 2001). Problem-solving and

reasoning skills are also stated among the thinking skills developed by mathematical communication. For example one teacher said, *"I ask what if questions and expect answers from the students to help them use their reasoning and problem solving skills"*, another teacher stated *"with mathematical communication, thinking skills, ability to see different viewpoints, critical thinking and problem solving skills become easier to develop"*. One other teacher asserted, *"there should be an interaction especially for the reasoning skills"*. Teachers also believe that by teacher's questions also students' critical thinking skills develop. For example one teacher stated that, *"when they start to think about my questions their critical thinking skills develop and they learn to ask "why" questions in other subjects."*

The relationship between mathematical communication and mathematical thinking skills is also widely discussed in the literature. Strategies such as questioning and discussion are related with the improvement of mathematical thinking skills of students (Cooke & Bucholz, 2005; Sfard, 2001).

One teacher reminded us the requirement of student motivation by using any form of communication to develop thinking skills. She stated clearly that "students do not think and read, they do not think why something happens. They mostly do not want to challenge themselves"

What the researcher observed during interviews is that although teachers stated that they believe mathematical communication is effective for developing mathematical thinking skills, they were not fully aware how communication enhances skills such as problem solving, reasoning and critical thinking. The responses they provided reflected weak relations with mathematical communication and mathematical thinking skills.

Theme Five-Application of mathematical communication in the classroom

Teachers asserted some (a) challenges and (b) suggestions for the use of mathematical communication on the classroom. Their responses are analyzed under these two sub-themes.

Participants mostly complained about the intensity of the curriculum as a challenge to employ mathematical communication strategies. For example one of them said, "*I don't have time and my class is very crowded. I cannot use it (mathematical communication) very often."* Another teacher asserted, *"I always defend child-centered education, but it does not always happen so. We have a very intense curriculum."* These are typical statements of teachers who wish to employ mathematical communication but mostly have no time. One teacher points out how stressful it becomes to be able to finish the curriculum on time:

We have to finish the curriculum at the end of the year. If we can't they (people in the management position) ask why. Or there is an exam and they ask us why your children's results are low? Therefore we cannot use it (mathematical communication) as much as we are supposed to.

Another teacher described how the curriculum should be organized in order to provide space for effective communication: "*the curriculum should give us an opportunity, for example, to discuss about one problem in a lesson. Then, every student will have something to say."*

Two participant teachers form a public school with crowded classes emphasized the challenge to have mathematical communication environment in over-crowded classes. One of them said:

Our class size is about 40-50. There is no opportunity for discussion. The teacher is speaking most of the time and short dialogues occur between the teacher and the students.

The second teacher stated that, *"I cannot use it (mathematical communication), because my class is over-crowded."*

Teachers also made some suggestions to foster mathematical communication in the classrooms. One teacher said, *"you should know the students very well and call them with their names to help students participate more confidently."* Some teachers pointed out the importance of participation of every student into discussions. One of them said, *"I pay attention not to choose the same persons (to talk)."* Another teacher stated, *"we do not want a discussion to turn into a fight and we have to prevent that a student talks too much and suppresses others."* One teacher also emphasized the importance of enhancing participation of all students by stating, *"I always ask if there is anybody else who wants to give an answer and I make everybody listen even if he gives the same answer as the others"*

Preparation for teaching is another suggestion teachers made repeatedly during the interviews. One teacher underlined that she even plans the questions she'll ask. She said:

You need to be fully prepared before you enter the classroom. It is important where and when I ask my questions. I always plan which questions to ask.

Another teacher focused on planning the activities she uses for mathematical communication, "*all strategies have to be planned very well. If you are doing a group study or peer learning activity you have to plan every detail to make them communicate in an effective way"*. Similarly, one teacher underlined giving clear directions as a facilitator of effective communication:

The questions have to be very clear. Students give importance to clearness. In order to make them talk and think they must not have any question marks. You need to give clear directions.

Starting mathematical communication strategies in lower grades is a common suggestion among the participant teachers. For example one teacher said, "*we need to start this kind of activities in lower grades to develop self expression skills.*" Another teacher asserted, *"we need to start (having mathematical communication) when students are younger. They will be more prepared to further grade levels if we make them discuss as much as possible."*

The final suggestion of the teachers' points out a very significant issue in mathematical communication. Teacher's positive attitudes towards the students are considered a strong facilitator of mathematical communication. Teachers believe personal communication comes before the mathematical communication. One teacher said, *"personal communication is also important; if the students like the teacher this also reflects itself in mathematics. Their questions change if they feel* you close to *themselves."* Another teacher said, *"if you communicate with the child effectively mathematical communication comes within"*. One teacher also emphasized close relationship by stating, "*there should be closer relationship between teacher and student. In this way, mathematical communication (in the classroom) will be more effective."* Some teachers also underlined the fact that mathematics teachers have a special authority in schools and this can make students feel unconfident during class discussions. One teacher pointed out this fact and described how the teacher should behave:

Mathematics teacher has a strong authority. It should be different. Teacher should be more relaxed and able to make her students confident in order to improve participation and encourage students to ask questions.

It is surprising that participant teachers working in public schools also emphasized this authoritarian nature of mathematics teachers. They suggest that mathematics teachers should spend extra effort to help their students gain confidence to participate in class discussions.

DISCUSSION

In this study teachers' perceptions and lived experiences about mathematical communication was explored through one research question. The research question focused on how teachers define mathematical communication, for which purposes they use mathematical communication strategies, what they think about the impact of mathematical communication on mathematical thinking, and which guidelines they suggest for classroom practice. Both literature and practice suggest that communication in mathematics lessons has an essential place to develop students' mathematical understanding. On the other hand, it is also emphasized that teachers mostly cannot see any connection between mathematics and communication, even if they see; they have little knowledge about how to integrate it in their lessons.

Five themes emerged from the interviews with the participants: (1) Concepts associated with mathematical communication, (2) Strategies to enhance mathematical communication, (3) Purposes and benefits of mathematical communication, (4) Impact of mathematical communication on mathematical thinking skills, (5) Application of mathematical communication in the classroom.

Teachers mostly use the term understanding by defining mathematical communication. The other themes emerged, which are using knowledge in real life contexts and using appropriate mathematical language, are also indicated as forms of understanding in the literature (Bruce, 2007; Van de Walle, Karp & Bay-Williams, 2010; Wiggins & McTighe, 2005). Moreover, teachers also underline the importance of mutual understanding for effective communication. From these points on, it can be concluded that mathematical communication is related mostly with mutual mathematical understanding by the participants.

Giving real life examples, question-answer technique, peer learning techniques and activities and games are among the strategies teachers use to facilitate mathematical communication. Strategies of question-answer and giving real life examples can also be found in the literature as the methods to enhance effective mathematical talk in classroom (Mooney, Briggs, Fletcher, Hansen & McCullouch, 2009; Chapin, O'Connor & Anderson, 2003). Halpern (1998), points out that thoughtful questions deepen student understanding related with a concept. Activities and games are also recognized among effective strategies to set a framework for communication between teacher and the students (Cook, 2006; Mooney, Briggs, Fletcher, Hansen & McCullouch, 2009) .

Teachers indicated a number of benefits related with mathematical communication. Teachers believe that mathematical communication is mostly beneficial for monitoring their students learning process. This finding is compatible both with previous research and literature. By talking, students give teachers clues about their understanding, errors and misconceptions (Mooney, Briggs, Fletcher, Hansen & McCullouch, 2009; Pape, Bell & Yetkin, 2003). It is also a common belief among the teachers that mathematical communication is useful for helping students understand abstract concepts. Students need more talking, explaining and connections to real life situations to construct meaning (Van de Walle, Karp & Bay-Williams, 2010). The last benefit mentioned is creating a positive learning environment. Mathematical communication is aligned with reducing mathematical fear and enhancing student motivation by the teachers. Teachers believe that by using mathematical communication students

perceive mathematics more accessible. Moreover, mathematical communication helps reducing fear in two ways: students find an opportunity to express themselves and teacher's positive encouragement helps them feel better about their ability to do mathematics (Siegle & McCoach, 2007).

Teachers believe mathematical communication is effective in developing mathematical thinking skills. Conceptual understanding, problem solving, reasoning and critical thinking are among the skills teachers focus on. The findings under this theme are also in line with literature and previous research where mathematical communication strategies are used to improve mathematical thinking skills (Cobb, Boufi, McClain & Whitenack, 1997; Cooke & Bucholz, 2005; Pape, Bell & Yetkin, 2003).

The last theme emerged in the study is related with how to apply mathematical communication in the classroom. Teachers consistently state that the intensity of mathematics curricula is a challenge for using mathematical communication strategies, because of the time constraints it creates. They feel the pressure to finish the curriculum, and therefore cannot focus on communication strategies to improve students' understanding. Enhancing all students' participation, planning and preparation, starting mathematical communication strategies in lower grades and improving personal communication with students are the suggestions of participants to foster effective mathematical communication. These suggestions are also widely emphasized in the literature (Chapin, O'Connor & Anderson, 2003; Michaels, O'Connor & Resnick, 2008; Mooney, Briggs, Fletcher, Hansen & McCullouch, 2009). Improving personal communication is related with the fear of mathematics of students. Participants expressed that mathematics teachers are mostly perceived as unreachable and distant in the school. To help students express themselves confidently, mathematics teachers have to spend extra effort to reduce prejudice against mathematics and the mathematics teachers.

Looking at the results of the study one can conclude that mathematical communication is used in several forms by the teachers. However, it remains as a concern whether teachers use mathematical communication consciously to improve student learning. In other words, teachers use communication strategies, but the relationship they establish between communication and learning is still weak in their practices. For this concern, Michaels, O'Connor and Resnick (2008) underline the concept of "accountable talk" to improve learning. The writers suggest that in order to use communication strategies effectively the interaction in the mathematics classroom should be accountable to three dimensions: (1) learning community, (2) standards of reasoning and (3) knowledge. Training teachers about this accountability dimensions for using communication in their classrooms could be beneficial to strengthen the impact of classroom talk and discussions on student learning.

Suggestions for Future Research

The present findings raise broad issues for future research. First of all, it is significant to explore the themes emerged in the present study with separate studies. For instance, the exploration of the impact of mathematical communication on mathematical thinking skills can provide valuable information for the field. Secondly, it is suggested that the questions of present study can be studied with larger samples and with different data collection methods. Further studies related with socio-cultural issues in learning mathematics will help strengthen the framework for improving mathematical understanding of students, which is among primary concerns of the scholars in the field.

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