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Enhancing Understanding through the Use of Structured Representations

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ABSTRACT

Mathematical representations are an essential tool in the study of mathematics and problem solving. They are also used in word problems to facilitate the transformation from textual to symbolic information. We proposed a stepwise, blocked, structured state transition graph (STG) based on the principles of instructional message design. In this study, we adopted a posttest-only non-equivalent group design to compare the performance of students who used either STG or matrix-like tables to learn to solve word problems via transition matrices. We also took into account the student's previous learning achievements in mathematics. The participants included four classes of senior students in a vocational high school, with two classes randomly designated as the experiment (STG) group and two designated as the control (Table) group. High-achieving students taught using STG outperformed their counterparts who were taught using matrix-like tables. The performance of low-achieving students appeared to be unaffected by the instructional method. These findings suggest that STG provides a clear representation of the relationships used in matrix calculation, which makes it easier to select and organize information. Nonetheless, alternative methods will be required to improve the performance of low-achieving students.

Keywords: instructional message design, representation, state transition graph, transition matrix

INTRODUCTION

K-12 students must learn to apply mathematical skills when solving everyday problems (Common Core State Standards Initiative, 2010; Mullis & Martin, 2013; OECD, 2016). Thus, word problems (representing everyday situations) are crucial to learning mathematics. Most word problems do not include superfluous or missing data; i.e., both the questions and answers are well defined. Nonetheless, even a simplified and direct description of reality can cause considerable difficulties for students (Bernardo, 1999; Jupri & Drijvers, 2016).

The complexities involved of solving word problems were detailed by Pólya (1945). This process can be broken down as follows: understand the problem, devise a plan, carry out the plan, and review the work. Understanding the problem is the most important step. Translating natural language into mathematical language is the principal difficulty (Cummins, Kintsch, Reusser, & Weimer, 1988; Sepeng & Sigola, 2013). Differences in semantic structure can be beguiling, particularly for students at lower grade levels (de Corte, Verschaffel, & de Win, 1985; Pavlin-Bernadrić, Vlahović-Štetić, & Arambašić, 2008; Riley, Greeno, & Heller, 1983). Students must also be able to identify the type of problem based on the information given to them (Lewis, 1989; Llinares & Roig, 2006). Due to their prior knowledge and learning topics they have already covered, students can often correspond the current problem with a previous solving pattern, leading them to find an appropriate solving strategy. Students must organize the information they use to derive patterns or devise a plan. Representations can be used to support mathematical

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Contribution of this paper to the literature

- This paper presents a stepwise, blocked, structured approach to multimedia instructional messages that promote learning.
- This paper outlines the usage of state transition graph to help students in the selection, organization, and integration of important information from textual descriptions.
- Most existing textbooks encourage students to transfer information into tables and then intuitively convert it into matrices for processing. We argue that state transition graphs are superior to tables in resolving transition matrix problems, particularly among high-achieving students.

reasoning and facilitate communications on mathematical topics (Kilpatrick, Swafford, & Findell, 2001). They are particularly effective in helping students to comprehend mathematical problems and learn new mathematical concepts.

The skills used in semantic translation can be applied to solving word problems in new situations. Cognitive load theory (Sweller, Ayres, & Kalyuga, 2011) posits that the presentation of instructional messages can help students to select, organize, and integrate information. This is particularly true in the use of multimedia to teach mathematics. For example, when solving a probability problem using a transition matrix; each element in the matrix describes the probability of moving from state to state in a one-time step. Matrix expressions impose complex operations, and they require a precise understanding of textual descriptions and an ability to encode those descriptions in the matrix. Most mathematics textbooks provide examples of working through transition matrices involving long textual descriptions and complex messages. Students are taught to compile information into matrix-like tables and then intuitively convert them into matrices for processing. Tables are a form of representation that allows for the display of textual information within a clear, systematic arrangement. Nevertheless, this requires that students possess the ability to alternate between rows and columns in order to interpret and integrate information for new situations. Students with extensive prior knowledge can keep pace with complex issues; however, students lacking prior knowledge are hindered by the need to select and organize information. The heavy cognitive load that this imposes makes it difficult for students comprehend the connotations of transition matrices.

The state transition graph (STG) is a graphical representation used to depict transition relationships using nodes and links. It is commonly used mathematical-based graphic in biology, engineering, computer science, and communications (Pretorius, 2008). Based on the principle of continuity (Ware, 2013, p. 183), STGs use connecting lines to identify sources and destinations, thereby augmenting the observation of the relationships between nodes. This makes it easier for students to extract essential information from the graphs and compare it with information from text, thereby freeing up cognitive resources for the integration of concepts represented in the transition matrices. Repeated analysis of relationships in each situation can induce the elaboration and automation of schemas, thereby enhancing learning efficiency. The spatial integration of textual and pictorial information (Sweller, 1994; Sweller, Chandler, Tierney, & Cooper, 1990) in conjunction with attention cueing (de Koning, Tabbers, Rikers, & Paas, 2010; Jamet, Gavota, & Quaireau, 2008) can enhance learning performance by reducing the need to conduct visual searches, particularly when using multimedia. In other words, cognitive and perceptual aspects must both be taken into account in instructional design.

The transition matrix plays an important role in advanced mathematics. Translating between textual descriptions and matrix expressions is difficult for many students. Our objective in this study was to identify mathematical representations that could be used to assist students in mastering transition matrices. Students were divided into groups depending on their previous achievements in mathematics. We hypothesize that using STG could help students select and organize pertinent information from textual descriptions. We also hypothesize that STG could guide students in integrating this information with prior knowledge.

LITERATURE REVIEW

This section provides an overview of the mathematical representations commonly used for solving word problems. We also present guidelines for the design of visual information contained in instructional messages given to students. Translating textual information to mathematical representation is a complicated process; well-designed instructional materials make the process more comprehensible to the learner.

Mathematical Representations in Word Problems

Real-world situations, manipulative aids, figures, and spoken and written symbols are five common representations typically found in mathematics learning and problem solving (Lesh, Post, & Behr, 1987). In multimedia learning environments, instructors use text, diagrams, equations, tables, graphs, sound, video, animations, and dynamic simulations to convey ideas. Illustrating the same idea using different representations

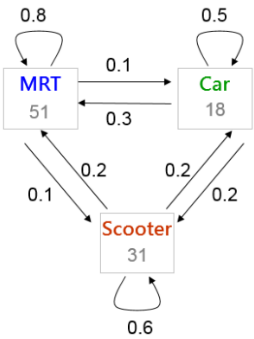


Figure 1. An example of state transition graph

enables the interpretation of problems from various perspectives. Ainsworth (1999, 2006) recommended the use of multiple representations to provide complementary information, constrain possible (mis)interpretations, and enable deeper understanding. The transformations between and within representations are the primary difficulties in learning mathematics (Duval, 2006). Students are able to solve simple word problems using informal strategies (e.g., guess-and-test, unwind) in order to derive the correct answer (Koedinger & Nathan, 2004). Nonetheless, transformations between textual information and symbolic representations remain the primary difficulty when solving these types of problems (Jupri & Drijvers, 2016).

Translating problems into tables and graphs allows students to represent and solve word problems, particularly with regard to the use of functions (Brenner et al., 1997; de Bock, van Dooren, & Verschaffel, 2013) and statistics (Friel, Curcio, & Bright, 2001). Tables are used to organize data and exhibit clear presentations when a variable has various values. For example, a table-like diagram can be used to solve time-distance-speed problems (Litvinova, 2014). This provides a stepwise, systematic method by which to extract information from a problem and present it in cells within a table. Tables can be used to present information in an explicit manner, emphasize empty cells, and highlight patterns and regularities (Ainsworth, 1999). This type of tabular representation is always used to present mathematical relations (e.g. functions) because it provides an ordered arrangement of rows and columns showing the corresponding relationship. However, the semantic relationship between the various dimensions can be a challenge for students to follow by themselves, particularly younger children (Underwood & Underwood, 1987).

"Diagrams are a frequent accompaniment to mathematical thinking" (Presmeg, 1986, p. 42); visual representations can be used to illustrate the organization of information. Graphical representations can be used to provide an overview of information with a clear indication of the direction, configuration, and relation of the objects. Using schematic spatial representations to illustrate the relationships between objects has been shown to enhance performance in solving mathematical problems (Ahmad, Tarmizi, & Nawawi, 2010; Hegarty & Kozhevnikov, 1999). Researchers have developed numerous types of schematic diagrams to make it easier to solve word problems. Lewis (1989) proposed a diagramming method for the representation of arithmetic word problem in which a number-line is used to indicate the values being compared. Students using such diagramming method showed a notable improvement in their comprehension of descriptions that included inconsistencies in language. Jitendra et al. (2007, 2011) proposed a schema-based instruction method that proved effective in emphasizing the mathematical structure of ratios, percentages, proportions, changes, and comparisons. Students were taught to classify problems by type and then select a schematic diagram appropriate to the representation of the features specific to that problem. For example, a fraction diagram can be used to illustrate a part-whole comparison, wherein students identify critical information and place it in areas of the diagram. Problems are easily solved by translating information in the diagram into a mathematical equation. Mathematicians often use graphs to illustrate structures. STG is a node-link diagram comprising nodes and links, each of which represents various entities and the relationships among them (Ware, 2013, p. 222). The rectangle boxes (nodes) in **Figure 1** show the initial state of various entities, and the connecting lines (links) represent the transition paths between them. When a probability appears close to the line, it is easier for students to understand the transition idea depicted in the textual description. Essentially, diagrams are an effective tool for connecting textual description to mathematical expression. They are particularly valuable in assisting students to comprehend overall concept.

Guiding Visual Attention

Graphs can be used to focus on the most important features while excluding extraneous information that could be distracting (Mayer, 2009). Humans receive most of their information through sensory (visual) memory. Attention is the primary selection mechanism determining which information will be processed and stored after it enters the working memory via the senses. This is particularly important in the presentation of instructional messages. The processing associated with working memory leads to the storage of knowledge in the long term memory. This process is commonly called the modal model of memory (Atkinson & Shiffrin, 1968). The auditory and visual systems are the main pathways for external information. Since the 1950s, visual search has been the key paradigm in research on attention (Müller & Krummenacher, 2006).

Treisman and Gelade (1980) conducted experiments on feature search tasks and conjunction search tasks. Participants conducted feature search tasks using single features (e.g. colors or shapes) on a tachistoscope. Conjunction search tasks are defined as those involving the conjunction of different features. Single visual features, such as shape, color, size, orientation, and motion, can be perceived in a parallel model (Pashler, 1988; Treisman, 1986; Wolfe, 1998). Parallel scanning of the entire field of vision involves automatic processing. For example, we can easily find a red dot within a pile of green dots without having to focus on each dot separately, such that the reaction time and number of objects are unrelated. In contrast, complex visual operations (e.g. combining two or more features) require greater attention resources to perform visual searches in a series. The design of teaching materials should include such features to guide the attention of students when dealing with information containing multiple graphical elements.

The organization of elements must also be taken into account in instructional design. The Gestalt approach, founded by gestalt psychologist Max Wertheimer (1880-1973), provides one approach to the problem of perceptual grouping. Goldstein (2014) summarized a number of principles related to perceptual organization. Objects that are similar, near, moving in the same direction, within the same region of space, or presenting at the same time are good candidates for grouping. Furthermore, connected areas with the same visual properties (e.g., lightness, color, texture, or motion) may also be perceived as a single unit. These ideas can be used to highlight important information in instructional design. For example, the split-attention effect based on cognitive load theory suggests that written text should be closely integrated with diagrams; i.e., using the nearness principle of perceptual organization (Plass, Homer, & Hayward, 2009; Sweller, 1994; Sweller et al., 1990). When written text is placed next to its referents, learner don't have to split their attention. This reduces the cognitive load associated with visual searches and facilitates learning.

Design of Instructional Messages

The limited cognitive resources of humans make it impossible to process a great deal of information at one time. Thus, the complexity of information and the means of presenting that information are the two main factors that must be taken into account in the design of instructional materials (Marcus, Cooper, & Sweller, 1996). When using transition matrices to solve word problems, some of the information must be clarified before it is transferred from text to the matrix. As mentioned earlier, problems can be translated into tables and graphs to make them more comprehensible; however, the information in tables and graphs can also be very complicated. We proposed three features to reduce the complexity of presenting information.

The first feature we call a *step*, which refers to a segmentation separating a general concept into several isolated and meaningful parts. For example, presenting procedural information first and conceptual information later helps students to understand tasks that require both types of information (Kester, Kirschner, & van Merriënboer, 2006). Similarly, Ayres (2006, 2013) separated terms in a polynomial (as single elements) to elucidate the meaning and operation of signs in each element. Separating multi-step mathematical problems into smaller steps is a common approach to reducing complexity. In a worked example on dice, Luzón and Letón (2015) showed that students who were taught using a step-by-step animation outperformed those who were taught using a static presentation that displayed all the information simultaneously. A segmented solution can be used to reduce the cognitive demand on students while seeking to grasp the instructional content.

The second feature we call a *block*, which refers to hints indicating a relationship among multiple elements. In multimedia design, designers commonly use cues or signals to assist in the selection, organization, and integration of related information without the need for further guidance (Mayer, 2009). Examples include arrows (Crooks, Cheon, Inan, Ari, & Flores, 2012), flashing entities (Hong, Thong, & Tam, 2004), abrupt visual onsets (Yantis & Jonides, 1990), and different colors (Jamet et al., 2008) to capture the attention of observers. These visual clues can also be used to emphasize the relationships among important entities. These and the gestalt principles mentioned earlier are used to block related information in order to direct the attention of students to the most essential elements.

The third feature we call *structure*, which refers to contextual information on which connotations are based. In the design of instructional messages, *steps* and *blocks* are used to reduce interactions among the elements (Sweller et al., 2011), thereby facilitating the selection and organization of essential information. Reducing interaction reduces cognitive load and thereby facilitates learning. Using cues in both time and space, structure and verbal guidance can help students generate a sense of context, which connects elements of the current problem with the prior experiences of students. Contextual integration triggers schemas for subsequent integration in the learning of concepts (Chen, Lee, Lei, Tso, & Lin, 2017). Two other indicators must also be taken into account: (a) communicability: the degree to which students can grasp the information necessary for selection and organization, and (b) connectivity: the linking of information with the prior experiences of students to form schemas for the integration of concepts.

In short, cognitive processes must be taken into account in the design of teaching materials. In this study, we integrated features within an STG to represent the operational relationships observed in transition matrices. The spatial arrangement of information in a graph can be used to facilitate the selection and organization of content, thereby leaving students more cognitive resources for integration and learning concepts.

METHODOLOGY

Participants and Design

One hundred and fifty-six students participated in this study. The participants comprised four classes of grade 12 students in a vocational high school in Taiwan. Two of the classes were randomly assigned to the experimental group to receive instruction based on STG, whereas the other two classes formed a control group to receive instruction based on matrix-like tables. The students were divided into high mathematics achievers and low mathematics achievers based on the average scores achieved in examinations the previous semester. The STG group contained 44 high achievement students and 41 low achievement students. The Table (control) group contained 28 high achievement students and 43 low achievement students.

We adopted a two-factor quasi-experimental design to compare the efficacy of representational methods (STG vs. Table) on learning performance among students with high or low achievements in mathematics. Before the lesson on transition matrices, a prior knowledge test was administered to evaluate the students' prior knowledge concerning matrix operations. An independent sample *t* test confirmed that there were no pre-existing significant differences between the experimental and control groups, based on their mock examination results from the previous semester (t = .263, df = 154, p = .793 > .05) and prior knowledge scores (t =.154, df = 154, p = .901 > .05). Thus, we adoped a posttest-only design using proxy pretest (Shadish, Cook, & Campbell, 2002) to characterize the performance of the STG group and the Table group in learning using transition matrices with or without a structured representation.

Procedures

All participants attended a 50-min instructional session on transition matrices. The same teacher taught both sessions. The day before this lesson, participants took a 10-min prior knowledge test that included five questions concerning the underlying concepts and calculation of matrices. Following the lesson, all participants also completed a 30-minute posttest.

The topic in this study was transition matrices. We prepared multimedia teaching materials based on the instructional message framework using PowerPoint with AMA add-ins as well as prompts and whole-class discussion (Lee & Chen, 2015, 2016). Both groups were shown slides with the same instructional content; however, the representations used in the worked examples differed between the groups.

Lesson Materials

Two worked examples were used to illustrate the utilization of transition matrices. Both examples were word problems involving probability. The examples were presented in the same style using the same number of steps; however, they were presented using transition matrices ordered in different ways. The question was presented at the top of each slide to allow students to read the descriptions whenever necessary. **Figure 2** presents one of the word problems concerning the proportion of Mass Rapid Transit (MRT) riders among the total number of commuters each month.

We adopted the same instructional design principles in drawing up the PowerPoint presentation, with the aim of reducing cognitive load and promoting visual search efficiency. **Figure 3** presents slides based on these principles. Irrelevant words and pictures were eliminated to reduce cognitive load, and the teacher presented textual descriptions orally. Including only key concepts on the slides was expected to help students to connect what they heard with what they read. We also specified colors to emphasize essential information and group related information. Finally, the elements were segmented so that they could be added to the slides in a step-by-step manner. Related information was presented closely to reduce split-attention (Sweller et al., 2011), which helped teachers interpret information about stepwise and associated image elements for their students. We used oral explanations to limit the cognitive resources that would otherwise be required for visual searching. Physical and temporal contiguity has been shown to reduce errors and save time.

Taipei City Government surveys the status of commuters' monthly usage on transportation as the following:
For commuters who take MRT this month, 80% of them will continuously take the MRT, 10% will change to drive a car and 10% will change to ride a scooter next month.
For commuters who drive a car this month, 50% will continuously drive a car, 30% of them will change to take the MRT and 20% will change to ride a scooter next month.
For commuters who ride a scooter this month, 60% will continuously ride a scooter, 20% of them will change to take the MRT and 20% will change to drive a car next month.
(1) Write the transition matrix A of this investigation.
(2) It is known that the current status of MRT commuters are 50%, car commuters are 10%, and scooter commuters are 40%. What is the proportion of MRT commuters to all commuters after one month?

Figure 2. One of the word problems used in this study

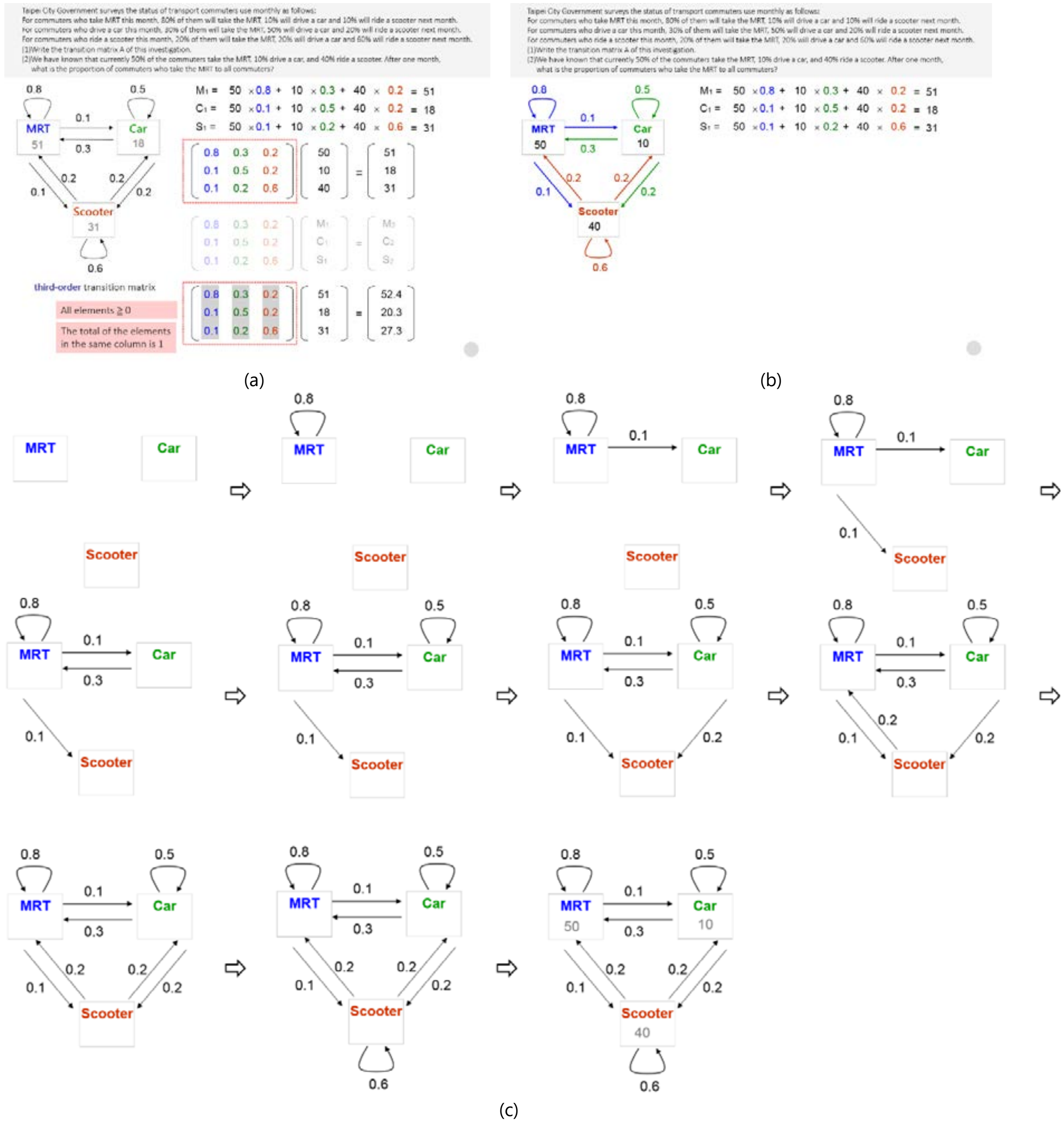
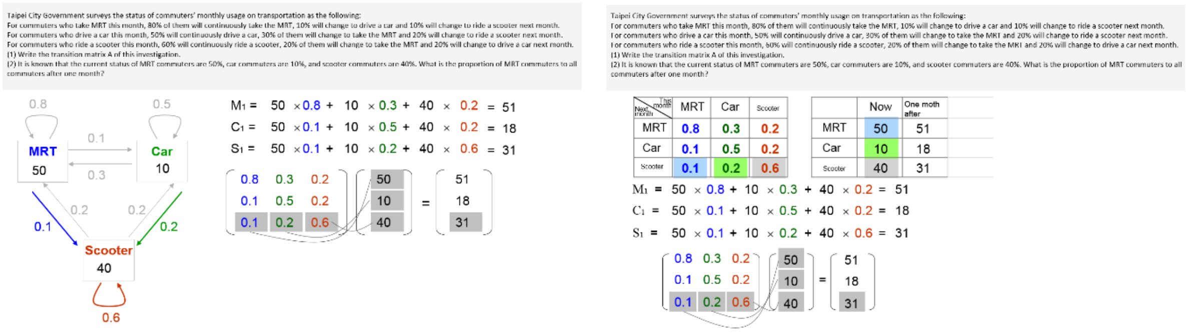


Figure 3. Screenshots showing design guidelines used in drawing up instructional slides (a) Irrelevant information was eliminated; i.e., only key concepts were displayed (b) Colors were used to identify classes (c) Elements were segmented and then added stepwise



(a) STG group (b) Table group

Figure 4. The different representations used in the STG and Table groups

Despite differences in the representations presented in the STG and Table groups, the manner of presentation was the same. As shown in **Figure 4**, we used similar instructional design principles in outlining the information in the STG and matrix-like tables; both groups received the same information but received it using different representations. The state transition graphs and tables were segmented and then presented in a step-by-step manner to help students form associations between the visual images and the oral descriptions provided by the teacher. Color blocking was used to connect related information. The state transition graph presented information based on spatial proximity with the aim of facilitating the selection and retrieval of information while allowing for visual rehearsals when necessary. In other words, STG and tables were both presented in a stepwise and blocked manner; however, STG organized the information within a structured presentation.

Measures

Students' prior knowledge was assessed using a five-question prior knowledge test, which included two questions on matrix multiplication, one question on the features of transition matrices, and two word problems. The Cronbach's α of 0.766 demonstrated that the internal consistency of the test paper was within the acceptable range.

The posttest comprised 8 questions (including 17 sub-questions) for a total score of 108 points. The problems dealt with (a) knowledge and comprehension and (b) problem-solving and application. Our analysis of problem difficulty was based on the average numbers of students with high and low achievement who answered the problem correctly. The average difficulty of the problems was 0.54, and the degree of discrimination ranged from 0.33 to 0.89. The test paper presented a Cronbach's α of 0.907, which indicates a high degree of internal consistency.

All of the problems were reviewed and revised by two experts in instructional design and three mathematics teachers averaging more than 10 years of teaching experience, indicative of high content validity.

RESULTS

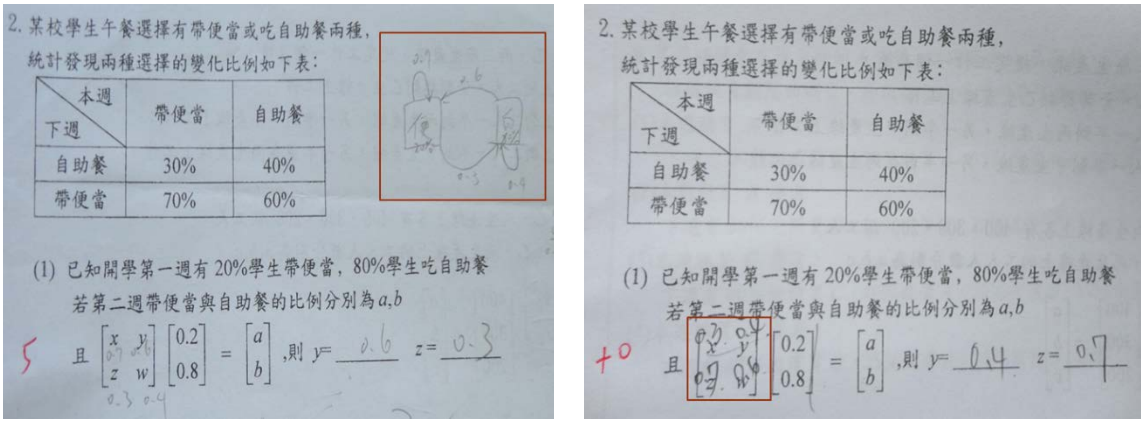
In the posttest, the students with high learning achievement in mathematics obtained higher average scores (M = 73.33, SD = 27.63) than did the students with low learning achievement (M = 44.98, SD = 27.16). Overall, the students in the STG group obtained higher average scores (M = 65.05, SD = 31.96) than did the students in the Table group (M = 49.70, SD = 21.17).

In the posttest, the high-achieving students in the STG group obtained higher average scores (M = 81.77, SD = 23.44) than did the high-achieving students in the table group (M = 60.07, SD = 28.87). In the posttest, the low-achieving students in the STG group obtained higher average scores (M = 47.10, SD = 30.28) than did the low-achieving students in the table group (M = 42.95, SD = 24.01); however, the difference was not significant.

Two-way ANOVA (Analysis of Variance) also revealed significant interaction effects between teaching materials and learning achievement (F = 4.127, p = .044, η^2 = .026). **Table 1** lists the results of simple main effects analysis. High-achieving students in the STG group as well as the Table group outperformed low-achieving students in both groups.

Table 1. Summary of two-way ANOVA of simple main effects of STG group and Table group and learning achievement groups in the posttest

Source of variation	SS	*df*	MS	*F*	Sig	η^2	Post hoc comparison
Different representation							
STG	25518.475	1	25518.475	35.122	.000	.297	High > Low
Table	4969.025	1	4969.025	7.341	.008	.096	High > Low
Learning achievement in mathematics							
High	8058.416	1	8058.416	12.229	.001	.149	STG > Table
Low	360.436	1	360.436	.485	.488	.006	STG = Table



a) STG group b) Table group

Figure 5. Problem-solving skills of high-achieving students in the two groups

The results are consistent with the central argument in that the use of STG reduces element interaction among presented information, thereby making it easier for high-achieving students to find information relevant to matrix calculations. It also leaves them with more cognitive resources to focus on deciphering the meaning of matrix calculations. This method of representation also forces students to perform the same thought process repeatedly while writing down the correct transition matrix. This type of repetition induces deeper understanding and automation of schemas, thereby enhancing the integration of information to facilitate learning. The use of tables to represent information requires students to search for and compare figures repeatedly in order to comprehend the meaning and connotations of the matrix calculations. This high degree of element interaction imposes greater cognitive load on the working memory. Furthermore, students sometimes regard tables as transition matrices without fully understanding the connotations of transition matrices or establishing associations. This makes it difficult for student to solve problems in a logical manner. As shown in **Figure 5**, the students solved the question below using a copy-and-paste approach in order to deal with the presentation of data in the table.

Students can bring their lunchbox or eat at the school cafeteria. The variation between these two choices was estimated as follows:

This week / *Next week*	*Lunchbox*	*School cafeteria*
School cafeteria	*30%*	*40%*
Lunchbox	*70%*	*60%*

20% of the students brought their lunchboxes and 80% of the students ate at the school cafeteria during the first week of a new semester. The proportions of students' bringing a lunchbox or eating lunch at the school cafeteria were a and b in the second week and $\begin{bmatrix} x & y \\ z & w \end{bmatrix}\begin{bmatrix} 0.2 \\ 0.8 \end{bmatrix} = \begin{bmatrix} a \\ b \end{bmatrix}$. *Identify the elements: y= _____, z = _____.*

The students in the STG group first drew the STG and then used it to solve the problem. In contrast, the student in the Table group (even high-achieving students) simply copied the table as a transition matrix and then set off on the wrong course in trying to solve the problem. This student confused the elements displayed in a table format with those presented in a matrix.

DISCUSSION

In this study, we examined the influence of different representations (STG vs. Table) on the learning performance of students learning to solve word problems via transition matrices. We also took into account the student's previous learning achievements in mathematics. High-achieving students who were taught using STG outperformed high-achieving students who were taught using tables. No significant differences were observed in the performance of low-achieving students taught using STG or tables.

The use of a transition matrix to solve word problems involves complex textual and procedural information. Most textbooks guide students to transfer information into tables and then intuitively convert it into matrices for processing. We employed a state transition graph as an example of multimedia instruction to exhibit problem-solving process in a stepwise, blocked, and structured manner to support effective learning. Detailed segmentation can be used to reduce cognitive load involved in matching visual images to oral descriptions. Gestalt theory posits that presenting materials in which connected ideas possess the same visual properties (e.g., lightness, color, motion) helps students to perceive related information as a single unit. Tables and STGs can both be used to promote learning; however, STG presents information in a graphical form; i.e., structured within a contextual scaffold. Integrating elements within a diagrammatic schema helps to reduce cognitive load and enhance comprehension (Marcus et al., 1996). As using the schema-based instruction is critical to success in problem solving (Jitendra et al., 2011).

Nonetheless, only high-achieving students were shown to benefit from the STG approach. In the current study, the effectiveness of learning was investigated using only a limited number of interventions. Further research will be needed to interpret how the cognitive behaviors of students influences reading comprehension, particularly the process of using STG or table representations influence student understanding. Furthermore, Sweller et al. (2011) reported that learning performance depends on the complexity of the learning materials, intrinsic cognitive load, and the expertise of learners. Low-achieving students possess less of the prior knowledge, such as matrix multiplication, which is required to solve the problems. Even though STG makes it easier to comprehend the operational meaning of transition matrices, students may still encounter difficulties in the selection and organization of information. For example, a failure to integrate multiple elements would leave them without the available resources required for integration and comprehension. The expertise of learners is based on schemas stored in their long-term memory, which means that a pre-training design (Clarke, Ayres, & Sweller, 2005) can be used to reduce cognitive load by strengthening key concepts as specific prior knowledge. Low motivation and a lack of willingness to participate may also contribute to low learning effectiveness. Low-achieving students possess less prior knowledge and are less adept at the automatic formulation of schemas. As a result, they tend to perceive learning tasks as difficult and must invest greater effort, which can hamper the selection and organization of information resulting in a loss of interest. Interviews could be conducted in groups based on learning achievement, and qualitative research methods could be used to clarify the influence of teaching materials on student impressions. The provision of appropriate instructional scaffolding and using simple but interesting examples could help to lessen the perceived difficulty as well as the effort that must be invested. Regardless, it would be worthwhile to examine alternative approaches to STG in order to make it easier for low-achieving students to learn transition matrices.

Mathematical representations provide complementary information and make it possible to develop a deeper understanding of concepts (Ainsworth, 2006). We propose the representation of information in a stepwise, blocked, and structured manner. Each feature can be presented discretely to enable an understanding of different concepts. STG and tables were both shown to help students in the development of skills pertaining to the application of transition matrices. A control group given no representations at all (i.e., the teacher guiding students to search for relevant information from a text-only description for transition matrices) could further clarify the influence of table representations on the teaching of transition matrices. Finally, it would be worthwhile to investigate the influence of STG and tables on retention. In the future, we suggest administering a delayed posttest to confirm the efficacy of the various representations in facilitating schema construction.

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