Estimation Of Spatial Dynamic Nonparametric Durbin Models With Fixed Effects

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Spatial panel data models have been widely studied and applied in both scientific and social science disciplines, especially in the analysis of spatial influence. In this paper, we consider the spatial dynamic nonparametric Durbin model (SDNDM) with fixed effects, which takes the nonlinear factors into account base on the spatial dynamic panel data models (SDPDM). Above all, we propose an iterative approach to estimate the spatial dynamic nonparametric Durbin model with corresponding hypothesis test and we find that convergence occurs since the second iteration. So we use a three stage iterative approach to improve the iterative approach. The results indicate that the three stage iterative approach is more reliable when T or N is large and the accuracy of the nonparametric components estimation is very important. We believe that the three stage iterative approach can be applied to other spatial dynamic nonparametric panel data models as well.

Keywords: Spatial Dynamic Panel Models, Iterative Approach, Nonparametric, Panel Data, Engineering Education

INTRODUCTION

Spatial panel data models have been widely used in many fields of economics to analyze the spatial interactions of different units. Recently, the spatial dynamic panel data model (SDPDM) draw more and more attention among spatial panel data models as it enables researchers to take into account the dynamic influences as well as the control of unobservable heterogeneity across units (Elhorst 2012).

In SDPDM study, Lee, Yu and other scholars make a lot of rigorous research on the estimate method for different spatial dynamic panel data models (e.g. the ML estimator (MLE) and the QML estimator (QMLE) and their asymptotic properties analysis for SDPDM with

ixed effects (Yu et al. 2008; Lee and Yu 2010), the QMLE for unit root SDPDM with fixed effects(Yu and Lee 2010), the QMLE of SDPDM with time varying spatial weights matrices (Lee and Yu 2012), the QMLE for SDPDM with random effects (Parent and Lesage 2012), GMME for SDPDM (Cleveland 1979). At the same time,

due to the statistic feature and application prospect of SDPDM, many researchers look SDPDM as a powerful research tool. For example, Parent and LeSage (2010) apply SDPDM with random effects to analyze the relationship between highway capacity and travel demand, Yu and Lee apply SDPDM in spatial cointegration and convergence study (Yu et al. 2012; Yu and Lee 2012), Hong and

Sun (2011) apply SDPDM to study the relationship between FDI and TFP in China, Zheng et al. (2013) apply SDPDM in central government's infrastructure investment study, and Baltagi et al. (2014) find the important application to new economic geography by forecasting with SDPDM.

However, all these studies are based on linear hypothesis, which means they focus on the linear relationships between endogenous variable and exogenous variables in these spatial panel data models. In practice, the true relationships between endogenous variable and exogenous variables are very complicated, which maybe contain not only simple linear relationships but also nonlinear relationships. Moreover, the traditional estimate method will fail and the estimators will be unreliable if we still estimate these spatial panel data models with nonlinear relationships under linear hypothesis. In order to deal with this problem, we choose an iterative approach which takes the spirit of the profile likelihood approach (Severini and Wong (1992). We separate the explanatory variables into two parts, linear components and nonparametric components, and then estimate them separately through an iterative process until convergence occurs. In this paper, we focus on one kind of the spatial dynamic panel data model, the spatial dynamic nonparametric

State of the literature

- Combine the spatial dynamic panel data model with nonparametric econometrics and include non-linear factors in the spatial dynamic panel data model to establish a spatial dynamic nonparametric panel data model.
- Study the estimation and test of the spatial dynamic nonparametric panel data model, expecting to acquire the conclusion with both theoretical meanings and application value.
- Precede scientific evaluation and improvement with Monte Carlo Method for the estimation and test of the spatial dynamic nonparametric panel data model.

Contribution of this paper to the literature

- Theoretically, the advantages of integrating spatial econometrics and nonparametric econometrics are integrated to expand the separate research fields and application domains.
- Non-linear relationship is included in the research field of the spatial dynamic panel data model in order to examine the non-linear effects between different spatial units.
- The logic sequence of traditional methods is changed to put non-linear effects in the first place when estimating and applying the spatial dynamic panel data model so as to reduce the effect of model specification errors.

Durbin model (Anselin 1988), which originally developed by Durbin (1960) in the context of time series analysis. The rest of the paper is organized as follows. Section 2 provides the model specification and the suggested estimation procedures. We begin with the spatial dynamic nonparametric Durbin model with dynamic effects, endogenous interaction effects, exogenous interaction effects and fixed time effects. Then we introduce the iterative approach based on the integration of maximum likelihood estimation and the partially linear model estimation. In Section 3, we propose two hypothesis tests for the nonparametric function and the linear spatial regression parameters separately base on their asymptotic properties. Section 4 discusses Monte Carlo experiments that we conduct and we propose a three stage iterative approach to reduce the computational burden of the iterative approach. Section 5 concludes the article with a brief discussion.

THEORY AND METHODOLOGY

The Spatial Dynamic Nonparametric Durbin Model with Fixed Effects

In this paper, we consider a spatial dynamic nonparametric Durbin model with the dynamic effects, the endogenous interaction effects, the exogenous interaction effects and the fixed effects, which takes the form

$$y_t = \rho W y_t + \alpha l_n + r y_{t-1} + \lambda W y_{t-1} + x_t \beta + W x_t \eta + M(\Lambda_t) + \varepsilon_t \tag{1}$$

Where y_t denotes *N*-dimensional vector of the dependent variable for all individuals in period *t* (*t* = 1, 2,…, *T*), y_{t-1} denotes the endogenous variable in period *t*−1 and its scalar parameters *r* and λ characterizes the dynamic effects. *W* is an *N*×*N* row normalized spatial weight matrix that models the interaction scheme between individuals, x_t is the *N*×*K* matrix of exogenous explanatory variables. α is a scalar of time effect and l_n is *N*-dimensional vector of ones. $M(\cdot)$ is a smooth function and Λ_t is an *N*-dimensional vector of exogenous explanatory variable. We assume that ε_t follows a multivariate normal distribution with zero mean and a constant scalar diagonal variance covariance matrix $\sigma^2 I_n$. y_t , y_{t-1} , x_t , Λ_t are independent with ε_t .

Method

To estimate the spatial dynamic nonparametric Durbin models with fixed effects, we introduce an iterative approach based on the integration of maximum likelihood estimation and the partially linear model estimation.

Following Lesage and Pace (2009), we define: $Z_t = \left[l_n, y_{t-1}, W y_{t-1}, x_t, W x_t \right]$ and $\delta = \left[\alpha, r, \lambda, \beta, \eta \right]'$, (1) can be changed into this form

$$y_t - \rho W y_t - M(\Lambda_t) = Z_t \delta + \varepsilon_t \tag{2}$$

The log likelihood function of (2), as if the disturbances were normally distributed, is

$$\ln L_{N,T}(\delta) = -\frac{NT}{2}\ln(\pi \sigma^2) + \ln|I_N - \rho W| - \frac{1}{2\sigma^2} e(\delta)' e(\delta) \tag{3}$$

Where $e(\delta) = y_t - \rho W y_t - Z\delta - M(\Lambda_t)$ and $\rho \in (\min(\omega)^{-1}, \max(\omega)^{-1})$. ω is the *N*×1 vector of eigenvalues of the matrix *W*. We assume that ω contains only real eigenvalues, because admissible values of ρ can become very complicated for *W* may have complex eigenvalues. Given ρ and $M(\Lambda_t)$, the ML estimators of δ and σ^2 can be solved from the first-order maximizing conditions, to get

$$\delta = (Z'Z)^{-1} Z' [(I_{NT} - \rho(I_T \otimes W))Y - M(\Lambda)] \tag{4}$$

$$\hat{\sigma}^2 = \frac{1}{NT} \sum_{t=1}^{T} e_t' e_t \tag{5}$$

Where $e_t = y_t - \rho W y_t - Z_t \hat{\delta} - M(\Lambda_t)$. Base on the approach of Pace and Barry (1997), the concentrated log-likelihood function of ρ can be written as

$$\ln L_{N,T}(\rho) = -\frac{NT}{2}\ln(\pi) - \frac{NT}{2}\ln(\hat{\sigma}^2) + T\ln|I_N - \rho W| - \frac{NT}{2}\ln(S(\rho)) \tag{6}$$

$$S(\rho) = e(\rho)'e(\rho), \ e(\rho) = Y - \rho WY - M(\Lambda) - Z\hat{\delta}.$$

Maximizing the concentrated log-likelihood function yields the ML estimator of ρ, given $\hat{\delta}$, $\hat{\sigma}^2$ and $M(\Lambda)$. So an iterative procedure may be used in which the set of parameters δ, σ^2 and ρ are alternately estimated until convergence occurs. During this iterative procedure, the key is to find out the estimator of $M(\Lambda)$. So we introduce an iterative approach based on the integration of maximum likelihood estimation and the partially linear model estimation to estimate ρ, δ, σ^2 and $M(\Lambda)$. The iterative process is as follows:

● In the first step, we can obtain the initial ML estimator of δ, σ^2 and ρ by estimate the spatial dynamic Durbin model with fixed effects, see (7) and (8). Then we calculate the residuals V_t as showed in (9).

$$y_t = \rho W y_t + \alpha l_n + r y_{t-1} + \lambda W y_{t-1} + x_t \beta + W x_t \eta + \varepsilon_t \tag{7}$$

$$y_t - \rho W y_t = Z_t \delta + \varepsilon_t \tag{8}$$

$$V_t = y_t - \hat{\rho} W y_t - Z_t \hat{\delta} \tag{9}$$

● Denote $\tilde{y}_t = y_t - \rho W y_t$, (2) can be convert to (10), which is in essence a semiparametric partially linear model.

$$V_t = y_t - \hat{\rho} W y_t - Z_t \hat{\delta} \tag{10}$$

To estimate the nonparametric part $M(\Lambda_t)$, we establish a nonparametric model of V_t and $M(\Lambda_t)$, which takes the form

$$V_t = M(\Lambda_t) + \varepsilon_t \tag{11}$$

Then we use the local-linear estimator of $M(\Lambda_t)$ as the estimator of nonparametric component based on the results of Stone (1977) and Cleveland (1979).

$$\hat{M}_{LL}(Z_t, h) = \frac{1}{NT} \sum_{i=1}^{NT} \frac{\{\hat{s}_2(Z_t, h) - \hat{s}_1(Z_t, h)(Z_i - Z_t)\} K_h(Z_i - Z_t) V_i}{\hat{s}_2(Z_t, h)\hat{s}_0(Z_t, h) - [\hat{s}_1(Z_t, h)]^2} \tag{12}$$

Where $\hat{s}_r(Z_t, h) = \frac{1}{NT} \sum_{i=1}^{NT} (Z_i - Z_t)^r K_h(Z_i - Z_t)^r$, $r = 0, 1, 2$. $K(\cdot)$ is the kernel function and in this paper, we choose the Gaussian kernel $K(u) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}u^2\right)$. h is the optimal bandwidth and we calculate it based on the optimal smoothing results of Bowman and Azzalini (1997):

$$h = \sqrt{h_Z \times h_V} \tag{13}$$

Where $h_Z = \left(\frac{4}{3NT}\right)^{1/5} \text{median}\{|Z_i - \text{median}(Z_i)|\}/0.6745$

and $h_V = \left(\frac{4}{3NT}\right)^{1/5} \text{median}\{|V_i - \text{median}(V_i)|\}/0.6745$.

● Make the local-linear estimator $\hat{M}_{LL}(Z_t, h)$ (12) as the estimator of $M(\Lambda)$ and put it into (1) to recalculate the ML estimator of δ, σ^2 and ρ.

● Repeat the second step to the third step until convergence occurs. Then we can get the final estimation results of δ, σ^2, ρ and $M(\Lambda)$.

Hypothesis Tests

Base on the estimation method described in the previous section, we propose two hypothesis tests. The first is for hypothesis testing the nonparametric function and the second is for hypothesis testing of linear spatial regression parameters.

Testing the Nonparametric Component

The nonparametric estimate of $M(\Lambda_t)$ provides us with descriptive information for exploratory data analysis and we can apply it to formulate a spatial panel model that takes into account the features which emerged from the nonlinear analysis. So we introduce the generalized likelihood ratio test (see Fan et al., 2001) to assess the appropriateness of a proposed spatial dynamic nonparametric Durbin model with fixed effects. Without loss of generality, we consider a simple linear null hypothesis. Accordingly, the null and alternative hypotheses are given as follows:

$$H_0 : M(\Lambda_t) = \gamma_0 + \gamma_1 \Lambda_t \text{ versus } H_1 : M(\Lambda_t) \neq \gamma_0 + \gamma_1 \Lambda_t \tag{14}$$

Where γ_0 and γ_1 are unknown constant parameters. Following the generalized likelihood ratio tests given by J. FAN (2001) and H. Liang (2010), let $\tilde{\rho}$, $\tilde{\delta}$ and $\tilde{M}(\Lambda)$ be the corresponding profile least-squares estimators of ρ, δ and $M(\Lambda_t)$ of H_0, while the estimator of $M(\Lambda_t)$ is $\tilde{M}(\Lambda_t) = \tilde{\gamma}_0 + \tilde{\gamma}_1 \Lambda_t$, where $\tilde{\gamma}_0$ and $\tilde{\gamma}_1$ are the ordinary least-squares estimators of γ_0 and γ_1. Meanwhile, let $\hat{\rho}$, $\hat{\delta}$ and $\hat{M}(\Lambda_t)$ be the corresponding profile least-squares and nonparametric estimators of ρ, δ and $M(\Lambda_t)$ of H_1. The resulting residual sums of squares under the null and alternative hypotheses are then

$$\text{RSS}(H_0) = \sum_{i=1}^{NT} (y_i - \tilde{M}(\Lambda_i) - Z_i \tilde{\delta} - \tilde{\rho} W y_i)^2 \tag{15}$$

$$\text{RSS}(H_1) = \sum_{i=1}^{NT} (y_i - \hat{M}(\Lambda_i) - Z_i \hat{\delta} - \hat{\rho} W y_i)^2 \tag{16}$$

To test the null hypothesis, we consider the following generalized likelihood ratio test:

$$F = \frac{r_K}{2} NT \ln \frac{\text{RSS}(H_0)}{\text{RSS}(H_1)} \tag{17}$$

Where $r_K = \dfrac{K(0) - 0.5 \int K^2(u) du}{\int [K(u) - 0.5 K(u) * K(u)] du}$ and $K(u) * K(u)$ denotes the convolution of $K(u)$. According the results of Fan et al. (2001), assume the regularity conditions in Fan et al. (2001) hold, then under H_0 in (14), as $nh^{3/2} \to \infty$ and $h \to 0$, F has an asymptotic X^2 distribution with df_n degrees of freedom, that is

$$F \overset{a}{\sim} X^2_{df_n} \tag{18}$$

Where $df_n = r_K c_K |\Omega| / h$. $|\Omega|$ stands for the support of Λ_i, $c_K = K(0) - 0.5 \int K^2(u) du$ and h is the optimal bandwidth.

Testing Parametric Components

Since the estimation of parametric components and nonparametric components are separated in the iterative process, the hypothesis testing of linear spatial regression parameters bases on the ML estimator of δ, σ^2 and ρ. In this paper, we mainly consider the significance testing of the linear spatial regression coefficients

$$H_0 : \varsigma_i = 0 \text{ versus } H_1 : \varsigma_i \neq 0 \tag{19}$$

Where $\varsigma = [\rho, \delta'] = [\rho, \alpha, r, \lambda, \beta, \eta]$, $i = 1, 2, \ldots, 2K + 4$. To provide a rigorous analysis of the MLE of of δ, σ^2 and ρ with their asymptotic properties, we assume that they obey the regularity assumptions of Lee and Yu (2010). Denote $\theta = [\delta', \rho, \sigma^2]'$ and its true value $\theta_0 = [\delta_0', \rho_0, \sigma_0^2]'$. The asymptotic distribution of the $\hat{\theta}_{ML}$ can be derived from the Taylor expansion of $\dfrac{\partial \ln L_{N,T}(\theta)}{\partial \theta} = 0$ at θ_0, where $\ln L_{N,T}(\theta)$ is the concentrated log-likelihood function of θ as follows:

$$\ln L_{N,T}(\theta) = -\frac{NT}{2}\ln(\pi\sigma^2) + T \ln|I_N - \rho W| - \frac{1}{2\sigma^2} e(\theta)' e(\theta) \tag{20}$$

Where $e(\theta) = y_t - \rho W y_t - Z\delta - M(\Lambda_t)$.

Base on the conclusion of Lee (2004), under the normal distribution assumption of ε and the regularity assumptions of Lee and Yu (2010), the asymptotic distribution of $\sqrt{NT}\left(\hat{\theta}_{ML} - \theta_0\right)$ is normal, $\sqrt{NT}\left(\hat{\theta}_{ML} - \theta_0\right) \xrightarrow{D} N\left(0, \Sigma_\theta^{-1}\right)$, where the information matrix $\Sigma_{\theta_0} = -\lim\limits_{N \to \infty} E\left(\dfrac{1}{NT}\dfrac{\partial^2 \ln L_{N,T}(\theta_0)}{\partial \theta \partial \theta'}\right)$.

Under H_0 ($\theta_0 = 0$), we can obtain the asymptotic variance matrix of the $\hat{\theta}_{ML}$ (21) for inference(standard errors and z-values) by the information matrix.

$$Asy.Var(\theta) = Asy.Var\left(\delta, \rho, \sigma^2\right) = \left[\lim_{NT \to \infty} \frac{1}{NT} \begin{pmatrix} \frac{1}{\sigma^2}Z'Z & \frac{1}{\sigma^2}Z'Wy_t & 0 \\ \frac{1}{\sigma^2}Z'Wy_t & \frac{1}{\sigma^2}(Wy_t)'Wy_t & 0 \\ 0 & 0 & \frac{NT}{2\sigma^4} \end{pmatrix}\right]^{-1} \tag{21}$$

RESULTS AND DISCUSSION

In this section, we conduct a Monte Carlo experiment to evaluate the performance of the iterative approach for the spatial dynamic nonparametric Durbin model with fixed effects. First of all, we consider a basic SDNDM with fixed effects, which samples are generated from the model as follows,

$$y_t = \rho_0 W y_t + \alpha_0 l_n + r_0 y_{t-1} + \lambda_0 W y_{t-1} + x_t \beta_0 + W x_t \eta_0 + M(\Lambda_t) + \varepsilon_t \tag{22}$$

Where $M(\cdot) = \sin(\cdot)$. We use $\theta_0 = [\delta_0', \rho_0, \sigma_0^2]' = [\alpha_0, r_0, \lambda_0, \beta_0, \eta_0, \rho_0, \sigma_0^2]'$

$= [0.1, 0.2, 0.2, 1, 0.2, 0.2, 1]'$ as the true value of θ. x_t, Λ_t and ε_t are generated from independent standard normal distribution. y_t is generated based on (22) with zero initial value. We generated the spatial panel data with $100+T$ periods and then take the last T periods for the Monte Carlo experiment. To compare the performance of

the iterative approach in different *N* and *T*, we consider nine cases with *N*=10, 30, 50 and *T*=10, 30, 50. The spatial weights matrix *W* is generated from Rook contiguity weight by the spatial distribution of different individuals as follows, which coordinates x_c and y_c follow standard normal distribution.

Then we estimate the spatial dynamic nonparametric Durbin model with fixed effects 1000 times by the iterative approach in the case with *N*=50, *T*=50 to check the performance of the iterative approach. The results of first 15 iterations are in Table 1. For each iteration, we report the empirical bias (Bias), the empirical standard deviation (E-SD) and the empirical root mean square error (RMSE) of ML estimator of linear spatial regression parameters.
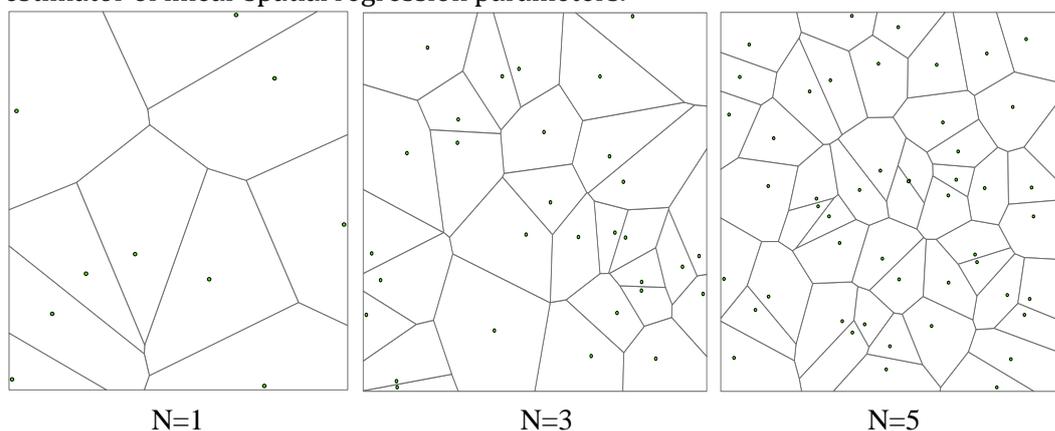

| N=1 | N=3 | N=5 |

Figure 1. The spatial distribution of different individuals

Table 1. The estimation of the SDNDM with fixed effects by the iterative approach (N=50,T=50)

iteration		α	r	β	Λ	η	ρ				$M(\Lambda)$
1	Bias	-0.005	0.007	-0.017	-0.004	0.007	-0.006				
	E-SD	0.000	0.000	0.000	0.000	0.000	0.000				
	RMSE	0.005	0.007	0.017	0.004	0.007	0.006				
2	Bias	-0.008	0.008	-0.008	0.004	-0.033	-0.006	h	0.243	MESD	0.060
	E-SD	0.002	0.000	0.006	0.000	0.019	0.000	df_n	8.071	F-test	244.792
	RMSE	0.006	0.008	0.013	0.004	0.024	0.006			$X^2_{df_n}$	20.204
3	Bias	-0.009	0.008	-0.009	0.003	-0.036	-0.004	h	0.244	MESD	0.059
	E-SD	0.002	0.000	0.005	0.000	0.016	0.001	df_n	8.043	F-test	244.769
	RMSE	0.007	0.008	0.012	0.003	0.029	0.005			$X^2_{df_n}$	20.159
4	Bias	-0.010	0.008	-0.009	0.003	-0.036	-0.004	h	0.244	MESD	0.059
	E-SD	0.002	0.000	0.004	0.000	0.014	0.001	df_n	8.043	F-test	244.770
	RMSE	0.008	0.008	0.011	0.003	0.031	0.005			$X^2_{df_n}$	20.159
5	Bias	-0.010	0.008	-0.008	0.005	-0.029	-0.010	h	0.244	MESD	0.059
	E-SD	0.002	0.000	0.004	0.001	0.012	0.002	df_n	8.043	F-test	244.743
	RMSE	0.008	0.008	0.011	0.004	0.030	0.006			$X^2_{df_n}$	20.159
6	Bias	-0.012	0.008	-0.009	0.003	-0.035	-0.005	h	0.244	MESD	0.059
	E-SD	0.002	0.000	0.003	0.001	0.011	0.002	df_n	8.039	F-test	244.752
	RMSE	0.009	0.008	0.010	0.004	0.031	0.006			$X^2_{df_n}$	20.153

Table 1. contiously

7	Bias	-0.013	0.008	-0.008	0.004	-0.034	-0.006	h	0.244	MESD	0.059
	E-SD	0.003	0.000	0.003	0.001	0.010	0.002	df_n	8.042	F-test	244.758
	RMSE	0.010	0.008	0.010	0.004	0.032	0.006			$X^2_{df_n}$	20.153
9	Bias	-0.015	0.008	-0.009	0.002	-0.039	-0.002	h	0.244	MESD	0.059
	E-SD	0.004	0.000	0.002	0.001	0.009	0.002	df_n	8.044	F-test	244.781
	RMSE	0.011	0.008	0.010	0.003	0.033	0.006			$X^2_{df_n}$	20.160
10	Bias	-0.016	0.008	-0.009	0.003	-0.036	-0.004	h	0.244	MESD	0.059
	E-SD	0.003	0.000	0.003	0.001	0.010	0.002	df_n	8.045	F-test	244.776
	RMSE	0.012	0.008	0.010	0.003	0.033	0.005			$X^2_{df_n}$	20.161
11	Bias	-0.016	0.008	-0.008	0.004	-0.033	-0.007	h	0.244	MESD	0.059
	E-SD	0.004	0.000	0.003	0.001	0.009	0.002	df_n	8.043	F-test	244.757
	RMSE	0.012	0.008	0.010	0.003	0.033	0.006			$X^2_{df_n}$	20.159
12	Bias	-0.017	0.008	-0.008	0.004	-0.030	-0.009	h	0.244	MESD	0.059
	E-SD	0.004	0.000	0.002	0.001	0.008	0.002	df_n	8.041	F-test	244.740
	RMSE	0.013	0.008	0.009	0.004	0.033	0.006			$X^2_{df_n}$	20.155
13	Bias	-0.019	0.008	-0.009	0.003	-0.037	-0.003	h	0.244	MESD	0.059
	E-SD	0.004	0.000	0.002	0.001	0.008	0.002	dfn	8.040	F-test	244.762
	RMSE	0.013	0.008	0.009	0.003	0.033	0.006			$X^2_{df_n}$	20.153
14	Bias	-0.020	0.008	-0.009	0.003	-0.037	-0.003	h	0.244	MESD	0.059
	E-SD	0.005	0.000	0.002	0.001	0.008	0.002	df_n	8.044	F-test	244.777
	RMSE	0.014	0.008	0.009	0.003	0.034	0.006			$X^2_{df_n}$	20.160
15	Bias	-0.021	0.008	-0.009	0.003	-0.036	-0.004	h	0.244	MESD	0.059
	E-SD	0.005	0.000	0.002	0.001	0.008	0.002	df_n	8.044	F-test	244.773
	RMSE	0.014	0.008	0.009	0.003	0.034	0.006			$X^2_{df_n}$	20.160

Moreover, we evaluate the empirical standard deviation of $M(\Lambda)$ (MESD), the optimal bandwidth h, F-test based on (17), the freedom of the chi-square statistics df_n and the right 0.01 quantile chi-square statistics $X^2_{df_n}$. Fig 2 depicts the variation of the Bias, E-SD and RMSE of ML estimators $\left[\hat{\alpha}_{ML}, \hat{r}_{ML}, \hat{\lambda}_{ML}, \hat{\beta}_{ML}, \hat{\eta}_{ML}, \hat{\rho}_{ML}\right]$, MESD and Likelihood function in the iterative process.

From the results, convergence occurs since the second iteration except the estimation of constant, which diffuses by systematic error at a very small amount. The accuracy of estimation of each statistics including the linear spatial regression parameters and nonparametric function also meet the qualification since the second iteration and we can find it more clearly in Fig 2. The likelihood function and the empirical standard deviation of $M(\Lambda)$ (MESD) also get to stable since the second iteration. Similar conclusion also occur in other N and T, so we can use a three stage iterative approach instead of the iterative approach to reduce systematic error and calculated amount while maintaining the accuracy.

The Improvement of Approach: Three Stage Iterative Approach

As the preceding analysis, convergence occurs since the second iteration, so we use three stage iterative approach to improve the iterative approach as follows:

● We estimate the spatial dynamic Durbin model with fixed effects (7) by ML estimation to obtain the initial estimator of δ, σ^2 and ρ. Then we calculate the residuals V_t as showed in (9).

● Repeat the second step to the third step until convergence occurs. Then we can get the final estimation results of δ, σ^2, ρ and $M(\Lambda)$. We establish the nonparametric model of V_t and $M(\Lambda_t)$ (11) to get the local-linear estimator of $M(\Lambda_t)$ (12). The definition and calculation of kernel function $K(\cdot)$ and the optimal bandwidth h are the same as the previous.

● Make the local-linear estimator $\hat{M}_{LL}(Z_t, h)$ (12) as the estimator of $M(\Lambda_t)$ and put it into (1) to recalculate the final ML estimator of δ, σ^2 and ρ.
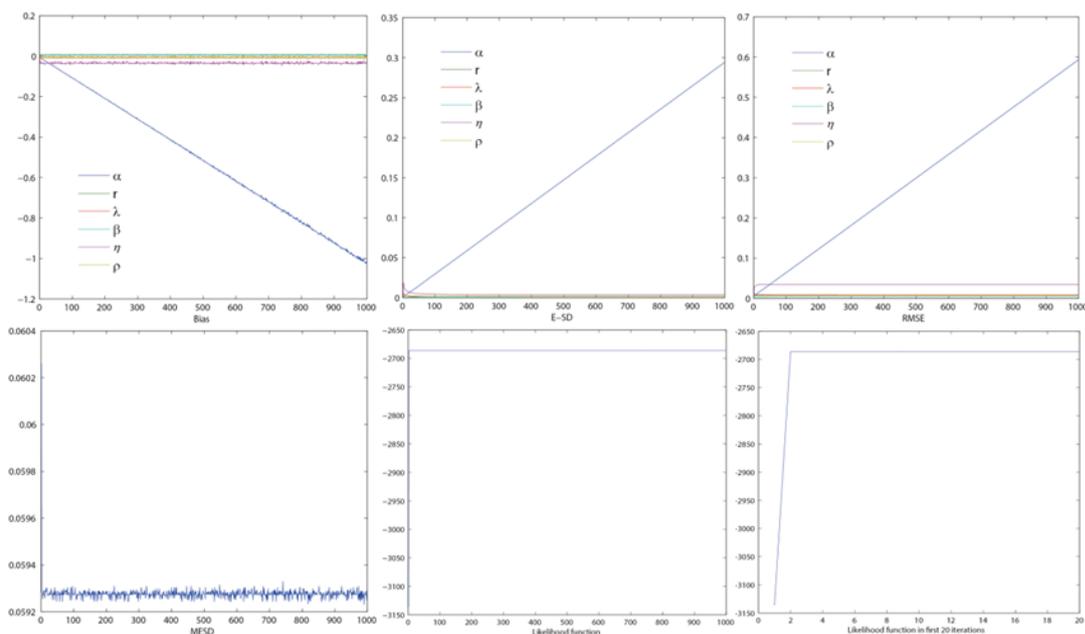

Figure 2. The Bias, E-SD, RMSE of parameters estimation, MESD and likelihood function of the SDNDM with fixed effects by the iterative approach under different iteration

The Application of Three-Stage Iterative Approach

We estimate the spatial dynamic nonparametric Durbin model with fixed effects by three stage iterative approach in nine cases with N=10, 30, 50 and T=10, 30, 50 to compare the performance of three stage iterative approach in different N and T, as shown in Table 2. For each case, we calculate the ML estimator of linear spatial regression parameters and their Z-probability, bias $\hat{\theta} - \theta_0$. Moreover, we evaluate the empirical standard deviation of $M(\Lambda)$, F-test based on (17) and the right 0.01 quantile chi-square statistics of $X_{df_n}^2$.

In order to analyze the accuracy of the estimator of $M(\Lambda)$, we take the first individual as an example and compare the local-linear estimator $\hat{M}_{LL}(Z_t, h)$ and $M(\Lambda_t)$ in different cases, as shown in Fig 3. It clearly shows that the local-linear estimator $\hat{M}_{LL}(Z_t, h)$ fits $M(\Lambda_t)$ better when N and T get larger in parallel with the findings of Zheng et al. (2013). We also make a comparison between dependent

variable Y and its fitted values of the first individual in different cases (the estimator under $M(\Lambda_t)$ nonlinear hypothesis and the estimator under $M(\Lambda_t)$ linear hypothesis). Since $M(\Lambda)$ is very close to Λ when Λ is small, the estimator under $M(\Lambda_t)$ linear hypothesis fits Y quite well, as shown in Fig 4.

Table 2. The estimation of the SDNDM with fixed effects by three stage iterative approach

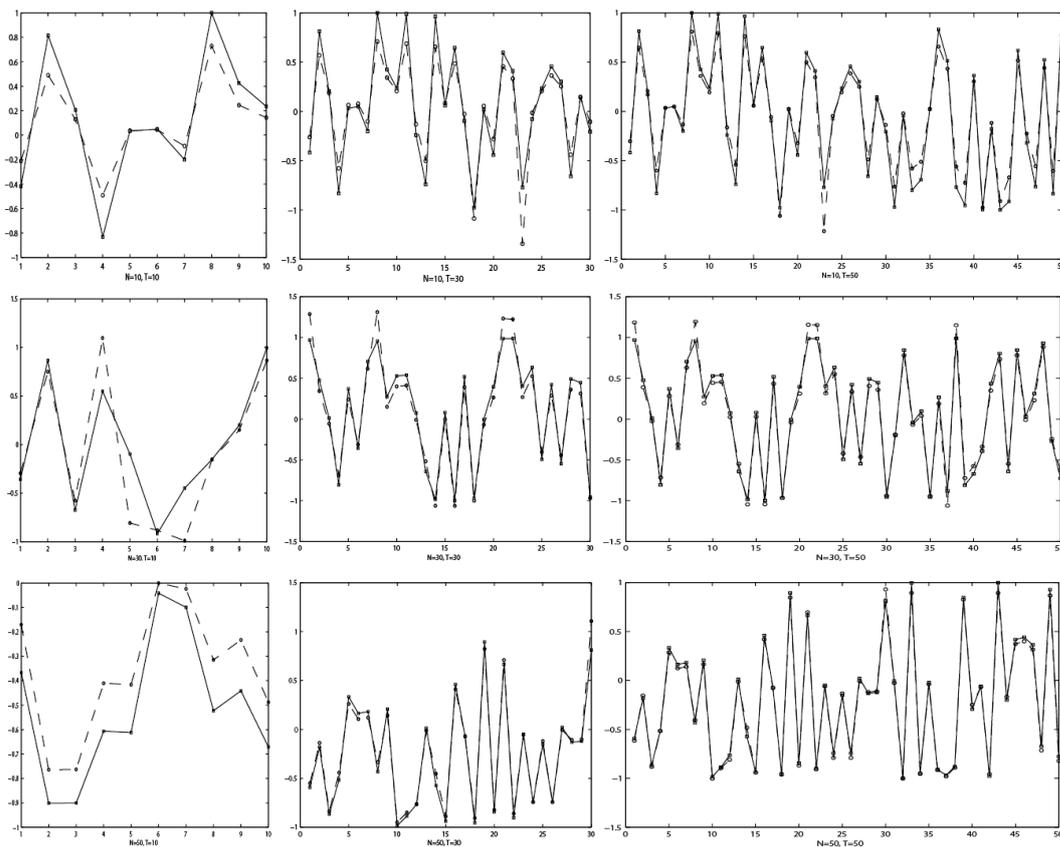| case	T	N		α	r	β	Λ	η	ρ	$M(\Lambda)$	
1	50	50	MLE	0.108	0.192	1.008	0.196	0.133	0.206	MESD	0.060
			z-prob	0.000	0.000	0.000	0.000	0.000	0.000	F	244.792
			Bias	-0.008	0.008	-0.008	0.004	-0.033	-0.006	$X^2_{df_n}$	20.204
2	50	30	MLE	0.103	0.189	1.037	0.192	0.099	0.212	MESD	0.106
			z-prob	0.036	0.000	0.000	0.000	0.002	0.009	F	180.929
			Bias	-0.003	0.011	-0.037	0.008	0.001	-0.012	$X^2_{df_n}$	18.968
3	50	10	MLE	0.125	0.174	1.071	0.142	0.230	0.067	MESD	0.154
			z-prob	0.003	0.000	0.000	0.008	0.056	0.305	F	50.001
			Bias	-0.025	0.026	-0.071	0.058	-0.030	0.133	$X^2_{df_n}$	17.150
4	30	50	MLE	0.057	0.190	1.019	0.238	0.163	0.227	MESD	0.076
			z-prob	0.212	0.000	0.000	0.000	0.000	0.003	F	97.978
			Bias	0.043	0.010	-0.019	-0.038	-0.063	-0.027	$X^2_{df_n}$	18.800
5	30	30	MLE	0.091	0.187	1.024	0.233	0.227	0.232	MESD	0.112
			z-prob	0.117	0.000	0.000	0.000	0.000	0.017	F	72.349
			Bias	0.009	0.013	-0.024	-0.033	-0.127	-0.032	$X^2_{df_n}$	17.956
6	30	10	MLE	0.137	0.193	1.028	0.246	0.203	0.174	MESD	0.144
			z-prob	0.059	0.000	0.000	0.004	0.223	0.025	F	13.605
			Bias	-0.037	0.007	-0.028	-0.046	-0.103	0.026	$X^2_{df_n}$	15.959
7	10	50	MLE	0.191	0.157	0.963	0.248	0.313	0.145	MESD	0.134
			z-prob	0.000	0.000	0.000	0.000	0.000	0.005	F	47.878
			Bias	-0.091	0.043	0.036	-0.048	-0.213	0.054	$X^2_{df_n}$	16.963
8	10	30	MLE	0.153	0.148	1.059	0.234	0.352	0.071	MESD	0.162
			z-prob	0.006	0.000	0.000	0.000	0.004	0.304	F	27.985
			Bias	-0.053	0.052	-0.059	-0.034	-0.252	0.129	$X^2_{df_n}$	16.604
9	10	10	MLE	0.128	0.171	1.190	0.253	0.419	-0.133	MESD	0.174
			z-prob	0.168	0.001	0.000	0.003	0.048	0.293	F	3.371
			Bias	-0.028	0.029	-0.190	-0.053	-0.319	0.334	$X^2_{df_n}$	14.343

Figure 3. The comparison between the local-linear estimator $\hat{M}_{LL}(Z_t, h)$ and $M(\Lambda_t)$ of the first individual in different cases (the solid line represents $M(\Lambda_t)$ and the dotted line represents $\hat{M}_{LL}(Z_t, h)$)

While the estimator under $M(\Lambda_t)$, nonlinear hypothesis perform even better. According our calculation, the residual sum of squares of the estimator under $M(\Lambda_t)$ nonlinear hypothesis is much smaller than the estimator under $M(\Lambda_t)$ linear hypothesis although they are very close in Fig 4.

From the results, three-stage iterative approach is more reliable when *T* or *N* is large, which successfully alleviates the "initial conditions" problem discussed in Neyman and Scott (1948) and Yu et al. (2012) as well as the "incidental parameters" problem (see Hsiao, 1986). The bias and the error' standard deviation of $M(\Lambda)$ (E-SD) tend to be smaller when *N* and *T* are large. But it gets worse when *N* and *T* are small due to the accuracy of estimators decline. Moreover, as the hypothesis tests are based on the asymptotic properties of statistics, the test of the nonparametric function and linear spatial regression parameters tend to fail when *N* and *T* are very small (see case 9, *N*=10, *T*=10). In general, the estimation of $M(\Lambda)$ is very important for the estimation of linear spatial regression parameters.
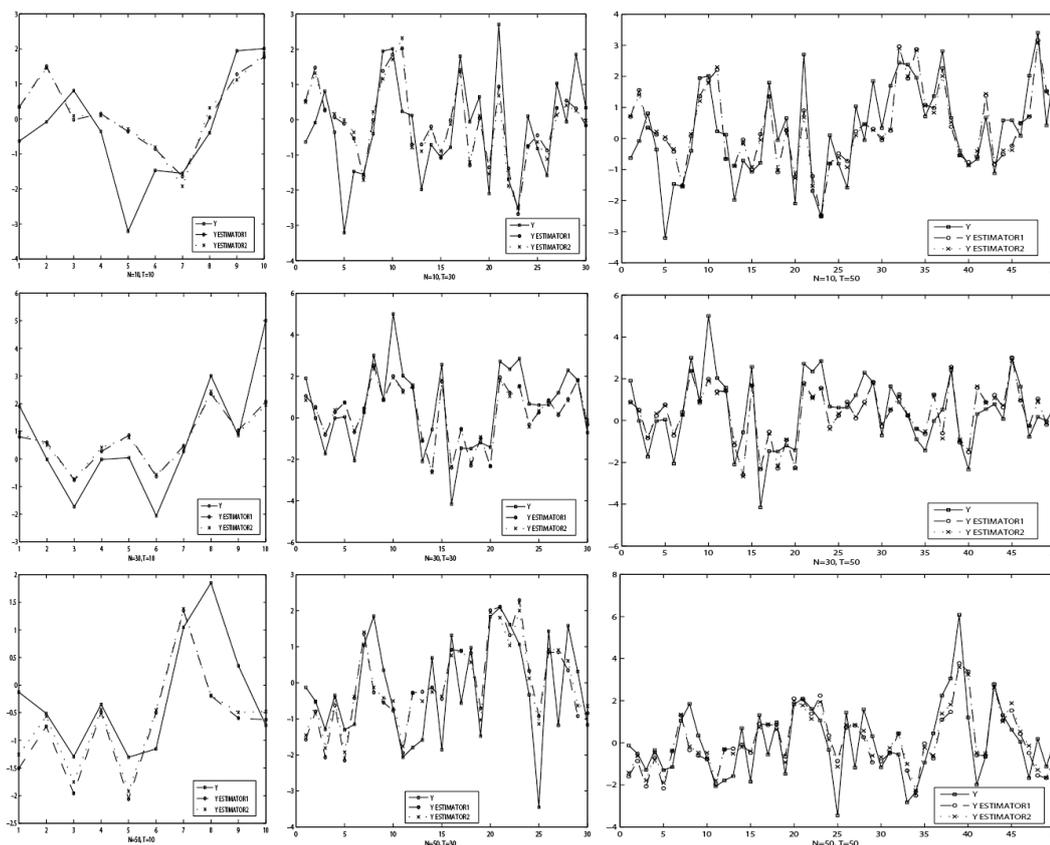
Figure 4. Y and its fitted values of the first individual in different cases(Y ESTIMATOR1 represents the estimator under $M(\Lambda_t)$ nonlinear hypothesis, sovled by three stage iterative approach and Y ESTIMATOR2 represents the estimator under $M(\Lambda_t)$ linear hypothesis, solved by ML method)

CONCLUSION

In the spatial dynamic nonparametric Durbin model with fixed effect, we propose using the three stage iterative approach to shrink parameters contained in both parametric and nonparametric components with corresponding hypothesis test. The resulting estimators indicate that the three stage iterative approach is more reliable when *T* or *N* is large and the accuracy of estimation of $M(\Lambda)$ is very important for the estimation of parametric components. We believe that the three stage iterative approach is reliable and effective estimation method for the spatial dynamic nonparametric Durbin models with fixed effects, improving the estimation accuracy of nonparametric components would enhance the reliability of three stage iterative approach in data analysis.

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