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Evaluating Students' Beliefs in Problem Solving Process: A Case Study

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Problem solving is not simply a process that ends when an answer is found; it is a scientific process that evolves from understanding the problem to evaluating the solution. This process is affected by several factors. Among these, one of the most substantial is belief. The purpose of this study was to evaluate the beliefs of high school students according to their explanations in the problem solving process. A case study was carried out with five students. Each of the students' problem solving processes was examined by means of clinical interviews and three problems were presented to the students. The results illustrate that the students who thought problem solving should be a short process and that they could address it by memorization of rules also believed that problem solving is difficult. This study concludes that not only beliefs affect the problem solving process, but also personal factors such as life experiences.

Keywords: belief, problem solving, problem solving process

INTRODUCTION

New social, economic, scientific and technological developments take place every day in the course of human life. As a reflection of these developments, individuals continuously encounter new problems and new demands; in order to go about their lives, they need to address the various obligations and difficulties that arise. To some extent, they struggle to solve the problems in their lives; this is not only the case in the past and present, but also a situation that they are likely to encounter in the future. However, people cannot predict the types of problems they will face. For this reason, contemporary education aims to develop the skills that people will need to overcome problems on their own and educate them with the required qualifications (Aladağ, 2009). As each new generation encounters more complex problems than the former generations, finding solutions to all the problems students may encounter is becoming increasingly difficult. Therefore, problem solving skills have an important place among the primary objectives of a contemporary curriculum. In this regard, one of the aims of mathematics education is to educate students to become good problem solvers (Baki, 2008). Being good at problem solving in mathematics requires a sufficient level of mathematics knowledge, as well as the ability to put this knowledge to practical use (Tertemiz & Çakmak, 2003). As Karataş (2002) notes, both understanding mathematics knowledge and awareness

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of the relationships between different aspects of mathematics are part of the problem solving process; in this respect, the problem solving process requires establishing a connection between the problem solver and his/her knowledge, efforts and thoughts (Schoenfeld, 1982). Thus, aside from providing students with basic mathematical concepts and skills, modern mathematics programs aim to develop consciousness of solving problems; sharing solutions and ideas; and thinking and deciding independently. For this reason, problem solving by establishing relationships between different aspects of mathematical knowledge is an indispensable part of mathematics education. Because problem solving is a comprehensive process which encompasses numerous human characteristics (e.g., as needs, aims, values, abilities, beliefs, attitudes), as well as creative thinking, cognitive ability, self-discipline and actions (Oğuzkan, 1989), factors such as experiences, perception, beliefs and motivation all affect this process.

State of the literature

- According to studies in the literature, beliefs have a strong and observable effect on the thoughts and behaviours of individuals. They are also effective in problem solving process.
- The beliefs are classified as positive and negative by researchers and students generally have negative beliefs about problem solving. Positive or negative beliefs affect students' behaviours in problem solving process.
- Positive beliefs motivate students to problem solving process, providing to make more effort to solving process. While these beliefs increase students' success in problem solving process, negative beliefs give rise to avoiding persevering to solve problems and decreasing their success.

Contribution of this paper to the literature

- Research studies about determining students' beliefs by observing their problem solving process are limited and most of them determine students' beliefs by questionnaires.
- Few studies have focused on observation of what certain individuals' beliefs were, or on the effects of these beliefs, by following the individuals through the problem solving process.
- This study enables to be informed more details about students' beliefs in problem solving process.

Factors affecting the problem solving process

Problem solving is a scientific process that a person passes through from understanding the problem to deciding on the information needed for a solution to solving the problem and evaluating the appropriateness of the solution (Williams, 2003). In the problem solving process, an individual must be able to combine the appropriate operations and apply them to the solution (Bernardo, 1999). Certain factors affect this cognitive process; these have been classified according to the perspectives of various researchers. For instance, Charles and Lester (1982) categorized the factors affecting the problem solving process as (1) cognitive (e.g., reasoning, reading and procedural skill, metacognition); (2) affective (e.g., self-confidence, stress, anxiety, motivation, interest); and (3) experience (e.g., age of student, previous knowledge, strategies and familiarity with the content of the problem). On the other hand, Schoenfeld (1992) outlined a framework of four different factors affecting students' problem solving processes: resources, heuristics, control, and belief systems. According to this framework, *resources* include formal and informal knowledge about facts and routines; *heuristics* refer to strategies and techniques for approaching a problem; and *control* involves the methods by which individuals check their own problem solving process and observe partial results to decide on further problem solving actions, as well as determining how and when to use available resources and heuristics. Finally, *belief systems* comprise individuals' mathematical world views, as well as the perspectives of the individuals towards themselves, their environment and the topic at hand. It has been concluded that mathematical knowledge, problem solving strategies, control and beliefs are important components of the problem solving process. Jonassen (2000) similarly elaborated the factors affecting problem solving performance as familiarity; domain and structural knowledge; constructivist

knowledge; cognitive controls and styles; general skills; metacognition; affective and conative effects; and epistemological beliefs.

Acquaince with the relevant mathematical features, algorithms and rules alone does not enable success in problem solving. The decisions and strategies used while controlling or regulating one's actions; the emotions experienced while performing a mathematical task; and the beliefs related to the applications of mathematical tasks also influence an individual's success in problem solving (Chiu, 2012; Garofalo, 1989; Viholainen, Asikainen & Hirvonen, 2014). In particular, Gómez-Chacón (2000) points out that the elements of affect *(emotions, attitudes, beliefs)* of students are key factors in understanding their behaviour in mathematics and adds that beliefs are one of factors play the central role in success or failure in mathematics. She also states that beliefs have the powerful impact how the pupils learn and use mathematics. Therefore, examining beliefs can enable understanding of how students learn about problem solving. Kloosterman and Stage (1992) likewise indicate that beliefs about mathematics influence the problem solving process, stressing that positive beliefs about mathematics are key elements in improving problem solving skills. Furthermore, Callejo and Vila (2009) determined that a dualistic belief system arising from student's school experiences and motivation has an effect on a student's approach to the problem solving process; while Schoenfeld (1983), in highlighting the importance of beliefs in terms of students' efforts to solve mathematical problems, stated that the problem solving process does not include cognitive applications alone. Both knowledge and beliefs affect problem solving.

Belief

The concept of belief is often undefined (e.g., Mason, 2004; Leder, 2008) and used synonymously with terms such as attitude, disposition, opinion, perception, philosophy, and value. Because these various concepts are not directly observable and have to be inferred, and because of their overlapping nature, it is not easy to produce a precise definition of beliefs. As an early definition of belief systems, Schoenfeld (1985) points out that belief systems are one's mathematical world view. In fact, this statement is a metaphor rather than a definition. The comparison provides an image of what belief systems are and how globally they operate. Schoenfeld (1992) also adds that beliefs are an individual's understandings and feelings shaping the ways that the individual conceptualizes and engages in mathematical behaviour. The definition stated by Schoenfeld (1992) stresses the dynamic of beliefs, i.e., how beliefs function. Lester, Garofalo and Kroll (1989) contend that beliefs constitute an individual's subjective knowledge about self, mathematics, problem solving, and the topics dealt with in problem statements. Lester, Garofalo and Kroll (1989) and Schoenfeld (1992), focusing on mathematical behaviour, assert similar definition of belief. Some researchers indicate that belief is a concept being associated with attitude (Eagly & Chaiken, 1993; Goldin, Rösken & Törner, 2009). In other words, they state that beliefs are some kind of attitudes. However, other researchers think that attitudes and beliefs are different concepts (e.g., Griffin & Ohlsson, 2001). Griffin and Ohlsson (2001) distinguish these concepts as: "Whereas attitudes refer to subjective evaluations of objects as 'positive' or 'negative', beliefs refer to the acceptance or rejection of propositions." Besides, it is possible to distinguish between belief and knowledge. Furinghetti and Pehkonen (2002) characterize beliefs as subjective knowledge and these researchers state when expressed as sentences they might be (or might not be) logically true. But, knowledge always has this truth-property (Lester et al., 1989). Whereas knowledge refers to the representation of a proposition, belief refers to the representation of a truth-value associated with a proposition (Griffin & Ohlsson, 2001). Besides, Thompson (1992) distinguishes beliefs from knowledge in terms of three

dimensions as: the degree of intersubjective consensus, the type of argument needed for the acceptance of beliefs and knowledge respectively, knowledge is related to truth and certainty, while belief is more associated with doubts and disputes. On the other hand, beliefs are associate with conceptions. Lloyd and Wilson (1998) pointed out that conceptions refer to a person's general mental structures that encompass knowledge, beliefs, understandings, preferences, and views. They are essentially metaphorical." For this study, beliefs are mental constructions representing the codification of individual's experiences, behaviours, understandings in problem solving process.

Beliefs in the problem solving process

A number of researchers have argued that beliefs reflect the decisions, preferences and behaviours that individuals will make throughout their lives (Deryakulu, 2004; Hofer & Pintrich, 1997; Pajares, 1992). As a part of individuals' lives, mathematics learning and instruction are influenced by cognitive as well as affective issues (Leder & Forgasz, 2002). One of affective issues is beliefs and they are important in understanding individuals' mathematical problem solving heuristics and strategies (Lester, Garofalo & Kroll, 1989; Schoenfeld, 1985). Problem solving is a process that requires making decisions until a solution is found; thus, beliefs are considered as variables affecting the problem solving process.

Attempting to overcome obstacles by trying different solutions represents the cognitive side of problem-solving; on the other hand, perseverance in the process, worrying, giving up the search for a solution, and hurrying to find a solution exemplify the affective side (Aksan & Sözer, 2007). In this process, the existence of belief cannot be ignored. Schoenfeld (1985), for instance, expresses that beliefs towards mathematics are effective in determining how individuals make choices while considering a problem, what kind of strategies they use or avoid, whether they consider the problem to be difficult or not, their level of anxiety, and how long it will take to solve the problem. Therefore, the beliefs are classified as positive and negative. Positive beliefs may be characterized as motivating to problem solving process, providing to make more effort to solving process. Negative beliefs may be described as giving rise to avoiding persevering to solve problems. The problem-solving process comprises stages such as determining how to approach the problem, which techniques to use, and how much effort and time will be expended in reaching a solution. Recognition of the problem solving process and obtaining information about the process contribute not only to effective learning, but also to the improvement of individual abilities. According to existing research, beliefs have a strong and observable effect on the thoughts and behaviours of individuals, as well as on their learning styles (Yılmaz, 2007).

How to determine students' beliefs in the problem solving process

Beliefs are measured using a variety of techniques in the studies reviewed. These include questionnaires, interviews, content analysis of journal entries, reflections, post lesson conferences, and observations. Leder and Forgasz (2002) also summarize various qualitative and quantitative methods for measuring beliefs as questionnaires (Likert-scales or checklists), physiological measures (like galvanic skin response), interviews and/or observations. On the other hand, they suggest another approach using the Experience Sampling Method (ESM) to measure the beliefs. In mathematics education, beliefs about problem solving are often measured by means of questionnaires and clinical interviews. The questionnaires are generally in the form of Likert-scale having items that are typically rated from "strongly agree" to "strongly disagree". Benefitting from these questionnaires, it is aimed to measure beliefs about problem solving in terms of different dimensions. However, there are

some disadvantages of the questionnaires. They are inadequate to understand exactly some forms of information (changes of beliefs, emotions, behaviour, feelings etc.). Besides, they have limited amount of information without explanation. Using clinical interviews to measure beliefs is criticized about reaching small samples, too. However, regarding to reach information in more detail, clinical interviews are useful. Therefore, this type of interview may be suitable for reflecting students' beliefs in the problem solving process comprehensively. In this study, it is aimed to determine beliefs giving rise to each of students' perseverance or not in problem solving thoroughly without generalizing. This situation also reinforces using clinical interview for the study.

Studies about determining students' beliefs in the problem solving process

Studies about beliefs toward problem solving are generally aimed at determining beliefs by means of questionnaires and examining whether beliefs change according to certain variables or after classroom implementations (Kloosterman & Stage, 1992; Mason, 2003; Stylianides & Stylianides, 2014; Uğurluoğlu, 2008, Vidic, 2014). However, few have focused on observation of what certain individuals' beliefs were, or on the effects of these beliefs, by following the individuals through the problem solving process. Uğurluoğlu (2008) conducted a study to examine whether the beliefs and attitudes of 7th and 8th grade students on mathematics and mathematics problem solving differentiated with respect to certain variables and to determine the relationship between these variables. In this investigation, mathematics problem solving attitude and belief scales were administered to 3556 students. The results revealed that students' attitudes and beliefs towards problem solving were affected positively with respect to their level of success, as well as their family's income and education levels. In addition, increased size of living space had a positive effect. It was also concluded that while students' attitudes towards mathematics and problem solving did not change according to gender, their beliefs towards mathematics and mathematics problems did change. Vidic (2014) focused his study to examine the differences between students' mathematics-related beliefs and beliefs about context problems. It was used a questionnaire consisting of five scales to determine first-year students' beliefs about mathematics and context problems. This study was carried out with two different groups in less (LD) or more demanding (MD) programme. According to the results, students from the MD have more negative beliefs about context problems than students from the LD. Contrary to this, students from the MD hold more positive beliefs about mathematics. Stylianides and Stylianides (2014) examined whether it was a positive impact on students' problem solving beliefs with an intervention of short duration. 39 undergradute students took the pre-post scales on beliefs. As a result of the study, they explored a different approach to the issue of impacting positively on students' problem solving beliefs. They focused on the problem solving process in terms of implementing the experimental study, but it is not for determining students' beliefs. Lerch (2004), focusing on the process of problem solving to a greater extent than other researchers, aimed to find out how control decisions played a role in solving routine and non-routine problems, as well as how beliefs affected these control decisions. In his study of 4 university students, it was discovered that the decisions made by the students during a solution attempt were dependent upon their content knowledge and personal belief systems. Specifically, he found that students' personal belief systems provided confidence that they would be able to solve the problem; furthermore, certain beliefs had a negative effect on their ability to find a solution. Generally, he concluded that beliefs affected students' approach to problem solving and had an effect on the process.

Purpose of the study

As a component of mathematics programs, the aim of problem solving (Howland, 2001), aside from finding a result, is to overcome any obstacles to finding a solution (Morgan, 1999). In this process; analyzing the problem, collecting necessary information, using the selected information appropriately, finding the best way, and evaluating results are vital (Chen, 2010). Whether solving problems related to daily life or mathematical problems, a comprehensive process involving multiple stages is required. As a result, it may be influenced by many factors, such as mathematical knowledge, attitude, beliefs, and so on. Through close analysis of the problem solving process, these factors can be examined in more detail. Studies about belief, a factor affecting problem solving, are mainly dependent on the use of belief scales. However, these studies are inadequate about reflecting beliefs in problem solving process thoroughly. On the other hand, the studies are deprive of individuals' explanations, it prevents from giving details about beliefs in the process. For this reason, it is significant to determine beliefs in the problem solving process by means of clinical interview. Thus, an individual's mathematical behaviour about problem solving can be observed and his/her beliefs in the problem solving process can be revealed by supporting with his/her explanations. Considering this situation, we proposed to evaluate the beliefs of high school students according to their explanations in the problem solving process.

METHODOLOGY

For the purposes of this study, a qualitative approach was applied, employing a case study method to investigate the beliefs according to the students' explanations in the problem solving process, as well as the effects of these beliefs on the process. The objective of the case study was to describe the current situation in detail rather than generalizations. For this purpose, three problems were presented to the students, and each individual's problem solving process was examined separately. Clinical interviews were used to elicit the students' beliefs, as well as the effects of these beliefs, in a detailed manner. Both the beliefs evidenced by the students during the problem solving process and the effects of these beliefs on the process are presented in this article.

Participants

The participants for this study were five ninth-grade students from a high school in Turkey. The selection of participants was based on voluntariness. The participants were classified as poor, middle and high level based on their academic achievement, success in problem solving, and the recommendations of their teacher. Students who had low academic achievement and were insufficient in terms of problem solving were labeled as poor; those who had high academic achievement and were sufficient in terms of problem solving were labeled as high level; and students who demonstrated one of these two features or were intermediate in terms of both features were labeled as middle level. Two poor (S1, S5), one middle (S2), and two high level (S3, S4) students were selected.

Instruments

During the study, three problems were presented to students. In selecting the problems, those that had more than one solution method and were suited for use of different strategies were selected. An academic with experience in carrying out studies on problem solving was consulted to ensure the validity of problems. The problems used in the study are illustrated below in Table 1.

Table 1. Problems used in the problem solving process

	Problems	Basis for Problem Selection
1	Damla made different necklaces for 25 friends in her office. She bought jewellery boxes to put them in. Damla needed wrapping paper to wrap the boxes. For each box, she needed 18x15 cm of paper. The wrapping paper was only available in 90x90 cm sheets, and it was very expensive. Therefore, she didn't want to buy too many sheets of paper. How many sheets did she need to buy to wrap all of the boxes?	• Suitable level for students • Multiple solutions • Observing the effect of the length of the problem expression. • Related to daily life • Clear and understandable expressions
2	In a hospital, medical examinations are performed in different polyclinics. Per hour, three fifths of the patients see the doctor and leave. After the fourth hour, a total of 32 patients are left in the hospital. How many patients were there at first in the hospital?	• Suitable level for students • Multiple solutions • Includes successive mathematical operations • Related to daily life • Clear and understandable expressions
3	Every hour, a train leaves from Istanbul to Ankara and from Ankara to Istanbul. The first train from Ankara to Istanbul leaves at 5:30. It takes 5 hours to travel in either direction. How many Ankara-Istanbul trains do we see on the way from Istanbul to Ankara?	• Suitable level for students • Multiple solutions • Observing whether the meaning of the time expression is distinguished • Related to daily life

Each student's problem solving process was examined via a clinical interview. One of the objectives of the use of the clinical interview was to make inferences about each student's mathematical understanding, knowledge structure and cognitive process, as well as the changes realized during the process (Goldin, 1998). During the clinical interviews, examinations were performed, with three problems given to each student and audio recording was made. It was acquainted the students with audio recording in their problem solving process and the purpose of the study. The time and place of the clinical interviews were also determined according to the students' requests. First, the students read the problems for comprehension in the clinicial interviews. After making sure they understood the problems thoroughly, the students were asked questions such as: *Did you understand the problem? Can you express it in your own words? What kind of solution can you produce? Why did you choose this solution? Can you verify your answer?* Apart from these, several other questions were asked to reveal the students' beliefs concerning the problem solving process: *Is problem solving based on a formula or on logic? Did you think, "I can solve this" when you read the problem? Are you interested in problem solving? If you dream about solving a problem, how do you feel? When problem solving activities are given, do you solve the problems willingly?* The questions enabled them to express the difficulties they encountered and their perspectives about problem solving during the clinical interview. The clinical interviews were carried out individually with each student.

Data analysis

The beliefs exhibited by the students during the process were examined by means of observation, as well as the clinical interviews that were carried out in the problem solving process. In this study, by observing sudents' problem solving processes, their understanding, strategies and solution plans were investigated. In addition, attention was given to analyzing their actions, including identifying the necessary information, following the mathematical operations, deciding how long these operations would take and evaluating the results they reached. Through observing these actions, the researchers attempted to reveal how much effect the students' beliefs had on their behaviour and what the effects of those beliefs were. In order to determine the students' beliefs, the researchers focused on their approaches to the problems, as well as their perspectives on the problems and on problem solving. The data obtained during the clinical interviews were coded, and

the beliefs that affected the problem solving process were identified according to students'explanations. In order to the reliability of data, it was used *"investigator triangulation"* (Johnson & Christensen, 2004) being one of the types of triangulation. Except for the researchers, an expert investigated the data and it was reached a consensus about the students' beliefs in problem solving process. At the end of the analysis, each student's problem solving process and beliefs were described in a detailed way.

RESULTS

The problem solving process of each student who participated in the study and their beliefs in the process are presented and discussed here. The results emerged from analysis of data generated by five students, S1, S2, S3, S4 and S5. Each student's problem solving and beliefs in the process are discussed separately, in five sections.

Student S1's problem solving process and beliefs about the process

Student S1 was asked to consider the problems. The conversation revolved around her experiences in the problem solving process. The aim of these conversations was to reveal all of the factors affecting the student in the process, rather than determining her success or failure in problem solving. One of the conversations carried out in the process is as follows:

R: Which statement might be the most helpful for finding the solution?
S1: The expression "25 friends" is important.
R: Why do you think so?
S1: We cannot find the answer without this information.
R: How can you produce a solution making use of this information?
S1: I haven't solved this kind of problem for a long time. It's strange.

After reading the first problem, she explained the given and requested information correctly and she determined the most helpful statement for finding the answer as "*25 friends*". Because of this inference, it might be said that she understood this problem. Although it was expected that she would produce a solution to the problem, when she was asked what kind of solution she would follow, she said she had not solved that type of problem for a long time. This opinion prevented her from producing a solution. On encountering the other problems, she began to use some expressions which reinforced this belief:

R: Is problem solving based on a formula or on logic?
S1: Based on a formula.
R: Did you think, "I can solve it" when you read the problem?
S1: I thought, "Maybe I can!"
R: Do you think problem solving is forgotten over time?
S1: Yes, it can be forgotten. I haven't solved this type of problem since I was in primary school.

Her explanations reflect the belief that solving problems is an action which requires memorization. During the process, she persisted in saying that she had not solved problems for a long time and that she had not solved such problems since her primary school years. These expressions support her belief that problem solving is forgotten over time. Believing that memorizing brings success in problem solving and that problem solving is forgotten after some time prevented her from approaching the problem and searching for a solution. Generally, she expressed that the problem solving process was difficult, and she believed that she could solve problems that were easy enough. Even though she identified the necessary information for a problem, she did not produce a solution. Her perspective on problem solving was generally negative. She also thought that it was of no use to try to solve a problem after a few attempts. The fact that she did not produce solutions

to any of the problems led to negative opinions and actions; therefore, she thought that the problems were difficult. In the end, she gave up on the problem solving process. In this entire process, she generally determined the required information for solving the problems and passed the step of the understanding the problem. But the beliefs such as *problem solving is forgotten*, *problems are solved with memorizing knowledge* prevented her from devising a plan for solving the problems.

Student S2's problem solving process and beliefs about the process

Student S2 produced a solution to only one of problems presented in the problem solving process. She did not have any difficulty in explaining what information was needed and why it was necessary for the first problem. In defining how she formulated a solution, she drew a diagram suitable for the solution of the problem. While drawing, she preferred to form part, rather than the whole. In other words, in forming each box to the existing box shape, she drew a square, and thereby she reached the correct result. She was unable to demonstrate the same success with the other problems. After encountering the other problems, she lost her belief that she could solve the problems. This situation can be inferred both from the student's tone and from her responses.

R: Which information is most helpful for finding the solution?
S2: I think all the information is important.
R: Can you find a solution, then?
S2: I will try.
R: Would evaluating each hour separately be helpful?
S2: Let me try by drawing shapes.(Draws a rectangle, divides it into 5)
R: What does each cell stand for in the diagram?
S2: Two of them represent the number of patients left: 32
R: So, can you find the whole?
S2: One cell is 16, all of them are 80.
R: What does that 80 mean?
S2: .....
R: Is that what you wanted to find the number of patients, or the number in only an hour?
S2: No, that's not what I wanted to find. It is in the fourth hour.
R: Can you find the number of patient in the 3rd hour?
S2: (Thinks for some time) It takes very long to find each hour separately. No, I can't find it.

Saying *"I will try."*, when it was asked whether she can produce a solution, it was shown that her belief about solving the problems were gradually decreasing. Besides, she had difficulty in expressing which information given in the problem was important for a solution. Saying *"I think all the information is important."*, she did not distinguish which information is more necessary to solve the second problem. But she stated unhesitantly that she could produce a solution. As she had reached a suitable solution to the first problem by drawing a diagram as a problem-solving strategy, she tried to use the same as the strategy for the other problems, as well; however, she could not produce the appropriate solution and got stuck on a particular step. She found the number of patients left in the 4th hour, but she could not find the number of patients present in the other hours. She stated "*It takes very long to find each hour separately. No, I can't find it."* Her expression referred that following mathematical operations for this problem takes a long time and reaching the answer is difficult. Besides, she thought the expression of the problem was too long. Among the reasons for such behaviour is the belief that the solution to a problem should be short and practical to be meaningful. Evaluating with regard to her expressions, these kinds of beliefs prevented her to solve the problem. As she

had difficulty in producing a solution after the first problem, she thought that the difficulty level of the problems was gradually increasing. She reflected this situation in her subsequent:

R: Did you understand the problem?
S2: It was a little bit complicated.
R: What is needed to solve the problem? What can you use?
S2: The problems are getting harder.

These opinions affected her behaviours in problem solving process and she did not produce any solutions. After the third problem, her opinion in this respect became particularly strong. She believed that the problems for which she produce a solution were easy, but that the ones she could not solve were difficult and complex.

In general, whenever she made progress toward a solution, she went about solving a problem eagerly. However, on not producing the solution or not understanding what to do for the next step, she gave up on solving the problem. For this reason, if she was unable to produce a solution after a long period of time, she believed that making a further effort was not necessary and that she would not be able to solve it after more time. On the other hand, she got bored with the problem solving process and tended to give up. Although she generally understood the problems, she did not determine the required information for solving the problems, except for the first problem. Thus, she could not pass beyond the step of devising a plan. The belief that prevented her from passing other steps is *the solution to a problem should be short and practical*.

Student S3's problem solving process and beliefs about the process

Student S3 was able to produce a suitable solution for only one problem. At the beginning of the problem solving process, as soon as she read the problem, she thought that she could solve it. However, she could not make any choices concerning what was the most important information for solving problem; she expressed that all of the information was important. When asked how she would produce the solution, she answered that she would use the given numbers and perform one of four mathematical operations. However, she had difficulty in expressing why the operations were needed and what the meaning of those operations was. Accordingly, the student was asked some questions to elicit the factors that affected her:

R: Are you affected by the length of the problem?
S3: Buying papers and using them… Arranging something in order was hard for me.
R: Maybe you can solve with a different strategy. If you want, you can try.
S3: (Starts to draw a diagram) The answer gets longer. I think I don't know how to solve it. In the diagram, you can cover 30 with a 90x90 square. But 5 are left outside.
R: What can you give as an answer?
S3: Then 1 is enough.
R: Are you sure that is right?
S3: Absolutely right, because it is clear with the diagram. When I didn't try solving it with a diagram; I didn't understand it clearly; but I get it now. From now on, I will solve the problems using a diagram.

The length of the problem and the need for successive operations affected her and led her to believe she would not be able to solve it. One of reasons for such an opinion may be a belief that the length of expression of a problem denotes that the solution for the problem is challenging. Furthermore, she thought that finding a solution would take a long time when she tried to solve the problem by drawing a

diagram. When she tried another problem solving strategy, she saw that her idea was incorrect. After that, she decided to use the drawing strategy and reached the correct answer. Then, she stated that she would use the drawing strategy in problem solving in general. Such an opinion reflects a belief that a strategy that is effective in solving one problem may useful for another type of problem, as well. Happy with solving the first problem, she started another problem with a highly positive attitude. After a brief look at the problem, she made sense of it and tried to produce a solution. The following dialog took place during the process:

R: What do you infer from the problem?
S3: There are medicial examinations. Each hour, three fifths of the patients see the doctor and leave. There are 32 left at the end of the 4th hour. They ask how many patients there are in total.
R: What do you think is the most helpful information for finding the solution?
S3: Every hour, three fifths of the patients see the doctor and leave.
R: What kind of a solution can you come up with?
S3: When it says three fifths, I think it means approximately. But there is certainly a solution.

When asked what kind of solution she would produce, she answered that the ratio stated in the problem looked like an approximate value. This indicates that she did not understand the problem correctly. She engaged in trying to solve the problem for quite some time, and she stated "*…But there is certainly a solution."* Her opinion provided to continue the problem solving process. Thus, she did not stop trying to solve the problem. Her belief that *every problem has a solution* and *every individual has problem solving abilities* led her to continue trying for a solution. However, while this belief prompted her to keep working, she remained unable to find a suitable answer after several attempts. When addressing the last problem, she made inferences about the expressions used in the problem, as shown in the following dialog:

R: What do you understand after reading the problem?
S3: In the problem, it says each hour and per hour; don't all these mean the same thing?
R: Do you think so?
S3: Both have the same meaning.
R: What information is requested in the problem?
S3: It asks how many trains pass by.
R: Can you solve this problem?
S3: Yes, I can. I won't give up!

Saying *"Yes, I can solve this problem. I won't give up!"*, it was shown that she would continue the problem solving process regardless of what happened. During the process, she hesitated about whether the meaning of "each hour" and "per hour" were the same in the problem. Despite reading the problem several times to resolve this ambiguity, she could not distinguish the meanings. As she spent a long time in making this distinction, she was not able to focus on the problem itself. However, her level of belief that she could solve the problem did not decrease. Such a belief enabled her to continue searching for a solution, but it did not help her to produce one. Towards the end of the problem solving process, she expressed that the reason she was unable to produce a solution was that she did not have a desire to solve the problem and that the difficulty level increased from easy to difficult. Although she did not give up searching for a solution, she expressed an opinion as soon as she realized that it was unnecessary to engage in the problem solving process any longer. When asked whether it was enough only to know the formula needed to solve a problem, she pointed out that questions about prisms can be solved by using

formulas, but that logic is also very important in the problem solving process. Her beliefs such as *every problem has a solution* and *every individual has problem solving abilities* provided to continue the process. But the belief such as *the difficulty of the problem affects the solution* prevented her from passing beyond devising a plan for the problems, except for the first problem.

Student S4's problem solving process and beliefs about the process

Beginning the problem solving process calmly, student S4 attempted to understand the problem and produce a suitable solution. The initiatives occurring in this process are reflected in the following dialog:

R: Can you express what you understand from the problem in your own words?
S4: There are 25 friends. She makes necklaces for them. After putting the necklaces in boxes, the boxes will be wrapped. Papers of 18x15 cm are enough to wrap them, and she needs to buy 90x90 cm papers. How many of these papers are needed?
R:What is the most helpful expression for finding the solution?
S4:15x18 cm boxes and 90x90 cm dimensions.
R: Can you solve the problem?
S4: Maybe. At first, I thought I could not.
R: Well, can you come up with a solution, then?
S4: As the mathematical operation, I will do this: 25x15x8. So, I could find how many sheets of paper are needed, but not anymore.

At first, she did not guess which a mathematical operation was continued and whether this operation was useful. Giving up this mathematical operation, she preferred using drawing strategy and reached the correct answer. Then, she stated "*I did not understand when I read it the first time, but I could understand it when I kept on. It happens that way a lot.*" Her explanation indicated, when she could not produce a solution, she believed that she could solve the problem by taking a short break and then refocusing on the problem. This belief enabled her both to continue working for a solution and to remain calm. Her belief that a person should focus on a problem closely and think deeply in order to solve it became more clear in her explanations about the other problems:

R: Did you think you would be able to solve the problem?
S4: When I focus on it a little bit more, I can tell.
R: Can you find a solution to the problem?
S4: (Starts to solve the problem and find an answer) Is the answer 20?
R: Can you check your answer?
S4: (Checks her answer by doing it in reverse) Yes, the answer is right.

On encountering the second problem, she expressed that she should focus on the problem more closely in order to produce the solution. She started to perform the mathematical operations after reading the problem several times and found an answer as a result. Although the answer she obtained for the total number of patients was smaller than the number of patients left, she did not realize that she had not found the correct total. She did not check the accuracy of the answer, and she left the solution as it was without understanding that it was wrong.

During the process of the final problem, she hesitated about whether the meanings of "each hour" and "per hour" were the same, but she managed to overcome her hesitation after realizing that they did not have the same meaning. She found different answers without performing any mathematical operations; however, she did not have any opinion about how she found those answers or whether they were correct. She believed that there was generally no problem she would not be able to solve; therefore, she was confident in her ability in terms of the problem

solving process. On the other hand, her self-confidence did not influence her ability to identify the appropriate steps to solve the problem or to follow those steps; it simply enabled her to continue working resolutely toward solving the problem. During the process, she approached each of problems calmly. Even if she could not solve a problem or had to spend too much time on it, she did not lose her calmness. Believing that *every problem absolutely has a solution* and *when close attention is given to a problem, a solution will be found* led her to continue working in whole process. But her beliefs were not effective in transition between the steps of problem solving.

Student S5's problem solving process and beliefs about the process

During the problem solving process, student S5 did not succeed in producing a solution. He attempted the first problem, but he did not do anything apart from reading the problem , as can be seen from the following dialog:

> R: Did you understand the problem?
> S5: Yes, but I can't concentrate on the problem, because I am reluctant [to work on it]. if I don't understand the beginning of the problem, I can't understand the rest. When scanning through, the rest of the problem occupies my mind.
> R: What information is given in the problem?
> S5: She wanted to make necklaces for her friends. The boxes she will put the necklaces in will be wrapped.
> R: What information is the problem asking for?
> S5: It asks how many sheets of paper will be purchased.
> R: Do you think you can solve the problem?
> S5: Yes, I can solve it.
> R: Can you produce a solution?
> S5: When one point of the problem gets stuck in my mind, I cannot solve it. I lose my motivation to solve the problem. I cannot concentrate on it anymore. I cannot associate [the parts of the problem].

He determined the given and requested information. But, he avoided solving the problem. He even did not attempt to solve it. He stated that the reason he could not solve the problem was based on focusing on a single point of the problem and an inability to connect the expressions in the problem to each other. He also revealed that his motivation for producing a solution decreased if he did not understand the problem on the first reading and did not visualize a solution. Therefore, it can be seen that he believed that a person must primarily be motivated to solve a problem. The belief that he would not solve the problem was more influential than his other beliefs. Because of this belief, he did not make any effort to solve the problems, nor did he perform any mathematical operations. When it was asked whether he understood the second problem, he stated "*It seems easier than the previous problem."* Despite this expression, he did not attempt to solve the problem again. Accordingly, the researcher questioned him about why he did not try to produce a solution or even to attempt to solve the problems. He said "*I didn't expect problems like these. I could solve it if it was about the subjects we have been studying."* Although he stated that the problems were not related with the subjects they have been learning currently, in the curriculum it was emphasized to give place to solve real life problems about each mathematical subject. The main reason behind his thinking could be a high level of belief that he would not be able to solve the problems. Besides, he said *"As new things are learned, problem solving is forgotten over time*. His explanation implied that he had a belief that learning new subjects had caused him to forget how to solve problems. He stated "*Not all of the problems are solved with formulas."* But he pointed out that the solutions to similar problems should be

known in order to successfully solve a problem. In other words, he believed that he could be successful by memorizing. On the other hand, he also indicated that logic was important in problem solving; such an opinion shows a contradiction in beliefs. Furthermore, the fact that he believed that solutions to problems can be forgotten and that problem solving is difficult led him to avoid attempting to solve the problems; he preferred not to solve any of the problems after attempting the first. He also maintained that he could solve his brother's/sister's math problems or easier problems to turn aside the belief that he could not solve any problems. He classified the problems that he would be able to solve as easy.

Overall, he expressed that the problems presented in the study were boring, and he did not want to solve them. He asserted that he avoided solving problems, and he generally hesitated at the beginning of the process. He sat doing nothing, rather than trying to search for a solution, and he was afraid that he could not solve them. He explained this attitude as follows: *"Yes, I am afraid I cannot solve problems. After our exams, my friends talk about how well they did. In such a situation, I feel badly, because I could not solve them; and I don't think about the problems I could not solve."* Such an opinion led him to avoid the problem solving process. As his self-confidence decreased, his belief that the problems were difficult solidified. In the problem solving process, he determined the given and requested information for all of the problems. But his beliefs such as *problem solving is a difficult activity* and *he cannot solve any problems* prevented him from passing beyond the step of understanding the problem.

In the course of the study, each student's problem solving process was examined, both through observations and through clinical interviews. As a result of these examinations, it was observed that the students were able to solve either only one problem or none of them at all. In the problem solving process, it was determined that the students had beliefs which either prevented them from solving the problems or enabled them to continue trying to solving. The beliefs affecting the students' problem solving process are outlined in Table 2.

According to Table 2, three of the five students believed that problems being solved are easy. As long as they solve a problem, they thought that it is an easy problem. In particular, student S5 did not engage in a problem that he could not solve. He did not attempt to solve the problem, because he believed he could not;

Table 2. The beliefs affecting the students' problem solving process

	Beliefs	Participants
1	If you stop solving problems for a long time, you forget how.	S1,S5
2	It is necessary to know the formulas to solve problems.	S1
3	Memorizing is the only way to solve problems.	S1,S5
4	Problems that can be solved are easy.	S1,S2,S5
5	If we cannot reach a solution for some time, there is no use in making an effort to find the solution.	S1,S2
6	If the solution is short and practical, it is meaningful.	S2
7	Every problem absolutely has a solution.	S3,S4
8	If the expression of a problem is long, more effort is needed to produce a solution.	S3
9	Every person has the ability to solve problems.	S3
10	A strategy that is effective in solving one problem is effective for other problems, as well.	S3
11	You need to keep trying until you produce a solution.	S3
12	For some problems, a formula is necessary, but logic is really important in problem solving.	S3,S5
13	If there is motivation to solve a problem, the solution will be found.	S3,S5
14	The difficulty of the problem affects the solution.	S3
15	When close attention is given to a problem, a solution will be found.	S4
16	If the expression of the problem is not understood in the first reading, there is no motivation to solve it.	S5
17	Fears about problem solving prevent solving problems.	S5
18	Problem solving is a difficult activity.	S1,S5

this belief was reinforced by his conviction that memorization was the only way to solve problems. Furthermore, both S1 and S5 felt that if they discontinued problem solving, they would forget how to do it. According to S1's belief, knowing the correct formulas made problem solving easier, and logical inferences were not of help in finding a solution. This belief is similar to the belief that problem solving techniques can be forgotten. Furthermore, although S5 believed that problem solving methods could be forgotten, he also expressed that most problems could be solved by means of logical inferences; these two beliefs are contradictory. On the other hand, some of the students expressed positive beliefs during the process. For instance, S3 and S4 believed that every problem definitely has a solution. This belief enabled them to continue working until a solution was produced. Another factor that motivated S3 to continue working to solve the problems was her belief that every person has problem solving ability. Considering the classification of students' levels, there was difference between the students in terms of continuing the problem solving process. While high level students (S3, S4) had the beliefs providing to continue the process, low level students (S1, S5) did not have these kinds of beliefs. For example, high level students thought that every problem absolutely had a solution and such beliefs prevented them to give up the problem solving process. However, low level students had the beliefs such as problem solving is a difficult activity, memorizing is the only way to solve problems etc., and they gave up the process easily. High and low level students agreed with regard to some beliefs. One of high and low level students stated the necessity of motivation for solving problems. But, low level student did not begin with high motivation for problem solving process. Accordingly, the students' behaviours changed in the process.

CONCLUSION AND DISCUSSION

As a result of the clinical interviews and observations carried out during the problem solving process, it was revealed that certain beliefs affected the behaviours of the students. There are numerous studies stating that we can make inferences by observing students in the problem solving process (Garofalo, 1989; Lampert, 1990; Schoenfeld, 1987). No matter how complicated the structure between the belief and the behaviour, determining people's beliefs provides an opportunity for predicting their decisions, as well as which behaviours they will perform in the process (Goldin, Rösken, & Törner, 2009). Furthermore, Kloosterman and Stage (1992) argued that students' beliefs affect their decisions, an argument which was supported by the findings of this study. While certain beliefs held by students promote their problem solving and mathematical achievements, other beliefs can lead to the oppoosite result (Cifarelli, Goodson-Espy & Chae, 2010). In the present study, while some students' beliefs had a positive effect on the process, others had a negative effect. Among the positive effects, we can consider persistence in looking for a solution; as a negative effect, we can consider giving up the problem solving process, failing to make an effort to solve the problem. Besides, as a negative effect on the process, it was concluded that students' beliefs prevented them from the transition between the steps of problem solving and the beliefs that prevented their transition between the steps changed according to the students.

Most of the participants thought that problem solving was an enjoyable activity as long as they were able to solve the problems they encountered. They believed that they needed to find solutions to the problems to increase their motivation for problem solving and to think of the problem solving process as gratifying. Therefore, it can be inferred that there was a direct correlation between their success in finding a solution and their appreciation for problem solving. It was also determined that students were motivated as long as they were successful; furthermore, if they could solve a problem, they believed that the problem was easy. On the contrary, it was

also revealed that some of the students engaged in problem solving without hesitation although they were unable to find a solution. It was determined that they did not give up because they believed they had the ability to find a solution. Moreover, one of the students approached the problem solving process with the belief that every person has the ability to solve the problem and that people should engage with a problem until they find a solution. Likewise, one of the students was motivated to keep trying due to a belief that a solution can be reached if enough attention is given to the problem.

It was also observed that those students who believed that formulas, algorithms, rules and even the solutions should be known avoided the problem solving process. Their beliefs that memorizing is the only method of problem solving and that methods of problem solving tend to be forgotten reinforced their avoidance. According to Garofalo (1989), students who demonstrate such a belief waste their time on memorizing formulas, rules and algorithms, rather than actually learning mathematics. In this respect, Underwood-Gregg and Yackel (2000) stress that most students think they can learn mathematics by memorizing the rules, while Carlson (1997) explains that they believe it is necessary to learn a new method for each problem encountered. These beliefs were mirrored by the participants who commented that they had not solved such problems for a long time, that solving problems required knowing the correct formulas, and that they generally preferred the problems that they had been able to solve previously. One of the students, who believed that problem solving is forgotten day by day and that memorizing is the only way to solve problems, contradicted himself by then expressing that logic is necessary for problem solving. His contradictory expressions indicated that he could not precisely internalize his opinion. In reality, a student who realizes that a problem is solvable using logic but who does not show this in his/her behaviour may be affected by personal factors such as educational experiences apart from his/her beliefs. For example, failure in problem solving throughout an entire educational experience and the psychology that "I can't solve any problem" can prevent a student from reflecting the necessary behaviours. Therefore, fears about problem solving may inhibit finding a solution or even attempting to find a solution.

In the case of students S3 and S4, it emerged that factors such as failure in problem solving, lengthy expression of a problem, and not understanding the problem on the first reading had a deep effect on the students' problem solving process. Specifically, it was observed that S3 believed that the solution would be challenging and time-consuming if the expression of the problem was long; this led to a biased approach. Moreover, it was discovered that the student's motivation for problem solving decreased and that she became bored with the process. On the other hand, her belief that every problem definitely has a solution encouraged her to concentrate, and therefore, she did not give up her efforts to solve a problem. Moreover, some of the students (S1, S2) felt that if a solution was not found for some time, or that the problem was not understood on the first reading, it was no use engaging in the problem further. For this reason, the students withdrew from the problem solving process, either after a certain period of time or on encountering the problem. This problem was emphasized by Schoenfeld (1992), who pointed out that even in high schools noted for providing a high level of mathematics education, students often expressed the belief that mathematics is a set of rules that should be applied by rote, and that problem solving should not take more than a few minutes. He stresses that this belief prevents students from approaching new problems with a positive attitude and leads them to give up on problems after an unsuccessful attempt. Using variety of different problems that are related to real-life situations in mathematics lessons, rather than a number of similar problems, may be of assistance in reducing this belief. Moreover, teachers should focus not only on checking the accuracy of students' answers, but also on the solution strategies they

use, as well as their reasons for attempting a given strategy. The fact that students will produce their own solutions to problems helps them to feel confident; students should be given the opportunity to experience the problem solving process on their own. By doing so, the students' motivation for problem solving may be increased, promoting positive beliefs toward the process.

In the present study, it was evaluated the beliefs of high school students according to their explanations in the problem solving process. The beliefs preventing or providing to continue the problem solving process were determined in accordance with the students' explanations. Considering these kinds of beliefs, it can be designed more productive learning environments for problem solving and consequently these environments can contribute students' problem solving. One of the aims of mathematics education, educating students to become good problem solvers, also may occur. The study was conducted with five students without generalization. It was intended to delineate students' beliefs in the problem solving process. However, the study can be conducted with larger sample. Thus, students' beliefs in the problem solving process can be presented in a wider range and generalized. Conducting similar studies with students from different cultures, it can be realized whether there is a difference between the beliefs evaluated in accordance with students' explanations. Thus, it may be taken attention to individual differences between students in design of learning environments for problem solving.

REFERENCES

Aksan, N., & Sözer, M. A. (2007). Üniversite öğrencilerinin epistemolojik inançları ile problem çözme becerileri arasındaki ilişkiler. *Ahi Evran Üniversitesi Kırşehir Fakültesi Dergisi, 8*(1), 31-50.

Aladağ, A. (2009). *İlköğretim öğrencilerinin orantısal akıl yürütmeye dayalı sözel problemler ile gerçekçi cevap gerektiren problemleri çözme becerilerinin incelenmesi* (Yüksek lisans tezi). Çukurova Üniversitesi, Sosyal Bilimler Üniversitesi, Adana, Türkiye.

Baki, A. (2008). *Kuramdan uygulamaya matematik eğitimi* (Genişletilmiş dördüncü basım). Ankara: Harf Eğitim Yayıncılık.

Bernardo, A. B. (1999). Overcoming obstacles in understanding and solving word problems in mathematics. *Educational Psychology: An International Journal of Experimental Educational Psychology, 19*(2), 149-163.

Callejo, L. M., & Vila, A. (2009). Approach to mathematical problem solving and students' beliefs systems: Two case studies. *Educational Studies in Mathematics*, *72*(1), 111-126. doi: 10.1007/s10649-009-9195-z

Carlson, M. (1997). Obstacles for college algebra students in understanding functions: What do high-performing students really know? *The AMATYC Review, 19,* 48–59.

Charles, R. T., & Lester, F. (1982). *Teaching problem solving; What, why & how*. Palo Alto, CA: Dale Seymour.

Chen, C. (2010). Teaching problem solving and database skills that transfer. *Journal of Business Research, 63*(2), 175-181.

Chiu, M. S. (2012). Identification and assesment of Taiwanese children's conceptions of learning mathematics. *International Journal of Science and Mathematics Education*, *10*, 163-191.

Cifarelli, V., Goodson-Espy, T., & Chae, J. (2010). Associations of students' beliefs with self-regulated problem solving in college algebra. *Journal of Advanced Academics*, *21*(2), 204-232.

Deryakulu, D. (2004). Epistemolojik inançlar. In Y. Kuzgun & D. Deryakulu (Eds.), *Eğitimde bireysel farklılıklar* (pp. 259–287). Ankara: Nobel Yayın-Dağıtım.

Eagly, A. H., & Chaiken, S. (1993). *The psychology of attitudes.* Fort Worth, TX: Harcourt Brace College Publishers.

Furinghetti, F., & Pehkonen, E. (2002). Rethinking characterizations of belief. In G. Leder, E. Pehkonen & G. Törner (Eds.), *Beliefs: A hidden variable in mathematics education?* (pp. 39–57). Dordrecht: Kluwer.

Garofalo, J. (1989). Beliefs and their influence on mathematical performance. *Mathematics Teacher*, *82*(7), 502-505.

Goldin, G. (1998). Observing mathematical problem solving through task-based interviews. In A. R. Teppo (Ed.), *Qualitative research methods in mathematics education* (Journal for Research in Mathematics Education: Monograph Number 9, pp. 40-62). Reston, VA: National Council of Teachers of Mathematics.

Goldin, G., Rösken, B., & Törner, G. (2009). Beliefs - no longer a hidden variable in mathematical teaching and learning processes. In J. Maaß & W. Schlöglmann (Eds.), *Beliefs and attitudes in mathematics education: New research results* (pp. 9-28). Rotterdam: Sense.

Gómez-Chacón, I. M. (2000). Affective influencesin the knowledge of mathematics. *Educational Studies in Mathematics, 43*, 149-168.

Griffin, T. D., & Ohlsson, S. (2001, August). Beliefs versus knowledge: A necessary distinction for explaining, predicting, and assessing conceptual change. In *Proceedings of the 23rd Annual Conference of the Cognitive Science Society* (pp. 1-4).

Hofer, B., & Pintrich, P. (1997). The development of epistemological theories: beliefs about knowledge and knowing and their relationship to learning. *Review of Educational Research, 67*(1), 88–140.

Howland, M. (2001). *Sixth-grade students use of schema knowledge in word problem solving* (Master's thesis). San Jose State University, San Jose, California/USA.

Johnson, B., & Christensen, L. (2004). *Educational research: Quantitative, qualitative, and mixed approaches* (2nd ed.). NY: Pearson/Allyn & Bacon.

Jonassen, D. H. (2000). Toward a design theory of problem solving. *Educational Technology: Research and Development, 48*(4), 63-85.

Karataş, İ. (2002). *8. sınıf öğrencilerinin problem çözme sürecinde kullanılan bilgi türlerini kullanma düzeyleri* (Yüksek lisans tezi). Karadeniz Teknik Üniversitesi, Fen Bilimleri Enstitüsü, Trabzon, Türkiye.

Kloosterman, P., & Stage, F. (1992). Measuring beliefs about mathematical problem solving. *School Science and Mathematics*, *92*(3), 109–115.

Lampert, M. (1990). When the problem is not the question and the solution is not the answer: Mathematical knowing and teaching. *American Educational Research Journal*, *27*(1), 29–63.

Leder, G. C. (2008). Beliefs: What lies behind the mirror. In B. Sriraman (Ed.), *Beliefs and mathematics* (pp. 39–54). Charlotte, NC: Information Age Publishing.

Leder, G. C., & Forgasz, H. J. (2002). Measuring mathematical beliefs and their impact on the learning of mathematics: A new approach. In G. C. Leder, E. Pehkonen, & G. Törner (Eds.), *Beliefs: A hidden variable in mathematics education?* (pp. 95-113). Dordrecht: Kluwer.

Lerch, C. M. (2004). Control decisions and personal beliefs: Their effect on solving mathematical problems. *Journal of Mathematical Behavior*, *23,* 21-36.

Lester, F. K., Garofalo, J., & Kroll, D. L. (1989). Self-confidence, interest, beliefs, and metacognition: Key influences on problem-solving behavior. In D. B. McLeod & V. Adams (Eds.), *Affect and mathematical problem solving: A new perspective* (pp. 75-88). New York, NY: Springer.

Lloyd, G. M., & Wilson, M. (1998). Supporting innovation: The impact of a teacher's conceptions on functions on his implementation of a reform curriculum. *Journal for Research in Mathematics Education, 29*, 248–274.

Mason, J. (2004). Are beliefs believable? *Mathematical Thinking and Learning, 6*(3), 343–352. doi:10.1207/ s15327833mtl0603_4.

Mason, L. (2003). High school students' beliefs about maths, mathematical problem solving and their achievement in maths: A cross-sectional study. *Educational Psychology: An International Journal of Experimental Educational Psychology*, *23*(1), 73-85. doi: 10.1080/01443410303216

Morgan, C. T. (1999). *Psikolojiye giriş* (13. Baskı). H. Arıcı & S. Karakaş (Çev.). Ankara: Meteksan Yayıncılık.

Oğuzkan, F. (1989). *Ortaokul dereceli okullarda öğretim*. Ankara, Türkiye: Emel Matbaası.

Pajares, M. F. (1992). Teachers' beliefs and educational research: Cleaning up a messy construct. *Review of Educational Research, 62*(3), 307–332.

Schoenfeld, A. H. (1982). Some thoughts on problem-solving research and mathematics education. In F. K. Lester & J. Garofalo (Eds*.) Mathematical problem solving: Issues in research* (pp. 27-37). Philadelphia, PA: Franklin Institute.

Schoenfeld, A. H. (Ed.) (1983). *Problem solving in the mathematics curriculum: A report, recommendations, and an annotated bibliography.* Washington, DC: Mathematical Association of America.

Schoenfeld, A. H. (1985). *Mathematical problem solving.* Orlando, FL: Academic Press.

Schoenfeld, A. H. (Ed.). (1987). *Cognitive science and mathematics education: An overview.* Hillsdale, NJ: Lawrence Erlbaum.

Schoenfeld, A. H. (1992). Learning to think mathematically: Problem solving, metacognition, and sense making in mathematics. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching* (pp. 334–370). New York: MacMillan Publishing.

Stylianides, A. J., & Stylianides, G. J. (2014). Impacting positively on students' mathematical problem solving beliefs: An instructional intervention of short duration. *The Journal of Mathematical Behavior, 23*, 8-29.

Tertemiz, N., & Çakmak, M. (2003). *Problem çözme: İlköğretim 1. kademe matematik dersi örnekleriyle.* Ankara, Türkiye: Gündüz Eğitim ve Yayıncılık.

Thompson, A. G. (1992). Teachers' beliefs and conceptions: A synthesis of the research. In D. A. Grouws (Ed.), *Handbook of research on mathematics learning and teaching* (pp. 127–146). New York: Macmillan

Uğurluoğlu, E. (2008). *Ilköğretim öğrencilerinin matematik ve problem çözmeye ilişkin inançlar ile tutumlarının bazı değişkenler açısından incelenmesi* (Yüksek lisans tezi). Osmangazi Üniversitesi, Eskişehir, Türkiye.

Underwood-Gregg, D., & Yackel, E. B. (2000). Supporting students'conceptualization of algebraic expressions and operations using composite units. In M. L. Fernandez (Ed.), *Proceedings of the 22nd Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 147-152). Columbus, OH.

Vidic, A. D. (2014). First-year students' beliefs about context problems in mathematics in university science programmes. *International Journal of Science and Mathematics Education*, 1-27.

Viholainen, A., Asikainen, M., & Hirvonen, P. E. (2014). Mathematics student teachers' epistemological beliefs about the nature of mathematics and the goals of mathematics teaching and learning in the beginning of their studies. *Eurasia Journal of Mathematics, Science & Technology Education*, *10*(2), 159-171. doi: 10.12973/eurasia.2014.1028a

Williams, K. M. (2003). Writing about the problem-solving process to improve problem-solving performance. *The Mathematics Teacher, 96*(3), 185-187.

Yılmaz, K. (2007). *Öğrencilerin epistemolojik ve matematik problemi çözümlerine yönelik inançlarının problem çözme sürecine etkisinin araştırılması* (Yüksek lisans tezi). Marmara Üniversitesi, İstanbul, Türkiye.