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# Examining the Differences of Linear Systems between Finnish and Taiwanese Textbooks

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The purpose of this study was to examine the differences between Finnish and Taiwanese textbooks for grades 7 to 9 on the topic of solving systems of linear equations (simultaneous equations). The specific textbooks examined were TK in Taiwan and FL in Finland. The content analysis method was used to examine (a) the teaching sequence, (b) application types, (c) representation forms, (d) response types, and (e) level of cognitive demand. The main difference between the Finnish and Taiwanese textbooks was that the Finnish textbooks introduced the topic of linear systems using a graphical approach, while the Taiwanese textbooks used an algebraic approach. Results also showed that the Taiwanese textbooks had fewer problems, but more challenging problems requiring a higher level of cognitive demand; the Finnish textbooks had more authentic application problems, and even more problems were displayed in visual forms. In addition, the Taiwanese textbooks had more open-ended problems, particularly problems asking students to explore or explain, whereas the Finnish textbooks did not have exploration problems. The topic of linear systems was also taught earlier in Taiwanese textbooks. Implications of this study and suggestions for future studies are discussed.

*Keywords*: algebra, linear equations, linear systems, problem types, textbooks

## INTRODUCTION

Currently, there is a growing attention on the cross-national textbooks comparison studies (e.g., Fan, Zhu, & Miao, 2013; Hong & Choi, 2014; Yang, Reys, & Wu, 2010; Zhu & Fan, 2006) .The results of TIMSS showed that there is a positive relationship between curriculum and math achievement (Schmidt et al., 2001). Törnroos (2002) also found that about 99% of 7th grade sample students depended on the textbooks to learn mathematics. Textbooks were also thought to be a crucial means for teaching (Pehkonen, 2004). In addition, comparing the similarities and differences of cross- national textbooks can provide meaningful information for

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improving the future textbook design, particularly in the representation of problems (Zhu & Fan, 2006). Törnroos (2005) suggested, "Even a quite simple analysis of textbooks can produce valuable information when looking for explanations for student achievement in mathematics" (p. 315). The quality of textbooks will influence the students' learning effect, as well as the teachers' teaching efficiency (Reys & Reys, 2006; Stein, Remillard, & Smith, 2007; Törnroos, 2005). This shows the importance of mathematics textbooks in students' learning (Floden, 2002; Sood & Jitendra, 2007)

Several studies have compared the textbooks from East Asian countries and the U.S. (Hong & Choi, 2014; Ni & Cai, 2011; Yang et al., 2010; Zhu & Fan, 2006). However, comparisons between textbooks from East Asian countries and European countries (such as Finland) have been relatively few, even though students in Finland have performed at the top on the PISA. It should be interesting to compare the differences between Finnish and Taiwanese textbooks. Moreover, this study selected the topic of systems of linear equations as the focus, because solving equations is always seen as an important topic in learning algebra (Cai, Nie, & Moyer, 2010). Nevertheless, there has been little research on this topic (Häggström, 2008). Therefore, the main purpose of this study was to investigate the content differences and problem types presented on the topic of linear systems in textbooks in Finland and Taiwan. The research questions are as follows:

1. What are the similarities and differences on the topic of linear systems between Taiwanese and Finnish textbooks?
2. What are the differences in the context types of problems presented (application problems and non-application problems) between Taiwanese and Finnish middle-grade mathematics textbooks?
3. What are the differences in the types of representation (pure mathematical forms, verbal form, visual form, and combined form) on the topic of linear systems between Taiwanese and Finnish middle-grade mathematics textbooks?
4. What are the differences in the response types of problems (open-ended problems and close-ended problems) between Taiwanese and Finnish middle-grade mathematics textbooks?
5. What are the differences in the cognitive demand types of problems (Memorization, Procedures without Connections, Procedures with Connections, and Doing Mathematics) between Taiwanese and Finnish middle-grade mathematics textbooks?

***State of the literature***

- The quality of textbooks will influence the students' learning effect, as well as the teachers' teaching efficiency.
- Textbooks in East Asian countries usually have fewer total amount of problems, fewer visual problems, fewer realistic problems but more challenging problems.
- Textbooks in East Asian countries usually introduce topics earlier than textbooks in Western countries.

***Contribution of this paper to the literature***

- There were two different approaches to introduce linear systems.
- In Taiwanese textbooks, linear systems were introduced by a situational problem and focusing on solving equations algebraically but in Finnish textbooks, linear systems were introduced by using equations and graphs and focusing on the connection between equations and graphs.
- Taiwanese and Finnish textbooks both have their merits. Integrating both merits may result in students' better understanding of linear systems. Textbooks should initially introduce linear systems by graphs to help students more easily grasp the topic but should extend students' thinking by focusing on algebraic methods and by providing more challenging problems.

## BACKGROUND

### Mathematics textbooks related studies

Several studies have shown that mathematics textbooks not only play an important role in transmitting major mathematics concepts, but also are an independent tool to help students learn mathematics (Brown & Edelson, 2003; Cai, 2008; Hudson, Lahann, & Lee, 2010; Martin, Mullis, Gonzales, & Chrostowski, 2004; Reys, Reys, & Chavez, 2004; Sood & Jitendra, 2007; Stein et al., 2007). At the same time, the quality of textbook contents will affect students' learning and, in this case, it will directly impact their performance (Haggarty & Pepin, 2002; Rittle-Johnson & Koedinger, 2005; Sood & Jitendra, 2007; Zhu & Fan, 2006). Moreover, the representation methods of mathematics, problem types, and the order and editing of contents will influence students' learning effect (Haggarty & Pepin, 2002; Sood & Jitendra, 2007; Zhu & Fan, 2006). Based on these arguments, this study will analyze the comparison of the representative mathematics textbooks in middle schools from Taiwan and Finland.

Textbook comparison studies have shown that textbooks in East Asian countries usually have some distinct features. Hong and Choi (2014) indicated that Korean textbooks contained more mathematics topics in a school year, and those topics were introduced earlier compared to other countries. Hong and Choi (2014) found that the topics related to quadratic equations are introduced relatively early in Korean textbooks compared to U.S. textbooks, and Korean textbooks included more topics than U.S. textbooks. In analyzing the problem types in Chinese and U.S. textbooks, Zhu and Fan (2006) found that U.S. textbooks had more visual forms of problems. They also found that U.S. textbooks included more "authentic application problems" than Chinese textbooks.

### Related studies in systems of linear equations

Compared to other areas of algebra, such as equations or solving equations with one unknown, relatively few studies have focused on systems of linear equations. What Kieran (2007) said is still true today: "Researchers have known very little about the ways in which students of this age range [6th to 9th grade] approach the solving of systems of equations" (p. 723). This may be because students have learned different concepts and methods in algebra before they start to learn to solve linear systems. However, it does not mean students have no problems solving linear systems. Several studies have shown that students who have been previously introduced to solving equations with one unknown still have difficulties in substituting one expression into another equation (Drijvers & van Herwaarden, 2000; Filloy, Rojano, & Solares, 2004). This is probably because they may not be able to apply the transitivity of the equal sign (e.g., A=C, B=C, then A=B) or not be able to see and operate on expressions as an object (e.g., y=6x+7; "6x+7" is an object to replace another y) (Sfard, 1991). Drijvers (2003) further found that the use of graphical representations can help students comprehend equations. In addition, students' difficulties in formulating algebraic equations and understanding variables and algebraic expressions may also impede their learning to solve equations (MacGregor & Stacey, 1993; Seeley & Schielack, 2007).

The use of multiple representations in learning algebra has been highly emphasized in the reformed curriculum (Brenner et al., 1997; Cooney, Beckmann, Lloyd, Wilson, & Zbiek, 2010; Hong & Choi, 2014; Kieran, 2007; Senk, Thompson, & Wernet, 2014). However, research continuously shows that understanding the relationships among symbolic and graphical representations is not easy for students (Kieran, 2007; Knuth, 2000; Panasuk & Beyranevand, 2010). In particular, symbolic notation is difficult for students to understand. For example, Nathan, Stephens, Masarik, Alibali, and Koedinger (2002) and Yerushalmy (2000) found students may more easily comprehend tables and graphs than symbolic equations and verbal expressions. This implies that the ways in which working examples used in textbooks deal with multiple representations affect students' learning. This provided the motivation to conduct this study.

## METHOD

### Selection of textbooks

Both Taiwan and Finland have national mathematics curriculum standards. Textbooks are developed based on the standards, and schools can select their preferred textbooks on their own. In this study, we selected the TK (a pseudonym) mathematics textbooks (Kang Hsuan Educational Publishing Group [TK], 2012) and the FL (a pseudonym) mathematics textbooks (WSOY, 2009) as representative textbooks from Taiwan and Finland, respectively.

#### *TK mathematics textbooks*

The TK middle-grade mathematics textbooks have about a 39% market share in Taiwan and are the most used middle-grade mathematics textbook series in Taiwan. TK includes six student textbooks for grades 7 to 9, and 11 out of 28 chapters deal with algebra. The topic of linear systems begins in grade 7 and includes two chapters with total 74 pages.

#### *FL mathematics textbooks*

Over 70% of Finnish schools adopted the FL textbooks. FL has three textbooks for grades 7 to 9 and 96 out of 245 units covering the topic of algebra. The FL textbooks include 10 units on linear systems for grade 9, with a total of 20 pages.

### Coding and analyzing

All of the problems in the student textbooks, including worked examples (with solutions), exercises (without solutions), and summary test problems were coded.

Both horizontal and vertical analyses (Charalambous, Delaney, Hsu, & Mesa, 2010; Hong & Choi, 2014; Li, 2000) were adopted in this study. The horizontal analysis gave a whole picture of the textbooks, including topics and teaching sequence. It provided the background information and overall structure of the textbooks. Complementary to the horizontal analysis, the vertical analysis offered in-depth understanding of mathematical content, such as problem characteristics and problem types. Accordingly, the textbooks were analyzed by their (a) topics and sequence, (b) context types, (c) representation forms, (d) response types, and (e) cognitive demand. The first item is related to the horizontal analysis and the rest are related to the vertical analysis.

Topics and sequence refers to how each topic was introduced and developed throughout the two textbook series (Hong & Choi, 2014), that is, how a lesson began and what sequence it followed. Based on Zhu and Fan's (2006) classifications, an application problem was posed under the context of a real-life situation; a non-application problem was a problem without any context. Among these application problems, they further distinguished two types of application problems. One was fictitious application problems (FAP), whose real-life situations were fictionally created by the textbook designers. In contrast, the authentic application problems (AAP) were created to reflect more realistic experiences. Figure 1 shows an example of a FAP, which is a fiction story.

*Today was the first day of the school for the clownfish, Nemo. He met lots of his friends there. When coming home, he told his daddy: "There were totally 11 octopus and turtles in my class. They had 68 legs in total (we call octopuses' arms as legs in Chinese)". How many octopus and turtles are there in his class (an octopus has eight arms)?*

**Figure 1.** A fictitious application problem in the Taiwanese textbooks (reproduction with English translation from TK, 2012, p. 49)

Zhu and Fan also defined four types of representation forms:

1. Purely mathematical form means mathematical expressions were included in a problem;
2. Verbal form means mainly written words described a problem;
3. In visual form, a problem is solved mainly by using its graph, chart, table, figure, or any other visual objects;
4. Combined form is a problem containing two or three of the forms above and there is no clearly distinction about which one is used.

In the coding criteria of the response type, we followed previous studies (Charalambous et al., 2010; Zhu & Fan, 2006). An open-ended problem means that the problem asks students to explain or justify the answers or process, or the problem can have many correct answers, while a close-ended problem can have only one exact answer.

Regarding the level of cognitive demand, the classifications were based on the studies of Stein, Grover, and Henningsen (1996) and Stein, Smith, Henningsen, and Silver (2000). They defined four levels of cognitive demand: "Memorization," "Procedures without Connections," "Procedures with Connections," and "Doing Mathematics." The first two are usually thought of as low-level cognitive demand, whereas the last two are considered high-level cognitive demand. Memorization means that students reproduced previously learned facts (e.g., formula, definition, etc.) in problems; Procedures without Connections means that students used algorithmic or procedural knowledge without having a connection to the concepts or meanings (pure application of procedures); Procedures with Connections means that students are required to attend to the concepts or meanings when using algorithmic or procedural knowledge (more than merely application of procedures). Doing Mathematics means that students use complex, non-algorithmic thinking to solve problems.

Examples of the application of coding criteria are in Table 1. Problem 1 is categorized as "Procedures without Connections," because students need to apply only the procedure to solve the problem. In contrast, in problem 2, students cannot directly apply the solving procedure. They must consider both the procedure and meaning of solving the linear system. Problem 3(a) and 3(b) are problems in the visual form, because students are required to solve the problem by using the information in the graph. Problem 4 is thought of as an "Application with APP" and "Doing Mathematics" problem, because comparing ages of people is realistic to students and students need to analyze the problem, set up equations, and determine the solution by applying procedures.

## Reliability

Two coders individually coded all the problems in the two countries' textbooks, and the third coder, who was also an experienced researcher in this field, served as a consultant when disagreement was found. These coders were fluent in both Chinese and English. Only the FL textbooks were translated into English at the beginning of the coding procedure. The percentage of agreement between the first two coders ranged from .81 to .96. Among the different codes, all disagreements were then reconciled by the third coder.

## RESULTS

### Topics and their sequence

The topic of systems of linear equations was introduced in different grades in Taiwan and Finland. This topic is covered much earlier by TK in Taiwan than by FL in Finland. As shown in Table 2, in the TK textbooks, it is introduced in the beginning

**Table 1.** Examples of applying coding criteria

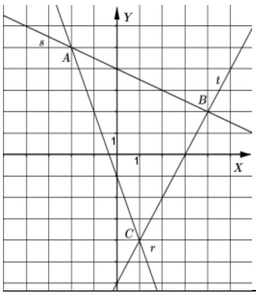
| | Problems | Coding |
|---|---|---|
| 1. | Example 10: solve the systems of linear equations $\begin{cases} \frac{x}{2}-\frac{y}{3}=3 \quad \cdots\cdots(1) \\ \frac{3x}{2}+2y=0\cdots\cdots(2) \end{cases}$<br>(reproduction with English translation from TK, 2012, p.34) | . Non-application<br>. Purely Mathematical<br>. Procedures without Connections<br>. Close-ended |
| 2. | Thinking: In addition to eliminate the fractional part by multiplying (1) by 6 and (2) by 2, do you have another approach to solve the above Example 10? What is your opinion?<br>(reproduction with English translation from TK, 2012, p. 34) | . Non-application<br>. Verbal<br>. Procedures with Connections<br>. Open-ended |
| 3. | <br>a) write equations for r, s, t<br>b) find the solution of point A, B and C. (reproduction with English translation from WSOY, 2009, p.133) | a) and b) both are<br>. Non-application<br>. Visual<br>. Procedures with Connections<br>. Closed-ended |
| 4. | In the school cafeteria, there are a number of round tables. If each table is seated by eight students, there will be ten students without seats. If each tables is seated by nine students, there will be one table with only three students. How many tables are there in the dining room, and how many students are there in the school? (reproduction with English translation from WSOY, 2009, p. 580) | . Application (AAP)<br>. Verbal<br>. Doing Mathematics<br>. Close-ended |

**Table 2.** Topics covered in Algebra and their sequence in ***Taiwanese and Finish textbooks***

| Grade | Taiwanese Textbooks | Finnish Textbooks |
|---|---|---|
| **Grade 7** | Linear Equations<br>Linear Systems<br>*Proportions*<br>*Linear Functions*<br>*Linear Inequalities* | Linear Functions (1st)<br>Linear Equations (1st) |
| **Grade 8** | *Polynomials*<br>*Quadratic Equations* | Polynomials (1st)<br>Linear Equations (2nd)<br>Quadratic Equations<br>Proportions (1st) |
| **Grade 9** | *Quadratic Functions* | Linear Functions (2nd)<br>Proportions (2nd)<br>Quadratic Functions<br>Linear Inequalities<br>Polynomials (2nd)<br>Linear Equations (3rd)<br>Linear Systems |

*Note. ( ), How many times the topic appears in the textbooks; Topics in italics are after the topic of linear systems*

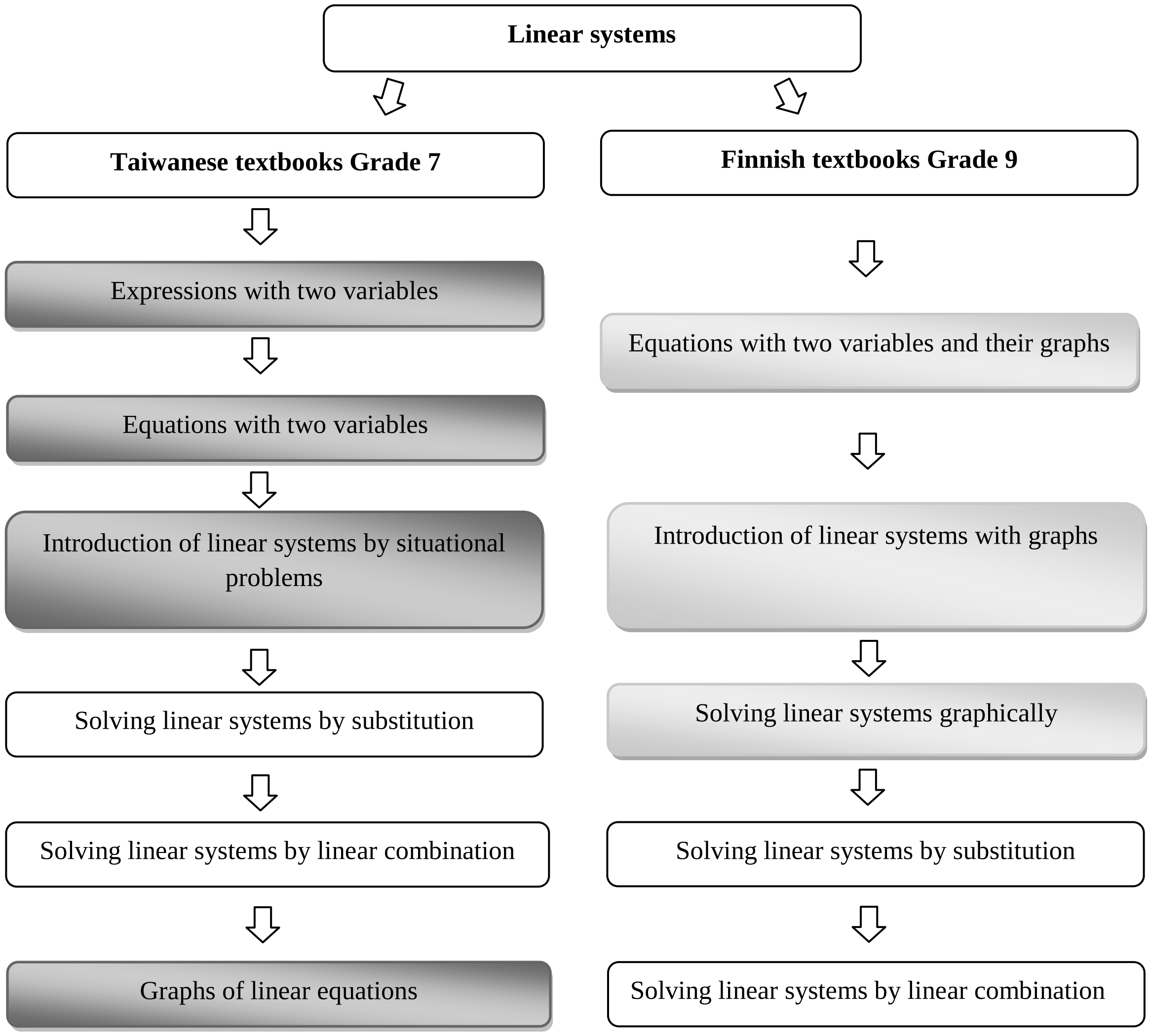



**Figure 1.** Content sequence in the Taiwanese textbooks and Finnish textbooks (differences are shaded in different level of color)

of the second semester of grade 7, whereas the same topic is introduced in a later section of grade 9 in FL textbooks. This means that the FL textbooks teach most of the topics in algebra prior to the topic of linear systems. These lessons include expressions, functions, equations, graphing and solving equations, polynomials, and proportions (see Table 2). However, in the TK textbooks, only the topic of "linear equations" (without graphs) was introduced prior to instruction of linear systems.

Figure 2 shows the content and sequence covered in the topic of linear systems for both textbook series. It was found that TK textbooks provided more content before linear systems were introduced than FL textbooks. The TK textbook introduced how to write, evaluate, or simplify expressions with two variables (covered in the topic of expressions of two variables) in the beginning of this section, and then how to write equations and identify their solutions was introduced in the following topic of equations with two variables. In contrast, the FL textbooks simply reviewed equations with two variables by graphing equations and determining which ordered pair represents a solution to a graph or an equation. When linear systems were introduced, the TK textbooks used a situational problem (Figure 3). However, the FL textbooks simply gave equations and their graphs (Figure 4). After that, three different methods for solving linear systems were introduced in the FL textbooks. Although the TK textbooks also introduced two algebraic methods to solve linear systems, the TK textbooks did not have the content of "solving linear systems graphically." The content related to graphs appeared only in the "graphs of linear equations" in the TK textbooks. However, this is about how to graph linear systems after students have solved equations algebraically. The connection between equations and graphs was not so emphasized by the TK textbooks in comparison to the FL textbooks.

## Context of problems

Table 3 shows the results of different application types. The FL textbooks include more application problems (24.4% > 17.2%) than TK textbooks.

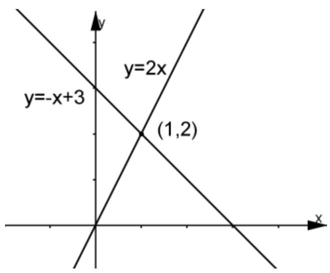
In addition, Table 3 also shows that there were 164 and 221 problems in the TK textbooks and FL textbooks, respectively. It shows that the FL textbooks have more problems than TK textbooks on linear systems. In further examining the distribution of application problems, the data showed that the FL textbooks include application problems distributed across the linear systems section. In contrast, the TK textbooks arrange most of the application problems in the same unit after the topic "solving linear systems algebraically" has been introduced. The TK textbooks lead students to learn how to solve application problems after the algebraic methods have been taught. However, the FL textbooks more evenly provide application problems among the topics of solving linear systems.

On the other hand, the FL textbooks include more authentic problems (AAP), which are more related to the students' daily life (37% > 14.3% in Table 3). The FL textbooks include 20 out of 54 application problems coded as APP, whereas the TK

*Yan bought three Adult tickets and one children ticket. It cost NT 300 and can be expressed by 3x+y=700. Yi bought five Adult tickets and two* Children *tickets. It cost NT 1200 and can be expressed by 5x+2y=1200. When using two equations to represent quantities in the problem, we can put them together. And we named it as a system of linear equations*

$$\begin{cases} 3x + y = 700 \\ 5x + 2y = 1200 \end{cases}.$$

**Figure 3.** A situation problem used to introduce linear systems in TK textbooks (reproduction with English translation from TK, 2012, p. 20)




*The system of linear equations* $\begin{cases} y = 2x \\ y = -x + 3 \end{cases}$ *consists of two equations. The ordered pair (x, y)=(1, 2) which satisfies both equations is the solution of the linear system. The intersection of two lines in the coordinate system represents the solution of the linear system.*

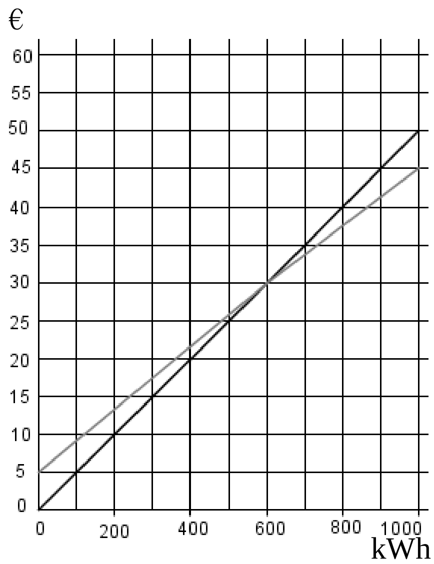
**Figure 4.** Introduction of linear systems by giving equations and graphs in the FL textbooks (reproduction with English translation from WSOY, 2009, p.130)

**Table 3.** Classifications by the different application types and the sub-types of application problem

| | Taiwan (n=164)* | Finland (n=221)* |
|---|---|---|
| | n (%) | n (%) |
| **Non-application** | 136 (83.4%) | 167 (75.6%) |
| **Application** | 28 (17.2%) | 54 (24.4%) |
| **AAP** | 4 (14.3%) | 20 (37.0%) |
| **FAP** | 24 (85.7%) | 34 (64.0%) |

*Note. AAP, Authentic application problem; FAP, Factious application problem; *The total number of problems*

| | Basic fee (€/kk) | Fee per kWh (€/kWh) |
|---|---|---|
| Flash Light Ltd. | 0.00 | 0.05 |
| Strength Mill Ltd. | 6.00 | 0.04 |




*Suominen's family has received a final offer for sale of electricity from two power companies: Flash Light Ltd changes only for power rate and Strength Mill Ltd also has the basic monthly fee. a) Compare their graphs. b) When consumption of 800 kWh / month, which company offers the better price and how much is their difference?*

textbooks have only 4 out of 28 APPs. Figure 5 shows an example that asks students to decide which power company offers a better price for a family. However, only a few similar problems of daily-life situations were found in the TK textbooks.

## Representation forms of problems

Table 4 reports the differences in the representation forms used in both textbook series. The prevalent representation form in both countries' texts is the purely mathematical form (TK: 76.7%; FL: 61.1%), but the TK textbooks have more purely mathematical form problems. For the visual form, the percentages of problems in both textbook series are quite similar (TK: 3.7%, FL: 3.6%). However, when we consider all the problems, including both visual form problems and combined form problems with visual data, the FL textbooks include more problems with visual representations (visual + combined form: 15.4% > 8.6%).

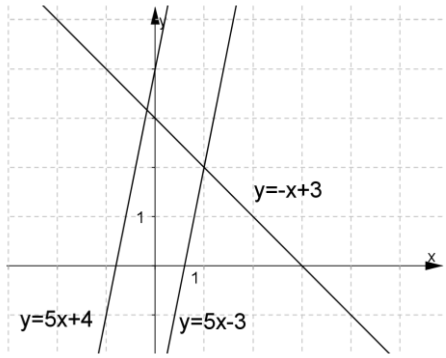
Figure 6 shows an example of a combined form problem with visual data in the FL textbooks. Both linear systems (purely mathematical form) and their graphs (visual form) are provided. However, the TK textbooks are less likely to have problems like this one. In fact, reading or using graphs to solve problems is not emphasized in the TK textbooks.

Table 4 also shows that the FL textbooks have a higher percentage of the verbal form than TK textbooks (23.5% > 15.3%). In the TK textbooks, the verbal form problems are almost equivalent to the application problems. However, the FL textbooks also have verbal form problems stating the relationship between x and y in words, instead of giving only algebraic expressions (Figure 7). In this type of problem, students are to write a linear system based on the relationship between x and y from the verbal description. However, this type of problem does not appear in the TK textbooks. When a linear system is given in TK textbooks, it always uses symbolic representation. The risk in doing so is that it increases the difficulty in understanding the meaning of equations for students, since students may more easily understand verbal descriptions than symbolic representation.

## Response types of problems

Table 5 shows the distribution of problems by different response types for both FL and TK textbooks. The majority of problems in both countries are close-ended.

(a) $\begin{cases} y = 5x - 3 \\ y = 5x + 4 \end{cases}$ (b) $\begin{cases} y = 5x - 3 \\ y = -x + 3 \end{cases}$




*How many solutions are there in the linear systems?*

**Figure 6.** A problem with the graph of linear systems in the Finnish textbooks (reproduction with English translation from WSOY, 2009, p. 133)

*The sum of the variable y and x is 8. The difference between them is 10. The two equations form a linear system and solve it graphically*

**Figure 7.** An example of the verbal form in the Graph section in the Finnish textbooks (reproduction with English translation from WSOY, 2009, p. 133)

**Table 4.** The difference of the representation forms in both countries

| | Taiwan (n=164*) | | Finland (n=221*) | |
|---|---|---|---|---|
| | n | % | n | % |
| **Pure mathematical** | 125 | (76.7%) | 135 | (61.1%) |
| **Visual** | 6 | (3.7%) | 8 | (3.6%) |
| **Verbal** | 25 | (15.3%) | 52 | (23.5%) |
| **Combined** | 8 | (4.9%) | 26 | (11.8%) |

*Note. *Total number of problems*

**Table 5.** Classifications by the different response types of problems

| | Taiwan (n=164)* | Finland (n=221)* |
|---|---|---|
| **Close-ended** | 151 (92.6%) | 216 (97.7%) |
| **Open-ended** | 13 (8.0%) | 5 (2.3%) |

*Note. *Total number of problems*

*(a) y = x + 5*
*(b) y = 2x - 1*
*Finding three ordered pairs (x, y) to satisfy the equation*

**Figure 8.** An open-ended problem with many different solutions in the Finnish textbook (reproduction with English translation from WSOY, 2009, p. 129)

**Table 6.** The difference of the cognitive demand of problems in both countries

| | **Taiwan (n=164*)** | | **Finland (n=221*)** | |
|---|---|---|---|---|
| | **n** | **%** | **n** | **%** |
| **Memorization** | 16 | (9.8%) | 8 | (3.6%) |
| **Procedures without connections** | 99 | (60.4%) | 159 | (71.9%) |
| **Procedures with connections** | 42 | (25.6%) | 48 | (21.7%) |
| **Doing Mathematics** | 7 | (4.3%) | 6 | (2.7%) |

*Note. *Total number of problems*

However, the TK textbooks have slightly higher percentage of open-ended problems (8% > 2.3%). Except for one, all the open-ended problems in the TK textbooks are “exploration” problems. These exploration problems are attached to worked examples that ask students to reason, think, and explore the given solutions. Usually the problems contain “What if…” in the prompts (Figure 8). In contrast, all the open-ended problems in the FL textbooks are problems with numerous solutions, but they are not categorized as exploration problems (Figure 8), which may require less cognitive demand.

## Cognitive demand of problems

Table 6 shows the percentages of cognitive demand in problems included in the textbooks from both countries. The majority of problems in both series require a lower level of cognitive demand (TK: 70.1% and FL: 75.5%, coded as Memorization or Procedures without Connections). However, the TK textbooks have more problems with a higher level of cognitive demand (TK: 29.9% and FL: 24.4%, coded as Procedures with Connections or Doing Mathematics). This result is consistent with an earlier study that indicated that East Asian countries’ textbooks have more challenging problems (Zhu & Fan, 2006), and many problems in algebra require simple algorithms or formulas (coded as Procedures without Connections, Hong & Choi, 2014).

In further examining the cognitive demand of problems, it was found that the higher level of cognitive demand in problems in the FL textbooks are most likely application problems, whereas the TK textbooks have more non-application problems coded as higher level of cognitive demand.

## DISCUSSION AND CONCLUSION

This study examined the differences between FL and TK textbooks for grades 7 to 9 on the topic of “solving systems of linear equations.” The results showed that when introducing linear systems, the TK textbooks highly focus on an algebraic approach, while the FL textbooks put more emphasis on a graphical approach. That is, TK textbooks provide less connection between equations and graphs; however, in the FL textbooks, graphs are integrated into the topic of solving linear systems in all sections.

The tendency of using an algebraic approach in the TK textbooks may result in a better performance in solving the process-constrained problems or routine problems that can be solved by applying algorithms or carrying out the procedure without understanding (Cai, 2000; Knuth, 2000; Reys & Yang, 1998). Moreover, research has shown that students in East Asian countries usually perform well on the process-constrained problems/routine problems but lack ability in graphing, higher-level thinking, and conceptual understanding (Cai, 2000; Wang & Lin, 2005).

Panasuk (2010) stated that students were less likely to develop conceptual understanding in algebra if they were limited to reading only one kind of representation. Panasuk also pointed out that, for some students, highly focusing on algebraic symbolic representation was too abstract to understand. In addition, Knuth (2000) argued that many students' work with functions was limited to the use of algebraic representations. This will result in students' lack of flexible ability in using and transforming algebraic and graphic representations. Therefore, the teaching sequence of "solving systems of linear equations" in TK textbooks will probably bring about that "many students either [perceive] the graphical representation as unnecessary or [use] it as a means to support their algebraic-solution methods rather than as a means to a solution in and of itself" (Knuth, 2000, p. 506). In fact, this supports the finding of Yang and Huang (2004): "Taiwanese students were highly skilled in written computation but their written skills were not equally transferred to use of non-computational paths that depended on symbolic representation, pictorial representation and number sense to solve similar problems" (p. 373).

Multiple representations, particularly graphical representations, are thought of as an important way to achieve the learning goal of algebra (Chesler, 2009); and when students use multiple representations, it can reduce the abstraction to a level that is more closed to their existing cognitive structure (Pape & Tchoshanov, 2001). The results also indicated that the TK textbooks have fewer application problems, especially AAP, as well as fewer problems that incorporate visual representation. The findings are consistent with earlier studies (Charalambous et al., 2010; Zhu & Fun, 2006). Cai (1995) also found that more visual representations in the textbooks result in students' better performance on visual representation problems and students using more visually related representation in problem solving, and a greater number of application problems can help students develop a higher level of understanding (Gu, Huang, & Marton, 2004).

The topic of linear systems introduced earlier in Taiwan is also similar to what Hong and Choi (2014) found—that concepts in the East Asian countries were usually introduced earlier. Results also showed that the majority of problems in both countries are close-ended problems. This result is consistent with the finding of Zhu and Fan (2006), that both U.S. and Chinese textbooks consisted mostly of close-ended problems. However, having too many close-ended problems but only few open-ended problems in the textbooks may cause students' difficulty in solving open-ended problems. Studies have indicated that students usually did not perform well on open-ended problems, due to students' infrequent exposure to open-ended problems in their textbooks (Cai, 1995; Zhu & Fan, 2006).

In terms of cognitive demand of problems, results revealed that the percentage of problems requiring a higher level of cognitive demand in the TK textbooks is about five percent higher than in the FL textbooks. In particular, a few of the types of higher-level cognitive demand problems do not even appear in the FL textbooks.

In sum, the findings showed that both textbooks have their own strengths and weaknesses in content and problem design. We suggest that both countries can learn from each other. In the TK textbooks, we can take note of the balance of multiple representations, particularly the connection between equations (symbolic representation) and graphs (pictorial representation). In fact, earlier studies suggested that the use of multiple representations can enhance students' meaningful understanding and critical thinking (Cai, 2001; Fennell & Rowan, 2001; NCTM, 2000; Panasuk, 2010). The TK textbooks should consider integrating graphs into solving linear equations. We do believe that introducing linear systems incorporated with graphs can help students to more easily access the concept of linear systems (Hollar & Norwood, 1999).

Regarding application problems, researchers have realized the importance of solving problems posed in realistic contexts (Cooper & Harries, 2002; Hough & Gough, 2007). Greeno (1991) and Inoue (2005) suggested that real-life knowledge plays an important role in mathematics learning. Researchers also believe that students are most likely to be motivated when problems are authentic and situated in real-world settings (Anderson, Reder, & Simon, 1996; Lombardi, 2007). Therefore, more application problems, especially APP, should be included in TK textbooks.

The finding of this study is consistent with earlier studies, that East Asian countries' textbooks have a fewer number of problems but a higher level of cognitive demand problems than Western countries' textbooks (Charalambous et al., 2010; Zhu & Fan, 2006). Studies have shown that using a high level of cognitive demand in problems or more exploration problems in the classroom will promote students' mathematics thinking and learning (Tarr et al., 2008). Furthermore, Silver and Stein's (1996) study found that by using high-level cognitive tasks and instruction and by emphasizing deeper understanding, students show better achievement. Because there are fewer challenging problems (both higher level of cognitive demand and open-ended problems), and, in particular, no exploration problems that appear in the FL textbooks, these textbook writers may consider incorporating more challenging problems. A good balance between high- and low-level cognitive demand problems may affect students' learning (Park, 2011).

Further research is needed to investigate questions such as: (1) How do middle-grade teachers in Finland and Taiwan implement their textbooks in the middle-grade classrooms? (2) How do mathematics textbooks in Finland and Taiwan affect middle-grade teachers' teaching and students' learning? (3) How do the cultures of Finland and Taiwan reflect on their textbooks? Finland is seen as a country to value equality, collaboration, individual children, applying skills out of classrooms, and less competition and fewer standardized tests (E. Pehkonen, 2009). Taiwan, on the other hand, is seen as a traditional East Asian country that emphasizes competition, studying hard, repeated practice, and has high expectations for children, using more standardized tests (Leung, 2001, 2006). Research has provided some information about how cultural differences may influence the textbooks' design (Fan, 1999; Leung, 2001). Future research can further investigate this issue to reveal the major differences in mathematics textbooks between East Asian and non-East Asian countries.

There existed some limitations in this study. First, we only analyzed one series of textbooks in each country. Although these selected textbooks were representative, it does not imply all the results found in this study are similar in other textbooks in both countries. Second, this study does not analyze how teachers use these textbooks. Although textbooks play a significant role in mathematics classes, we cannot assume that all teachers teach linear systems in exactly the same ways. Finally, textbooks is one of the factors that influenced students' mathematics learning. Researchers or policymakers should treat our results very carefully and not overwhelmingly use these results.

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## AUTHORS' NOTE

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