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Formation of Schoolchildren's Creative Activity on the Final Stage of Solving a Mathematical Problem

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ABSTRACT
The aim of the research is to study the possibilities of the final stage of working with a mathematical problem as a means of forming schoolchildren's creative activity. The leading method of investigating this problem is to establish the correspondence between the components of the final stage of working with the mathematical problem and the procedural features of the student's creative activity. The study resulted in defining the structure of the final stage of working with a mathematical problem, which made it possible to identify a certain set of activities that make up the ability to work with the problem on the final stage of its solution. The article establishes the relationship between actions appropriate to this stage of work with the task and signs of the student's creative activity. It is proved that in the process of working with the problem on the final stage of its solution, students develop procedural features of creative activity. The author's method of forming students' creative activity suggested in the article can be used by the teachers of mathematics in school practice, by the authors of methodological manuals for students and teachers, and also can be used as the basis for a special course for students of pedagogical universities.
Keywords: mathematical problem, signs of creative activity, the formation of creative activity

INTRODUCTION

Relevance of research

The current stage of the development of society is characterized by an increasing demand for well-trained specialists who are able to show creative initiative, to sort out the growing flow of knowledge, to choose from the flow of information or the knowledge that is required to solve a specific problem, who possess skills of non-traditional thinking. The importance of this provision leads to the necessity of forming the creative experience in the next generations. Hence the main goal of general education is "the formation of a diversified, creative personality capable of realizing creative potential in dynamic social and economic conditions" (Conception, 2000) Mathematical education, being a part of general education, is included in the process of revising value and target guidelines. Creative activity occupies one of the key positions in the value system of modern mathematical education. It is universally recognized that teaching mathematics has a high potential for including schoolchildren

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State of the literature

- The introduction of new generation standards in the mathematical education system, as well as the implementation of the ideas of the Concept of the development of mathematical education, shifts the emphasis in teaching mathematics towards the development of the creative potential of the student by means of the subject, which is not sufficiently reflected in studies on the methodology of mathematics today.
- In the scientific and methodological literature on the theory and methodology of teaching mathematics, questions on the formation of experience in the creative activity of schoolchildren in the mathematics lessons have been little studied.
- Most of the studies devoted to finding ways to include students in creative activities are either of a rather general nature, which does not allow us to bring the proposed methods to the level of a lesson, or they describe narrow specific tools that require additional study time and are not applicable in the daily practice of teaching mathematics.

Contribution of this paper to the literature

- A technique is proposed for the formation of procedural features of the creative activity of schoolchildren by working with a mathematical problem at the final stage of its solution, which allows the mathematics teacher to involve the pupils in the experience of creative activity within the framework of a regular lesson in mathematics, that is, systematically.
- For the first time, the structure of the final stage of the task's solution has been determined, actions adequate to each of its components have been singled out, correspondences has been established between these actions and the procedural features of the creative activity of schoolchildren.
- The technique described in the article makes it possible to include tasks from school textbooks of mathematics in the work with pupils, rather than specially selected tasks of a certain type, which was not mentioned in the traditional method of teaching mathematics.

in creative activity. Analysis of the goals of modern mathematical education shows that involving students in the experience of creative activity and developing the ability to apply this experience is a priority goal of teaching mathematics in terms of implementing the Conception of the development of mathematical education in the Russian Federation (Conception, 2013). This emphasizes the relevance of finding opportunities to familiarize schoolchildren with the creative activity in the process of teaching mathematics. The creative activity for students is understood as an activity in the course of which the student creates a fundamentally new way of solving a problem or constructs it from known techniques, or obtains new knowledge in the process of solving. Since educational creativity seldom leads to an objectively new result for society, the description of creativity as a manifestation of certain procedural features, intellectual properties, or mental formations is important. Didactics attribute the following features to the creative activity in the educational process: independent transfer of knowledge and skills to a new situation; seeing new problems in familiar, standard conditions, situations; seeing a new function of a familiar object; seeing the structure of the object to be studied; ability to see an alternative solution; ability to combine previously known methods of the activity in a new technique; ability to create an original way of solving with availability of others (Lerner, 1974). The signs of creative activity are characterized by one common feature - they are not acquired as a result of obtaining verbal information or viewing the mode of action. The designated characteristics of creative activity cannot be conveyed except by the inclusion of a person in a feasible activity that requires certain creative features, and thereby shaping these features. Each component of creative activity can be successfully formed when working with a mathematical problem on the final stage of its solution. The aim of our research is to show the interrelation between actions that are appropriate to the structural components of the final stage of working with a mathematical problem and the components of creative activity (Zelenina, 1998, 2002, 2004). This approach of familiarizing schoolchildren with creative activity has certain advantages: it does not require developing a special content of mathematical education, special forms and methods of instruction; does not require additional mathematical knowledge from the student; can be implemented at every lesson of mathematics that is used systematically.

Aims and objectives of the study

The aim of the research is to develop theoretical and methodological foundations for using the final stage of working with a mathematical problem as a means of forming schoolchildren's creative activity. The main tasks are: to define the structure of the final stage of solving the problem, and on its basis to single out actions that make it up; to establish interrelations between actions that are appropriate to the final stage of working with the mathematical problem, and procedural features of schoolchildren's creative activity singled out in didactics; to ground the possibility of using the final stage of working with a mathematical problem as a means of forming students' creative activity; to develop methodical recommendations for implementing research results.

LITERATURES REVIEW

In scientific and methodological literature, several approaches for the formation of students' creative activity in teaching mathematics are singled out. One of them is based on the inclusion of specially-selected problems imitating scientific research in the learning process. This approach is described in the works by Hadamard (1970), Poya (1991), Weber, Radu, Mueller et al. (2010), Czarnocha & Dias (2016) and others. Gnedenko (1979), Kolyagin (1977), Krupich (1995), Bahar & June Maker (2015), Collis, Watson & Campbell (1993) and others link the development of students' creative activity in teaching mathematics with the application of problem-search problems. Another approach is based on systematic use of heuristic methods in teaching mathematics. Its authors are Artemov (1973), Balk (1969), Semenov (1995), Trinchero & Sala (2016), Liljedahl, Santos-Trigo, Malaspina, Bruder (2016) and others. Liashchenko (1988), Schwarzburd (1964) and others consider students' independent work as priority educational activity to be the basis for the formation of creative activity. Schwarzburd (1964), Shvartsman (1987) and others suggest using the potential of extra-curricular classes in mathematics to introduce schoolchildren to creative activity. Without belittling the value of these approaches, it should be noted that their authors use philosophical or psychological definitions of creativity that are too general for the methodology of teaching mathematics, and, as a consequence, offer fairly general ways of including schoolchildren in creative activity. The analysis of psycho-pedagogical and scientific-methodical literature, the experience of mathematics teachers shows that over years individual aspects of involving students in creative activity in the process of teaching mathematics are the subject of scientific research. Let us single out their main directions. Gorbachev (2001) considers the use of the analogy method in teaching pupils elements of spherical geometry to be a means of familiarizing schoolchildren with creative activity in the process of studying mathematics. Aksyutina and Shuklin (2013, 2014) speak of applying stereo-metric material in the course of planimetry. Goryaev (1997) proposes to form creative activity when teaching in the system of consolidated didactic units. Tyuina (2003) considers the formation of analysis through synthesis as a method of creative activity. In the studies of Kuznetsova (1997) and Azarova (2012) means of forming junior schoolchildren's creative activity when teaching mathematics are entertaining tasks as, Maslova's (1996) – the search for regularities, Novoselova and Kochneva's (2012) – situations of natural science content and their mathematical models, Sullivan & Clarke (1992) suggest using open-ended tasks. The influence of mathematics learning styles on the students' cognitive features is investigated by Kablan & Kaya (2013). Napalkov and Gusev (2014), Kharitonov (2015) connect the involvement of schoolchildren in creativity with the use of WEB-technologies. Santos & Silva (2016) pay special attention to the research method of learning with the use of case technologies. Modeling the process of scientific creativity in teaching mathematics is examined in the works of Rosa & Orey (2012). The works of orev (2011, 2012, 2016) are devoted to the formation of creative abilities of schoolchildren in additional to mathematical education. The variety of the above studies emphasizes the importance of the problem of forming creative activity in the process of teaching mathematics. Undoubtedly, the proposed tools and techniques contribute to the development of intelligence, the ability to find ways to solve a variety of problems, the ability to navigate and think quickly, to show proactive attitude and initiative.

MATERIALS AND METHODS

Methods of research

The following methods were used to carry out the research: analysis of psychological, pedagogical and mathematical-methodical literature on the research topic, analysis and generalization of the teachers' experience and the author's own experience in conducting training sessions, analysis of educational products, method of mental experiment, systematization and generalization of facts and concepts, modeling, method of expert assessments, analysis of the results of educational activity, development and application of educational materials.

Experimental research base

Approbation, generalization and implementation of research results are carried out:

- during teaching experience of the author with schoolchildren, in practical classes on the methods of teaching mathematics and during the period of pedagogical practice with students of the Faculty of Mathematics of VyatSHU, the Faculty of Computer and Physics and Mathematics Sciences of VyatSHU, in the practice of teachers of mathematics in Kirov and Kirov region;
- in the form of reports and speeches at scientific conferences and seminars of various levels, including international, publications in collections of scientific articles and scientific and methodical periodicals.

Stages of research

The study was conducted in three stages.

On the first stage, the state of the problem in the theory and practice of teaching students was revealed. For this purpose, research and analysis of psychological-pedagogical and mathematical-methodical literature on the research problem was carried out, observation and analysis of the experience of teachers of mathematics in implementing the work at the mathematics lessons with the problem on the final stage of its solution.

The second stage developed methodical recommendations for using the final stage of solving a mathematical problem as a means of forming schoolchildren's creative activity. The discussion of implementing methodical recommendations has been and are carried out through feedback from teachers of mathematics, as well as during reports at conferences and seminars of various levels, which leads to a consistent improvement of the proposed methodology.

In parallel with the second, the third stage was carried out and continues to be implemented, during which the author and teachers of mathematics at schools in Kirov and Kirov region, conducts experiential teaching and approbation of the proposed recommendations.

RESULTS

To establish the correspondence between the procedural features of creative activity and actions appropriate to the final stage of working with a mathematical problem, let us turn to its structure. Depending on the nature of the students' activities at the final stage of working with the task, we, as a result of the theoretical study, identify two stages in its structure, which we call reflexive and transformative (Zelenina, 2004, 2005, 2006). The pupil's activity at the reflexive stage is concentrated on considering the condition, the search, the course and the result of solving the problem - "look back" (Poya, 1991). At this stage, the student is "inside" the problem, that is, returns to individual stages of its solution, analyzes them, fixes new results obtained in the course of working with the problem: facts, formulas, properties, signs, theorems, methods, techniques, ways of solving problems; correlates the problem with the types of problems known to him and available theoretical knowledge; allocates and formulates heuristic prescriptions; realizes and outlines the ways of further development of the problem. As a result, there is a redesign, re-evaluation, systematization, increment of knowledge and skills available to the student. The knowledge obtained in this way is not digested from outside but built by the student himself, which emphasizes the creative nature of such activity. The reflexive stage of the final stage of solving a mathematical problem can be carried out in several different directions. Let us represent these directions in the form of diagrams reflecting the way of the student's activity, using the following notation: PC is the problem condition; SPS – the search for the problem solution; PS – the problem solution; RPS is the result of problem solved; FSSP – the final stage of the solved problem; NP is a new problem, the NWSP is a new way of solving the problem.

1. **PC → SPS → PS → RPS → FSSP**

2. **PC → SPS → PS → RPS → FSSP**

3. **PC → SPS → PS → RPS → FSSP**

4. **PC → SPS → PS → RPS → FSSP**

Their various combinations are also permissible.

At the transformative stage, the student's activity is aimed at developing the task - "looking ahead" (Sarantsev, 1995, 2002). The student "goes beyond" the problem, that is, going back to the individual components of the solution and analyzing them, formulates new problems based on the initial one, integrates them into blocks, cycles, "chains", a series of interrelated tasks, and finds new solutions. This process is represented by a partial change in the condition of the problem, the application of the basic methods of cognition: observation, comparison, analogy, generalization, concretization, analysis, synthesis, and formulation, proof or refutation of the hypotheses put forward. The pupil acts as a researcher. His activities include heuristic, logical components and, without a doubt, creative. The transformative stage of the final stage of solving the mathematical problem, like the reflexive one, can be carried out in several different directions, which are also represented by diagrams reflecting the mode of the students' activity.

1. **PC → SPS → PS → RPS → FSSP NP**

2. **PC → SPS → PS → RPS → FSSP NP**

3. **PC → SPS → PS → RPS → FSSP NP**

4. **PC → SPS → PS → RPS → FSSP NP**

5. **PC → SPS → PS → RPS → FSSP NWPS**

Each of the stages identified in the structure corresponds to certain components of working with the problem on the final stage of its solution. Each component includes a block of actions that are appropriate to it. Blocks are interrelated, interdependent and part of the ability to work with the problem on the final stage of its solution. Let us give a detailed description of the actions that make up each of the blocks.

Reflexive stage

Understanding the problem condition: analysis of the problem condition for the ambiguity of its interpretation; analysis of the problem condition from the point of view of lack or excess of data; understanding the ways of partial change; reconsideration of mathematical objects or their elements in terms of new mathematical concepts; understanding the ways to find new solutions.

Understanding the course of the problem solution: checking the correctness of calculations and transformations in the problem solution; understanding the theoretical basis for solving the problem; identifying the main idea of the problem solution; defining the scheme for the problem solution; defining the method or technique for the problem solution; identifying new mathematical facts, formulas, as well as properties, features of the mathematical objects considered in the problem; establishing links of the solved problem with previously solved problems, available theoretical knowledge; formulating reference and auxiliary tasks; singling out available and new heuristic techniques; establishing intra-subject connections; establishing, if possible, of inter-subject links; systematizing knowledge, as well as methods and techniques for solving problems.

Understanding the result of the problem solution: estimating the plausibility of the result from the point of view of common sense; checking the result of the problem solution by dimension; checking by the condition; interpreting the obtained result from the point of view of practical application and application value; using the same pattern in different situations; understanding the possibilities of using the result of solving the problem when solving and setting other problems; understanding the ways to change the conclusion of the problem; constructing a mathematical model.

Conversion stage

Partial change in the problem condition: taking some problem elements for variables (problems with parameters); replacing a part of the original problem data with other data without changing the conclusion; replacing problem data while retaining its conclusion; replacing the problem conclusion while saving its data; reformulating the problem conclusion and partial modification of the data; formulating the inverse problem; singling out the elements of the mathematical object considered in the problem and including them in new connections; introducing additional elements or relations and their inclusion in new relations, considering the limiting case.

Application of cognition methods: analysis, synthesis, comparison, generalization of the data or the conclusion of the problem; specifying the data or the conclusion of the problem; applying analogy.

Search for new ways to solve the problem: rethinking mathematical objects or their elements in terms of new mathematical concepts; introducing additional elements or relations and their incorporation into new links; transferring the content of the problem into the language of a particular theory.

The above defined and described functions and the structure give grounds to assert that the final stage of solving the problem is an effective means of involving schoolchildren in creative activity. The student learns to see and formulate problems, discover new ones and incorporate his knowledge into new connections, build a whole block of new problems around this one, applying generalization, concretization, analogy, consider several different ways of solving the problem, establish intra-subject and inter-subject links, that is, to some extent reproduces the path of knowledge in mathematics. In addition, the activities on the final stage of solving the problem form the procedural features of creative thinking singled out in didactics. Let use show that each of the components of creative activity can be successfully formed when working with a mathematical problem on the final stage of its solution.

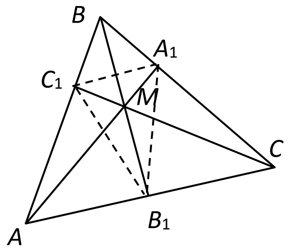
Let us illustrate this idea by the example of the last stage of working with the following problem.

Problem. In a triangle ABC with an acute angle C there are heights AH and BD. Prove that $\Delta ABC \backsim \Delta HDC$.

Proof

From the similarity of the right-angled triangles AHC and BDC (on two angles) we conclude that $\frac{HC}{DC} = \frac{AC}{BC} = \frac{AH}{BD}$ (*). In triangles ABC and HDC, the angle C is a common angle and $\frac{HC}{AC} = \frac{DC}{BC}$. Hence, $\Delta ABC \backsim \Delta HDC$ by two proportional sides enclosing equal angles.

The proof is complete. In the process of understanding the problem condition and the way to solve it, there arises a question: "And if the angle C in the triangle ABC is obtuse, will these triangles be similar?" **Independent use of knowledge** about the types of angles of the triangle leads to the hypothesis, and thus to **seeing and**



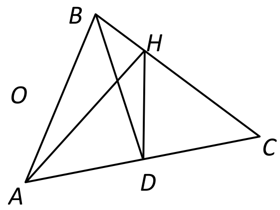
formulating a new problem in an already familiar situation. Let us verify whether the fact proven in Problem 1 if the angle C is obtuse. We formulate the problem:

1.1. In the triangle ABC, the angle C is obtuse, AH and BD - heights. Are the triangles ABC and HDC similar?

This problem is interesting because its solution is analogous to the solution of the problem.

We pay attention to the ratio obtained in the course of the solution (*), in recording we use the legs and hypotenuses of the rectangular triangles AHC and BDC. We note that the basic property of the proportion can be applied to this relation. Then, $\frac{HC}{AC} = \frac{DC}{BC} = k = \cos C$, where k is the similarity coefficient. In this situation, the **independent transfer of knowledge** about the relationships between sides and angles in a right-angled triangle and the **ability** to apply the basic property of proportion lead to the discovery of a new useful fact: *the similarity coefficient of the triangles ABC and HDC is equal to the cosine of their common angle C if the angle C is acute, and $|\cos C|$ if the angle C is obtuse.*

Then, returning to the condition and solution of problem 1 ($\angle C$ is acute), we consider rectangular triangles AHB and ADB. Triangles AHB and ADB have a common hypotenuse AB. The center of the circle described near the right triangle is the middle of the hypotenuse. Triangles AHB and ADB under consideration are inscribed in the same circle with the center at the point O, which is the midpoint of AB. Thus, the quadrangle $ADHB$ is inscribed in a circle. **There are manifestations of such signs of creative activity, as independent transfer of knowledge to a new situation, seeing of the structure of the object and understanding its new function.** What follows? A new way of proving the similarity of the triangles ABC and HDC (on two angles), that is, manifestation of one more property of creative activity - **seeing an alternative solution.** By the property of a quadrangle inscribed in the circle $\angle BAD + \angle BHD = \angle ABH + \angle ADH = 180^\circ$. Next $\angle BAD + \angle BHD = 180^\circ$ and $\angle BHD + \angle DHC = 180^\circ$. Consequently, $\angle BAD = \angle DHC$. Similarly, $\angle ADH + \angle ABH = 180^\circ$ and $\angle ADH + \angle HDC = 180^\circ$. So, $\angle ABH = \angle HDC$, that is, $\Delta ABC \backsim \Delta HDC$ by two angles.



A new way of solving the problem allows us to re-**understand the structure of the object** in question and, first, draw another useful conclusion from the work done: the segments connecting the middle of the side of the triangle with the bases of heights drawn to the other two sides are equal, and secondly, enter into consideration not yet used the third height of the triangle.

We formulate a new, more general problem

1.2. Altitudes AA_1, BB_1, CC_1 were made in the acute-angled triangle ABC. Prove that each of the triangles A_1B_1C, A_1BC_1, AB_1C_1 is similar to the triangle ABC.

Since the solution of Problem 1.2 is analogous to the solution of Problem 1, we make the following conclusions without special difficulty: $\Delta ABC \backsim \Delta A_1B_1C$ with the similarity coefficient $k = \cos C$, $\Delta ABC \backsim \Delta A_1BC_1$, $k_1 = \cos B$, $\Delta ABC \backsim \Delta AB_1C_1$, $k_2 = \cos A$. In addition, the cut-off triangles are similar to each other: $\Delta A_1CB_1 \backsim \Delta A_1C_1B$, $\Delta A_1CB_1 \backsim \Delta AC_1B_1$, $\Delta AB_1C_1 \backsim \Delta A_1BC_1$.

The question arises: is the number of pairs of such triangles exhausted by the list above? We consider triangles, for which one of the vertices is the intersection point of the heights of the triangle ABC. In this case, a **separate element** (the point of intersection of heights) **is singled out from the structure of the considered object** (triangle ABC), **seeing a new function of this element** (to be the vertex of a triangle) and **posing a new problem in a familiar situation**. The simplest analysis of the problem posed makes it possible to formulate the following (already third in the list) problem obtained at the final stage of the work with problem 1.

1.3. In the acute-angled triangle ABC, the altitudes AA_1, BB_1, CC_1, intersecting at the point M are drawn. Prove that $\Delta AMC \backsim \Delta C_1MA_1$, $\Delta BMC \backsim \Delta C_1MB_1$ $\Delta AMB \backsim \Delta B_1MA_1$. It should also be noted that the process of formulating and solving this problem illustrates well such a feature of creative activity as **the ability to combine previously known ways of solving the problem in a new way**. Note also that problem 1.3 can be solved in the same ways as task 1.

Studying the configuration shown in the figure, we obtain the following problem describing the property of the orthocenter M of the non-rectangular triangle ABC:

1.4. Point M is the orthocenter of ΔABC. Prove that each of the four points A, B, C, M is the orthocenter of the triangle formed by three other points.

We draw attention to the fact that in proving the similarity of triangles, the equality of angles was often used. Can we now find their values? There is an assumption: if the angles of ΔABC are known, you can try to find the angles $\Delta A_1B_1C_1$. This situation indicates that further self-sufficient study of the structure of the object occurs, as a result of which new ways of searching for problems are outlined. We formulate a problem corresponding to the situation described above.

1.5. In the acute-angled ΔABC, heights AA_1, BB_1, CC_1 are given. Find the angles $\Delta A_1B_1C_1$, knowing the angles of ΔABC

Suppose that $\angle A = \alpha,\ \angle B = \beta,\ \angle C = \gamma$ then $\angle A_1 = 180^{\circ} - 2\alpha,\ \angle B_1 = 180^{\circ} - 2\beta,\ \angle C_1 = 180^{\circ} - 2\gamma$.

Analysing the solution of Problem 1.5, a hypothesis appears that the altitudes of ΔABC are the bisectors of the angles $\Delta A_1B_1C_1$ (**seeing a new function of the object**), the validity of which is easily established: $\angle AA_1B_1 = 90^{\circ} - \alpha = \angle AA_1C_1$, hence A_1A is the bisector $\angle C_1A_1B_1$. Similarly for B_1B and C_1C. Thus, another new task is born.

1.6. Prove that the bisectors of $\Delta A_1B_1C_1$ in Problem 1.3 lie on the heights of ΔABC. We note that problems 1.5 and 1.6 are presented in the school textbook as problems of increased complexity. However, the above example convincingly shows that on the final stage of working with the original problem, problems 1.5 and 1.6 can not only be easily solved by the students, but also formulated independently. Analysis of problem 1.6 allows us **to see a new function** of the point of intersection of the heights of the triangle ABC: the point of intersection of heights ΔABC is the center of the circle inscribed in $\Delta A_1B_1C_1$ and then **formulate a new problem**.

1.7. Heights AA_1, BB_1, CC_1 were given in the acute-angled triangle ABC. Prove that the center of the circle inscribed in the triangle $A_1B_1C_1$ is the intersection point of the heights of the triangle ABC.

The facts obtained in solving problems 1-1.7 are applied in solving problems 1.8 - 1.10.

1.8. The lengths of segments 5, 12, 13, connecting the bases of heights A_1, B_1, C_1 of the acute-angled triangle ABC are known. Find the value of angle C.

1.9. The lengths of segments 5, 12, 13, connecting the bases of heights A_1, B_1, C_1 of the acute-angled triangle ABC are known. Find the length of segment AC.

1.10. The lengths of segments 5, 12, 13, connecting the bases of heights A_1, B_1, C_1 of the acute-angled triangle ABC are known. Find the area of the triangle ABC.

We examined the possibilities of the last stage of solving on the example of one problem. Meanwhile, the experience of teaching shows that a methodically competent organization of the final stage of working with the problem and its careful planning turns almost every task into a good ground for the development of schoolchildren's creative activity. The analysis of the investigated stage of mathematical problems shows that the above-mentioned signs of creative activity are formed in the students' activity on the final stage of solving a mathematical problem. Thus, the final stage is a necessary and essential part of solving the problem and contains a huge potential for teaching, developing and educating students, improving the methodology of working with a mathematical problem.

DISCUSSIONS

The study showed that issues related to forming students' creative activity are widely discussed in scientific and methodological literature. Many researchers see a great potential of mathematics and offer quite a variety of ways and means of involving schoolchildren in creative activity in the process of teaching the subject. The problem is that the means for including the student in such activities must have a kind of universality, that is, be incorporated in almost every ordinary lesson. Fulfillment of this requirement will allow to form the experience of creative activity systematically, and not sporadically with using special content of training and special techniques. In order to avoid this kind of difficulties, we propose to take procedural features of such activity identified in didactics as the basis for developing creative activity experience and form them on the final stage of working with the mathematical problem through the actions which we have identified and which are appropriate to this stage of solving the problem. Our research has shown that the competent organization of the final stage of working with the problem and its careful planning turns almost every problem into a good testing ground for the development of schoolchildren's creative activity. This problem will become completely resolvable if the teachers of mathematics realize the importance of the final stage of working with the mathematical problem, its great opportunities for the student's development. This requires teachers' theoretical training and a workshop on the use of the proposed methodology. This can be the subject of discussion in the educating students of pedagogical training (profile of Mathematics) in the study of theory and methodology of teaching mathematics, elementary mathematics, and geometry. There is a possibility for the coursework within the framework of improving qualification of teachers of mathematics.

CONCLUSION

Based on the analysis of normative documents, scientific, pedagogical and methodological literature, the study of teachers' experience and the author's own teaching experience, a methodology for forming schoolchildren's creative activity has been developed and implemented through working with a mathematical problem on the final stage of its solution. Two stages are distinguished in the structure of the last stage of working with the problem: reflective and transformative. On the reflexive stage, a return to the already realized stages of the problem solution and their comprehension take place, on the transformative stage the student's activity is aimed at further development of the problem. Each stage assumes the existence of certain components of working with the problem on the final stage of its solution. Each of the identified components includes a block of actions appropriate to the final stage of working with the mathematical problem. The identified actions correspond to the procedural features of creative activity. Thus, by implementing the final stage of solving the problem at the lesson of mathematics, the student is included in the creative activity and forms procedural features of such activity. Approbation of the developed methodology shows both the possibility of its implementing in the process of teaching mathematics and the appropriateness of its use at the lessons of mathematics. This is confirmed by the results of students' learning activities, observations of teachers of mathematics, as well as the author's own observations. The proposed methodology does not require a special organization of the learning process, adjustments in the content of teaching mathematics, additional time for classes. Forming signs of creative activity is due to a specially organized work with the mathematical problem on the final stage of its solution, which, undoubtedly, serves the means of student's development, improving the quality of knowledge, the effectiveness of the learning process.

RECOMMENDATIONS

The materials of the article can be useful for teachers of mathematics, teachers of additional mathematics education, and teachers of higher educational institutions who are interested in substantially increasing the level of their students' development, to support interest in the subject, to carry away the prospect of making their own discoveries. We see the ways to improve the proposed methodology in creating sets of tasks on various topics in the course of mathematics, where the final stage of the solution largely ensures forming schoolchildren's creative activity.

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