Geometrical Sense Making: Findings of Analysis Based on the Characteristics of the van Hiele Theory among a Sample of South African Grade 10 Learners

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This paper reports on one part of a large study which attempted to identify the linguistic and hierarchical characteristics of van Hiele theory amongst grade 10 learners. The sample consisted of a total of 359 participants from five purposively selected schools from Mthatha District in the Eastern Cape Province of South Africa. The performance of these learners on a written geometry test was analysed. On the basis of the analysis, 30 learners were selected to be the subjects for interviews. This paper focuses on the analysis of the interview transcripts to discern the van Hiele characteristics among learners' geometrical tasks. This paper reports particularly on linguistic and hierarchical characteristics of the van Hiele theory. The ethical requirements were met. The findings from the study affirmed the existence of the linguistic and hierarchic characteristics of the van Hiele theory.

Keywords: geometry, van Hiele theory, linguistic characteristics, hierarchical nature

INTRODUCTION

The van Hiele theory of geometrical thinking was developed and structured by Pierre van Hiele and Dina van Hiele-Geldof in the period from 1957 to 1986. As experienced teachers in Montessori secondary schools, the van Hieles were concerned about the difficulties which their students encountered with secondary school geometry. According to Fuys, Geddes and Tischler (1988, p. 4), the van Hieles believed that secondary school geometry involved thinking at a relatively "higher level" and the students had not had sufficient experiences in thinking at prerequisite "lower levels" and they investigated the prerequisite reasoning abilities needed to

successfully engage a logical-deductive system of thought. Van Hiele (1957) described the model of geometric thinking using three aspects: the existence of levels, properties of the levels and the movement from one level to the next higher level (Haviger & Vojkůvková, 2014). The van Hiele theory comprises of two main components: (a) levels of geometric thinking and their characteristics and (b) phases of learning (Crowley, 1987).

A recent writing on van Hiele theory by Fujita and Jones (2007, p. 5) suggests the following as the description for the five levels of geometrical thinking.

- Level 1: Visual - identifying shapes according to their concrete examples
- Level 2: Descriptive/analytic - identifying shapes according to their properties
- Level 3: Abstract/relational/informal deduction - identifying relationships between shapes and producing simple logical deduction
- Level 4: Formal deduction - understanding logical deduction
- Level 5: Rigor/meta-mathematical - axiomatic systems of geometry are understood.

According to Clements and Battista (1992, p. 429), "the bulk of the evidence from the van Hiele-based research along with research from the Piagetian perspective, indicated the existence of thinking more primitive than, and probably prerequisite to, van Hiele's level 1". They called the level as Level 0 (pre-recognition level) (see Alex & Mammen, 2012). The existence of Level 0 was taken into consideration in the present study. According to van Hiele, the levels have five distinctive properties (van Hiele, 1986, 1999). Some others (Usiskin, 1982; Mayberry, 1983; Burger & Shaughnessy, 1986; Senk, 1989; Pegg, 1995) also have affirmed the same through their research. The characteristics are:

- *Intrinsic and extrinsic properties:* At each level, there appears in an extrinsic way which was intrinsic at the preceding level. At the base level (level 1), figures were in fact also determined by their properties, but someone thinking at this level is not aware of these properties (van Hiele, 1986).
- *Hierarchic arrangement:* The ways of thinking of the levels have a hierarchic arrangement. "Thinking at the second level is not possible without that of the first level; thinking at the third level is not possible without thinking at the second level" (van Hiele, 1986, p. 51). The levels are hierarchical in that a student cannot operate with understanding on one level without having been through the previous levels. This can be taken as one of the reasons for a learner not being able to understand the teacher (van Hiele, 1986).
- *Discontinuity:* According to van Hiele (1986, p. 49), "the most distinctive property of the levels of thinking is their discontinuity, the lack of

State of the literature

- The van Hiele theory of geometrical thinking proposed five distinct hierarchical levels of geometrical thinking, each characterised by certain properties and characteristics including the linguistic characteristic.
- The linguistic characteristic plays a significant role in the progress from one level to the next higher level.
- Most research reports are based on studies carried out outside Africa and South Africa or in urban schools and hence, reporting research results in rural South African schools is pertinent.

Contribution of this paper to the literature

- Using qualitative interviews, the lead researcher carried out 40-60 minutes interviews with each of the 30 Grade 10 members of the sample, six each from five different rural schools.
- The interviews were based on learners' responses to tasks on triangles and quadrilaterals.
- Results based on analysed transcripts confirmed that language competency is a barrier and teachers need to recognise that learners' competency in the teaching language is important for progress to higher levels of geometrical thinking.

coherence between their networks of relations". He observed that at certain points during instruction, the learning process has stopped and later on it would continue itself as it was and a student having reached a given level remains at the level for a time, as if maturing (Pegg, 1995). At that time, the teacher does not succeed in explaining the subject.

- *Linguistic character:* Each level has its own linguistic symbol and its own system of relations connecting these signs and a relation which is "correct" at one level can reveal itself to be incorrect at another level (van Hiele, 1986). Two people operating at two different levels speak a very different language (van Hiele, 1986). This is what often happens between a teacher and a student (van Hiele, 1986). Neither of them can manage the thought process of the other and their dialogue can only proceed if the teacher attempts to form for himself an idea of the student's thinking and to match to it (van Hiele, 1986). Van Hiele suggests that a teacher beginning the teaching of geometry, should address himself in a language that his students understand and that he must not use a language of a higher level, which his students have not achieved (van Hiele, 1986).

- *Advancement:* "The transition from one to the next higher level is not a natural process but takes place under the influence of a teaching-learning program. The transition is not possible without the learning of a new language" (van Hiele, 1986, p .50). The maturation which leads to the higher level happens in a special way. Adequate and effective learning experiences are required at lower levels in order to learn how to think and reason at the higher levels.

In addition to the above characteristics, researchers such as Usiskin (1982) and van de Walle (2004) also identified a few other features and characteristics of the levels from their studies. For example, van de Walle (2004, p. 348) affirmed the following additional feature as suggested by van Hiele (1986): "The levels are not age dependent in the sense of the developmental stages of Piaget. A third grader or a high school student could be at level 1. In fact, some students and adults remain forever at level 1 and a significant number of adults never reach level 3. But age is certainly related to the amount and type of geometric experiences that we have".

THE IMPORTANCE OF LANGUAGE IN THE VAN HIELE THEORY

> "Language is very important to thinking. Without language, thinking is impossible. Without language, there is no development of sciences."
> (van Hiele, 1986, p. 9)

Van Hiele (1986) states that when he began teaching, there were parts of subject matter, that he could explain and re-explain, but students could not understand it even after the repeated tries. In the years that followed, he changed his explanation many times, but the difficulties remained. He felt that he was speaking a different language and the solution to this concern was the different levels of thinking (van Hiele, 1986). Again, van Hiele speaks of an unavoidable situation in class, such as finding a group of learners having started homogeneously not passing the next level of thinking at the same time that half of the class might speak a language which the other half may not understand (van Hiele, 1986). Van Hiele stressed the importance of the introduction and the use and attainment of language at each level of thinking (Genz, 2006). According to van Hiele, each level has its own language and a teacher beginning to teach geometry should address himself to the learners in a language they understand and by doing that the teacher inspires the learners' confidence and they will understand him (van Hiele, 1986). In stressing the importance of language, van Hiele notes that many failures in teaching geometry result from a language

barrier-the teacher using the language of a higher level than those of the students (Fuys, et al., 1988). Van Hiele believed in the importance of language in moving from one level to the next, but according to van Hiele, Piaget did not see it (van Hiele, 1986, p. 5). Van Hiele points out that when it was occasionally mentioned to Piaget that the children did not understand his question, he said they understood it and could be read from their actions. According to van Hiele, although the actions might be adequate, one cannot read from them the level at which they can think (1986, p. 5). Pace (1991) comments that Piaget clearly tied language to the figurative aspect of knowledge and thus took a definite position against any such necessity of language for thought, while for van Hiele, "without language, thinking is impossible". Lai and White (2012) also suggest that communication and language are important aspects of the van Hiele theory. Clements and Battista (1992) state that the existence of the unique linguistic structures at each level has been supported by other researchers (for example, Mayberry 1983; Burger & Shaughnessy, 1986; Fuys, et al., 1988).

The research question that is addressed in this paper is: What is the evidence from the interviews with learners on the characteristics of the van Hiele theory?

THE CHARACTERISTICS OF THE VAN HIELE THEORY – EVIDENCE FROM RESEARCH

The van Hiele theory has precipitated a large body of work using, evaluating and modifying the theory itself. It has deepened and expanded research in the learning and teaching of geometry. It has served as a theoretical backbone in a wide range of related topics. It is a model of connections among theory, research, practice of teaching and students' thinking and learning. Because of its wider applicability in any imaginable field of teaching and learning, it has gathered some responses from researchers around the world.

A considerable amount of research projects was focused on testing the validity of the theoretical underpinning of this theory with all its properties (King, 2003). Besides a significant amount of research studies into students' understanding of geometric proofs, the van Hiele theory stands out as one of the best recognised frameworks for the teaching and learning of geometry (Dindyal, 2007). With the said interest, a number of research studies were and still are conducted in different parts of the world. Since the proposal of the van Hiele theory, studies have focused on various components of this learning model at different grade levels. The van Hiele theory has been extensively investigated with different research groups such as learners from different grades and also with pre-service teachers and in-service teachers in various parts of the world since the early 1980's (Hoffer, 1981; Smith, 1987; Clements & Battista, 1992). According to Pusey (2003), there are three different lines of work, namely, research that focused on the testing of the van Hiele theory, research that focused to find appropriate ways to assess the levels and implications of these assessments and research that examined the validity of the van Hiele theory in terms of curricula and research that focused on the effects of interventions on van Hiele model with students and teachers. King (2003) suggests that some of them have been recognised and acclaimed as ground-breaking projects in the field (For example, Usiskin, 1982; Mayberry, 1983; Burger & Shaughnessy, 1986; Fuys, et al., 1988; Clements & Battista, 1992). Of these, Burger and Shaughnessy (1986) examined three specific questions related to the van Hiele theory of learning in geometry, such as knowing whether these levels were useful for classifying students' thinking in geometry, whether there could be any specific indicators in students' reasoning that might be aligned with each of the levels and whether an interview procedure that could reveal predominant levels of reasoning

on specific geometry tasks, instead of a written test. Their subjects varied in age from primary school to college level. Their findings did support much of van Hiele's description and characteristic of the levels, they could assign certain behaviours to each level and the van Hiele levels were useful in describing students' reasoning process for polygons. They were also successful in designing an interview protocol and they developed a series of indicators for each level for assessing students' geometric understanding in task based interviews. This paper rests on the interview tasks and level descriptors as suggested by Burger and Shaughnessy (1986, pp. 43-45).

The van Hiele theory is particularly relevant in South Africa, where mathematics remains a problematic learning area, as Fuys, et al., (1988) suggest that "its emphasis on developing successively higher thought levels appears to signal direction and potential for improving the teaching of mathematics"(p. 191).

METHODOLOGY

This paper emanates from a larger study which was conducted to determine the effectiveness of van Hiele theory based geometry instruction in Grade 10. The sample consisted of a total of 359 participants from five purposively selected schools from Mthatha District in the Eastern Cape Province: 195 learners in the experimental group and 164 learners in the control group. For the larger study, a quasi-experimental design was implemented to check the effectiveness of the instructional framework. In the main study, a learner's level of thinking was determined mainly by his/her responses in the van Hiele Geometry Test (see Alex & Mammen, 2014). During the analysis process, in cases where there appeared to be certain trends in a learner's understanding, it was decided to conduct a structured interview with the learners in order to explore these features in greater detail and to obtain clarification on his/her understanding in geometric shapes. So as to confirm the levels, the lead researcher interviewed 30 learners individually through one-on-one interviews taken from the five different schools which participated in the study (6 learners each from a school - 3 from the experimental group and 3 from the control group). The interviewer described the interview process and its purpose to the cooperating educators and then asked the educators to select some of their learners according to their performance in the van Hiele Geometry Test as one from each level. The selection was therefore purposive. The number of learners in each gender was specified as to have a gender ratio of 2:1 of girls and boys as the number of girls were more in the sample. The interview tasks were first piloted in the lead researcher's own school prior to the commencement of the interviews. All the ethical requirements were met.

Instruments

The authors adapted an interview protocol which consisted of giving the learners seven open ended tasks dealing with geometric shapes. These tasks were developed by Burger and Shaughnessy (1986), which were also used by Genz (2006, p 57-58), which were designed to reflect the descriptions of the van Hiele levels. According to Burger and Shaughnessy (1986), these tasks were developed to evaluate learners' basic geometric skills. The tasks involved drawing triangles and quadrilaterals, identifying and defining shapes, sorting shapes and engaging in informal and formal reasoning about geometric shapes (Alex & Mammen, 2014). These tasks were expected to draw out the characterisations of van Hiele levels 1 to 3 (Burger & Shaughnessy, 1986). Investigation on van Hiele level 4 was not done as none of the learners in the entire sample were at level 4 (Alex & Mammen, 2014). Two sets of drawing, identifying and sorting tasks were administered: one set for triangle

shapes and one set for quadrilateral shapes. The tasks were open ended and were designed to provide interpretation at several different van Hiele levels as the learners were at different levels of geometric understanding according to the van Hiele theory (Genz, 2006, p57-58). Examples of one set of tasks for triangles are described below. The tasks for quadrilaterals were similar.

Drawing task

Each learner was asked to draw different triangles, one after the other, by drawing a new triangle each time which was different from the previous one in some way. Then the learner was asked how the figures differed and how many different triangles he/she could draw. The task investigated the properties that the learners varied to make different figures and explored whether they could draw many or only a few triangles (Burger & Shaughnessy, 1986, p. 34).

Identifying and defining tasks

Each learner was given triangles drawn on a sheet of paper and was asked to put a T on each triangle and then to justify his/her marking and if necessary and why some of the other figures had been omitted. And further to elicit properties the learner perceives as necessary for a figure to be a triangle, the learner was asked, "What is the shortest list of things you tell someone to look for to pick out all the triangles on a sheet of figures?" the same was repeated for different quadrilaterals, namely square, rectangle, parallelogram and rhombus by asking the learner to put an S on all the squares, R on all rectangles, P on all parallelograms and so on. Thus the activity explored the learners' definitions and class inclusions.

Sorting task

A set of cut out triangles was spread out on the table. The learner was asked, "Can you put some of these together that they are alike in some way? How are they alike?" These kinds of questions were repeated until he or she could come up with new sorting properties. The same was repeated for quadrilaterals also. To further determine the learner's ability to distinguish common properties of pre-selected triangles, the interviewer selected a set of triangles that have some common property: For example, all isosceles triangles, all right angled triangles, all obtuse angled triangles and the learner was asked "All of these shapes are alike in some way. How are they alike?" The same was repeated for all the quadrilaterals.

The interview procedure

The interview tasks were administered to each learner by the lead researcher in an audio taped one-on-one interview in the learners' classrooms after school under the guidance of the mathematics educators. The learners were informed about the interview such that there would be some questions in the interview on triangles and quadrilaterals. Pencils, papers, erasers and mathematics instrument boxes were made available. The learners were encouraged to use any of these instruments at any time during the interview. The interviewer presented the tasks to each learner in the same order according to the script written by Burger and Shaughnessy (1986). On completion of each task the interviewer was free to follow up on any response. Only the learner and the interviewer were present at each interview. Each interview took about 40 to 60 minutes to complete. The interviews were made as similar as possible among the 30 learners. The structured, conversational interview format enabled the interviewer to gather similar information from each learner while at the same time to explore how each learner has come to his/her current understanding The data for the interview consisted of the audio files, the learners'

drawings and the interviewer's notes for each interview that contained annotations about surprising or unexpected responses and indicators about learner confidence.

RESULTS

During the analysis, the audio data and the learners' written responses and the researcher's notes were reviewed often to find the relevant dialogue and examples that reflected the findings and to check the accuracy of the findings.

The interviews yielded a number of particularly interesting aspects of the van Hiele theory. Even though a total of 30 learners were interviewed from the two different groups concerned, only three interview scripts were chosen from the experimental group for inclusion in this paper. The only criterion used was that each represented one level of van Hiele thinking. As such, the quasi-experimental design used for the larger study has no implication for the results reported in this paper. In order to ensure anonymity, they were named as Andiswa (18 year old female), Mila (16 year old male) and Nana (16 year old female). The analysis of the interviews is done under two major headings namely, triangle activities and quadrilateral activities. The anecdotes between the interviewer and the learners are not included in the paper due to page constraints.

Analysis of triangle activities

Learner 1: Andiswa

Andiswa was classified as being on the pre-recognition level (level 0) from the van Hiele Geometry Test. It appeared that drawing triangles other than the one with usual names that she was familiar with was of concern to her. When required to draw as many different triangles as possible, she provided four triangles, of which two triangles were identical in shape and orientation. Identifying and naming triangles was a problem for her. Andiswa could not identify certain triangles and she did not use the properties when she focused on identifying them. This showed that she had not reached visual, analysis and informal deduction levels of thinking. For her, the properties that she perceived as necessary for a figure to be a triangle was not clear. Anything that 'looks like a triangle' was a triangle for her.

Learner 2: Mila

Mila was classified as being on the recognition level (i.e., van Hiele level 1) from the van Hiele Geometry Test. When required to draw as many different triangles as possible, Mila drew four of them which were similar in size but different in orientations.

Identifying and naming triangles was not a problem for him. The use of precise language was a problem for him. He could identify certain triangles and he did not use the properties when he focused on identifying them. This indicated that he had not reached the analysis level of thinking and informal deduction level of thinking. The use of correct terminology was a problem for him. He also referred 'same shape' as 'equal size'. 'both have small bases' and 'heights' are equal , 'bases are longer' , 'this triangle is 'thinner' than the other one', were some of the terminology that he used. The use of descriptive, imprecise language like "long", "thinner", "heights" and "longer" showed that he was looking at the visual characteristics of the figures.

Learner 3: Nana

Nana was classified as being on the analysis level (i.e., van Hiele level 2) from the van Hiele Geometry Test. When required to draw as many different triangles as possible, Nana provided four of them in which she used a ruler to measure the sides

as she was drawing and indicating the equal sides. They were all in different sizes. She could draw all of them correctly and gave the correct name to it. She used very precise, short sentences to explain her triangles. Identifying and naming triangles was not a problem for her. She could identify triangles and used the properties when she focused on identifying them. This indicated that she had reached analysis level of thinking for the concept of triangles. Nana also knew the necessary condition for a figure to be a triangle. It was noted that Nana used the correct terminology and precise language in her explanations.

Analysis of quadrilateral activities

Learner1: Andiswa

In quadrilaterals, when asked to draw as many quadrilaterals as possible, she drew five different quadrilaterals. It appeared that her figures, the names of the figures and the explanations for them were not corresponding to each other. When asked for identifying quadrilaterals, she said she knew squares, rectangles, parallelograms, kite, trapezium and rhombus, and when asked to put an S on all squares and R on rectangles, and so on, she could not mark them properly. It appeared that she had serious problems identifying quadrilaterals. At times she seemed to understand a square, but got confused between rectangles and parallelograms due to "two sides being equal".

It looked like Andiswa did not know the necessary condition for a figure to be square, a rectangle and a parallelogram. For her, parallelogram was sometimes 'two long sides and two short sides' (mentioned it two times) and another time it was 'two sides which are equal and two sides which are not equal, which are not straight'. A square is a figure with 'four straight sides', which she mentioned it also two times. 'All sides are same length' is a 'quadrilateral' for her – which she used when she drew the triangles also.

The listing of properties in those items designed to assess analysis and informal deduction level thinking did not make sense and Andiswa used inappropriate vocabulary in almost all the tasks. It looked like, according to her, 'quadrilateral' stands for 'all sides equal 'whether it was a triangle or a quadrilateral, and she used it instead of "equilateral" and 'all sides are straight' was a square for her. Such a response suggested that this vocabulary did not have any meaning to her but that she had simply memorised the words without the conceptual understanding. Her use of vocabulary and properties was owing to memory rather than understanding.

Learner 2: Mila

In quadrilaterals, when asked to draw as many quadrilaterals as possible, Mila drew five different quadrilaterals. It was noted that while talking about the quadrilaterals that were drawn by him, Mila did not talk about the specific name of the quadrilaterals; rather he used the lengths of the sides to differentiate them. When asked to identify quadrilaterals, he said he knew squares, rectangles, parallelograms and trapezium, and when asked to put an S on all squares and R on rectangles, P on parallelograms and T on trapeziums and so on, when it was asked about marking trapezium in the sheet of figures, he asked the interviewer whether she could give the definition of a parallelogram so that he could think of trapezium. The interviewer did mention it, but even after thinking for a while he could not mark any trapeziums. This showed that he resorts to rote learning when defining shapes. When it was asked about rhombus, he said he had forgotten the definition of rhombus and rechecked and confirmed that he did not mark any rhombus in the given set of quadrilaterals. His explanations were rather messy and long to express a particular figure. It was noted that Mila was not sure of the necessary properties of many quadrilaterals. Class inclusion seemed to be a bit of a problem to him as he

could not remember most of the definitions. It appeared that the definitions or the names he got for different shapes were all mixed-up. Any angle that was less than 90^0 was 45^0 for him. Rote learning could have been the underlying reason behind this.

The listing of properties in those items designed to assess analysis and informal deduction level thinking did not make sense and Mila used inappropriate definitions for quadrilaterals like parallelogram and rectangles. It was also evident that even though he was at level 1 for the concepts of triangles he had not mastered the level for quadrilaterals.

Learner 3: Nana

In quadrilaterals, when asked to draw as many quadrilaterals as possible, she drew five different quadrilaterals in which she used a ruler to measure the sides as she was drawing and indicating equal sides. They were all in different sizes. It was noted that Nana spoke about her quadrilaterals in terms of the sides and angles. The term 'diagonal' was also mentioned in her explanation about squares. When asked about identifying quadrilaterals, she said that she knew squares, rectangles, parallelograms, kite, trapezium and rhombus, and when asked to put an S on all squares and R on rectangles, and so on, she marked them correctly and gave sufficient explanations for each of them. It also appeared that her definitions were very precise in terms of the sides and angles. It appeared that class inclusion was not a problem for her. The listing of properties in those items designed to assess analysis and informal deduction level thinking did make sense and Nana used appropriate definitions and vocabulary that matched to van Hiele level 2.

DISCUSSION

These interviews yielded a number of particularly interesting aspects of the van Hiele theory and are discussed below.

Linguistic character of the van Hiele theory as evident from learners' interviews

Andiswa was classified as being on the pre-recognition level (level 0) from the van Hiele geometry Test. During the interview, the interviewer felt that Andiswa was unaware of many characteristics and features of figures she drew and many other concepts that were linked to geometry learning that should be predominant to a grade 10 learner. Initially it felt to the researcher as if they were speaking different languages. The interviewer had to adapt the level of her language accordingly and could only speak about the shapes with which Andiswa was familiar and got used to the wrong terminology (language) that she had put for each concept. Andiswa used imprecise language in most of her explanations. This conversation seemed to confirm van Hiele's (1986) suggestion that each level has its 'own language' and people need to be speaking the same language in order to understand one another. Each level has its own linguistic symbol and its own system of relations connecting these signs. A relation which is "correct" at one level can reveal itself to be incorrect at another level (van Hiele, 1986).

Mila was classified as being on the recognition level from the van Hiele Geometry Test. When the interviewer and Mila started off the interview, the interviewer felt that Mila was operating at level 1 as he could draw triangles in different orientations and talked about making more triangles by changing the angles and sides. For Mila, concerning the concept of triangles, he had the language ability of a level 1 learner. But as the interview progressed he had problems with the naming and identification of most of the quadrilaterals except squares. He used descriptive imprecise language in most of his explanations. The interviewer had to adapt the level of her language

accordingly and could only speak about the shapes with which Mila was familiar and the interviewer got used to the wrong terminology (language) that Mila was putting for each concept. This also seemed to confirm van Hiele's suggestion that each level has its 'own language' and people need to be speaking the same language in order to understand one another (1986). The interviewer thus encountered a similar problem to that experienced with Andiswa. Although Mila's use of the properties of shapes suggested level 1 thinking, his use of vocabulary indicated that he might still be in transition to this level.

Nana was classified as being on the analysis level (level 2) from the van Hiele Geometry Test. It was evident from the interview that Nana was operating at level 2 even though she did not draw triangles in different orientations. The interviewer could see Nana's confidence when she used a ruler to draw the triangles, to make sure that she drew what she meant. Concerning the concept of triangles, Nana had the language ability of a level 2 learner. At this level, language is important for describing shapes. It was also felt that Nana was aware of many characteristics and features of figures she drew and many other concepts that are linked to geometry learning that should be dominant to a grade 10 learner. It was felt to the interviewer as if they were speaking the same language. It was easy for the interviewer to adapt to the level of her language as Nana could speak about all the shapes which she was familiar with the correct terminology (language) that she had put for each concept. She used precise language in most of her explanations. The conversation between Nana and the researcher seemed to confirm van Hiele's suggestion that each level has its 'own language' and people need to be speaking the same language in order to understand one another (1986).

Earlier studies also noticed the same findings about language in different levels (e.g. Mayberry, 1983; Burger & Shaughnessy, 1986; De Villiers & Njisane, 1987; Fuys, et al., 1988; Senk, 1989; Genz, 2006).

Hierarchical characteristic of levels of thinking as evident from learners' interviews

"Thinking at the second level is not possible without that of the first level; thinking at the third level is not possible without thinking at the second level" (van Hiele, 1986, p. 51). The van Hiele theory is hierarchical in that a student cannot operate with understanding on one level without having been through the previous levels. Mayberry (1983), Senk (1989) and Pegg (1995) confirm that a student who has not attained level n may not understand thinking of level n +1 or higher. Therefore, as Hoffer (1981) suggested, for Andiswa to function adequately at one of the advanced levels in the van Hiele hierarchy, she must have mastered large portions of the lower levels.

When challenged with tasks set at level 2 thinking, Mila showed some transition towards level 2 thinking but definitely not higher than that. When asked whether a square can be a rectangle, he said it is not possible because 'a rectangle is made up of two sides that are equal and because a rectangle cannot have four sides equal, a square cannot be called a rectangle'. This showed that even though he could list some of the properties of squares and rectangles, he could not see that these were sub-classes of one another which were one of the characteristics of level 2 thinking. When asked questions from the van Hiele Test, he could answer only the one from level 1 and could not answer any of the other ones correctly. This also gives more evidence that Mila's thinking was that of more appropriate to level 1. This also provides evidence on the hierarchical nature of the van Hiele levels. Therefore, as mentioned in the case of Andiswa, for Mila also, to function adequately at one of the advanced levels in the van Hiele hierarchy, he must have mastered large portions of the lower levels.

The van Hiele theory is hierarchical in that a student can operate with understanding on one level if he/she has been through the previous levels. On the same grounds as Hoffer (1981) suggested, Nana could function adequately at one of the advanced levels in the van Hiele hierarchy as she had mastered large portions of the lower levels. When asked whether a square could be a rectangle, the answer given was 'yes' because 'opposite sides are equal and angles are 90^{0}'. A rectangle could be a parallelogram because 'opposite sides are equal and opposite sides are parallel'. Also a square could be a rhombus because 'all sides are equal all angles are equal'. This kind of thinking was that of level 3 thinking as she could give the relationships between figures or it could be assumed that class inclusion could be possible at level 2.

Findings from the interviews

Learner 1: Andiswa

The above interpretations of the van Hiele theory would suggest that Andiswa was operating in the pre-recognition level for most of the concepts in triangles and quadrilaterals. In certain concepts it looked like she was in transition from pre-recognition to recognition level.

In stressing the importance of language, van Hiele notes that many failures in teaching geometry result from a language barrier- "the teacher using the language of a higher level than is understood by students" (Fuys, et.al, 1988, p. 7). Andiswa's repeated mention of "quadrilateral triangle" also points out the fact that she might have learned (heard) it from her teacher. Atebe (2010) suggests that during teaching, teachers may use certain words in a sense peculiar to the subject that may not be precisely understood by the learners and the learners' proficiency in the teaching language is important for learning geometry specifically and mathematics in general.

Learner 2: Mila

The interpretations that were discussed of the van Hiele theory would suggest that Mila was operating in the recognition (visual) level for most of the concepts in quadrilaterals even though some key indicators such as visual prototype to charactarise shapes were not dominant in him. He could recognise a square by its form and a square seemed different to him than a rectangle. Even though he could identify, name, compare and operate geometric shapes such as triangle and square, he could not operate well on trapezium, rhombus and kite and he could not explicitly identify the properties of these shapes. He could operate on common shapes such as rectangle and parallelogram to a certain extent without explicitly regarding to properties of its components such as angles being 90^{0} or equal angles. He could sort and classify shapes based on some characteristics other than their appearances. In certain concepts it looked like he was in transition from recognition level to analysis level. It could be concluded that he had made some progress towards level 2 with familiar shapes such as triangles and squares but encountered difficulties with unfamiliar figures and that progress was marked by frequent instability and oscillation between levels.

It was also noticeable at this point that Mila's level of thinking was also not that expected at the secondary school level. As explained by other researchers like Teppo (1991) and Genz (2006), that systematic geometry instruction in the middle grades is necessary to prevent students from entering high school at low levels of geometric concept development.

Learner 3: Nana

From the discussions, it could be concluded that Nana's level of thinking was the thinking level that was expected of a learner at the secondary school level.

The interpretations that were discussed of the van Hiele theory would suggest that Nana was operating at the analytic level for most of the concepts in quadrilaterals. She could identify, name, compare and operate geometric shapes such as triangles and quadrilaterals and she could explicitly identify the properties of these shapes. She could sort and classify shapes based on characteristics other than their appearances.

In certain concepts it looked like she was in transition from analytic level to informal deductive level as she could identify class inclusion and gave abstract definitions to all the shapes that were presented.

Summary of findings

In general it was found that the interviews were useful in analysing learners' responses. It might not be possible to conclude that the features that emerged in the interviews were the result of the nature of the activities, the result of the particular interpretation used, or features of the van Hiele theory itself. Several features of the levels emerged during the course of the interviews.

The inability of Andiswa to identify common shapes, the misconceptions of Mila, the language of Nana were typical of levels 0, 1 and 2 thinking respectively.

Even though the learners were classified into discrete van Hiele levels as from the discussion of the interviews of these learners under levels of thinking, it has been noted that the levels are not discrete, more continuous than static, and the learners seem to move back and forth between levels and they were in different levels for different concepts. Occasionally, they seemed to be attaining a higher level than their predominant level. Previous research studies such as international studies like Usiskin (1982); Mayberry (1983); Burger and Shaughnessy (1986), Fuys et al. (1988), and Wu and Ma (2006) have also noticed the same behavioural patterns in their studies.

The interviews from this study also support the claim of Mayberry (1983) that high school learners do not perceive the properties of shapes. It also supports Burger and Shaughnessy (1986) who stated that a number of secondary school learners in their study were not sufficiently grounded in basic geometric concepts and relations. The interviews also support the claim of Burger (1985), that many learners rely on imprecise qualities to identify shapes (like, 'pointy triangles' and 'slanted squares').

The present study found that learners attempting definitions of concepts were influenced by their levels of understanding. Learners at level 1 gave visual definitions and learners at level 2 gave correct definitions. This is consistent with the study by Govender and De Villiers (2004). This study also found that many learners included irrelevant attributes when identifying and describing shapes, like orientation of the shape on the page (like 'turning a square to make a rhombus'). It was consistent with the studies of Burger (1985).

Findings from the study also suggest that language competency in general is a barrier to the attainment of higher levels of understanding. Coupled with the language feature of the van Hiele levels and as noticed in the learners, it can be assured that language is a barrier for learners who speak English as a second language. Setati and Barwell (2006) point out the use of the learners' main languages as a support whilst learners continue to develop proficiency in the language of learning and teaching at the same time as they learn mathematics.

According to the van Hiele theory people at different levels speak, use and understand geometrical terms differently. Wirszup (1976) noted that most of the

terms used by teachers can only be understood by learners who have progressed to the third or fourth van Hiele level. Therefore, in a class while the teacher is trying to explain, he or she might be completely misunderstood. This was particularly noted in the interviews with Andiswa and Mila. Thus it is very important that teachers investigate the levels of the learners to provide meaningful instruction in the classroom.

Pegg and Davey (1998) suggest that the descriptions of the levels are content specific and the levels are actually stages of cognitive development. Van Hiele (1986, p. 41) asserts that "the levels are situated not in the subject matter but in the thinking of man". Progression from one level to the next is not the result of maturation or natural development. It is not age dependent as the stages of Piaget. It was also evident from the interviews. Andiswa who was 18 years old, the oldest in the group which was mentioned here, was at the lowest level and even though Mila and Nana were of the same age (16 years), they were also not operating at the same level even though all these learners had been through the same subject matter.

Previous research studies such as international studies like Usiskin (1982), Mayberry (1983), Burger (1985), Burger and Shaughnessy (1986), Fuys et al. (1988), Renne (2004), Genz (2006), Özerem (2012) and South African Studies like Feza and Webb (2005) and King (2003) point out that many learners in the middle school have severe misconceptions concerning some important geometric ideas. In South Africa, studies like, De Villiers and Njisane (1987); Siyepu (2005) and Atebe (2008) indicate that high school learners in general and more especially, Grade 12 learners are functioning below the levels that are expected of them, i.e., they are at concrete and visual levels than at abstract level in geometry. This was noted predominantly at level 0 and level 1 thinkers in the interviews. De Villiers points out that this may be due to the fact that the transition from concrete to the abstract level of thinking poses "specific problem to second language speakers" and success in geometry involves the acquisition of the technical terminology (1987). It is essential that connections between relationships of mathematical concepts and terminology should be established.

According to de Villiers (2010), in traditional teaching, learners are introduced to rectangles, parallelograms and other geometric figures as static geometric objects – as an example, a rectangle might be introduced by comparison to a shape of a door or a static picture in a book, but the door or a picture in a book cannot be transformed into a square unless parts are cut off. Thus the concept of rectangle is completely disjoint from the concept of a square. This was very evident in the present interviews.

When given a set of quadrilaterals and when the learners were asked to mark all the parallelograms, they marked only the parallelogram, simply not knowing or realising the intention of the question that all special cases (e.g., rectangles, squares and rhombi) had to be marked as well. This is in line with the finding of Mayberry (1983) where only 3 out of the 19 pre-service mathematics teachers indicated squares are also rectangles. Findings from a study conducted by Duatepe-Paksu, Pakmak and Iymen (2012) on pre-service elementary teachers also showed that only 22 % of pre-service teachers mentioned more than necessary properties which characterise a square and a rhombus which showed that the pre-service teachers did not reach 2nd van Hiele geometric thinking level. According to Fujita (2012), for learners to reason successfully about whether a rhombus is a (special type of) parallelogram (i.e., there is an inclusion relation between parallelogram and rhombus), they need skills in manipulating its image in their minds and also to examine its properties both in a conceptual way and by grasping theorems associated with it. It was evident from this study that the learners involved in the study lack such skills.

All the aspects that are discussed for each learner are of importance to instruction as it is a big concern which affects the understanding of mathematics in general and geometry in particular. It also appeared that the learners in different schools involved in the study had varied exposure to geometric figures and their characteristics.

Just knowing the definition of a concept does not at all guarantee the understanding of the concept. De Villiers (2010), observed that although a student may have been taught and he is able to recite, the standard definition of a parallelogram as a quadrilateral with opposite sides parallel, the student may still not consider rectangle, square and rhombus as parallelograms since the learners' concept image of parallelogram is that not all angles or sides are allowed to be equal. The present research also observed the same in the interviews.

The language competency in general is a barrier to the attainment of higher levels of understanding. Atebe (2010) points out that learners' proficiency in the teaching language is important for learning mathematics generally and geometry specifically. Language is important for learning and thinking and that the ability to communicate mathematically is central to learning and teaching school mathematics (Setati, 2008). The present research is strengthened by these earlier studies which inferred the same conclusion through their studies.

According to the van Hiele theory, understanding of formal textbook definitions only develop at level 3, and that the direct provision of such definitions to learners at lower levels would be doomed to failure (De Villiers, 2010). On the other hand, if we think of the constructivist theory also, learners ought to be engaging in the activity of defining and be allowed to choose their own definitions. According to De Villiers (2010), the meaningful definition of a rectangle for a learner operating at level 1 can be called visual definition, where a rectangle is a figure which looks like this (draws or identifies a quadrilateral with all angles 90^0 and two long and two short sides) and that of a level 2 thinker is called uneconomical definition, where a rectangle is a quadrilateral with opposite sides parallel and equal, all angles 90^0, equal diagonals, two long sides and two short sides as that of a level 3 thinker, is called correct, economical definitions , where a rectangle is a quadrilateral with two axes of symmetry through opposite sides. The present interviews clearly substantiated the above kind of definitions.

It appears that class inclusion is difficult to accomplish with geometric figures. The learners' spontaneous definition at van Hiele levels 1 and 2 as shown above, would also tend to be in such a way that they would not allow the inclusion of squares among the rectangles, because rectangles have two long and two short sides. On the contrary, level 3 thinker will allow the inclusion of squares among the rectangles. This also validates De Villiers' (2010) claim.

The interviews with the learners at level 0, 1 and 2 had showed some common characteristics. These were confirmatory findings when compared with those of Burger and Shaughnessy (1986) and Clements and Battista (1992).

CONCLUSION AND RECOMMENDATIONS

In general, it was found that the interviews were useful in analysing learners' geometrical levels of thinking. It might not be possible to conclude that the features that emerged in the interviews were the result of the nature of the activities, the result of the particular interpretation used, or features of the van Hiele theory itself. It is felt, however, that a number of interesting features of the van Hiele theory and its interpretations have emerged, for example, the existence of pre-recognition level which is lower than level 1, the transition between levels, learners at different levels for different concepts of basic figures, and the importance of the linguistic property.

Progression from one level to the next is not the result of maturation or natural development. It also appeared that the learners in different schools involved in the study had varied exposure to geometric figures and their characteristics.

Triangle tasks were found to be easier and in the case of quadrilaterals, providing information for unfamiliar shapes such as rhombus, kite and trapezium was found be a problem for many learners.

 Learners' attempt on definitions of concepts were influenced by their levels of understanding. Coupled with the linguistic character of the van Hiele levels, it is concluded that the language competency in general is a barrier to the attainment of higher levels of understanding. The learners' competency in the teaching language is important for learning mathematics generally and geometry specifically.

All the aspects that were discussed for each learner are of importance to instruction as it is a big concern which affects the understanding of mathematics in general and geometry in particular. Teachers need to investigate the levels of the learners to provide meaningful geometry instruction.

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