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Intended mathematics curriculum in grade 1: A comparative study

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Abstract

Learning mathematics in grade 1 as the formal starting point for learning mathematics in many countries can significantly impact students' subsequent learnings. One of the most critical factors influencing teacher teaching and student learning is the written intended curriculum materials (official curricula). Despite the importance of this topic, there is little research on how many mathematics topics should be taught in grade 1 and to what depth students should learn these topics until the end of the first grade. In this study, we investigated and compared the grade 1 intended mathematics curriculum of Australia, Iran, Singapore, the Province of Ontario in Canada, and New York State in the USA. Indeed, we sought to examine how curriculum developers and decision-makers in education in these jurisdictions prepared the content of the first-grade mathematics in the curriculum writing materials. To do this, by examining the official curricula for grade 1 of these countries and using a procedure called general topic trace mapping, we found a list of 14 topics. The findings of the current paper showed similarities and differences in the topics intended in the mathematics curriculum of these countries. Ontario, Australia, Singapore, New York, and Iran curricula cover 13, 11, 9, 9, and 7 topics of 14 topics, respectively. We also considered five content strands and examined and compared the progress of each intended curriculum in these strands at the end of grade 1. We found that the learning progression in some content strands is different among countries. The results demonstrate the nuanced complexity of these comparisons and the importance of cross-national comparisons. We concluded this article with suggestions for curriculum developers, textbook writers, and teachers.

Keywords: intended mathematics curriculum, grade 1, focus, topic coverage, learning progression

INTRODUCTION

In many countries, children start to learn mathematics formally through school and in grade 1. Preparing students for this formal start in mathematics is essential in curriculum studies (Díaz et al., 2021; Tanase & Wang, 2013). Indeed, consideration of the number of mathematical topics in different grades and the depth of progress of each topic in those grades are questions that have occupied many researchers (Hirsch & Reys, 2009; Johnson et al., 2013; Wang & McDougall, 2018). Teachers usually use mathematics textbooks for their teaching (Van Steenbrugge et al., 2013). Mathematics textbooks are developed according to the official curricula (Nathan et al., 2002; Schmidt & Houang, 2012; Van Steenbrugge et al., 2013). Therefore, mathematics curriculum writing materials will potentially impact student learning of mathematics.

Many processes and variables affect mathematics learning outcomes in grade 1, such as teaching approaches, teacher characteristics, student characteristics, class climate, etc. In this context, the curriculum writing materials and mathematics textbooks related to the intended curriculum play a key role in learning and teaching processes that affect learning outcomes. Some researchers argued that when the intended curriculum is different, it will affect students' knowledge and cognition and lead to different learning achievements in students (Bieda et al., 2020; Breda et al., 2021; Wang & McDougall, 2018).

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Contribution to the literature

- This study extends the findings on existing literature of intended mathematics curriculum in the first-grade.
- Exploring the ways in which policymakers in Australia, Iran, Singapore, the province of Ontario in Canada and New York State in the USA put weight on different topics in first-grade mathematics.
- This study explores the important factors influencing the intended mathematics curriculum in the first-grade.

Much research has been done on first-grade students' understanding of mathematics concepts in curriculum syllabuses for this grade (Kamii et al., 2005; Rasberry, 1995; Tanase & Wang, 2013). However, there is little research on how many mathematics topics should be taught in grade 1 and to what depth these topics should be learned by school students until the end of the first grade (Wang & Dougall, 2018).

Since 1995 the third international mathematics and sciences study (TIMSS) results in the mathematics section have enabled researchers to find top-performing countries helping them to estimate similarities or dissimilarities in the mathematics curriculum between top-ranked countries and countries with low performance. One general answer to research studies about TIMSS and program for international student assessment (PISA) is to reproach teachers. However, teachers in all countries are performing what their curriculum developers, policymakers, and decision-makers have inquired of them to carry out. Studying TIMSS achievement outcomes has aroused policymakers in all countries to ponder merely what it would mean to have a world-class science or mathematics curriculum.

In most countries, there is a high public interest in cross-national comparisons. Despite the countless problem intrinsic in these analogies, news media envoys, policymakers, politicians, and instructors generally take the conclusions at face value as accommodating rigid theoretical attestation of students' attainment in the countries under study. This frequently conduces in turn to cursory exegesis and unhelpful commendation. Our role as scholars of mathematics curriculum and education is to indicate the nuanced intricacy of such analogies. The outcomes of international analogies are often caught at face value by tutors, statesmen, policymakers, and news media envoys as equipping accurate theoretical witnesses on students' success in the countries under study. However, the objectives and practices of such users of international analogies usually remain on simplistic conjectures about the communication between the several interactive essential components of the educational setting in a country: teaching practices in schools, textbooks by instructors, teacher arrangement, and growing abutment, and student understanding conclusions. In particular, distinctions in the contextual situations of the mathematics curriculum in different countries are barely constructed explicit in comparative investigations.

Some research has suggested that a notable factor pertained to the low efficiency of some countries at international examinations like PISA and TIMSS is the nature of the curriculum (in terms of content coverage) both in its intended and enacted forms. Both forms of the curriculum are samples of the more significant notion of opportunities for students to learn that studies have shown to be related to academic achievement (Bieda et al., 2020; Kouropatov & Dreyfus, 2013; Sahin, 2009; Schmidt & Maier, 2009). Indeed, in terms of the curriculum matters, what we teach is what we get. Schmidt and Houang (2012) reported 1995 TIMSS and the common core state standards in mathematics. They found 'whole number meaning', 'whole number operation' and 'measurement units' as three topics intended in the grade 1 curriculum by two-thirds or more of the top-achieving nations in the 1995 TIMSS. According to their results, common core standards considered eight topics in the grade 1 curriculum, which were: whole number meaning, whole number operations, properties of whole numbers operations, fractions, measurement units, polygons and circles, data representation and analysis, and 3D geometry. In a similar study, Wang and McDougall (2018) studied and compared the intended mathematics curricula in grades 1-11 within China and the Province of Ontario (Canada). Based on their results, Ontario covered 13 topics for grade 1 while China covered only five topics. Both curricula introduced whole number meaning, whole number operations, measurement units, data representation and analysis, and the basics of 2D geometry in grade 1. Furthermore, Ontario covers eight topics more than China in grade 1, including common fractions and operations, polygons and circles, estimating computations, estimating quantity and size, measurement estimation and errors, creating and building 2D & 3D shapes with concrete objects, 3D geometry, and patterning and algebra.

By following the previous studies in the mathematics curriculum, we focused only on grade 1 in order to get more insights into the beginning. Our goal in this study is to employ the grade 1 intended mathematics curricula from Singapore, Australia, Iran, New York State in the USA, and the Canadian Province of Ontario to study the differences and similarities of the content of this grade in terms of focus in topic coverage and learning progression in content strands. By learning progression, we mean expectations associated with strands that each curriculum has from students to learn by the end of the grades.

The rank of these countries in the mathematics section of grade 4 in TIMSS 2015 among 57 countries was: Singapore as number 1, the USA as number 14, Australia as number 28, Canada as number 29, and Iran as number 42. The rank of Ontario State in TIMSS 2015 was 30 (Brochu et al., 2017; Provasnik et al., 2016). In TIMSS 2019, their rank in the mathematics section of grade 4 among 58 countries was as follows: Singapore in rank 1, the USA in rank 15, Australia in rank 27, Canada in rank 32, and Iran in rank 50. The rank of Ontario in TIMSS 2019 was 18. The results of the US states are not reported separately (Mullis et al., 2021). We chose the countries:

(1) based on their performance in TIMSS,

(2) possibility to analyze their official curricula, and

(3) the language of the official curricula of the countries to be analyzable for the authors.

Indeed, there is a diversity in the ranking of these countries from low to high performing in the last two exams of TIMSS (2015 and 2019). We had access to these countries' official mathematics curricula, and the official language of the four curricula was English.

The research questions of this study were, as follows:

1. How is the intended mathematics curriculum of grade 1 in Singapore, Australia, Iran, New York state of the USA, and the province of Ontario in terms of focus and learning progression in strands?
2. What are the differences and similarities in the intended mathematics curriculum between these jurisdictions?

INTENDED MATHEMATICS CURRICULUM

In education, the term "curriculum" is broadly delineated as the sequence of learning opportunities or experiences that take place in the instructional process and is incarnated in written curriculum documents and materials. The word usually refers to a designed sequence of guidance and instruction or insight into students' experiences regarding the instructors' or schools' educational aims. Designing written curriculum materials involves choices, selections, arrangements, organizations, and using learning experiences. Tyler (1949) considered five generic principles for the choice of learning experiences. The first one says that for a given goal to be achieved, students should have experiences that allow them to work out the type of behavior comprised the goal. The second foundation is that the learning experience should be to gain acquiescence from keeping the type of behavior comprised the goal. The third principle is the feedbacks pertinent experience inside the area of possibility for the participation of students. The fourth is that numerous specific experiences and practices can be utilized for achieving the same instructional and educational goals. The fifth principle says that the same learning practice and experience will conduct various consequences.

Four levels of the classification of the curriculum are: Standards or official curriculum, textbooks, the content implemented by instructors, and learning of students (Hirsch & Reys, 2009; Schmidt et al., 2001; Van Steenbrugge et al., 2013). Since textbooks illustrate specific educational purposes for a particular frame of students and are utilized by instructors and teachers, they can demonstrate the intended curriculum, which contains the textbook and official curriculum (Nathan et al., 2002; Schmidt et al., 2001). Researchers have considered three forms of curriculum: intended, enacted (implemented), and assessed (attained) (Cai, 2014; Husen, 1967; Schmidt et al., 2001).

The intended curriculum is considered the documents written formally and explains learning expectations for each grade. It consists of expectations and goals with curriculum standards and official syllabi, and officially accepted textbooks in some countries. Scholars usually use specific concepts to study the intended curriculum: focus and learning progression in strands. The number of topics intended for each grade is the focus index. Learning progression in strands shows expectations that the curriculum has from students to learn specified contents by the end of each grade. The enacted (implemented) curriculum is defined as the instructional and educational practice instructors, and teachers utilize in the class. The enacted curriculum also includes teachers' materials, teachers' experiences, beliefs, and values about teaching. The assessed (attained) curriculum refers to what students learn and show in their attainments, attitudes, and achievements (Hirsch & Reys, 2009).

The relationships between these three curricula (intended, enacted, and assessed) are shown in **Figure 1** (Wang & McDougall, 2018). The dashed lines represent the definitional connections between curriculum zones, and the solid line arrows represent the possible observational connections between the components (Bieda et al., 2020; Schimidt et al., 2001). **Figure 1** shows that the intended curriculum has a foundational role in teacher teaching and student learning. The official curriculum supplies direct and attributive descriptions of students' performance and content levels (Schmidt et al., 2001; Wang & McDougall, 2018).

It gives direction, guidance, and instruction to curriculum developers, teachers, and textbooks writers associated with what content should be taught and when the process and content should be devoted to students

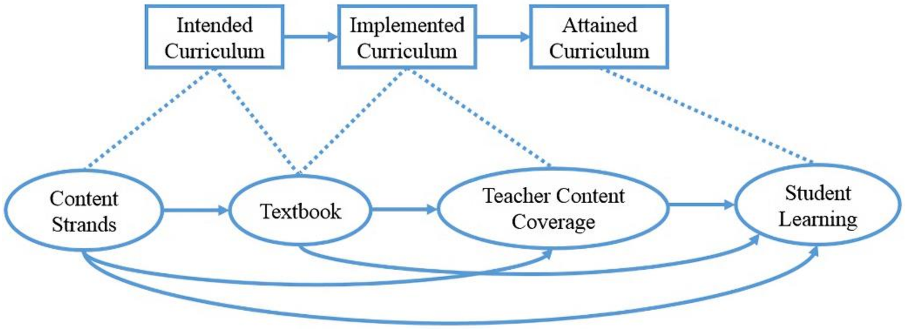

Figure 1. Relation between three types of curriculum (Schmidt et al., 2002)

and conducting the appraisal and evaluation developed for students' learning (Hirsch & Reys, 2009; Sahin, 2009; Schmidt et al., 2001, 2002).

Textbooks are like a bridge between teacher content coverage and the official curriculum and contain content for learning. Students usually learn mathematics according to learning experiences and opportunities created by teachers who execute and use textbooks. A growing body of research literature shows that instructors use textbooks (Nathan et al., 2002; Schmidt et al., 2002). Indeed, teachers use and dress the content designed in the textbook and track the progress, length and continuity of topics in the textbook (Nathan et al., 2002; Schmidt et al., 2002; Van Steenbrugge et al., 2013). Accordingly, student achievement could be investigated in the assigned topics within the characteristic of the intended content. Scholars studying the intended curriculum usually consider two items: official curricula and textbooks (Schmidt & Houang, 2012). The comparison of official curricula has been examined in some research works (Schmidt et al., 2002; Schmidt & Houang, 2012). Schmidt et al. (2002) and Schmidt and Houang (2012) compared curriculum standards for examining the intended curriculum between high-performing countries in TIMSS exams and the US common core state standards. They concentrated on what mathematics is highlighted, the level of cognitive request, and when and how specific topics are presented. In Schmidt et al. (2002), authors studied TIMSS 1995 and identified the six top-performing countries in mathematics. After examining in detail, the constitution of the curriculum standards of these top-performing countries, Schmidt and his colleagues created a curriculum model named A+ composite. Their framework contained 32 core topics in grades 1–8 mathematics. These 32 core topics are used by at least two-thirds of the top-performing countries in their curriculum standards.

METHOD

The data for this research was chosen from the Singapore mathematics syllabus primary one (Ministry of Education, Singapore, 2019), the Ontario mathematics curriculum for grade 1 (Ontario Ministry of Education, 2005), sequence of mathematics content in the Australian Curriculum for year 1 (Australian Curriculum, Assessment, and Reporting Authority, 2017), New York mathematics curriculum for grade 1 (New York Common Core Curriculum, 2014), and Iranian mathematics curriculum (Ministry of Education of Iran, 2013). As the Iranian curriculum does not represent detailed mathematics demands for each grade, the data sources also contained the mathematics textbook for grade 1 published by Organization for Educational Research and Planning (2016).

Examining the above materials using the procedure of the general topic trace mapping (GTTM) (Schmidt et al. 1997b), the model of A+ composite (for grade 1) extended by Schmidt et al. (2002), and also research done by Wang and McDougal (2018), we found a list of 14 topics that include all of the mathematics topics of grade 1. Based on GTTM procedure, one needs to divide the content standards into small parts called blocks. After determining the blocks, the actual instructional material in each block is described using sections from the given curriculum framework, i.e. coders described each block's content in terms of the topic(s) contained. More complex standards can be determined with more than one topic as appropriate. We consider the case of Ontario to show, using an example, how we reduced the data to find the final framework. The Ontario curriculum expects students to "identify common two-dimensional shapes and three-dimensional figures and sort and classify them by their attributes" (Ontario Ministry of Education, 2005, p. 37). Based on the five strands, we considered this piece of the Ontario curriculum in the category of geometry and spatial sense. In our negotiation, we extracted 2D and 3D shapes as the two concepts for grade 1 derived from this part of the official curriculum.

The topics from Singapore, Australia, Iran, New York, and Ontario intended mathematics curriculum were then coded and reconciled with the 14 topics of this study. We investigated how 14 topics were organized in the grade 1 mathematics curricula within these countries. To match the A+ composite pattern used by Schmidt et al. (2002) and to compose obvious pictures of the topic structure, a two-dimensional table was made for coding topics and their distribution. We put topics in

Table 1. Topic coverage of grade 1 in Singapore, Australia, Iran, Ontario, and New York

Topic	Singapore	Australia	New York	Iran	Ontario	Frequency
Whole numbers meaning	*	*	*	*	*	5
Adding & subtracting whole numbers	*	*	*	*	*	5
Multiplying & dividing whole number	*					1
Fractions (only name)		*	*		*	3
Money & financial mathematics	*	*	*		*	4
Length measurements	*	*	*	*	*	5
Area (non-standard units)					*	1
Patterns	*	*		*	*	4
Time (clock)	*	*	*	*	*	5
2D shapes	*	*	*	*	*	5
3D shapes		*	*		*	3
Location, movement, & transformation		*			*	2
Data representation & interpretation	*	*	*		*	4
Symmetry				*	*	2
14 topics	9	11	9	7	13	

the first column and countries and provinces in the first row of the table. If a topic is intended in grade 1 of a country, a "*" is placed at the intersection of the topic column and the row of the country. We also added a last column to the designed table that shows the number of countries covering the considered topic. The table was utilized to analyze and compare the topic coverage of grade 1 in the countries as they are reflected in the internal rational system of mathematics (Schmidt et al., 2002). Data analyses were applied on how the five intended curricula construct the grade 1 mathematics topics and content in terms of learning progression and focus of the topics.

Each researcher separately used GTTM procedure to find all the topics intended in the curricula for grade 1 of the jurisdictions. We then discussed our findings, and the differences were negotiated to grasp an entire agreement. We ended up with 14 topics presented in the paper. Furthermore, two professors of the mathematics curriculum, who were experts in the curricula of the elementary grades, validated the data and the findings. These professors were thoroughly familiar with the official mathematics curricula of these countries.

FINDINGS

The focus and learning progression in strands for each curriculum will be discussed in the following sections.

Focus Index in the Intended Curriculum

The mathematics topic coverage in grade 1 designated in Singapore, Australia, Iran, Ontario, and New York are listed in **Table 1**. The total number of topics is 14. The number of topics in grade 1 for each curriculum is as follows: Ontario 13 topics, Australia 11 topics, Singapore nine topics, New York nine topics, and Iran 7 topics.

As shown in **Table 1**, there are five common topics in all the five curricula: whole numbers meaning, adding and subtracting whole numbers, length measurements, time, and 2D shapes. Money and financial mathematics and data representation and interpretation are covered in the four curricula except for Iran. The concept of patterns is covered in the four curricula except for New York. Fractions and 3D shapes are covered in the three curricula except for New York and Iran. Symmetry is covered in Iran and Ontario, while the content related to location, movement, and transformation is covered in Australia and Ontario. Multiplying and dividing whole numbers is covered only in the Singapore curriculum, and the concept of area in non-standard units is covered only in Ontario curriculum.

Learning Progression in Strands

The Singapore curriculum organizes the mathematics syllabus into three content strands: number and algebra, measurement and geometry, and statistics. The Australian curriculum classifies mathematics content into three strands: number and algebra, measurement and geometry, statistics and probability. The curriculum of Iran considers three content strands: number and algebra, geometry and measurement, and statistics. The Ontario curriculum classifies mathematics content into five strands: number sense and numeration, measurement, geometry and spatial sense, patterning and algebra, and data management and probability. New York curriculum organizes mathematics content of grade 1 into four strands: operation and algebraic thinking, number and operations, measurement and data, and geometry.

By examining the content strands of the mathematics curricula, we defined five content strands: number sense and numeration, measurement, geometry and spatial sense, patterning and algebra, and data management and probability. In the following subsections, we examine and compare each curriculum's expectations for each content strand by the end of grade 1. In this paper, we will not study how each of the curricula considered the trajectory of constructing each particular concept in the written curriculum materials.

Number Sense and Numeration

Singaporean students in grade 1 learn numbers up to 100, adding and subtracting within 100, multiplying within 40, and dividing within 20. They practice counting the amount of money in cents up to $1, in dollars up to $100. They solve 1-step word problems involving the adding and subtracting of money in dollars only (or in cents only).

Australian curriculum expects grade one students to establish confidence with number sequences to and from 100, represent and solve simple addition and subtraction. Regarding the concept of fractions, students recognize and describe one-half as one of two equal parts of a whole. They also recognize, describe and order Australian coins according to their value.

All Iranian students in the same grades have the same textbooks nationwide designed by Organization for Educational Research and Planning (2016). The official mathematics textbook for grade 1 considers numbers up to 100 and adding and subtracting whole numbers within 20.

Grade 1 students in New York count to 120, add and subtract within 20. They work with part-whole relationships within composite shapes like halves and quarters of rectangles and circles to get acquainted with the fraction concepts. Students use halves to tell time.

Ontario mathematics curriculum expects students in grade 1 to indicate the learning of magnitude by counting forward to 100. It also expects students to solve various problems containing the adding and subtracting of whole numbers to 20, divide whole objects into small parts, and describe and identify, through investigation, equal-sized sections of the whole, using fractional names (e.g., halves; fourths or quarters).

While Australia, New York, and Ontario curricula start fundamental understanding of fractions in grade 1, Singapore and Iran shift it to upper grades. Iran does not involve practicing with money in grade 1.

Measurement

For grade 1, the Singapore curriculum expects students to measure length using various non-standard units and tell time to the hour/half hour.

Australian curriculum expects students in grade 1 to measure and compare the lengths and capacities of pairs of objects using uniform informal units, tell time to the half-hour, and describe duration using months, weeks, days, and hours.

Based on the mathematics curriculum of Iran, Iranian students in grade 1 practice to measure the length of objects using informal units, telling the time in the exact clock hours, or simply just less than or just more than an exact hour without using half, quarter, etc.

In New York, students in grade 1 convey the length of an object as a whole number of length units and write and tell the time in hours and half-hours with digital and analog clocks.

The Ontario curriculum expects students to appraise, measure, and characterize time, length, area, capacity, mass, and temperature, with non-standard units of the same size. Grade 1 students in Ontario write and tell time to the hour and half-hour in everyday experiences. They name the months of the year in order and read the date on a calendar.

As a comparison, Ontario covers more quantities for measuring, such as area, mass, temperature, and capacity. This is while the other countries may consider these quantities in their science curriculum.

Geometry and Spatial Sense

Based on the Singapore curriculum, students in grade 1 identify, name, describe, and classify 2D forms and shapes such as squares, triangles, rectangles, and circles.

Australian curriculum expects grade 1 students to recognize and classify familiar 2D shapes and 3D objects using apparent features, present and follow directions to familiar locations.

The official mathematics textbook for grade 1 covered identifying, naming, describing, and classifying 2D shapes. Students work to draw horizontal and vertical symmetry for 2D shapes.

Based on the Iranian official mathematics textbook for grade 1 (Organization for Educational Research and Planning, 2016), students name and describe 2D shapes (i.e., triangle, square, rectangle, and circle). They also learn about symmetry and draw horizontal and vertical symmetry lines for some polygons.

The New York mathematics curriculum for grade 1 expects students to recognize defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., overall size, orientation, and color) and create and draw shapes to possess defining attributes. Students compose 2D and 3D shapes to create a composite shape and compose new shapes from the composite shape.

Based on the Ontario curriculum, grade 1 students identify common 2D shapes and 3D shapes. They place shapes in a setting that has symmetry and then describe the symmetry. They find the relative locations of people and materials with positional language (e.g., under, over, etc.), and explain the relative locations of objects on concrete maps.

Although we can see similarities in the strand of geometry and spatial sense intended in the curriculum of these countries, there are also variations and differences. Some of them expect their students to work only with 2D shapes. In contrast, some others also consider practicing with simple 3D shapes in grade 1. The concepts related to location and position are not involved in the grade 1 curriculum of some of these countries.

Patterning and Algebra

The mathematics curriculum of Singapore for grade 1 allows students to work with patterns in number sequences. Students characterize a considered number sequence using language such as ‘1 more/less’ or ‘10 more/less’ before continuing the sequence and exploring the missing number and follow patterns with 2D figures using one or two of characteristics like shape, size, orientation, and color. Regarding early algebra, students work in groups of three or four students to make addition and subtraction stories with concrete figures and describe an addition or subtraction equation for a given story; compare two numbers within 20 to understand how much one number is greater (or smaller) than the other by subtraction.

Australian mathematics curriculum expects grade 1 students to investigate and explain number patterns resulting from skip counting by 2s, 5s, and 10s. Students continue simple sequences involving numbers and objects and represent and solve easy addition and subtraction questions using various strategies, including counting on, partitioning, and rearranging sections.

The official mathematics textbook for grade 1 in Iran covers geometric and numeric repeating patterns. Students work to create adding and subtracting story problems using concrete figures, describe an addition or subtraction equation for a given context, and compare two numbers within 20.

Mathematics curriculum in New York expects students to utilize addition and subtraction within 20 to solve story questions including situations of putting together, adding to, taking from, taking apart, and comparing, with unknowns in all situations, e.g., by using figures and shapes, drawings, and equations with a symbol for the unknown number to represent the question. It also expects students to learn the meaning of the equal sign, recognize if equations involving addition and subtraction are true or false, and determine the unknown whole number in an addition or subtraction equation relating three whole numbers.

Based on the Ontario mathematics curriculum, grade 1 students identify, describe, extend, and create numeric and geometric repeating patterns represented in various ways (e.g., pictures, actions, colors, sounds, numbers, and letters). The curriculum expects grade 1 students to demonstrate an understanding of the concept of equality, using concrete materials and addition and subtraction to 10, and determine, through investigation using a “balance” model and whole numbers to 10, the number of identical objects that must be added or subtracted to establish equality.

It seems all curricula intend some concepts of early algebra using numbers and concrete objects. However, the way of implementing the content might be different between countries.

Data Management and Probability

Based on the Singapore mathematics curriculum, students in grade 1 collect data and utilize the data to create a figural drawing for representation. Students read and interpret data from figural drawings using language such as ‘least’, ‘most’, ‘smallest’, ‘greatest’, ‘as many as’, and ‘as much as’. They draw picture graphs in both horizontal and vertical shapes and create a story using information from a drawing.

The Australian mathematics curriculum expects grade 1 students to choose simple questions, gather responses, and make simple inferences. Indeed, students collect data by asking questions, draw simple data representations and make simple inferences. They show data with materials and drawings, where one material or drawing represents one data value. Grade 1 students identify results of familiar events involving chance and explain them using everyday language such as ‘will happen’, ‘will not happen’, or ‘might happen’.

The New York mathematics curriculum expects grade 1 students to represent, organize, and interpret data with up to three groups. By the end of grade 1, students explain the likelihood that everyday events occur using mathematical language (i.e., impossible, unlikely, less likely, more likely, certain).

Based on the Ontario mathematics curriculum, grade 1 students organize and collect categorical data and represent the data using concrete drawings, without regard to the order of labels on the horizontal axis. Regarding probability, students explain the likelihood that everyday events will happen.

The Iranian mathematics curriculum does not consider any topics related to data management and probability in grade 1. Except for Iran, the other curricula implement contents associated with elementary concepts of data collection and chance in the mathematics curriculum of grade 1.

DISCUSSIONS AND CONCLUSIONS

Considering the importance of written intended curriculum materials, we, in this study, contribute to those parts of mathematics education that can affect directly or indirectly the process of teaching and learning. We studied the grade 1 intended mathematics curriculum in five jurisdictions: Singapore, Australia, Iran, the Province of Ontario in Canada, and New York State in the USA. The results will illustrate five different curriculum models in the five educational jurisdictions and make the setting of the subject visible. The heart of this paper is the investigation of the curriculums for grade 1. To answer the research questions of this study, we consider both focus and learning progression. The results showed that although five curriculum models require instructors to teach a similar group of core mathematics topics (whole numbers meaning, addition and subtraction whole numbers, length measurements, time, and 2D shapes, which are five of the total 14 topics) in grade 1, there are differences in other topics. The other nine topics are distributed unevenly in the curricula. Ontario has the highest number of topics (13), and Iran has the lowest with seven topics. Australia considered 11 topics while Singapore and New York intended the mediocrity of the number of topics (nine). Some topics are implemented only in one or two countries, such as symmetry (in Iran and Ontario), the concepts related to location, movement, and transformation (in Australia and Ontario), multiplying and dividing whole numbers (in Singapore), and area in non-standard units (in Ontario).

Our results also showed similarities and differences in the progression in the five content strands (i.e., number sense and numeration, measurement, geometry and spatial sense, patterning and algebra, and data management and probability) among the five curricula in grade 1. There were different types of quantities that each curriculum expects its students to familiarize themselves with and measure with formal or informal units. However, measuring some quantities like mass and temperature has been considered in the science curriculum in some countries (e.g., Iran). Regarding number sense and numeration, all curricula considered learning whole numbers (usually up to 100) and operation (usually within 20). Iran and Singapore start the concept of fractions in grade 2, and the other three countries take the fundamental understanding of this concept using concrete objects into account in their grade 1 curriculum. In the strand of geometry and spatial sense, 2D shapes, 3D shapes, location, and symmetry were intended in the curricula. It seems the progression in patterning and algebra strand is somewhat the same among countries. They consider patterns using numbers, objects, and colors in sequences. Working with straightforward addition or subtraction equations is also intended in this strand. Iran does not include data representation and probability in its grade 1 curriculum. The other four curricula in grade 1 expect students to work with data collection and representation and elementary concepts related to chance.

This comparison research shows the advantage and importance of studying the number of topics in grade 1 for effective learning and teaching. If teachers in grade 1 have to teach many topics, they have to move between topics at a fast speed. This may cause students and teachers to not remain on one topic long enough to construct the content more profound and consequently have a conceptual understanding. On the other hand, if teachers in grade 1 teach an average quantity of topics, they can remain on topics longer and teach topics deeply. When teachers in grade 1 start to teach a new topic, they should help children recall their real-life experiences providing rational opportunities for children to create coherent connections between new formal content and their experienced and professional skills in real life (Rasberry, 1995; Tanase & Wang, 2013).

In the highest achieving countries at international examinations, like TIMSS and PISA, there are logical and consistent principles and guidelines for teachers in the content of a national curriculum (Schmidt & Houang, 2012; Wangn & McDougall, 2018). They also profit from related and suitable tools and training and educating teachers' guidelines, workbooks and textbooks, and teacher training education–that make them ready to teach the curriculum contents and create opportunities for curriculum-based professional progression. "Market-driven textbooks containing something for anyone but very few training, tools, and guidance" and "long lists and contents of ideas about what should be taught" may cause low performance for those countries who follow this approach in their educational system. Researchers in countries with low performance should be concerned if their instructors and teachers must work a little harder to comprehend what they will teach (Bieda et al., 2020). Literature research shows that teachers and students in low-performing countries are significantly harmed by their country's absence of a coherent, and standard curriculum and the materials, texts, training, and instruction that complement it (Hirsch & Reys, 2009). Some research studies somehow address the number of topics in different grades and in different countries and their relation to speed and the learning progression of the topics. For example, in a similar study, Wang and McDougall (2018) studied and compared the intended mathematics curricula in grades 1-11 within the Province of Ontario and China. Their results showed that the detailed topic framework of the two curricula differed markedly. The curriculum in China included a few topics in each year, a short continuation or spread of each topic, and a fast-paced topic progression. The Ontario curriculum, in contrast, comprises more topics each year, a longer duration of many topics, and a small pace of topic progression in grades 1-8 and a fast pace of topic progression in grades 9-12. Due to different curriculum designs in grades 1-12, the intended curriculum may affect teachers' professional development, students' cognitive constructions of mathematics, learning behavior and thinking, and learning efficiency and achievement.

The naïve idea is that some people believe the goal of an international study is to find which country has a high performance and then invite other countries to imitate its educational system and curriculum. Hence, we should consider cultural differences. So, for example, lifting something from one cultural context and expecting it to work elsewhere does not seem to be a rational solution. However, international comparison helps curriculum developers criticize and challenge some of their shared presumptions of education and curriculum and think about other possibilities for what they are performing.

Students' opportunities in learning should be improved by the advantages that go along with a common and coherent curriculum (Paterson & Sneddon, 2011). Furthermore, teachers and instructors should work together with a common language and common goals. It is also important that new teachers acquire clear and understandable directions about what they need to teach. In general, professional curriculum development can be anchored in the educational system that teachers teach. The research findings highlight that textbooks should be more focused so that more in-depth coverage can be provided with a lesser set of topics (Nathan et al., 2002; Van Steenbrugge et al., 2013). Having a coherent and common curriculum can help transient teachers and students to adapt more quickly to new schools. All of these cases contribute to more excellent stability and quality in schools and thus better learning.

As we said before, it is seldom possible to import other nations' practices and their educational systems as whole-cloth. Countries are culturally and institutionally different. However, we can learn from other countries' curricula and educational systems. Indeed, we should find ways and methods to adapt effective practices from other countries to our use. However, nations should move in different directions and areas to create a more coherent curriculum and transfer its benefits to their students, teachers, and schools who merit no less than the quality of education experienced by students in high-performing countries at international examinations.

If instructors in different countries present mathematical concepts in early grades differently, students may create different mental constructions that affect their learning process (Farsani et al., 2020). In addition, different curricula conduct instructors in different paths in their professional development because they need to help children understand content based on children's previous knowledge.

We consider this comparison study necessary. The knowledge base developed through research on mathematics education and mathematics curriculum should be the basis for developing a research-based designing curriculum. We encourage mathematics education and curriculum researchers to think and further study defined lists of topics and content progressions in each topic for different grades to refine their curriculum to increase teacher teaching and student learning opportunities.

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