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Intertwining Lexical and Conceptual Learning Trajectories - A Design Research Study on Dual Macro-Scaffolding towards Percentages

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Monolingual or multilingual students with low academic language proficiency need to acquire conceptual understanding for percentages and the language to communicate about them. The design research study explores how these two learning goals can be fostered by a macro-scaffolding approach for seventh grade students. The dual hypothetical learning trajectory starts from students' prior experiences and guides them, via several levels, towards the mathematical models and meanings (conceptual learning trajectory) as well as towards the necessary vocabulary for thinking and communicating about them (lexical learning trajectory), linked by the percent bar. For lexical scaffolding, the processes of offering, building and reflecting lexical means on different levels are focused. The qualitative empirical investigations of the design experiments show in depth how students' conceptual understanding evolved together with the lexical means. For this purpose, the interpretative reconstruction of semiotic chains is instrumental.

Keywords: percentages, design research, macro scaffolding, language learners, conceptual scaffolding

INTRODUCTION

Many students and adults have difficulties to communicate about percentages adequately. This general finding concerns two facets, a conceptual and a linguistic facet: Several empirical studies in mathematics education research report on *limited conceptual understanding* of percentages (Hafner, 2012; Kouba et al., 1988; Lamon, 2007; Parker & Leinhardt, 1995). Additionally, some studies give evidence for the lack of necessary lexical means for reading, speaking or writing about percentages which can be traced back to linguistic difficulties (Parker & Leinhardt, 1995; Pöhler,

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Prediger, & Weinert, 2015). Especially for monolingual or multilingual students with low language proficiency, the learning arrangements should hence offer learning opportunities for both, conceptual learning and lexical learning (Prediger & Wessel, 2013). This aspect deserves increasing attention in a world of increasing migration and multilingualism (Barwell et al., 2015). That is why this article focuses on intertwining lexical and conceptual learning trajectories.

For intertwining the learning trajectories, the design is based on the instructional approach of macro-scaffolding. Scaffolding in general is increasingly discussed as an important approach for supporting students' learning pathway in mathematics classrooms (Smit, van Eerde, & Bakker, 2013; Gibbons, 2002). Whereas micro-scaffolding in interactions has attracted the attention of many mathematics education researchers (Smit, Bakker, & Wegerif, 2016 for a research overview), too few empirical insights exist about effects and conditions of macro-scaffolding, i.e. to those aspects of scaffolding that require the longer term consideration already in the design of the learning arrangements.

The design research study presented in this article promotes a macro-scaffolding approach to percentages by intertwining conceptual and lexical learning trajectories, two key terms which require to be defined in detail as a major theoretical contribution. A possible design realization, its effects and conditions of success are investigated in iterative design experiment cycles. By this, the study intends to contribute to instructional theories for mathematics learning for students with low academic language proficiency.

The article first discusses the theoretical background of this duality (Section 1: Duality of lexical and conceptual scaffolding) and the mathematical topic percentages (Section 2: Learning trajectories towards percentages.). After sketching the methodology of the design research study (Section 3: Methodological framework of the design research study), its design outcome is presented, namely the learning arrangement on percentages (Section 4: Design outcome: A dual learning trajectory for fostering low achievers' pathways to percentages) and selected research outcomes of the qualitative investigation of students' learning pathways (Section 5: Research outcome: Empirical insights into students' pathways along the dual trajectory).

State of the literature

- Empirical studies show obstacles for monolingual and multilingual students with low language proficiency while acquiring conceptual understanding for mathematical topics. Thus, many authors claim the necessity of integrating language learning and mathematics learning.
- Although some instructional approaches exist for integrating language and mathematics learning, e.g. the macro-scaffolding approach, there exist very limited empirical insights into their functioning for acquiring conceptual understanding for specific mathematical topics.
- For the chosen mathematical topic of percentages, a well-established conceptual learning trajectory has been developed for approaching percentages with understanding. But so far, the language support was not systematically integrated.

Contribution of this paper to the literature

- The paper presents a design research study on integrating topic-specific learning and language learning in a dual macro-scaffolding approach.
- The practical outcome is a teaching-learning arrangement for a dual learning trajectory towards percentages for seventh graders ready for use in classrooms. Additionally, deep empirical insights into the students' learning pathways along the intertwined conceptual and lexical learning trajectory are given.
- Theoretically, the design research study makes contributions to topic-specific local instruction theories on learning percentages as well as to topic-transcending knowledge on integrating language learning and mathematics learning.

DUALITY OF LEXICAL AND CONCEPTUAL SCAFFOLDING

Extending micro-scaffolding to macro-scaffolding

Many different approaches exist how to overcome difficulties arising in multilingual schools (cf. Barwell et al., 2015, for an overview), among which this study chose the approach of macro-scaffolding (Gibbons, 2002). The main idea of scaffolding was first formulated by Wood, Bruner and Ross (1976, p. 90): enabling learners to "carry out a task […] beyond his unassisted efforts" (p. 90) by supporting them in one-to-one interactions, e.g. by prodding (keeping the learner in pursuit of an objective), pointing out discrepancies, modeling the solution, etc. As these early ideas mainly concerned the scaffolding given by one more knowledgeable person (mother, teacher, …) in one-to-one interaction, they can be classified as *interactional micro-scaffolding.*

Later, various authors have widened the idea of scaffolding to different contexts, especially to multilingual classrooms (e.g. whole-class interactional micro-scaffolding in Smit et al., 2013). For transferring the scaffolding approach to classrooms, Hammond and Gibbons (2005) have emphasized the importance of complementing the interactional micro-scaffolding by a longer-term so-called macro-scaffolding that guides the instructional design of learning arrangements.

Macro-scaffolding in this sense comprises the reconstruction of students' prior experience and the learning goals for sequencing intermediate learning goals and instructional tasks, i.e. the learning trajectory (Gibbons, 2002; Hammond & Gibbons, 2005) as will be explained in more detail in the following sections. The intended learning trajectories, together with support means for students to progress along them, serve as scaffold for students to reach the specified mathematical and linguistic learning goals with several intermediate steps. As the aim is to achieve students' independence, the construction metaphor of scaffold is still valuable.

Learning trajectories are not only necessary to define and to reach mutual zones of proximal development. In design research in general, they are considered as the core of a learning arrangement (Confrey, 2006). In this article, the term 'trajectories' is used for the hypothetical classroom trajectories (conceptualized as wider corridors rather than narrow trajectories, cf. Confrey, 2006) and 'pathways' for the empirically reconstructed individual trajectories.

Lexical scaffolding and conceptual scaffolding on different levels

Lexical scaffolding requires conceptual scaffolding

Although scaffolding can address all areas of language learning (lexical, semantical, grammatical, pragmatical, and discursive, cf. Morgan et al., 2014 for a wider view), the efforts on language learning in many mathematics classrooms mostly focus on the *lexical area*, i.e. on acquiring lexical items like words and phrases from the technical mathematical register. For this focus, instructional approaches with word lists or vocabulary posters have been elaborated and disseminated. We call that *lexical scaffolding* as it aims at extending the students' individual lexicon (i.e., vocabulary actively used or understood).

However, rather than acquiring only words and phrases for mathematical concepts, students need to construct the meaning of these mathematical concepts (*semantic area*) and learn for which problems and purposes they are applied (*pragmatic area*), as already Freudenthal (1983) has emphasized. Lexical scaffolding therefore requires to be underpinned by scaffolding in the semantic and pragmatic area.

Of course, also discursive and grammatical aspects are always touched during processes of language learning (Morgan et al., 2014; Moschkovich, 2010). However, the research *focus* is on the lexical aspects and its interplay to the pragmatic and

semantic areas, the latter two are hereby subsumed as *conceptual scaffolding*.

Conceptual scaffolding based on the RME-approach

In analogy to lexical scaffolding, an instructional approach for conceptual macro-scaffolding should support students' development of conceptual understanding in a *conceptual learning trajectory* starting from everyday experiences towards sustainable and flexible mathematical concepts (Hiebert & Carpenter, 1992, for an overview).

One well established instructional approach in mathematics education research has elaborated these ideas for a long time (without naming it conceptual macro-scaffolding), the Realistic Mathematics Education approach (Freudenthal, 1983; Gravemeijer, 1999; van den Heuvel-Panhuizen, 2003). In RME, the hypothetical conceptual learning trajectories are constructed starting from imaginable context problems which allow students to re-invent mathematical concepts. Thereby they mentally construct their meanings in a guided process of emergent modelling, first as *models for* context problem situations, later used as *models of* abstract mathematical concepts (Gravemeijer, 1999).

One central design principle in RME is the level principle according to which a hypothetical learning trajectory is to be sequenced over four levels of increasing deepness of understanding (that is why the levels are here ordered from top to bottom in Fig. 1):

1. The situational level, "where domain-specific, situational knowledge and strategies are used within the context of the situation",
2. The referential level, "where models and strategies refer to the situation described in the problem"
3. The general level, "where a mathematical focus on strategies dominates over the reference to the content", and
4. The level of formal mathematics, "where one works with conventional procedures and notations" (Gravemeijer, 1998, p. 286f)

After going through the four levels in this order (and then flexibly switch between them), students should be able to apply the acquired concepts to new problems in other contexts. Some mathematical topics thereafter require vertical mathematization (with innermathematical abstraction, schematization or generalization processes), but not the chosen topic of percentages.

Although the RME approach traditionally includes systematic reflections on language learning and especially on visualizations models, the lexical trajectory is analytically separated from the conceptual learning trajectories in Table 1 (in order

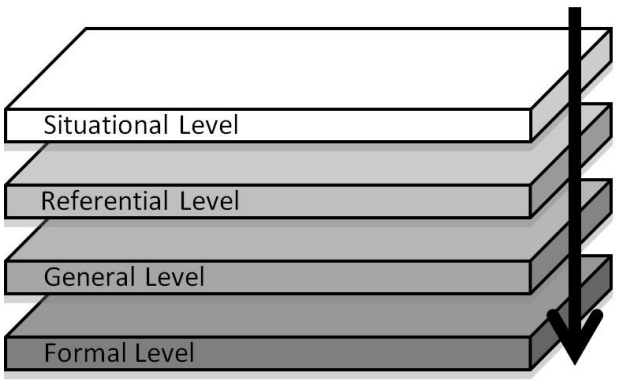


Figure 1. Gravemeijer's (1998) levels for a conceptual learning trajectory

to receive a language by which their strong connection can be shown in the empirical part).

Lexical scaffolding for students with low academic language proficiency

Students with low language proficiency often develop only weaker conceptual understanding than their more language proficient peers (cf. Ellerton & Clarkson, 1996, for an overview; Pöhler et al., 2015, for percentages). This empirical finding applies for monolingual as well as multilingual students and can be explained by a well-established linguistic theoretical background in which limitations in the academic school register are thoroughly described (addressed as "language of schooling" by Schleppegrell, 2004 and Thürmann, Vollmer, & Pieper, 2010; or as "CALP - academic language proficiency" by Cummins, 2000): These linguistics describe that academic language has an important epistemic function as it shapes the thinking, so that limitations might restrict cognitive as well as discursive processes (Schleppegrell, 2004; Cummins, 2000). For the specific case of meaning construction in mathematics, qualitative investigations have confirmed this theoretical assumption and unfolded how students' construction of relational meanings for mathematical concepts requires linguistic means of the academic school register (Prediger & Wessel, 2013; Prediger & Krägeloh, 2015).

That is why the conceptual learning trajectory must be underpinned by a lexical learning trajectory from students' everyday resources via the academic school register (required for meaning construction) towards the technical register (Pimm, 1987; Prediger, Clarkson, & Bose, 2015; cf. three upper levels in Fig. 2). This lexical trajectory for supporting conceptual understanding encompasses four levels (as defined in Prediger, 2016): *Students' individual resources* in the everyday register (including gestures and facial expressions), the *basic meaning-related vocabulary* in the academic school register, the *formal vocabulary* in the technical register, and the *extended reading vocabulary* again in the academic school register (cf. Fig. 2).

In spite of early assumptions in mathematics education research on mathematics and language (Pimm, 1987), students' individual resources in the everyday register are often not sufficient for constructing meanings (Prediger & Krägeloh, 2015). Hence, phrases from the academic school register are required to grasp mathematical relations and meanings. We call the relevant topic-specific part of the register the *basic meaning-related vocabulary* (Prediger & Wessel, 2013), e.g., "old price" for the base, "new price" for the amount and "the part of the new price from the old price" for explaining the rate in percent as part of a whole. Usually, this basic meaning-related vocabulary is situated in the context of the situational and referential level being chosen for constructing meanings in the conceptual learning trajectory.

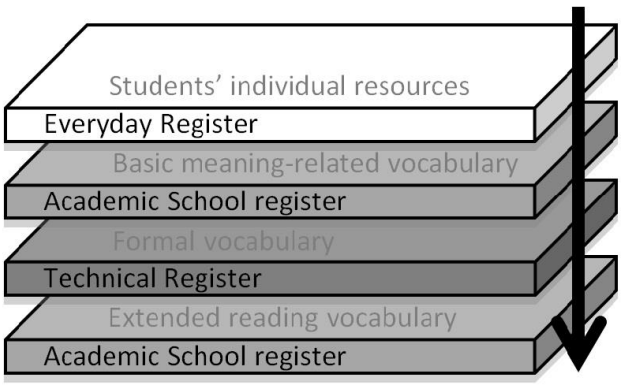


Figure 2. Levels for a lexical learning trajectory with increasing deepness

In the next step, *formal vocabulary* can be introduced for transcending the first context (e.g. the symbols p, G, P, and the formal terms base, amount, rate which appear more formal in German: Grundwert, Prozentwert, Prozentsatz). Usually, the formal vocabulary is well-defined by the curricula.

Once the conceptual understanding is achieved and the meanings of mathematical concepts are constructed, the initial context must be extended to reading further contexts and problems, e.g. in journals or tests. As they are expressed with other (possibly synonymous) expressions (for example "additional costs in €" for the amount which the students have to determine and to add to the regular price for calculating the new price), this can raise new linguistic challenges in the academic school register. These possibly synonymous words must be collected in an *extended reading vocabulary* of the academic school register for the lexical learning trajectory (cf. upper level in Fig. 2), for example by inventorizing typical word problems of a mathematical topic with respect to most frequent vocabulary (Niederhaus, Pöhler, & Prediger, 2015).

Whereas the basic meaning-related vocabulary addresses the epistemic role of language (i.e. the language needed for constructing new knowledge and meanings, cf. Schleppegrell, 2004), the extended reading vocabulary addresses the communicative role of language (i.e. the language used for communicating ideas) once the relevant epistemic processes of developing conceptual understanding for the new mathematical concepts have been finalized.

For the lexical macro-scaffolding, both are sequenced in the lexical learning trajectory in Figure 2. Especially in the levels 1 to 3, the conceptual and lexical learning trajectories are deeply intertwined, as the empirical analysis will show. That is why we talk about dual scaffolding along the dual learning trajectory.

Research and design gap and analytic approach by semiotic chains

Although the macro-scaffolding approach with its interplay of lexical and conceptual trajectories is shortly mentioned by several authors, only few design realizations (cf. Gibbons, 2002; Smit et al., 2013) and little empirical insights into the details of this duality exist. For closing this design and research gap, the design research study presented in this paper aimed at iteratively designing a dual macro-scaffolding by intertwining the conceptual and lexical learning trajectory towards percentages and investigating its functioning and the students' individual learning pathways along both.

Investigating the interplay of students' lexical and conceptual learning pathways on the micro-level requires a theoretical approach which connects both parts. For this purpose, we adapt Presmeg's (1998) approach of investigating individual processes of meaning construction by *identifying students' semiotic chains*. Classically in semiotic, the relation between signified (here: expressions to make sense of) and signifiers (here: used by the individual for explaining the meaning) is considered. Semiotic chains appear when the signifiers become signifieds in later levels, and this typically happens in learning trajectories covering several levels. Lexical means from earlier levels then serve as signifieds for the meaning construction on later levels.

LEARNING TRAJECTORIES TOWARDS PERCENTAGES

Difficulties with percentages

The relevance of the focused topic percentages is given due to its major role in middle school mathematics and everyday academic life, being present in many curricula and wide-spread areas.

In spite of this relevance, many empirical studies have shown students' difficulties with percentages (e.g., Kouba et al., 1988; Hafner, 2012; for overviews cf. Lamon,

2007; Parker & Leinhardt, 1995). Parker and Leinhardt (1995, p. 473ff) specified four reasons for students' difficulties; the first three of them focus on conceptual understanding:

1. The complexity of the mathematical content,
2. The diversity of different relations which can be described by percentages (cf. Fig. 3), which
3. Are – except for part of whole – often not explicitly treated in the curricula,
4. Linguistic challenges in cracking word problems with percentages appear because the relevant mathematical relations are often "invisible in the language." (ibid, p. 473)

This intertwinement of linguistic and conceptual challenges specifically calls for dual scaffolding.

A conceptual learning trajectory towards percentages

A convincing conceptual learning trajectory towards percentages has been developed by van den Heuvel-Panhuizen (2003). The learning trajectory starts with some first explorative activities in terms of everyday situations in which percentages play a role as descriptors of so-many-out-of-so-many structures. For example, the students describe how occupied a theater will be for different performances (by coloring parts of the theater hall's picture). Based on this experience, the students invent percent bars (cf. Fig. 4) as a *model of* situations of the fullness of parking lots and then further develop it into a *model for* concepts (such as base, amount, and rate) in multiple contexts.

Along this learning trajectory, the percent bar gradually changes its role from the concrete context-connected representation to a more abstract representational model (replacing the real parking lot by an occupation meter which visualizes the percentage of occupied spaces for the students). Moreover, the bar functions as an estimation model for approximating percentages and then as a calculation model with 1 % benchmarks. Subsequently the students deal with more complex problems like

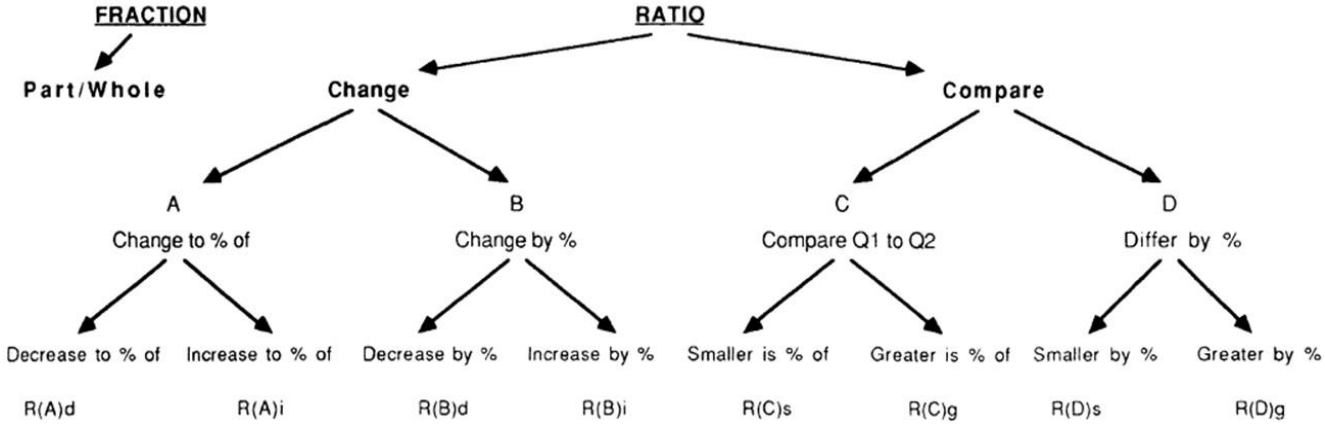


Figure 3. Diversity of relations described by percentages (Parker & Leinhardt, 1995, p. 439)

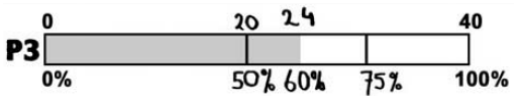


Figure 4. Percent bar as a structural base in a conceptual learning trajectory to percentages (van den Heuvel-Panhuizen, 2003, p. 22)

situations of change (in the context of price reductions), percentages as operators (reductions and enlargements of a photocopier), situations which ask for backward reasoning (find the base) and exponential increases (context of an interest-bearing savings account). The percent bar supports the process of understanding such complex situations as a thought model for getting a grip on the problems.

In general, the percent bar with its different roles allows structuring the relations between the concepts by its double scale that supports the underlying proportional reasoning.

For the macro-scaffolding design, central elements of van den Heuvel-Panhuizen's learning trajectory were adapted, with a shortened beginning and the restriction to the contexts of download bars and shopping. For extending it by a lexical learning trajectory, an empirical specification is required.

Missing research on the lexical learning trajectory towards percentages

Analogically, the lexical learning trajectory towards percentages has to start from *students' individual resources* in the everyday register, elaborating the *basic meaning-related vocabulary* in the academic school register and the *formal vocabulary* in the technical register (cf. Fig. 2). On the last level, the *extended reading vocabulary* is built. So far, no empirical investigation of this learning trajectory exists. However, the design research project can rely on some existing, not yet connected findings.

Traditionally, the *formal vocabulary* for percentages in the technical register is well specified by traditional curricula and textbooks (e.g. base, amount, rate, discount, difference, cf. Parker & Leinhardt 1995). For specifying the other three levels, empirical investigations are required.

For specifying the *extended reading vocabulary* (e.g. 'interest', 'loss of profits'), the design research study can refer to a corpus linguistic analysis in which typical reading challenges around percentages were identified. By investigating excerpts of four textbooks and assessments, most common words and phrases for expressing percentage relations in these kinds of texts were identified (Niederhaus et al., 2015). An extension to newspaper texts is currently conducted.

For specifying typical *individual students' resources* (e.g. 'this here' or 'what we pay' for amount) and the necessary *basic meaning-related vocabulary* (e.g. 'reduced by 30 %' for the discount / difference between base and amount), the study builds upon a prior empirical study of processes of conceptual development towards parts and wholes (Prediger & Wessel, 2013) and the described design research of van den Heuvel-Panhuizen (2003). However, further design experiments are required for specifying students' typical lexical resources and needs during the individual processes of meaning construction (cf. Section 5: Research outcome: Empirical insights into students' pathways along the dual trajectory).

Guiding questions for this design research study

Based on the existing literature on learning trajectories towards percentages, the research questions for the design research study were formulated as follows:

- Q1 How can the intended hypothetical conceptual and lexical learning trajectories towards percentages be intertwined and initiated by a sequence of instructional tasks?
- Q2 Which students' learning pathways can be reconstructed along the hypothetical trajectories?
  - Q2a Which role does the percent bar play for students' conceptual and lexical learning pathway?
  - Q2b Which lexical means do students activate and require along their pathways?

Q2c How are students' conceptual and lexical learning pathways towards percentages intertwined in the process of individual meaning construction?

Question Q1 refers to the design outcome, the questions Q2a-Q2c to its effects on students' learning pathways, with an increasing deepness of investigation.

METHODOLOGICAL FRAMEWORK OF THE DESIGN RESEARCH STUDY

Project framework in topic-specific didactical design research

The design research study for developing and investigating the dual scaffolding towards percentages is embedded in a long term curriculum project for low achieving middle school students (Prediger, Selter, Hußmann, Nührenbörger, 2014). This article focuses on the design research in context of a remediating percentage course, comprising three to five sessions of 60-90 min. each.

The iterative interplay between design, design experiment and analysis was conducted within the methodological framework of Topic-Specific Didactical Design Research (cf. Prediger et al., 2012; Prediger & Zwetzschler, 2013; following main ideas of Gravemeijer & Cobb, 2006).

The framework relies on the iterative and intertwined interplay between *four working areas* (cf. Fig. 5): (a) Specifying and structuring learning goals and contents by developing hypothetical learning trajectories, (b) developing the design of the teaching-learning arrangement, (c) conducting and analyzing the design experiments by systematically comparing the hypothetical learning trajectories to students' individual learning pathways, and (d) (further) developing local theories on teaching and learning processes.

Expected design outcomes comprise a further elaboration of the specified and structured learning content (here: conceptual understanding of percentages and necessary vocabulary on different levels), the refined design principles (here for dual scaffolding along the conceptual and lexical trajectories) and the prototypic teaching-learning arrangement.

The *research outcomes* consist of empirical insights and contributions to local theories on learning and teaching processes of the treated topic (here concerning the interplay of lexical and conceptual scaffolding and students' pathways).

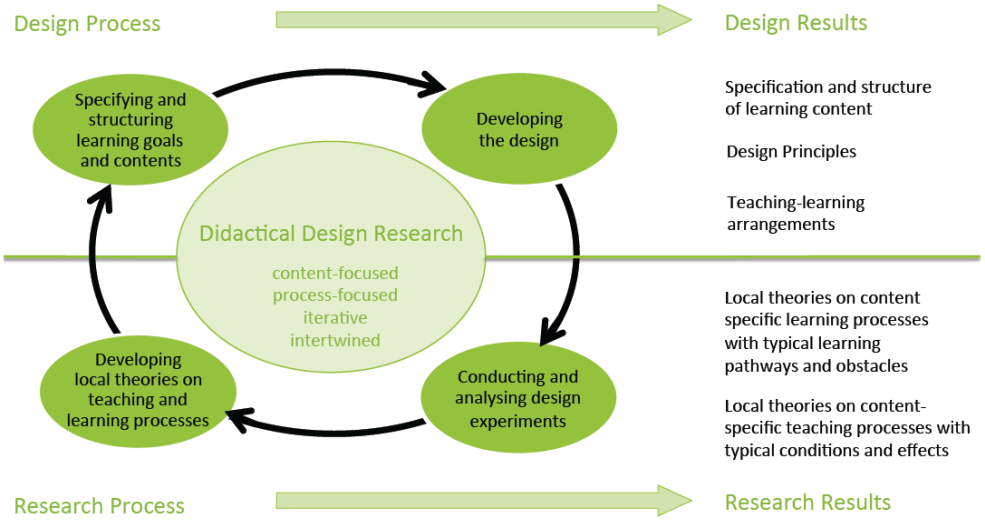


Figure 5. Working areas in Didactical Design Research (Prediger et al., 2012; Prediger & Zwetzschler, 2013)

Design experiments as method for data collection

Design experiments are considered to be the methodological core of design research studies as they allow in-depth investigations of teaching learning processes (Gravemeijer & Cobb, 2006).

For this project, four design experiment cycles were conducted with seventh graders low achieving in mathematics and in German academic language (age 12-14 years), including monolingual as well as multilingual learners (selected according to a percent test and a language test, cf. Pöhler et al., 2015). Three design experiment cycles were conducted in laboratory settings by the first author and experienced prospective teachers with university exam. They worked with two students each, in sum n_1=19 students. The *first and second design experiment cycle* was mainly used for specifying and structuring the necessary vocabulary and constructing the intertwined dual trajectory. Whereas the formal vocabulary ('base', 'amount', 'rate', 'reduced the base by 40 %') and the extended reading vocabulary were specified prior to the design research study, these cycles served for refining the inventory of typical students' resources (e.g. 'this here' or 'what we pay' for amount) and the necessary basic meaning-related vocabulary (e.g. 'reduced by 30 %' for the discount / difference between base and amount). The *third design experiment cycle* focused on reconstructing students' learning pathways in a process perspective. All design-experiments in laboratory settings were video-taped (altogether about 1200 minutes of video, therefrom about 590 min in the third cycle) and partly transcribed.

In the *fourth cycle*, design experiments were conducted in three whole classes with their regular math teachers (and n_2= 82 students with varying achievement) for testing the suitability of the design for whole classes and the practicality under field conditions. These design-experiments in classroom settings were partly video-taped and documented by students' written products and teachers' reflections.

Qualitative methods for data analysis

For the deeper investigation of students' conceptual and lexical pathways in relation to the learning arrangement, a qualitative analysis was conducted in three steps. The data for the in-depth analysis mainly stemmed from the third design experiment cycle because in this cycle, the learning trajectories and tasks were optimized after the cycles of iterative refinement of designs.

Step I. For reconstructing the students' conceptual pathways, a sequential analysis was carried out by *systematic extensive interpretation* of the individual and interactional comprehension processes and the individual steps of making sense of information and relations in the text (cf. Prediger & Wessel, 2013). As the role of the percent bar is in focus of Question Q2a, students' use of the percent bar is systematically related to the reconstructed pathway.

Step II. The students' lexical pathways were reconstructed by *inventorizing* all lexical means (here words and phrases) when first used by the students on the different levels of the dual hypothetical trajectory. The lexical means were further categorized as self-invented or adopted from oral or written offers.

Step III. For getting access to the interplay of lexical and conceptual learning pathways in the data of the third cycle, the individual processes of meaning construction were reconstructed by *identifying students' semiotic chains* (following the method of Presmeg, 1998). For this purpose, the signified (here: expressions to make sense of) and signifiers (here: used by the individual for explaining the meaning) are reconstructed for each step of the learning pathways and combined in the semiotic chains.

For each of the steps, each pair of students was first analyzed separately and then systematically compared the cases for finding structures in the phenomena. Typical cases are presented in Section 5.

DESIGN OUTCOME: A DUAL LEARNING TRAJECTORY FOR FOSTERING LOW ACHIEVERS' PATHWAYS TO PERCENTAGES

During the first three design experiment cycles, the original conceptual learning trajectory towards percentages (van den Heuvel-Panhuizen, 2003) was adapted to the specific requirements of a remediating course of seventh graders with low academic language proficiency. The necessary adaptations for evolving it into a macro-scaffolding approach specifically concerned its coordination with a systematic lexical learning trajectory following the lexical levels (Fig. 2).

Without accounting for this iterative process in three cycles, this section presents the design *outcomes* which are understood as answers to research question Q1. These concern the two intertwined learning trajectories (Section 4.1: Linking the levels of two intertwined learning trajectories) and the sequence of instructional tasks for activating students along the trajectories (Section 4.2: Sequence of instructional tasks on the dual learning trajectory).

Linking the levels of two intertwined learning trajectories

After three design experiment cycles, the conceptual and the lexical learning trajectory towards percentages are now coordinated on six levels as shown in Table 1. The conceptual learning trajectory starts with constructing meanings and inventing strategies for rates and amounts on Levels 1 to 3; then introduces the third problem type "find the base". On Level 4, the problem types are enriched by other problems ("find the base after reduction") before simultaneously considering all problems and further more complex ones. Hence, the Level 5 and 6 transcend the four levels presented in Section 1 (Fig. 1). In most tasks, the conceptual and the lexical learning trajectory are addressed in a combined form. However, their mutual dependence is a challenge for coordinating the lexical and the conceptual learning trajectory (Gibbons,

Table 1. Dual learning trajectory and structure-based scaffolding

Levels	Conceptual learning trajectory: Mathematical conceptions	Structure-based scaffolding by percent bar (different functions)	Lexical learning trajectory through different vocabularies
Level 1: Informal thinking starting from students' resources	Constructing meaning for percents by representing and estimating rates (in context download bar)	Introduce percent bar as *model of* a familiar context (download bar)	Intuitive use of *students' resources in everyday register*, limited offer of new lexical means
Level 2: First informal strategies and basic meaning-related vocabulary	Developing informal strategies for determining rates, amounts and later bases (shift to shopping context)	Percent bar as *model of* situations, used to find informal strategies and to structure the relations of concepts and context elements	Establish *basic meaning-related vocabulary* in the academic school register for constructing meaning for rates, amounts, bases in shopping context
Level 3: Procedures for standard problem types	Calculating amounts, rates and later bases (in shopping context)	Percent bar as *model for* calculating and structuring the relations of meanings and formal elements	Introduce *formal vocabulary* in the technical register
Level 4: Extending the repertoire	Widening to other problem types: change and comparison (in shopping context)	Percent bar as *model for* constructing more complex relations	Enrich the *basic meaning-related vocabulary* to more complex problem types
Level 5: Identification of different problem types	Identifying problem types of (also non-)standard problems (in diverse contexts)	Percent bar as structural base for identifying problem types	Explicit use and training of formal and basic meaning-related vocabulary
Level 6: Flexible use of concepts and strategies	Cracking more complex context problems flexibly (in non-familiar contexts)	Percent bar as structural base for identifying problem types	Introduce extended reading vocabulary

2002): words and phrases are empty without their meaning, but the process of meaning construction necessitates a language to express the thoughts (Presmeg, 1998). Hence, additional semiotic resources are required to mediate both trajectories structurally. For this, the *percent bar* was established as structural scaffold which had already proven its relevance for the conceptual developments by allowing the students to structure situations and interpretations visually (van den Heuvel Panhuizen, 2003 and Fig. 4).

The empirical analysis in Section 5.1 (Students' dual learning pathways - The role of the structural scaffold for intertwining the lexical and the conceptual pathway) will show that the percent bar can also offer linguistic support as it allows the use of non-verbal and deictic means for expressing ideas ("this", "there") about meanings. On later levels, it offers a structural base for putting different words and phrases into relation. And finally, it can serve as reference structure and thought model for mathematizing unknown problem situations (middle column of Table 1).

Sequence of instructional tasks on the dual learning trajectory

In the following, each level of the dual learning trajectory (i.e. each row of Table 1) is presented. Table 2 presents eight (out of 20) shortened tasks *on* or *between* different levels which combine the lexical and conceptual trajectories by means of the percent bar (note that the columns in Table 2 do not stand for the different trajectories).

On **Level 1**, students' informal thinking is activated starting from students' resources. Task 1 on Level 1 illustrates how students are fostered to construct the meaning of percentages in a familiar context – the download bars (cf. Prediger, 2013). This context allows the students to activate everyday resources for the conceptual and the lexical learning trajectory. On this level, only few new lexical means are offered because the students' individual resources are to be elicited, including gestures and other modalities. Here, students spontaneously use gesturing and phrases like "… % is already loaded", "it is .. %", "the light stripe of the bar".

The shift from Level 1 to **Level 2** is initiated by adding the second scale on the double scale of the percent bar (cf. Task 3 in Table 2). In this way, the percent bar as *model of* download situations can be used to find informal strategies and to structure the relations of concepts and context elements, which mark the shift to Level 2. In this shift, teachers offer lexical means like "at the top and at the bottom of the bar" or "have loaded … GB of … GB". Transcending pure estimation of Level 1, the informal strategies are elaborated to calculating rates, amounts and later bases, in the new shopping context of Level 2 (cf. Task 5 in Table 2). This development is structurally scaffolded by portioned percent bars like in Figure 6. Furthermore, the shopping context allows to introduce the basic meaning-related vocabulary for base (old price), amount (new price) and rate (rate to be paid), as well as phrases like "discount in %", "money saved", "money to be paid". In Task 7, the percent bar is used as a structural base for systematizing the vocabulary. Task 8 fosters the use of the vocabulary (see Table 2) and furthermore the flexible use of the percent bar.

On **Level 3**, students are challenged to develop procedures for standard problem types. The abstraction from the informal strategies in the shopping context is introduced by an operative series of tasks in Task 9. In the same task, the formal

Table 2. Eight out of twenty tasks in the dual learning trajectory (shortened and translated from German)

Task 1. Download Bars (Level 1)

a) …

b) The friends download films and look at the download bars.

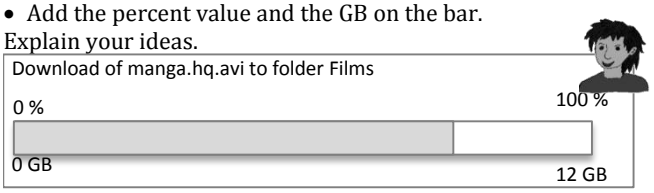
- What percent of the film has been already downloaded? (Tip: Portion the download bar suitably.)
- What percent of the film still has to be loaded?
- Can you express that also with a fraction? Explain your ideas for all questions.

Task 3. Already downloaded (Level 1 → 2)

Kenan downloads a film of 12 GB.

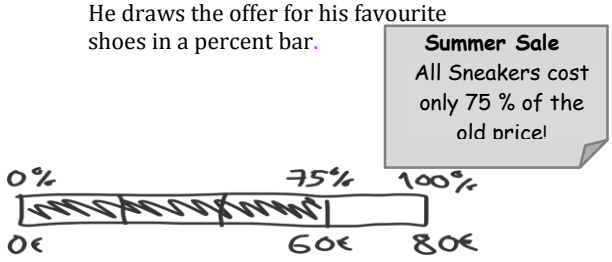
- How much GB has he already downloaded, approximately?
- Add the percent value and the GB on the bar.

Explain your ideas.



Task 5. Different offers for the favourite shoes (Level 2)

a) Maurice discovered an offer in the city. He draws the offer for his favourite shoes in a percent bar.



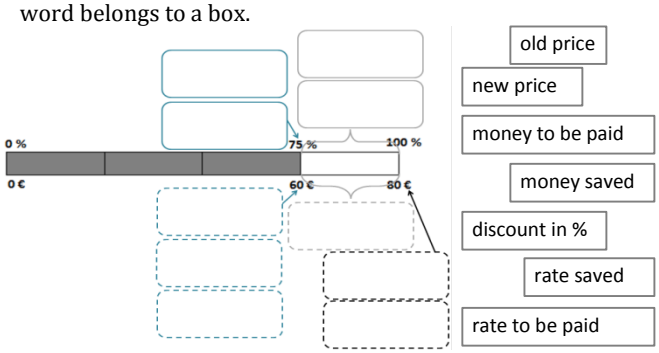
- What can you see in this bar?
- How has Maurice found the new price of 60 €?
- Compare to Kenan's bar in Task 3.

b) …

Task 7. Vocabulary for percent problems (Level 2)

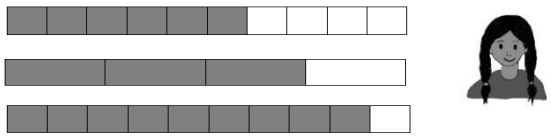
The words on the word cards can help you to describe offers and calculations like in Task 5.

- But which word belongs to what?
- Fill the boxes in the percent bar. Sometimes, more than one word belongs to a box.



Task 8. Different offers for the favourite shoes II (Level 2)

Taras favourite shoes used to cost 120 €. She receives three offers for discounts. How much does she has to pay?



- Complete all six values on the percent bars.
- Describe with the words from Task 7, what you can see on the percent bar.
- How much discount does she get in each offer?

Task 9. Filling gaps (Level 3)

a) Fill the gaps. You can use the percent bar. What do you discover? Explain your ideas.

(1) 5 % of 40 € are ____ €.
15 % of 40 € are ____ €.
25 % of 40 € are ____ €.

(2) 1 GB of 20 GB are ___ %.
2 GB of 20 GB are ___ %.
8 GB of 20 GB are ___ %.

(3) 30 % of 20 € are ___ €.
30 % of 30 € are ___ €.
30 % of 40 € are ___ €.

(4) 30 % of ___ € are 9 €.
30 % of ___ € are 18 €.
30 % of ___ € are 27 €.

b) Explain what is given and what is to find in the problems (1) – (4). Use the concepts base, amount, rate, and write them on the percent bar of Task 7.

Task 13. Sales discount (Level 4)

Tara has found these offers in a shop.

a) Tara buys shorts for 28 €. Complete a percent bar. What did the shorts cost before?

b) Complete these sentences and explain how to see it in the percent bar:

- The price of the shorts has been reduced by ___ %.
- Tara has saved __ €.

b) Tara buys a T-shirt for 15 € and a dress for 30 €. Complete two percent bars: What did the clothes cost before?

Task 16. Systematizing percent problems (Level 5)

a) Which word problem (1), (2), (3) belongs to which problem type? (Find amount, find base, or find rate?) Write the information of each problem in a percent bar and add a question mark.

(1) In a tombola 45 % of all lots shall win. These are 90 winning lots. How many lots were sold?

(2) Salami has a fat content of 40 %. How many grams of fat are in 200 g salami?

(3) 195 of 300 students of a primary school go by bus. What percent are these?

b) Solve the three word problems in a) by means of the percent bar. Write the solutions under the question marks.

Figure 6. Beren's labelled percent bar for Task 5 (Translation from German: "25 % cheaper", "20 € cheaper")

vocabulary base, rate and amount is systematically introduced and used in simple and complex phrases.

On **Level 4**, the repertoire of problem types is enriched by the problem type "Find base after reduction" which is contrasted with other problem types for three situations (in Task 13 a), and even as different perspectives on the same situation (in Task 13b). This task allows the students structuring the formal concepts in complex relations. In addition, Task 13 yields new lexical challenges, namely extended reading vocabulary like "reduced by" and "reduced to". Their meaning is constructed by contrasting formulations for the diverse relations and subtle prepositions.

On **Level 5**, students are supposed to identify different problem types of a variety of problems in different contexts. The percent bar now serves as a structural base for cracking the relations of the word problems. Once the vocabulary from the shopping context (old and new price) is experienced as not adequate for the new context, the use of the formal more context-independent vocabulary becomes more and more relevant.

On **Level 6**, students are challenged to make flexible use of concepts and strategies in further, non-familiar contexts and extend their reading vocabulary by finding synonyms. This level is not treated in this paper, because the focus lies on the concept development.

The first to third design experiment cycles were useful for iteratively refining this teaching-learning arrangement along the dual trajectory. The fourth design experiment confirmed that the instructional sequence of tasks is also suitable for whole class teaching.

However, deeper empirical investigations are required in order to understand not only *that* the learning arrangement works, but *how* it supported students' conceptual and lexical learning pathways in an individualized form, as shown in Section 5.

RESEARCH OUTCOME: EMPIRICAL INSIGHTS INTO STUDENTS' PATHWAYS ALONG THE DUAL TRAJECTORY

This section provides empirical insights into the complexity of students' pathways. For illustrating how the intended learning trajectory functioned in the third design experiment cycle, Section 5.1 (Students' dual learning pathways - The role of the structural scaffold for intertwining the lexical and the conceptual pathway) shows excerpts from students' learning pathways for answering question Q2a. In order to give a first answer to question Q2b, the Section 5.2 (Students' lexical learning pathway - The case of Beren and Manik) points out the case of Beren and Manik as an example of a lexical learning pathway, showing which lexical means they use at different levels of the learning trajectory. Whereas Section 5.1 gives an overview of typical learning pathways of several students, Section 5.3 (Students' dual learning pathway along the semiotic chain of meaning construction - The case of Beren) outlines deeper results from their systematic investigation with respect to the meaning constructions: The case of Berens' dual learning pathway along the Levels 1-3 allows accounting for the

interplay of lexical and conceptual pathways in the semiotic chain of meaning construction and helps to formulate a first answer to research question Q2c.

Students' dual learning pathways – The role of the structural scaffold for intertwining the lexical and the conceptual pathway

In this section, brief excerpts from students' learning processes provide insights into the individual learning pathways along the hypothetical trajectories. They especially show the role of the structural scaffold percent bar for intertwining the lexical and the conceptual pathway.

Level 1: Informal thinking starting from students' resources

The familiar context of download bars allows the activation of everyday resources for students' conceptual and lexical learning pathways. For example, the excerpt from Beren's (B) and Manik's (M) learning process of Task 1 shows how the girls estimate the given part.

127 B I think this second [bar] looks like 80 %, what is loaded. And 20 which must still be loaded.

… … …

129 M I say about 30, I believe.

130 T Must be loaded? [M nods] You mean? How much is loaded, then?

131 M 70.

Beren and Manik use linguistic means (#127 "what is loaded" and "which must still be loaded") which are similar to the ones offered by the task formulation. Both intuitively develop the idea that the two parts sum up to 100 %, as teacher's (T) (#130) question makes explicit.

Like Beren and Manik, all students in the design experiments can spontaneously make sense of the download bars and find (often non-verbal or deictic) means to communicate about it. Hence this context serves as an adequate resource for introducing the percent bar and for activating first lexical means.

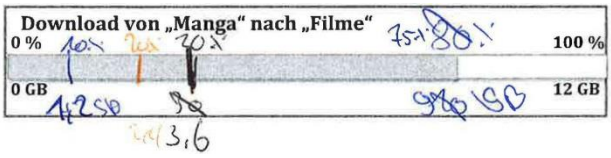
Level 2: First informal strategies and basic meaning-related vocabulary

During the shift from Level 1 to Level 2, the second scale on the double scale of the percent bar is added: The students are asked to estimate the state of charge in percent and in gigabyte (cf. Task 3 in Table 2).

The excerpt from Kevin's (K) conversation with the teacher (T) shows how he activates intuitive strategies for estimating the amount of gigabyte. By portioning the percent bar, Kevin handles the percent bar as *model of* the download situation.

302 K Well, I believe it is about 76 %. Because, it looked similar to 75 % [hints to the percent bar], but anyway a bit more, I believe. Eh, by estimating so about.

… … …



309 T Ok. And how did you find the 9 gigabyte, then?

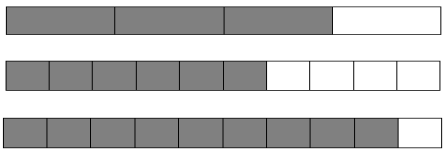
… … …

312 K Yes, well, I have looked [hints to the white part of the percent bar] of, eh, 10 gigabyte, and therefore we can, well, 76, eh 7.6 gigabyte and 2 gigabyte again. About this distributed, so 9.

Transcending pure estimation of Level 1, the initial informal strategies evolve towards calculating rates, amounts and later the bases, in the new shopping context. This development on Level 2 is structurally scaffolded by portioned percent bars (like in Figure 6). On this level, the percent bar supports structuring the relations of the mathematical concepts and context elements. On this pathway, portioning the percent

bar in different manners fosters the students' flexible use of the percent bar, as the excerpt from Kevin's work on Task 8 can illustrate:

803 K Eh, well, I have here [hints to the first percent bar] here was one of the things, was 30 €, mainly, eh, 120. Because 30 plus 30 plus 30, hence 4 times 30, these are 120, though. And here [hints to the bar] three times 30 are 90, then.



… … …

808 K But I believe here [hints to the second percent bar] you have one thing in the value of 12 €, one box. Eh, are 72 €.

809 T How did you find out that one box, eh, has 12 €?

810 K Well, and eh, 120 divided by 10 are 12, though.

… … …

812 K [to his partner Melina] Yes, you can easily here [hints to 120] 120 minus 12, these are 108.

In order to find the value of one box, Kevin first uses a repeated addition, then a multiplicative strategy (#803: "here was one of the things, was 30 € […], because 30 plus 30 plus 30 […], hence 4 times 30, these are 120, though.") and later a division (#810: "Well, and eh, 120 divided by 10 are 12, though."). Then, for finding the amount (i.e. the new price of the shoes), he uses different strategies. Whereas he multiplies the value of one field with the number of marked fields in the first example (#803: "And here three times 30 are 90, then."), he subtracts the value of the non-marked field from the base in the third example (#812: "Yes, you can easily here 120 minus 12, these are 108). His repeated use of non-verbal and deictic means ("here" in #803 and #808, "these" in #803 etc.) is typical for the linguistic relief supplied by the percent bar for many students. This illustrates how the linguistic demands can be decreased by gestures and deictic means when the students can refer to the visual model of the percent bar and its structures.

Like Kevin, also the other 18 students in the design experiment cycles 1-3 develop flexible strategies, most of them referred to the percent bar.

Level 3: Procedures for standard problem types

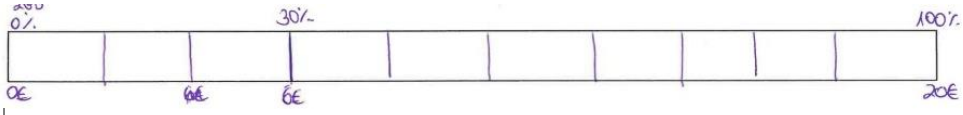
The operative series of tasks in Task 9 aims at a first abstraction from the shopping context for developing formal strategies. The same situation is used for introducing the formal vocabulary base, rate, and amount.

The excerpt from Melina's learning process shows how she transfers her strategies developed on Level 2 to the context-free tasks and even to the new problem type "Find the base" in the series (4) of Task 9. The percent bar offers the structural base again, here for grasping the relations between formal concepts. While solving the task "30 % of 20 € are __€" in series (3), she says:

906 M Yes. Here, I had [hints to her worksheet] you must first portion it in 10 boxes. Then, you must, easily, eh, where do I have it – eh [hints to the percent bar]. […] 20 divided by eh 10 are 2, though.

… …

908 M Then you have to with 3 2, accordingly.



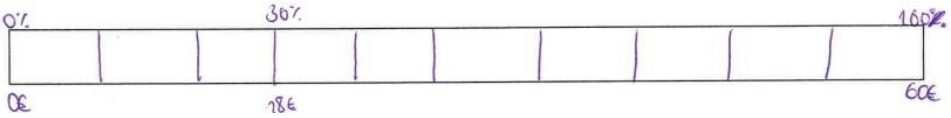
(3) 30 % of 20 € are _____ €.
30 % of 30 € are _____ €.
30 % of 40 € are _____ €.

(4) 30 % of _____ € are 9 €.
30 % of _____ € are 18 €.
30 % of _____ € are 27 €.

Melina explains her correct solution for the known problem type "Find the amount" with many deictic means towards the percent bar and only few explicit formulations. This again shows the role of the structural base as a linguistic relief. The partitioning of the percent bar helps Melina to grasp the relations and to solve the task.

Also for the new problem type "Find the base" in series (4) "30 % of __ € are 18 €" (which is explicitly treated later in Task 10), she succeeds in transferring the strategies:

938 M Yes, well here [hints to the percent bar] it was, that, that 30 % are 18 €. These were three boxes. Then you have to 18 eh, yes, to find to which multiples it belongs, for example in multiples of 6. Then, you must make 6 times 10, because 6 times 10 are 10 boxes, though.



… …

940 M Then you have 60 €.

With the structural and deictic support of the percent bar as linguistic relief and tool for thinking, Melina succeeds in developing and communicating her ideas and successfully receives the result 60 €.

Like many other students in the design experiments, Melina refines her linguistic means in the further course of the learning trajectory, for expressing ideas by formal and basic meaning-related vocabulary.

Level 4: Extending the repertoire

Level 4 aims at extending the repertoire of the problem types "Find the amount", "Find the rate" and "Find the base" to the more complex problem type "Find base after reduction". Task 13, the first task of Level 4, initiates that the students reflect on distinctions between three situations of similar problem types "Find the base" and "Find the base after reduction" (in Task 13a and c) and on different perspectives (different linguistic formulations) on the same situation (in Task 13b).

As the excerpt from Kevin's and Melina's learning process (Task 13b) shows, this task allows the students to structure the formal concepts in complex relations:

1301 T Kevin, what have you written for "the price of the shorts…"?
1302 K Eh, was reduced by 30 %.
1303 T How can you see that? How can you see that on the percent bar?

1304 K [hints to his percent bar] Because, the shorts has cost, eh, although – the shorts have cost 70 % before.
1305 T Mmh.
1306 K Ah, it has cost 100 %. And now, it costs only 70 %.
1307 T Yes.
1308 K And in between, between 70 and 100 are 30, though.

While solving Task 13, the students additionally have to overcome new lexical challenges, namely distinguishing extended reading vocabulary like "reduced by" and "reduced to". Their meaning is constructed by contrasting subtle prepositions. Again, they clarify the subtle differences by means of the structural scaffold, the percent bar.

Like Kevin and Melina, most students make use of the percent bar as model for constructing more complex relations and then as structural base for identifying different problem types.

Level 5: Identification of different problem types

On Level 5, students are asked to identify different problem types in a variety of contexts. The percent bar as structural base supports the students in cracking the relations hidden in the word problems. This becomes visible in the example of Kevin who has also developed very flexible ways of calculating:

> 1601 K Well, for the tombola, there were 200 lots and 45 % well 90, eh 90 of it always win. And, eh, the question was, how many lots were sold. 100 % are 200 lots, then.
> 1602 T Mmh, ok. And, what you have written down now, what problem type is it, then?
> 1603 K Eh, find the base.
> 1604 T Ok. And you have found out 200 lots, how did you mark that in your percent bar and how did you do that?
> 1605 K Well, first, I had here [hints to the half of the percent bar] about 45 %. These are 90, then. And then, I have here, eh, first, 10, first eh 9 lines, these are 10, no 8. Anyway, so that it resulted in 10 fields. And then, I have then, again, the left of 45 % mmh I have portioned them [counts the colored fields in his bar]. That is was 9 fields, well left to 45 %. And then, there was one field, was though 10, eh 10 lots big, and then, I found the 90. And for 50 %, there were 100 lots and…

On this level, Kevin makes use of the percent bar as reference structure and thought model for mathematizing unknown problem situations (in #1605). The vocabulary he uses does not belong to the shopping context anymore. Instead, it is strictly related to the percent bar (in #1605: "lines", "resulted in 10 fields" and "portioned them").

Making aware that the vocabulary from the shopping context (old and new price) is not adequate for the new context, the teacher initiates the use of the formal, more context-independent vocabulary acquired on Level 4:

> 1663 T Okay, can you again explain it with the words old price, new price, reduce to, reduce by? Then we try to repeat it. Kevin, you try it with the first bar?
> 1664 K Huh, but how it is possible? It isn't Euro and so on.
> 1665 T Okay.
> 1669 T So it doesn't work well with new and old price.
> 1670 K Because. There is none.
> 1671 T Mm. What could you say instead of new and old price?
> 1672 K Yes. The base.
> 1673 T Okay, then we make it like this.
> 1674 K Well, the base is 200 lots. And eh I have the amount of 90, and the rate for 45 %.

It is quite typical for many students' pathways that the teacher must explicitly invite students' active use of formal vocabulary.

In the end of the conceptual learning trajectory, Kevin, Melina, and the other students have acquired all necessary concepts and phrases along the dual learning trajectory and apply calculation strategies on the percent bar flexibly and successfully. Kevin's experience nicely shows the distinction between the epistemic role of the basic meaning-related vocabulary and the communicative role of language being expressed in different registers.

Students' lexical learning pathway – The case of Beren and Manik

For investigating question Q2b (Which lexical means do students activate and require along their pathways?), the lexical pathways of the students were reconstructed by inventorying all individually used lexical means, i.e. all words and

Table 3. Inventory of all lexical means used by Beren and Manik in Level 1 to 3

	Lexical means used by students Beren and Manik (*italics*: *self-invented words or phrases* / adopted from written material or teachers' speech)			
Level	… for base	… for amount	…. for rate	… for comparing
Level 1 Students' individual resources	• bar • long bar	• dark bar • … % is already loaded, *… % is fully loaded* • *be … %*	• the light stripe • … % still must be loaded	[not yet required]
Shift from Level 1 to Level 2	• *at GB / at percent (at the bar)* • bar • *other GB (for another movie size)*	• be … %, *already have … % / GB* • *has been charged up to … %* • … % are … GB / … • GB are …, *… GB at … %*	• *still need / load … GB*, • … GB still must be loaded, … GB / % still missing	• *be more* / larger than
Level 2 Basic meaning-related vocabulary	• bar • *at the end of the bar* • old price (is) (T / S) • cost… € before • *how much it has cost before* • grey bar	• be loaded … GB • money to be paid • money *still must* be paid • *money you give away* • rate to be paid [sic] • new price is • *cost money* • *cost … € (now / then)* • be … € / % • *would cost … €* • *still remain* • … % are *like / is … €* • *… % cost … €*	• saved / *could save* • become … % / € cheaper • *become (… %) lower price* • *be reduced (to)* • discount • give discount of … % • money saved • rate saved • *save … % (discount / money)* • *be made cheaper* • *be made down*	• be cheaper • *be more expensive* • *be smaller / equal*
Level 3 Formal vocabulary	• base	• amount	• rate	[no new phrases required]

phrases for the relevant relations and concepts which a pair of students uses on each level of the dual hypothetical learning trajectory.

Table 3 shows the inventory for the two girls Beren and Manik during their conceptual pathways from Level 1 to 3. The rows separate the Table 1 into the Levels 1 to 3 (and the period of shift from Level 1 to 2). The columns assign the used lexical means for the concepts rate, amount, base and comparisons. Lexical means in italics are invented or modified by the students, those without italics are adopted from written (formulation of the task) or oral offers (in teacher's speech).

The inventory in Table 3 shows a rich repertoire of lexical means used by the girls during their pathway through Level 1 to 3 with interesting tendencies (cf. Prediger & Pöhler, 2016 for more details):

- *Level 1.* Students mainly use lexical means with *reference to graphical aspects* of the bar (e.g. the adjectives long, dark and light) and sometimes to the download context ('is already / fully loaded' and 'still must be loaded').
- *Shift from Level 1 to Level 2.* The graphical references disappear. As the tasks now focus on relations of the double scale (for which the students find words like 'at GB / at percent'), the use of lexical means for *relations* ('… % are … GB' or '… GB at … %') or *comparisons* ('other GB' or 'be larger than') increases.
- *Level 2.* The references to graphical aspects of the bar slightly reappear during the shift to the shopping context. The rate of self-invented expressions decreases significantly on this level. Instead, students adopt or slightly modify the offered basic meaning-related vocabulary (e.g. 'be cheaper' being offered, the students say 'be made down').
- *Level 3.* The formal vocabulary is much less rich and not self-invented by students. However, students can make use of it when asked to.

Altogether, the character of expressions changed while descending deeper in the levels of the dual trajectory from single words to more complex phrases, expressing relations and comparisons. The source of the expressions change from mainly self-introduced to mainly adopt from written or oral offers.

The exemplary reconstruction of the lexical pathway of Beren and Manik by an inventory of the used lexical means contributes to the empirical investigation of the lexical trajectory which had previously been lacking before (cf. Section 2.3 "Missing research on the lexical learning trajectory towards percentages"). The results of this and other pairs of students emphasize the relevance of offering lexical means on Level 2 and 3 orally and insisting in its use, hence the high relevance of micro-scaffolding. However, it does not give an account on how meanings are constructed.

Students' dual learning pathway along the semiotic chain of meaning construction – The case of Beren

For investigating research question Q2c (How are students' conceptual and lexical learning pathways intertwined in the process of individual meaning construction?), semiotic chains are reconstructed as a tool for accounting for the individual processes of meaning construction.

Before presenting the whole reconstructed semiotic chains for the case of Beren (Fig. 7 at the end of this section), some excerpts of transcripts are presented to show how the semiotic chains for rate, amount, base, and reduction / remainder are derived from the analysis of transcripts.

On **Level 1**, the dual learning trajectory starts with asking students to estimate percentages in Task 1a) intuitively. As long as linguistic means are not offered, Beren describes the relations on the graphical level with gesturing and with reference to the percent bar:

106 B 55 percent.

107 T Well. Hm. And how do you see this in the download bar?

…

109 B Well, that is such a long bar, though [hints to the bar].

…

111 B Then, this must result in, eh, 100 percent.

…

113 B …Well, that is, it [hints to the grey part of the bar] looks like the half exactly. […] a bit more…

Without mentioning the context of download bars, Beren activates '55 percent' (in turn #106), 'that [*hints to the bar*]' (#113) and 'like the half' (#113) for the ***rate*** of the loaded film and 'a long bar' (#109) and '100 percent' (#111) for the ***base***, here the complete film (cf. semiotic chains for rate and base in Fig. 7).

For reconstructing her semiotic chains, the *direction of signification* is important: 'that' (#109) is explained by 'the long bar' (#109) and quantified by '100 percent' (#111). Hence, 'the long bar' is a signifier for the signified 'that', the complete bar. In contrast, '55 percent' is the signifier for 'that' (#113), the signified grey part of the bar. 'Like the half' (#113) is another name given for 'that'.

Only later in Task 1b), Beren adopts the expressions offered in the download context:

158 B I think for this second [refers to the bar of birthday.mp4], it looks like 80 percent, what is already loaded. And 20 must still be loaded.

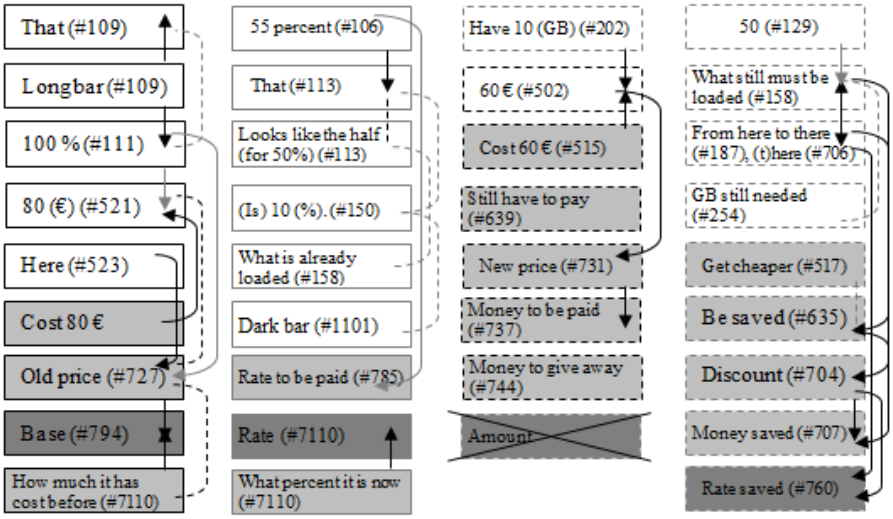
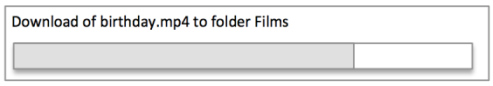


Figure 7. Four semiotic chains for Beren's individual construction of meaning of base and rate



187 B That is from here to there, though, these are 20 percent [hints to the white part of the bar birthday.mp4]

...

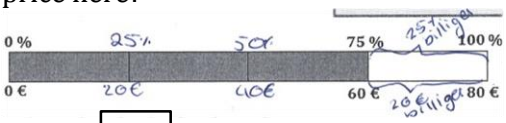
1101 B Yes. Well, I think the dark bar is about 25 percent, now [refers to the bar horse-love.mp4]

Here, Beren directly refers to the 'dark bar' (#1101), but also to the context: 'what is already loaded' and 'must still be loaded' (#158) and similar 'have be loaded' (#177). Some of her links to previous lexical means are indirect, e.g. for 'It looks like 80 percent, what is already loaded' (#158) which is related to the previous 'looks like the half' (in #113).

The ***remainder*** is first addressed in the context by 'still must be loaded' (#158, #177) and later explained by deictic means in the percent bar, 'from here to there' (#187).

Proceeding to **Level 2**, basic meaning-related vocabulary in the shopping context is offered in the formulation of Task 5.

520 T Okay, first, the mmh, it is about the old price. What is the old price here?



521 B 80.

522 T 80. What makes you think that?

523 B Because it stands here [hints to the end of the percent bar].

...

551 T [...] Could you [Beren] now better understand the numbers? Why then are 60 and 80?

552 B Thus when there now stand, mh, the shoes costs 80 € and there stand it get 25 percent cheaper, then it is, then they cost 60 €.

The teacher asks Beren to explain the 'old price' (#520), and Beren explains it with the isolated number '80' (#521). For filling it with more meaning, she refers the new signifier 'old price' to the signified 'here [*end of the bar*]' (#523), with the explicit

explanation ‘because it stands here’. Although she does not explicitly mention ‘100 %’ or ‘the long bar’, both is non-verbally addressed by hinting to it on the bar.

After a longer negotiation of meaning, Beren enters the shopping context with ‘cost 80 €’ for the base and ‘get 25 percent cheaper’ for the discount (#552), and ‘then they cost 60 €’ (#552) for which the meaning is clear by ‘then’.

Although Beren uses the offered vocabulary in Task 5 selectively, she and her partner Manik cooperatively succeed in assigning the cards to the right places when demanded to systematize the complete list of words in Task 7 on **Level 3** (deviant from Table 2, also the formal words base, rate, amount were included in the task in this design experiment cycle).

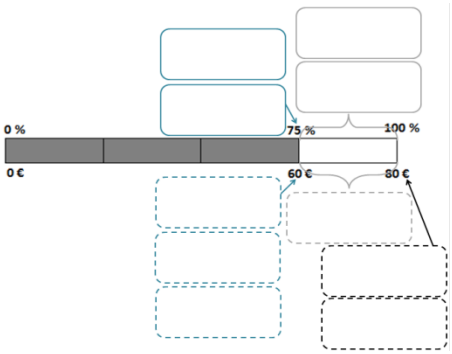
704 B Well, I would put that, eh ‘discount’, here [puts the word card ‘discount’ on the right place].

…

706 B Because, here is such a bracket and then there is ‘discount’, though.

707 M I believe ‘money saved’ must be here [puts the word card to the place of ‘amount’].

708 B I believe, it [refers to the word card ‘money saved’] must be here [hint to the correct place], because, it is ‘discount’, that was saved and then the money must be saved, too.



…

760 B This [hints to ‘rate saved’] would fit here, because here is this bracket, wait! ‘Rate saved’. I would put that here above, then [shows the correct place]. Because this is above that, and this bracket is always where we have ‘discount’.

…

784 B Rate to be saved

785 M To be paid.

786 B Yes. I would say this.

…

788 M We have to put it here [hints to the correct place].

789 B Here, right, because it is the 75.

Beren assigns ‘discount’ to the right place on the percent bar (#704), and justifies it by the bracket for the white part of the bar (#706), a signified she constructed earlier. The shopping expressions ‘money saved’ (#707-708) and ‘rate saved’ (#760) are explained by the concepts already treated, namely the signified ‘discount’ and ‘be saved’ in #708 respective #760, not vice versa. Her reference to the bracket again activates the implicit link to ‘from here to there’. In contrast, the expressions ‘old price‘ and ‘rate to be paid‘ are not explained in the shopping context but by the numbers on the percent bar: ‘80 €‘ (#727: „Because the old price was 80 €‘) and ‘75 %‘ (#789: „Here, right, because it is the 75“.) for which she had constructed a meaning in Task 5 (#521) and Task 1 (#150).

For the formal concepts ‘base’ and ‘rate’, she refers to the meaning-related vocabulary:

7110 B And base is the old price of, well, of how much it has cost before […] And rate is, then, what percent it is now.

The semiotic chain for base hence refers to ‘old price’ (first in #796, also in #7110), whereas ‘rate’ is explained by the word ‘percent’ (#7110). In both cases, this is complemented by a context reference (explicit for base ‘how much it has cost before’ as a synonym of ‘old price’, implicit for ‘what percent it is

now', both in #7110) that refers to the temporal relation old-new. The assignment is affirmed by reformulating 'base' into 'how much it has cost before' (#7110, temporal relation old-new).

This kind of analysis (from only selected parts were printed here) resulted in four semiotic chains for each of the concepts base, rate, amount (a word she does not use at all), and discount (cf. Fig. 7). The frames of the rectangles in the transcript symbolize the chains to which they belong; the expressions in the chains appear in their chronological order of appearance in the transcript. Additionally, their level in the lexical trajectory is marked by the fillings: white filings signify the individual resources (Level 1), light grey the meaning related vocabulary (Level 2) and dark grey the formal vocabulary (Level 3). The different kinds of links in the pathway of meaning construction are marked by the lines and arrows: An arrow with continuous line symbolizes the link from the signified to the signifier, with the signified being used to explain the signifier; a dotted line symbolizes the synonymous use of expressions without explicitly explaining one by the other; and a grey line signifies an implicit assignment of meaning from the signified to the signifier.

To sum up, the analysis and the identified semiotic chains show how Beren is able to construct meanings along the hypothetical conceptual and lexical trajectory *in many steps of a non-unidirectional pathway.* The deviances between individual learning pathway and hypothetical trajectory mainly concern the micro-order of meaning construction: sometimes later expressions are used to assign meaning expressions mentioned shortly before.

On all levels, the process of meaning construction is supported by the possibility of hinting to the percent bar as this supports also non-verbal modalities. This structural base also smooths the shift from the download context to the shopping context in which Beren approaches the thinking in a proportional double scale with different units. The basic meaning-related vocabulary in the shopping context supports the approach to the formal concepts although the relation between signifier and signified is sometimes reversed (cf. semiotic chain of base in Fig. 7). All in all, the semiotic chain proved to be a very helpful analytic tool to reconstruct the processes on the micro-level.

CONCLUSION AND OUTLOOK

How can students with low mathematics and language proficiency be supported in acquiring conceptual understanding for percentages and the lexical means to communicate about them? The reported design research study offers first answers to the raised research questions.

For research question *Q1 (How to intertwine the intended hypothetical conceptual and lexical learning trajectories towards percentages?),* the study has produced a well-functioning *design outcome* after three iterative design experiment cycles. The dual learning trajectory for fostering low achievers' pathways to percentages intertwines a conceptual learning trajectory with a systematic lexical learning trajectory (consisting of necessary vocabulary) on six levels of increasing deepness of conceptual understanding and language use (cf. Table 1). By these levels, the RME level principle (van den Heuvel-Panhuizen, 2003) can be combined with sequencing from informal resources to school academic and technical registers (Gibbons, 2002; Schleppegrell, 2004).

This dual learning trajectory is realized in a sequence of instructional tasks (cf. eight examples in Table 2). The main link between both trajectories is the percent bar as a structural scaffold which activates also students' resources in non-verbal modalities (Schleppegrell, 2004). The third design experiment cycle (on which the detailed empirical analysis focused) and the fourth design

experiment cycle (in regular classrooms) provided qualitative insights into the functioning of the learning trajectory, as was often claimed necessary (e.g. Schleppegrell, 2004). In a future study, a quantitative measurement of effectiveness in a quasi-experimental control trial is to be conducted.

The main *research outcomes* dealt with research questions Q2 (about students' learning pathways). In a deeper investigation of research question *Q2a* (*role of percent bar for students' conceptual and lexical learning pathway*), the role of the percent bar was reconstructed to be changing along the learning trajectory. Along the conceptual trajectory, it first allows the students to structure the relations of the mathematical concepts and interpretations visually; later it can serve as a reference structure and thought model for identifying different problem types and for mathematizing unknown problem situations (as shown by van den Heuvel-Panhuizen, 2003). For the lexical trajectory, the percent bar can serve as linguistic support as it allows the use of deictic means for expressing ideas about meanings. Later, it offers a structural base for grasping the relations of different words and phrases and for cracking the relations hidden in word problems. That is why the percent bar can successfully bridge between both trajectories.

Concerning the reconstruction of students' lexical pathways in research question *Q2b (Which lexical means do students activate and require along their pathway?),* inventorying all lexical means used on different levels by a pair of students yielded a remarkably rich repertoire of expressions. The used lexical means changed with the levels of the dual trajectory, from isolated words to phrases expressing relations and comparisons (which replicates progressions described for other topics e.g. by Schleppegrell, 2004; Gibbons, 2002; Prediger & Wessel, 2013). Furthermore, the use of lexical means varied from mainly self-invented or modified to mainly adopted from written or oral offers (a new finding not mentioned in the literature so far). These traces in the individual inventories allowed on the one hand to refine the lexical offers in the intended learning trajectory. On the other hand, it showed that oral offers of the teachers seem to function as major condition of success which is not yet well understood. This raises a further research questions about the functioning of the interactional micro-scaffolding along the learning trajectory (Prediger & Pöhler, 2016).

An in-depth insight into the processes was gained by pursuing *research question Q2c (intertwinement of the students' conceptual and lexical learning pathways in the process of individual meaning construction)*. The reconstructed semiotic chains unpack students' non-unidirectional pathways, sometimes deviating from the intended learning trajectories, with especially the meaning-related vocabulary being of major importance for approaching the mathematical concepts and relations. The developed method of extended semiotic chains (following Presmeg, 1998) is promising to be transferred also to other projects.

As a whole, the design research study has resulted in substantial design and research outcomes. Of course, the number of investigated students is still too limited to generalize. But already, the study offers important theoretical and empirical contributions to the ongoing discourse on content- and language related learning which is of upper importance in a world with increasing multilingualism and migration. Even if the study is restricted to the limited mathematical topic of percentages, we are optimistic that our approach of a dual conceptual and lexical learning pathway is promising also for other topics in mathematics or science.

Further research in our research group will investigate the possibilities to transfer the design approach and research outcomes to other mathematical

topics. We hope to also pursue the question of sustainable learning outcomes by a quasi-experimental control trial, and the question of role and functioning of the micro-scaffolding by investigating the interaction processes in more detail.

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