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Intuitionistic Linguistic Multiple Attribute Decision-making Based on Heronian Mean Method and Its Application to Evaluation of Scientific Research Capacity

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ABSTRACT

The main focus of this paper is to investigate intuitionistic linguistic information fusion based on Heronian mean. Two new intuitionistic linguistic aggregation operators called intuitionistic linguistic generalized Heronian mean (ILGHM) and intuitionistic linguistic generalized weighted Heronian mean (ILGWHM) operators, are introduced. The ILGHM and ILGWHM operators are characterized by the ability to deal with the intuitionistic linguistic multiple attribute decision making problems in which the attributes are interactive. Some desired properties and special cases with respect to the different parameter values in the developed operators are studied. Furthermore, a method based on the proposed operators is developed to deal with multiple attribute group decision making (MAGDM) method problems. Finally, an illustrative example concerning evaluation of scientific research capacity is provided to illustrate the decision-making process and to discuss the influences of different parameters on the decision-making results.

Keywords: intuitionistic linguistic set, Heronian mean, multiple attribute group decision making, scientific research capacity

INTRODUCTION

Because the objects are often fuzzy and uncertain, the available information involved in multiple attribute group decision-making (MAGDM) problems are not always expressed as real numbers, and sometimes it is better suited to use another approach to deal with this information such as fuzzy set (Zadeh, 1965), linguistic information (Atanassov, 1986) and intuitionistic fuzzy set (IFS) (Herrera and Herrera-Viedma, 2000). Among all the tools, the intuitionistic fuzzy set (IFS) proposed by Atanassov (Herrera and Herrera-Viedma, 2000), is a useful tool to describe and deal with vagueness. A prominent characteristic of IFS is that it assigns to each element a membership degree and a non-membership degree, and thus, it is more powerful to deal with uncertainty and vagueness in real applications than fuzzy set which is only assigns to each element a membership degree (Hsieh and Chan, 2016). The IFS has received more and more attention since its appearance (Xu, 2007; Yu, 2015; Boran and Akay, 2014; Wan, Wang and Dong, 2016; Zeng and Chen, 2015; Zeng and Xiao, 2016; Zeng, Su and Zhang, 2016).

However, in real decision-making problems, it is difficult for decision makers to provide exact numbers for the membership and non-membership degrees of an intuitionistic fuzzy set while it is easy to provide linguistic assessment values. On the basis of the intuitionistic fuzzy set and the linguistic assessment set, Wang and Li (2010) proposed the concept of intuitionistic linguistic set (ILS), whose basic elements are intuitionistic linguistic numbers (ILNs). As a generalization of intuitionistic fuzzy numbers and linguistic variables, the ILN is able to handle the vague characters of things more accurately than linguistic term sets and intuitionistic fuzzy numbers. Wang and Li (2010) also proposed the score function, accuracy function and some operational laws of the intuitionistic linguistic set. The intuitionistic linguistic information fusion and aggregation method has received more and more attention. For example, based on the ordered weighted average (OWA) operator (Yager, 1988), Liu (2013) proposed the intuitionistic linguistic generalized dependent aggregation operator and applied it to group decision making. Liu

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Contribution of this paper to the literature

- This study explores the intuitionistic linguistic Heronian mean method, which can fully capture the interrelationship of the input arguments.
- This study develops an approach to group decision-making based on the ILGWHM operator, which is able to deal with the interrelationships between the attributes with intuitionistic linguistic information.
- The developed evaluation method for scientific research capacity provides a lot of different scenarios for decision makers to select the best one(s) by picking the particular values according to their interests.

and Wang (2014) proposed some intuitionistic linguistic power aggregation operators. Su et al. (2014) presented the intuitionistic linguistic OWA dis-tance (ILOWAD) operator. Ju et al. (2016) extended Maclaurin symmetric mean aggregation operators to intuitionistic linguistic environment.

The generalized OWA (GOWA) operator introduced by Yager (2004) is a very common aggregation method, which uses generalized means in the aggregation process. It generalizes a wide range of aggregation operators such as the generalized mean, the OWA and the ordered weighted geometric (OWG) operator. The GOWA operator has been studied by various authors (Beliakov, Pradera and Calvo, 2007; Merigo and Yager, 2013; Peng, Gao and Gao, 2013; Zeng, Chen and Li, 2016). Another interesting aggregation operator is the Heronian mean (HM), which is developed to deal with the exact numerical values (Beliakov, Pradera and Calvo, 2007). The desirable characteristic of the HM is that it can capture the interrelationship of the input arguments, which makes it very useful in decision-making.

Motivated by the idea of GOWA operator and Heronian mean, in this paper, we propose two new intuitionistic linguistic aggregation operators: intuitionistic linguistic generalized Heronian mean (ILGHM) and intuitionistic linguistic generalized weighted Heronian mean (ILGWHM) operator. Furthermore, some desirable properties of the ILGHM and ILGWHM operators are studied. At the same time, some special cases of the generalized parameters in these operators are analyzed. To do this, the remainder of this paper is organized as follows. In Section 2, we briefly review some basic concepts. Section 3 presents the ILGHM and ILGWHM operators and analyzes a wide range of particular cases. In Section 4, we develop a method for multiple attribute decision making based on the ILGWHM operator. Section 5 presents an illustrative example about scientific research capacity evaluation and Section 6 summarizes the main conclusions found in the paper.

PRELIMINARIES

This section briefly reviews the intuitionistic linguistic set, the GOWA operator and the HM operator.

The Intuitionistic Linguistic Set

The linguistic method is an approximate technique, which represents qualitative aspects as linguistic values by means of linguistic variables (Atanassov, 1986). For convenience, let $S = \{s_\alpha | \alpha = 0,1,\ldots,l\}$ be a finite and totally ordered discrete term set, where s_α represents a possible value for a linguistic variable, $l+1$ is the cardinality of S. For example, a set of seven terms S could be given as follows:

$$S = \{s_0 = extremely\ poor, \quad s_1 = very\ poor, \quad s_2 = poor, \quad s_3 = fair, \quad s_4 = good, \quad s_5 = very\ good, \quad s_6 = extremely\ good\} \tag{1}$$

In these cases, it is usually required that there exist the following (Xu, 2015):

(1) The set is ordered: $s_i > s_j$, if $i > j$.

(2) There is the negation operator: $\mathrm{nes}(s_i) = s_j$, such that $j = l - i$.

In order to preserve all the given information, Xu (2015) extended the discrete term set S to a continuous term set

$\bar{S} = \{s_\alpha | \alpha \in [0, l]\}$, where, if $s_\alpha \in S$, then we call s_α the original term, otherwise, we call s_α the virtual term.

Consider any two linguistic terms $s_\alpha, s_\beta \in \bar{S}$, and $\mu > 0$, some operational laws are defined as follows:

(1) $s_\alpha \oplus s_\beta = s_{\alpha+\beta}$;

(2) $\mu s_\alpha = s_{\mu\alpha}$.

Wang and Li (2010) first proposed the ILS and gave the definition of ILS.

Definition 1. An ILS A in X is defined as

$$A = \{\langle x, s_{\theta(x)}, (\mu_A(x), v_A(x))\rangle | x \in X\} \tag{2}$$

Here $s_{\theta(x)} \in \bar{S}$, and numbers $\mu_A(x)$ and $v_A(x)$ represent, respectively, the membership degree and non-membership degree of the element x to linguistic index $s_{\theta(x)}$, $0 \le \mu_A(x) + v_A(x) \le 1$, for all $x \in X$.

For each ILS A in X, if

$$\pi_A(x) = 1 - \mu_A(x) - v_A(x) \tag{3}$$

then $\pi_A(x)$ is called the indeterminacy degree or hesitation degree of x to linguistic index $s_{\theta(x)}$.

For computational convenience, we denote an ILN by $\tilde{a} = \langle s_{\theta(a)}, (\mu(a), v(a)) \rangle$, where $s_{\theta(a)}$ is a linguistic term, $\mu(a), v(a) \ge 0$, $\mu(a) + v(a) \le 1$.

Let $\tilde{a}_1 = \langle s_{\theta(a_1)}, (\mu(a_1), v(a_1)) \rangle$ and $\tilde{a}_2 = \langle s_{\theta(a_2)}, (\mu(a_2), v(a_2)) \rangle$ be two ILNs and $\lambda \ge 0$, then the operations of ILNs are defined as follows (Wang and Li, 2010; Liu, 2013b):

1) $\tilde{a}_1 + \tilde{a}_2 = \langle s_{\theta(a_1)+\theta(a_2)}, (1 - (1 - \mu(a_1))(1 - \mu(a_2)), v(a_1)v(a_2)) \rangle$;
2) $\tilde{a}_1 \otimes \tilde{a}_2 = \langle s_{\theta(a_1)\times\theta(a_2)}, (\mu(a_1)\mu(a_2), v(a_1) + v(a_2) - v(a_1)v(a_2)) \rangle$;
3) $\lambda\tilde{a}_1 = \langle s_{\lambda\times\theta(a_1)}, \left(1 - (1 - \mu(a_1))^{\lambda}, (v(a_1))^{\lambda}\right) \rangle$;
4) $\tilde{a}_1^{\lambda} = \left\langle s_{(\theta(a_1))^{\lambda}}, \left((\mu(a_1))^{\lambda}, 1 - (1 - v(a_1))^{\lambda}\right) \right\rangle$.

In order to compare two ILNs, Wang and Li (2010) defined the score function and the accuracy function of ILN as follows.

Definition 2. Let $\tilde{a}_1 = \langle s_{\theta(a_1)}, (\mu(a_1), v(a_1)) \rangle$ be an ILN, the score function $S(\tilde{a}_1)$ and accuracy function $H(\tilde{a}_1)$ of an ILN $\tilde{a}_1$ can be represented as follows:

$$S(\tilde{a}_1) = \mu(a_1) + 1 - v(a_1) \times \theta(a_1) \tag{4}$$

$$H(\tilde{a}_1) = (\mu(a_1) + v(a_1)) \times \theta(a_1) \tag{5}$$

Definition 3. If $\tilde{a}_1 = \langle s_{\theta(a_1)}, (\mu(a_1), v(a_1)) \rangle$ and $\tilde{a}_2 = \langle s_{\theta(a_2)}, (\mu(a_2), v(a_2)) \rangle$ are any two ILNs, then:

1) If $S(\tilde{a}_1) > S(\tilde{a}_2)$, then, $\tilde{a}_1 > \tilde{a}_2$;
2) If $S(\tilde{a}_1) = S(\tilde{a}_2)$, then

If $H(\tilde{a}_1) > H(\tilde{a}_2)$, then, $\tilde{a}_1 > \tilde{a}_2$;

If $H(\tilde{a}_1) = H(\tilde{a}_2)$, then, $\tilde{a}_1 = \tilde{a}_2$.

The GOWA Operator

The generalized OWA (GOWA) operator is an extension of the OWA operator with generalized means (Yager, 2004). It provides a wide range of aggregation operators including the OWA operator with its particular cases. The GOWA operator can be defined as follows.

Definition 4. A GOWA operator of dimension n is a mapping GOWA: $R^n \to R$ with an associated weighting vector W of dimension n with $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$, such that:

$$GOWA(a_1, a_2, \ldots, a_n) = \left(\sum_{j=1}^{n} w_j b_j^{\lambda}\right)^{1/\lambda} \tag{6}$$

The GOWA operator is commutative, monotonic, bounded and idempotent. If we look to different values of the parameter λ, we can obtain other special cases such as the usual OWA operator and the OWGA operator.

The Heronian Mean (HM)

The Heronian mean (HM) operator (Beliakov, Pradera and Calvo, 2007) is an important aggregation operator which can capture effectively the relevance between the aggregated arguments. It can be defined as follows.

Definition 5. Let $a_i (i = 1, 2, \ldots, n)$ be a collection of nonnegative numbers. If

$$HM(a_1, a_2, \ldots, a_n) = \frac{2}{n(n+1)} \sum_{i=1}^{n} \sum_{j=1}^{n} \sqrt{a_i a_j} \tag{7}$$

then HM is called the Heronian mean (HM).

The desirable characteristic of the HM is that it can capture the interrelationship of the input arguments, which makes it very effective in decision making. However, the existed HM is only suitable to aggregate input data taken by the forms of crisp numbers rather than any other types of arguments, which restricts the its potential applications to more extensive areas. In next section, we should the HM to process intuitionistic linguistic environment.

INTUITIONSITIC LINGUISTIC GENERALIZED HERONIAN MEAN

Based on the Heronian mean and the GOWA operator, in this section we develop the intuitionistic linguistic generalized Heronian mean (ILGHM) operator. The main advantage of the ILGHM operator is that that it can capture the interrelationship of the input arguments, which makes it very suitable in decision making with intuitionistic linguistic information. It can be defined as follows.

Definition 6. Let $\tilde{a}_i = \langle s_{\theta_i}, (\mu_i, v_i) \rangle$ $(i = 1,2,\ldots,n)$ be a collection of ILNs, $p, q \geq 0$ and p, q do not take the value 0 simultaneously, if

$$ILGHM^{p,q}(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) = \left(\frac{2}{n(n+1)} \sum_{i=1}^{n} \sum_{j=1}^{n} \tilde{a}_i{}^p \tilde{a}_j{}^p \right)^{1/(p+q)} \tag{8}$$

then ILGHM is called the intuitionistic linguistic generalized Heronian mean (ILGHM).

Based on the operational laws of the ILNs described earlier, we can derive the result shown as the Theorem 1.

Theorem 1. Let $\tilde{a}_i = \langle s_{\theta_i}, (\mu_i, v_i) \rangle$ $(i = 1,2,\ldots,n)$ be a collection of ILNs, $p, q \geq 0$ and p, q do not take the value 0 simultaneously. Then the result aggregated value by the ILGHM is still an ILNs, and even

$$ILGHM^{p,q}(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) = \left(\frac{2}{n(n+1)} \sum_{i=1}^{n} \sum_{j=1}^{n} \tilde{a}_i{}^p \tilde{a}_j{}^q \right)^{1/(p+q)}$$

$$= \left\langle s_{\left(\frac{2}{n(n+1)}\sum_{i=1}^{n}\sum_{j=1}^{n}\theta_i^p\theta_j^q\right)^{\frac{1}{p+q}}}, \left(1 - \left(\prod_{i=1,j=1}^{n}\left(1 - u_i^p u_j^q\right)\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{p+q}}, \right.$$

$$\left. 1 - \left(1 - \left(\prod_{i=1,j=1}^{n}\left(1 - (1 - v_i)^p \left(1 - v_j\right)^q\right)\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{p+q}} \right\rangle.$$

Proof. Since $(\tilde{a}_i)^p = \left\langle s_{\theta_i^p}, u_i^p, 1 - (1 - v_i)^p \right\rangle$, $(\tilde{a}_j)^q = \left\langle s_{\theta_i^q}, u_j^q, 1 - (1 - v_j)^q \right\rangle$,

and $(\tilde{a}_i)^p(\tilde{a}_j)^q = \left\langle s_{\theta_i^p\theta_j^q}, u_i^p u_j^q, 1 - (1 - v_i)^p(1 - v_j)^q \right\rangle$,

Then we have

$$\sum_{i=1}^{n}\sum_{j=1}^{n}(\tilde{a}_i)^p\,(\tilde{a}_j)^q = \left\langle s_{\sum_{i=1}^{n}\sum_{j=1}^{n}\theta_i^p\theta_j^q}, 1 - \prod_{i=1,j=1}^{n}\left(1 - u_i^p u_j^q\right), \prod_{i=1,j=1}^{n}\left(1 - (1 - v_i)^p(1 - v_j)^q\right) \right\rangle,$$

$$\frac{2}{n(n+1)}\sum_{i=1}^{n}\sum_{j=1}^{n}(\tilde{a}_i)^p\,(\tilde{a}_j)^q$$

$$= \left\langle s_{\frac{2}{n(n+1)}\sum_{i=1}^{n}\sum_{j=1}^{n}\theta_i^p\theta_j^q}, \right.$$

$$\left. 1 - \prod_{i=1,j=1}^{n}\left(1 - u_i^p u_j^q\right)^{\frac{2}{n(n+1)}}, \left(\prod_{i=1,j=1}^{n}\left(1 - (1 - v_i)^p(1 - v_j)^q\right)\right)^{\frac{2}{n(n+1)}} \right\rangle,$$

So,

$$ILGHM^{p,q}(\tilde{a}_1,\tilde{a}_2,\ldots,\tilde{a}_n)=\left(\frac{2}{n(n+1)}\sum_{i=1}^{n}\sum_{j=1}^{n}\tilde{a}_i{}^p\tilde{a}_j{}^q\right)^{1/(p+q)}$$

$$=\left\langle s_{(\frac{2}{n(n+1)}\sum_{i=1}^{n}\sum_{j=1}^{n}\theta_i^p\theta_j^q)^{\frac{1}{p+q}}},(1-\prod_{i=1,j=1}^{n}\left(1-u_i^pu_j^q\right)^{\frac{2}{n(n+1)}})^{\frac{1}{p+q}},\right.$$

$$\left.1-(1-(\prod_{i=1,j=1}^{n}(1-(1-v_i)^p(1-v_j)^q))^{\frac{2}{n(n+1)}})^{\frac{1}{p+q}}\right\rangle.$$

which completes the proof of Theorem 1.

Moreover, the ILGHM also has the following desirable properties:

Property 1 (**Idempotency**). Let $\tilde{a}_i=\langle s_{\theta_i},(\mu_i,v_i)\rangle$ $(i=1,2,\ldots,n)$be a collection of ILNs, $p,q\geq 0$ and p,q do not take the value 0 simultaneously. Then

$$ILGHM^{p,q}(\tilde{a}_1,\tilde{a}_2,\ldots,\tilde{a}_n)=ILGHM^{p,q}(\tilde{a},\tilde{a},\ldots,\tilde{a})=\tilde{a}.$$

Property 2 (**Monotonicity**). Let $(\tilde{a}_1,\tilde{a}_2,\ldots,\tilde{a}_n)$ and $(\widetilde{a'}_1,\widetilde{a'}_2,\ldots,\widetilde{a'}_n)$ be two collections of ILNs, if $\widetilde{a'}_i\leq\tilde{a}_i$ for all $i(i=1,2,\cdots,n)$, then

$$ILGHM^{p,q}(\widetilde{a'}_1,\widetilde{a'}_2,\ldots,\widetilde{a'}_n)\leq ILGHM^{p,q}(\tilde{a}_1,\tilde{a}_2,\ldots,\tilde{a}_n).$$

Property 3 (**Bounded**). The $ILGHM$ operator lies between the max and min operators, i.e.,

$$\min(\tilde{a}_1,\tilde{a}_2,\ldots,\tilde{a}_n)\leq ILGHM^{p,q}(\tilde{a}_1,\tilde{a}_2,\ldots,\tilde{a}_n)\leq\max(\tilde{a}_1,\tilde{a}_2,\ldots,\tilde{a}_n).$$

Property 4. (**Permutation**) Let $\tilde{a}_i=\langle s_{\theta_i},(\mu_i,v_i)\rangle(i=1,2,\ldots,n)$be a collection of ILNs, then

$$ILGHM^{p,q}(\tilde{a}_1,\tilde{a}_2,\ldots,\tilde{a}_n)=ILGHM^{p,q}(\dot{\tilde{a}}_1,\dot{\tilde{a}}_2,\ldots,\dot{\tilde{a}}_n).$$

where $(\dot{\tilde{a}}_1,\dot{\tilde{a}}_2,\ldots,\dot{\tilde{a}}_n)$ is any permutation of $(\tilde{a}_1,\tilde{a}_2,\ldots,\tilde{a}_n)$.

By assigning different values to the parameters p and q, we can get some special cases of the ILGHW, such as:

(1) If $p=q=1/2$, then the $ILGHM^{p,q}$ reduces to

$$ILGHM^{1/2,1/2}(\tilde{a}_1,\tilde{a}_2,\ldots,\tilde{a}_n)$$

$$=\left\langle s_{\frac{2}{n(n+1)}\sum_{i=1}^{n}\sum_{j=1}^{n}\theta_i^{\frac{1}{2}}\theta_j^{\frac{1}{2}}},1-\left(\prod_{i=1,j=1}^{n}\left(1-u_i^{1/2}u_j^{1/2}\right)\right)^{\frac{2}{n(n+1)}},\right.$$

$$\left.\left(\prod_{i=1,j=1}^{n}\left(1-(1-v_i)^{1/2}\left(1-v_j\right)^{1/2}\right)\right)^{\frac{2}{n(n+1)}}\right\rangle.$$

which we call the intuitionistic linguistic Heronian mean.

(2) if $p=q=1$, then the $ILGHM^{p,q}$ reduces to

$$ILGHM^{1,1}(\tilde{a}_1,\tilde{a}_2,\ldots,\tilde{a}_n)$$

$$=\left\langle s_{\frac{2}{n(n+1)}\sum_{i=1}^{n}\sum_{j=1}^{n}\theta_i\theta_j},\left(1-\left(\prod_{i=1,j=1}^{n}\left(1-u_i\ u_j\ \right)^{\frac{2}{n(n+1)}}\right)\right)^{\frac{1}{2}},\right.$$

$$\left.1-\left(\left(\prod_{i=1,j=1}^{n}\left(1-(1-v_i)\left(1-v_j\right)\right)\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{2}}\right\rangle.$$

which we call the intuitionistic linguistic generalized interrelated square mean.

It is easy to see that in the $ILGHM^{p,q}$ operator, we only consider the input parameters and their interrelationships, and do not consider the importance of each aggregated datum. However, in many practical situations, the weight of input data is also an important parameter. So, we should define an intuitionistic linguistic generalized weighted Heronian mean (ILGWHM) operator.

Definition 7. Let $\tilde{a}_i = \langle s_{\theta_i}, (\mu_i, v_i)\rangle (i = 1,2,\dots,n)$ be a collection of ILNs, $p, q \geq 0$. $w = (w_1, w_2, \dots, w_n)^T$ is the weight vector of $\tilde{a}_i$ $(i = 1,2,\dots,n)$, where indicates the importance degree of $\tilde{a}_i (i = 1,2,\dots,n)$, satisfying $w_i \geq 0$ and $\sum_{i=1}^{n} w_i = 1$. If

$$ILGWHM(\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_n)^{p,q} = \left(\frac{2}{n(n+1)}\sum_{i=1}^{n}\sum_{j=1}^{n}(nw_i\tilde{a}_i)^p \otimes \left(nw_j\tilde{a}_j\right)^p\right)^{1/(p+q)}. \quad (9)$$

then ILGWHM is called the intuitionistic linguistic generalized weighted Heronian mean (ILWGHM) operator.

Similar to Theorem 1, the theorem 2 can be derived easily.

Theorem 2. Let $\tilde{a}_i = \langle s_{\theta_i}, (\mu_i, v_i)\rangle (i = 1,2,\dots,n)$ be a collection of ILNs, $p, q \geq 0$. $w = (w_1, w_2, \dots, w_n)^T$ is the weight vector of $\tilde{a}_i (i = 1,2,\dots,n)$, satisfying $w_i \geq 0$ and $\sum_{i=1}^{n} w_i = 1$. Then the aggregated value by using the $ILGWHM(\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_n)^{p,q}$ is also an ILNs, and

$$\begin{aligned}
ILGWHM(\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_n)^{p,q} &= \left(\frac{2}{n(n+1)}\sum_{i=1}^{n}\sum_{j=1}^{n}(nw_i\tilde{a}_i)^p \otimes \left(nw_j\tilde{a}_j\right)^p\right)^{1/(p+q)} \\
&= \left\langle s_{\left(\frac{2}{n(n+1)}\sum_{i=1}^{n}\sum_{j=1}^{n}(nw_i\theta_i)^p(nw_j\theta_j)^q\right)^{\frac{1}{p+q}}}, \left(1 \right.\right. \\
&\quad \left. - \prod_{i=1,j=1}^{n}\left(1-(1-(1-\mu_i)^{nw_i})^p\left(1-\left(1-\mu_j\right)^{nw_j}\right)^q\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{p+q}}, \\
&\quad \left. 1-\left(1-\prod_{i=1,j=1}^{n}\left(1-(1-(v_i)^{nw_i})^p\left(1-(v_j)^{nw_j}\right)^q\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{p+q}}\right\rangle
\end{aligned}$$

Similarly to the $ILGWHM(\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_n)^{p,q}$operator, by using a different manifestation in the parameters p and q, we are able to obtain a wide range of particular types of $ILGWHM(\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_n)^{p,q}$ operator.

Theorem 3. The ILGHM operator is a special case of the ILWGHM operator.

Proof. Let $w = \left(\frac{1}{n}, \frac{1}{n}, \dots, \frac{1}{n}\right)$, then

$$ILGWHM(\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_n)^{p,q} = \left(\tfrac{2}{n(n+1)}\textstyle\sum_{i=1}^{n}\sum_{j=1}^{n}(nw_i\tilde{a}_i)^p \otimes \left(nw_j\tilde{a}_j\right)^p\right)^{1/(p+q)} =$$
$$ILGWHM(\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_n)^{p,q} = \left(\tfrac{2}{n(n+1)}\textstyle\sum_{i=1}^{n}\sum_{j=1}^{n}\left(n\frac{1}{n}\tilde{a}_i\right)^p \otimes \left(n\frac{1}{n}\tilde{a}_j\right)^p\right)^{1/(p+q)} =$$
$$\left(\tfrac{2}{n(n+1)}\textstyle\sum_{i=1}^{n}\sum_{j=1}^{n}(\tilde{a}_i)^p \otimes \left(\tilde{a}_j\right)^p\right)^{1/(p+q)} = ILGWHM(\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_n)^{p,q}$$

which completes the proof of the theorem.

AN APPROACH TO GROUP DECISION-MAKING BASED ON THE ILGWHM OPERATOR

In many actual decision problems, there exist the interrelationships between the attributes (Chang & Wang, 2016). At the same time, because of fuzziness of the attributes, they can be easily expressed by the intuitionistic linguistic variables. Thus, it is necessary to propose a decision-making approach based on the ILGWHM operator to deal with the interrelationships between the attributes with intuitionistic linguistic information.

Let $A = \{A_1, A_2, \dots, A_m\}$be a discrete set of alternatives, and $C = \{C_1, C_2, \dots, C_n\}$ be the set of attributes, whose weight vector is $w = (w_1, w_2, \dots, w_n)^T$, satisfying $w_j \geq 0$ and $\sum_{j=1}^{n} w_j = 1$. Let $E = \{e_1, e_2, \dots, e_t\}$be the set of decision makers (whose weight vector is $\lambda = (\lambda_1, \lambda_2, \dots, \lambda_t)^T$, $\lambda_k \geq 0, A^k = \left(\tilde{a}_{ij}^k\right)_{n\times m}$). Suppose that $A^k = \left(\tilde{a}_{ij}^k\right)_{n\times m}$ is the decision matrix, where $\tilde{a}_{ij}^k = \left\langle s_{\theta(a_{ij}^k)}, \left(\mu(a_{ij}^k), v(a_{ij}^k)\right)\right\rangle$takes the form of the ILN, given by the decision maker e_k, for alternative A_i with respect to the criterion C_j, and $s_{\theta(a_{ij}^k)} \in S$, $0 \leq \mu(a_{ij}^k) \leq 1$, $0 \leq v(a_{ij}^k) \leq 1$, $\mu(a_{ij}^k) + v(a_{ij}^k) \leq 1$.

Based on the ILGWHM operator, an approach is given for multi-criteria decision making under intuitionistic linguistic environment:

Table 1. Decision matrix-Expert 1

	C_1	C_2	C_3	C_4
A_1	$\langle s_5,(0.2,0.7)\rangle$	$\langle s_2,(0.4,0.6)\rangle$	$\langle s_5,(0.5,0.5)\rangle$	$\langle s_3,(0.2,0.6)\rangle$
A_2	$\langle s_4,(0.4,0.6)\rangle$	$\langle s_5,(0.4,0.5)\rangle$	$\langle s_3,(0.1,0.8)\rangle$	$\langle s_4,(0.5,0.5)\rangle$
A_3	$\langle s_3,(0.2,0.7)\rangle$	$\langle s_4,(0.2,0.7)\rangle$	$\langle s_4,(0.3,0.7)\rangle$	$\langle s_5,(0.2,0.7)\rangle$
A_4	$\langle s_6,(0.5,0.4)\rangle$	$\langle s_2,(0.2,0.8)\rangle$	$\langle s_3,(0.2,0.6)\rangle$	$\langle s_3,(0.3,0.6)\rangle$

Table 2. Decision matrix- Expert 2

	C_1	C_2	C_3	C_4
A_1	$\langle s_4,(0.1,0.7)\rangle$	$\langle s_3,(0.2,0.7)\rangle$	$\langle s_3,(0.2,0.8)\rangle$	$\langle s_6,(0.4,0.5)\rangle$
A_2	$\langle s_4,(0.4,0.5)\rangle$	$\langle s_5,(0.3,0.6)\rangle$	$\langle s_3,(0.2,0.6)\rangle$	$\langle s_3,(0.2,0.7)\rangle$
A_3	$\langle s_4,(0.2,0.6)\rangle$	$\langle s_4,(0.2,0.7)\rangle$	$\langle s_2,(0.4,0.6)\rangle$	$\langle s_3,(0.3,0.7)\rangle$
A_4	$\langle s_5,(0.3,0.6)\rangle$	$\langle s_4,(0.4,0.5)\rangle$	$\langle s_2,(0.3,0.6)\rangle$	$\langle s_4,(0.2,0.6)\rangle$

Table 3. Decision matrix- Expert 3

	C_1	C_2	C_3	C_4
A_1	$\langle s_5,(0.2,0.6)\rangle$	$\langle s_3,(0.3,0.7)\rangle$	$\langle s_4,(0.4,0.5)\rangle$	$\langle s_4,(0.2,0.7)\rangle$
A_2	$\langle s_4,(0.3,0.7)\rangle$	$\langle s_5,(0.3,0.6)\rangle$	$\langle s_2,(0.1,0.8)\rangle$	$\langle s_3,(0.1,0.8)\rangle$
A_3	$\langle s_4,(0.2,0.7)\rangle$	$\langle s_5,(0.3,0.6)\rangle$	$\langle s_1,(0.1,0.8)\rangle$	$\langle s_4,(0.2,0.7)\rangle$
A_4	$\langle s_3,(0.2,0.7)\rangle$	$\langle s_3,(0.1,0.7)\rangle$	$\langle s_4,(0.3,0.6)\rangle$	$\langle s_5,(0.4,0.5)\rangle$

Table 4. The collective decision matrix A

	C_1	C_2	C_3	C_4
A_1	$\langle s_{4.68},(0.17,0.68)\rangle$	$\langle s_{2.56},(0.31,0.66)\rangle$	$\langle s_{4.08},(0.39,0.58)\rangle$	$\langle s_{4.24},(0.27,0.59)\rangle$
A_2	$\langle s_{3.60},(0.20,0.67)\rangle$	$\langle s_{4.28},(0.23,0.67)\rangle$	$\langle s_{2.52},(0.29,0.69)\rangle$	$\langle s_{4.08},(0.23,0.39)\rangle$
A_3	$\langle s_{4.32},(0.37,0.59)\rangle$	$\langle s_{4.36},(0.34,0.56)\rangle$	$\langle s_{3.04},(0.13,0.73)\rangle$	$\langle s_{3.40},(0.39,0.59)\rangle$
A_4	$\langle s_{4.84},(0.36,0.53)\rangle$	$\langle s_{2.92},(0.25,0.66)\rangle$	$\langle s_{4.96},(0.26,0.60)\rangle$	$\langle s_{3.88},(0.30,0.57)\rangle$

Step 1. Utilize the intuitionistic linguistic weighted average (ILWA) operator in Eq.(10) to aggregate all the decision matrices $A^k(k=1,\ldots,t)$ into a collective decision matrix $A=(\tilde{a}_{ij})_{m\times n}$, where $\lambda=(\lambda_1,\lambda_2,\ldots,\lambda_t)^T$ is the weight vector of decision makers.

$$\begin{aligned}\tilde{a}_{ij} &= \left\langle s_{\theta(a_{ij})}, \left(\mu(a_{ij}), v(a_{ij})\right)\right\rangle = ILWA\left(\tilde{a}^1_{ij}, \tilde{a}^2_{ij}, \ldots, \tilde{a}^t_{ij}\right) \\ &= \left\langle s_{\sum_{k=1}^t \lambda_k \theta(a^k_{ij})}, \left(1-\prod_{k=1}^t (1-\mu(a^k_{ij}))^{\lambda_k}, \prod_{k=1}^t (v(a^k_{ij}))^{\lambda_k}\right)\right\rangle.\end{aligned} \tag{10}$$

Step 2. Utilize the ILGWHM operator

$$\tilde{a}_i = \langle s_{\theta(a_i)}, (\mu(a_i), v(a_i))\rangle = ILGWHM(\tilde{a}_{i1}, \tilde{a}_{i2}, \ldots, \tilde{a}_{in})^{p,q}, i=1,2,\ldots,m$$

to derive the collective overall preference values $\tilde{a}_i$ $(i=1,2,\ldots,m)$ the alternative $A_i(i=1,2,\ldots,m)$, where $w=(w_1,w_2,\ldots,w_n)^T$ is the weight vector of $\tilde{a}_i$, satisfying $w_j\geq 0$ and $\sum_{j=1}^n w_j=1$.

Step 3. Calculate the score function $S(\tilde{a}_i)$ and accuracy function $H(\tilde{a}_i)$ (if necessary) of the collective overall values $\tilde{a}_i$ ($i=1,2,\ldots,m$)) to rank all the alternatives A_i $(i=1,2,\ldots,m)$) and then to select the best one(s).

Step 4. Rank all the alternatives A_i $(i=1,2,\ldots,m)$) and select the best one(s) in accordance with $S(\tilde{a}_i)$.

Step 5. End.

EXAMPLE

Next, we give an example concerning evaluation of scientific research capacity to illustrate the proposed method. The scientific research capacity evaluation is crucial to the development and planning of discipline, thus raising the ability of scientific research is a central task of discipline construction. Currently, Zhejiang Gongshang University will increase the intensity of investment in scientific research, and tries to select a subject from all disciplines for pilot study. Four alternatives $(A_i, i=1,2,3,4)$ will be evaluated to confirm which is fitted best. Three experts $(e_i, i=1,2,3)$ from administration establishes the panel of decision makers that will take the whole responsibility for this evaluation (the weight vector of experts is $\lambda=(0.4,0.28,0.32)^T$). They evaluate the subjects $(A_i, i=1,2,\ldots,m)$ according to following four aspects, size of the research team C_1, scientific influence of research output C_2, research platform construction level C_3 and achievement transformation C_4, respectively, the intuitionistic linguistic decision matrices $A^k=\left(\tilde{a}^k_{ij}\right)_{4\times 4}(k=1,2,3)$ as shown in **Tables 1-3**.

Step 1. Utilize the ILWGA operator to aggregate all the decision matrices $A^k=\left(\tilde{a}^k_{ij}\right)_{4\times 4}$ $(k=1,2,3)$ into a collective decision matrix $A=\left(\tilde{a}_{ij}\right)_{4\times 4}$ (**Table 4**).

Table 5. Ordering of the alternatives by utilizing the different p and q in ILGWHM operator

p,q	Ordering
$p=q=\frac{1}{2}$	$A_1 \succ A_4 \succ A_3 \succ A_2$
$p=1, q=0$	$A_3 \succ A_4 \succ A_1 \succ A_2$
$p=0, q=1$	$A_3 \succ A_4 \succ A_2 \succ A_1$
$p=2, q=1$	$A_3 \succ A_4 \succ A_2 \succ A_1$
$p=1, q=2$	$A_4 \succ A_3 \succ A_1 \succ A_2$
$p=2, q=2$	$A_4 \succ A_3 \succ A_1 \succ A_2$

Step 2. Suppose that the weight vector of four attributes is $w=(0.32,0.26,0.18,0.24)^T$. Utilize the ILGWHM operator in Eq. (8) (suppose $p=q=1$) to derive the collective overall preference values $\tilde{a}_i$ of alternative A_i ($i=1,2,3,4$). We can get

$$\tilde{a}_1=\langle s_{4.966},0.274,0.634\rangle, \tilde{a}_2=\langle s_{4.677},0.257,0.648\rangle,$$
$$\tilde{a}_3=\langle s_{4.907},0.389,0.610\rangle, \tilde{a}_4=\langle s_{5.227},0.299,0.601\rangle.$$

Step 3. Calculate the expected values $S(\tilde{a}_i)$ ($i=1,2,3,4$) of the collective overall intuitionistic linguistic preference values $\tilde{a}_i (i=1,2,3,4)$

$$S(\tilde{a}_1)=3.178, S(\tilde{a}_2)=2.848, S(\tilde{a}_3)=3.823, S(\tilde{a}_4)=3.648.$$

Step 4. Rank all the alternatives A_i ($i=1,2,3,4$) in accordance with the expected values $S(\tilde{a}_i) (i=1,2,3,4)$ of the collective overall intuitionistic linguistic preference values $\tilde{a}_i (i=1,2,3,4)$, we can get

$$A_3 \succ A_4 \succ A_1 \succ A_2,$$

and thus the most desirable alternative is A_3.

If we use the different values p and q in above methods to rank the alternatives, the ordering of the alternatives may be different. The ranking results are listed in **Table 5**.

As we can see, depending on different p and q in this example, the ranking of the alternatives may be different. Comparing with evaluation method proposed by Yu (2013), the main advantage of the method in this paper is that it can deal with the interactions between the attributes. Moreover, it provides a lot of different scenarios for decision makers to select the best one(s) by picking the particular values p and q according to their interests.

CONCLUSION

In this paper, we have extended the traditional Heronian mean operator to process the intuitionistic linguistic information. We have proposed the intuitionistic linguistic generalized Heronian mean (ILGHM) and the intuitionistic linguistic generalized weighted Heronian mean (ILGWHM). The prominent char-acteristic of the ILGHM and ILGWHM operators is that they can not only accommodate the intuitionistic linguistic environment, but also consider the inter-dependent phenomena among the criteria. Based on the developed operators, we have also presented an application of the new approach to a group decision-making problem about the evaluation of scientific research capacity. The example shows that the proposed method is very flexible because it can provide the decision makers more choices to select the particular cases by assigning different parameter values for the operator.

In future research we expect to develop further extensions by adding new characteristics in the problem such as the use of order-inducing variables. We will also consider other decision-making applications such as human resource management and product management.

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