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Investigating Preservice Mathematics Teachers' Manipulative Material Design Processes

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Students use concrete manipulatives to form an imperative affiliation between conceptual and procedural knowledge (Balka, 1993). Hence, it is necessary to design specific mathematics manipulatives that focus on different mathematical concepts. Preservice teachers need to know how to make and use manipulatives that stimulates students' thinking as it is a crucial competency, which they will need during their careers. The goal of the present study was to investigate the manipulative material design processes of preservice mathematics teachers. Data were gathered by using interviews and questionnaires to explore how the preservice teachers devise new manipulatives. It was concluded that the preservice mathematics teachers are struggling to develop new ideas for an appropriate manipulative material design. They encountered structural difficulties when they attempted to transform their ideas into concrete models. Moreover, their ideas and designs sometimes can be very different from manipulatives designed by experts.

Keywords: teacher education, preservice secondary mathematics teachers, manipulative material

INTRODUCTION

The National Council of Teachers of Mathematics stipulates representation as one of the process standards of manipulatives in teaching mathematics (NCTM, 2000). According to the representational view of the mind, mathematical understanding alludes to the manifestation of innate representations of mathematical ideas. (Puchner et al., 2008). Goldin and Shteingold (2001) clarifies that an individual assigns the internal representation to a mathematical idea or process while incorporating problem solving, language usage, and visual imagery. They also assert that an external representation is a substance that is not easily comphrended; a mathematical idea incorporates visuals aids such as graphs, symbols and equations.

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State of the literature

- The manipulatives support student's learning/understanding.
- Several curriculums recommended to use of manipulatives at all levels.
- Preservice teachers should to learn how to use manipulatıves.

Contribution of this paper to the literature

- This paper described the preservice teachers' experiences in the process of manipulative development.
- In this research, the difficulties of preservice teachers in manipulative development and ways of getting rid of these difficulties are shown.
- In this research it's shown that manipulative design process could help preservice teachers' development of knowledge about students.

We use various manipulatives for helping students form internal representations. Students give meaning to a mathematical idea by using these concrete tools.

Bellonio (2012) defined manipulatives as "small, usually very ordinary objects that can be touched and moved by students to introduce or reinforce a mathematical concept". They can be in many forms, like geo-boards, tangrams, graph paper, empty egg cartons, etc. They represent explicitly mathematical ideas that are abstract. Manipulatives possesses not only visual but also physical appeal, to learners which in turn can gain hands on experiences. (Moyer, 2001). Heddens (1986) states that students who have experience with solid objects begin to better grasp and develop clearer mental images as opposed to those with limited concrete experiences. Furthermore, children who learn mathematics with manipulatives are better equipped to integrate the abstract world of mathematics with the everyday world they live in (Dienes, 1960).

Educational research has shown that when manipulatives are practically applied, students begin to construct their own mathematical understanding which in turn allows for an invaluable learning experience. (Boggan, Harper & Whitmire, 2010). They provide the opportunity for experiences related to mathematics and the construction of mathematical knowledge specifically allowing students to make a vital linkage between conceptual and procedural knowledge. Moreover, they help students identify the connections between different branches of mathematics, along with viewing mathematics as one entity (Balka, 1993).

Numerous research studies support (Kelly, 2006; Raphael & Wahlstrom, 1989; Sowell, 1989) and use manipulatives in professional development workshops for teachers. Bellonio (2012) believes that manipulatives are particularly important in helping children make a fluid transition from the concrete to the abstract level. Manipulatives are beneficial in their ability to provide cognitive models for not only abstract mathematical ideas but also abstract mathemical processes. Ojose & Sexton (2009) said that the instruction which uses manipulative materials that students can relate to, such as in the form of money (magnetic coins), has greatly aided in the achievement of students. Students have even gone as far as becoming self-directing when given the ability to choose teacher provided manipulatives then other more traditional aids. (e.g., Moyer & Jones, 2004, as cited in Ojose & Sexton, 2009).

In mathematics, current studies show students are active participants who frequently create and decipher information from their experiences. As a pedagogical device, manipulatives serve as representations for students to express a concept. So the structure of the manipulative should give the opportunity for achieving the true expressions and constructions. Mathematical learning is a constructive process and teachers should prepare the suitable tools and materials for their students since certain manipulative materials can be subjective and open to each student's own misinterpretation (Ball, 1992). Ball (1992) states that teachers could take the initiative of guessing the value of manipulative materials due to their familiarity with the concepts being illustrated. They would have deep understanding about the representations and they do not need prior understanding like a child. As a result, developing and choosing suitable manipulatives is crucial for students' understanding.

Stacey et al. (2001) discussed the contribution of instructional physical materials to students' understanding of mathematical ideas. They considered the transparency of instructional materials as a product of three factors: the specific processes of engagement with the materials by individual students in individual classrooms and settings, the epistemic fidelity of the materials, which is not dependent on the use by the students but more importantly on the mathematical domain and materials themselves, and the accessibility of the materials which is not student specific but more focused on social or psychological factors that occur in the use and representations of these materials by students. They claim that the effectiveness of physical materials in classrooms cannot be predicted on the basis of epistemic fidelity alone, but this does not mean that it depends entirely on individual processes. The accessibility of the materials, working across general groups of students can also be taken into account.

The researchers also emphasize the long-term use of manipulatives which is more effective than short-term use (Ojose & Sexton, 2009; Ruzic & O'Connell, 2001; Sowell, 1989). For example, Sowell (1989) studies kindergarten through post-secondary students to investigate the effectiveness of various types of manipulatives. By involving a meta-analysis of 60 additional research studies, she concludes that achievement can be increased through long-term use of manipulatives.

To sum up manipulatives have very important role in helping students to develop mathematical concepts and ideas (Swan & Marshall, 2010). Thus, several curriculums in the world in particular the National Council of Teachers of Mathematics (NTCM) encourages the use of manipulatives at all levels (NCTM, 2000). Similarly, in Turkey use of materials have been recommended to be used at all levels. Hence, mathematics teachers should learn how to use these materials in their teaching (Cakiroglu & Tuncay, 2007). In fact, Standard 4 (Knowing Mathematical Pedagogy) of Standards for the Professional Development of Teachers of Mathematics states that:

> The preservice and continuing education of teachers of mathematics should develop teachers' knowledge of and ability to use and evaluate instructional materials and resources, including technology. (NCTM, 1991, p. 151)

In a similar vein, the General Directorate of Teacher Training (GDTT) in Turkey put forward some competencies and standards on evaluating teachers' knowledge about usage of materials under the heading "Implementing and developing appropriate sources, materials and technologies". This involves the following:

(1) The teacher uses the materials (compass, ruler, protractor, etc.) that are necessary for mathematics teaching.

(2) The teacher uses attainable sources and materials by making some changes if necessary.

(3) The teacher develops required materials related to mathematics teaching (GDTT, 2011).

These standards show that the GDTT places emphasis on developing and producing new materials. Besides their effective use, it is necessary for the teacher to select and present the appropriate materials to support the teaching process. The teachers not only know how to use materials effectively, but they also should develop and produce new materials if necessary. Hence prospective teachers should be trained according to these professional requirements. This training might be in the context of developing their professional knowledge base (Cakiroglu & Tuncay, 2007).

Subject Matter Knowledge for Teaching and Designing a Manipulative

Researchers have proposed several categories about the subject matter knowledge of teachers: content knowledge (organization and amount of knowledge in the teachers' mind), pedagogical content knowledge (the knowledge used for translating subject matter knowledge to students), and curriculum knowledge (awareness about the methodization and organization of topics within a school year and the use of curriculum resources) (Shulman et al., 1986). Grossman (1990, as cited in Kahan, Cooper & Bethea, 2003) reorganized this categories: subject matter knowledge, general pedagogical knowledge, pedagogical content knowledge and knowledge of context. Along with these categories, practical knowledge is seen as an important component of the knowledge base that underlies all actions by teachers (Carter, 1990; Elbaz, 1983; Verloop, 1992). According to Van Driel, Verloop and Vos (1998), pedagogical content knowledge is a specific type of a practical knowledge. Practical Knowledge (PK) represents the knowledge that influences teachers' actions in practice (Johnston, 1992). It includes teachers' knowledge of classroom situations and the practical dilemmas they face in carrying out purposeful action in these settings (Carter, 1990). A teacher uses PK to adapt themselves to situations, or to shape situations in teaching environments. Also, they make selections among available choices. In this study, Johnston (1992) believes that practical knowledge counsels teachers' actions in a classroom. This particular category includes teachers' knowledge of classroom situations and the real-world dilemmas they face in carrying out purposeful action in a classroom environment (Carter, 1990). PK is used to aid a teacher through certain conditions or alter the conditions in a classroom.

Hence we need to provide opportunities for prospective teachers to develop their PK during their university education. They should encounter situations where they need to give certain decisions about materials that support the development of specific mathematical concepts. They need to think about how certain materials support learning. While they are thinking about certain materials they also think about the related concept, this concept's place in the curriculum, students' difficulties about the concept etc. (Marshall & Swan, 2008). As a result prospective teachers develop their PCK while they think about materials usage and development.

Such thinking on materials also help prospective teachers about incorporating materials proposed by curriculum developers in their teaching environments. Thus, it is important to prepare prospective teachers about choosing and developing manipulatives as a teaching material. It is imperative to learn what teachers choose as a manipulative or how they prepare a manipulative in order to build knowledge.

This study focuses on the teacher's perspective of designing and preparing concrete manipulative materials. It is aimed at discussing the manipulative choice of teachers and discovering a mathematical relationship for the concrete modeling of abstract mathematical ideas through the development of an appropriate manipulative. The aim is to discuss teachers' manipulative choices and to establish the mathematical correlation through the development of an appropriate manipulative for the concrete modeling of abstract mathematical ideas.

While it is virtually impossible to make a perfect manipulative, which shows the mathematical concept directly, it might be possible to provide the learner with a meaningful task environment. Since experience is an important source of PK, previous experiences that involve preparing teaching materials are crucial for mathematics teachers. In this study, the researcher allows prospective teachers to elaborate further on the concepts of theory and practice by developing materials. The researcher also asked prospective teachers to produce classroom vignettes in which they use their prepared materials. In this way the researcher aimed to help prospective teachers' develop their knowledge about using materials in the classroom.

METHODS

Context of the Study

The participants of this study were 20 preservice secondary mathematics teachers who were enrolled in the Instructional Technologies and Material Development course. This course was given to fourth year Mathematics Education department's students. The participants in this study completed their fundamental subject matter courses such as Calculus, Advanced Calculus, Algebra I-II, Linear Algebra, Topology, Set Theory and Topology etc. and they also completed fundamental pedagogical courses such as Introduction to Educational Sciences, the Psychology of Development, System of Turkish Education and School Management, Classroom Management, Approaches and Theories of Teaching and Learning etc. In the fifth year prospective teachers take School Experience and School Practice. In the semester when the study carried out participant were also enrolled to Teaching Methods in Mathematics Education course.

The researcher was also instructor of the participants. And the researcher designed the course content according to research aims. More information will be provided in the following section. To monitor preservice teachers' processes of material development, basic qualitative research was conducted over 14 weeks. Participants were separated into groups of five and they took notes when the activities were conducted. In addition, preservice teachers' reports were examined in detail and noted at the conclusion of the semester.

Procedure

In the first two weeks of the course, the researcher reviewed examples of materials with participants and discussed what is expected in materials. Later, the preservice teachers required to develop original material. Materials that were prepared by participants had to be appropriate according to the secondary mathematics curriculum. With this in mind, the researcher also asked the preservice teachers to explain the material that they designed based on the following criteria:

- originality: the material proposal should be original that is it has not been used by previous semesters' prospective teachers or it should not be copied from the Internet or books.
- the specific concept being taught: it should be related teaching of a particular concept
- the concept images that students had to develop: the material should help students to develop appropriate concept images,
- misconceptions that may arise: the material should not cause any misconceptions or misunderstandings,
- flexibility in different activities: although the material should be related to only one concept different activities should be developed using the material,
- ability to modify: the material should be modified so that different activities could be developed,
- cost: it should not be too expensive,

Throughout the 12 weeks, the researcher and group discussed the processes of developing material. To achieve this goal, discussions were conducted about the materials that were prepared by the preservice teachers. These discussions took 20 minutes on average. In these discussions, materials created by preservice teachers were examined by both the researcher and other preservice teachers based on above criteria. After the researcher and preservice teachers agreed that the materials fit the criteria, the participants started to prepare a demonstration of the materials. Next, the researcher and the preservice teachers examined the feasibility of the demonstrations. After discussing the demonstrations, the preservice teachers began to prepare the materials. During this process, the researcher asked the preservice teachers a variety of questions. Questioning them was necessary ensure the materials used would match up accordingly to the intended demonstration. The line of questioning included “Where did this idea come from?”, “What do you exactly do?”, “How did you plan this material?”, “How will this work?”, “What materials would you use?”, and “Is this material matching your idea?”. After completing the material development processes, each participant was asked to prepare a video and guidelines on the use of the material. Such a request was necessary to see how the material works and what it does. These guidelines and videos were examined in detail by the researcher.

Data Analysis

Since the aim of this research was to describe prospective teachers’ experiences during the process of concrete material development, the researchers’ interview and observation notes analysed qualitatively by descriptive analysis technique. The data is organized firstly considering its related phase in the development process. The development process had two phases: “Developing the idea (thought process)” and “Production process”. The analysis of the first phase focused on the important themes in the experiences of prospective teachers’ idea development. This focus produced three main themes. Similarly, analysis of the second phase is carried out by focusing on determining the common themes in the production process which resulted in three main themes. All of these themes and their descriptions will be presented in the next section.

FINDINGS

Findings about Material Developing Process

After the participants were separated into groups of five, they were asked to negotiate about the material that they had prepared together. During these negotiations, notes were taken concerning their ideas and opinions about the materials. Two categories and sub-categories were subsequently assigned as a result of the analysis of the notes. These categories are related to the processes that the participants experienced. During the first phase the participants focused on suggesting an idea and in the second phase they attempted to put this idea into practice.

A. Developing the Idea (Thought Process)

First, the participants sought to develop an idea that is appropriate for the suggested conditions. After coming up with an appropriate idea, they questioned how realistic and authentic the idea was. During this processes it was observed that prospective teachers passes through three stages:

Raw ideas

The first opinions were developed without examining the curriculum. These were the opinions of the participants’ experiences and difficulties they encountered previously in learning mathematics. The opinions were mainly about solving a problem rather than about a concept or teaching that concept.

Some of the ideas of the participants were not appropriate for high school. Although the ideas were supposed to be based on secondary education curriculum, the participants proposed ideas from primary school curriculum. As a result, some of the opinions were not appropriate for the cognitive enhancement of the children because the suggestions were more relevant to primary school students.

Additionally, the initial opinions of the participants were mostly about other objects that could be used in the class rather than a material (in the context of the requested specialties). Most of the ideas were regarding activity-designing, not about a material. Their thoughts were to use it as a teaching tool in the class. But that was an activity design, not a material design. They were not totally aware of the difference between designing an activity and a material. Almost all of the ideas (every student had more than one opinion) have been eliminated due to the reasons mentioned above. The participants did not elaborate on their first ideas much before hand.

The following are excerpts from the questioning process:

Participant (P): I wanted to use it while designing it by this means: (He designed a speed problem activity and thought of preparing it as a tangible material). The car will go from here to there and the student will see (two cars are going to go from right to left on a cardboard shaped as road).

Researcher (R): Okay, but isn't that an activity? Isn't your material moving two cars? Can we design different activities by using this?

P: Yes, but… This was supposed to be used by students in the class. I did not think of it to use alone.

R: What is the different and authentic side of it?

P: I didn't research, anyway every one could think of this material.

Practicalities/Feasibility

After thinking of an opinion, the next question that the participants dealt with was whether their opinions were practical or not. Some of the supplies of the thought material could not be produced/ found or made. The participants said that they did not think of this factor very much at first. The participants thought mostly about the opinion and had not planned how they could materialize their opinions.

P: Here, the student will pour water from this cylinder to this globe and will see the relation between the volume of the cylinder and the volume of the globe with the help of the lines on them.

R: Where are you going to buy these supplies?

P: I can buy, I suppose. Can't I find them anywhere?

R: How will you make water pass through the cylinder to the globe?

P: I will put a pipe here and the water can pass via the pipe! Wouldn't it pass? (It is impossible physically.)

Authenticity

Another question that the participants come across was whether they were one of the materials produced in the Class Tools Building Centre or were they produced by combining some other materials. Although the issue of authenticity had been emphasized from very beginning of the class, the students had not been testing their opinions by researching over the past 4-5 weeks. The fact that their opinions were not accepted because of this reason prompted the participants to better research the sources.

P: I put a mirror on a pole. I will use it to find the reflection of the spot I want on the coordinate axis.

R: Do you think this idea is authentic? Someone else could have made this discovery before?

P: I don't know. Is it authentic?

R: Weren't you supposed to check if this idea had been discovered before? CTBC has semi translucent plastic mirrors for this.

The development of ideas was different for each participant. Some participants developed their ideas in 2-3 weeks and passed to the second phase. But more than half of the participants completed this process at the end of the 7th or 8th week. Some participants completed this process during the last two weeks. Other participants could not manage to develop an idea in 12 weeks. Although two of them found an idea almost every week, they were unsuitable. Some other participants passed to the next phase (production phase) but gave up when they realized that their idea couldn't be put into practice.

B. Production Process

The other phase was to begin producing the developed idea. The participants first prepared the demonstration and then organized the production when they moved to the making phase. After preparing the demonstration they searched for the most appropriate material for the construction.

Preparing the Demonstration

In the materializing process, the first phase was to prepare the demonstration. The incompletely developed opinion was being tested via a demonstration. Preparation of the demonstration was to give an idea of how it would look and how it would be used by the participants. The mistakes in the design helped the participants to develop their ideas. These demonstrations enabled the participants to realize several issues, such as: some materials would not work properly, the physical impossibilities of the materials, and design flaws that would confuse the students or result in structural problems. In the design it was observed that there were some issues that could lead to the users acquiring misconceptions as well as result in structural problems for the material that was going to be produced. Participants didn't think of possible misconceptions or misunderstandings while they were forming their ideas and preparing the demonstration of the material. They reviewed their ideas after the interviews about the demonstrations as indicated below.

The participants finalized their materials with the help of the notes taken from the demonstrations; they tried to choose the appropriate material or gave up the idea to find an alternative one.

P: A Field measurement can be calculated in integral with this material. I will show the footprint by lining up those pieces here. (She is talking while showing the demonstration material made of paper, which is why the thickness has been ignored.)

R: How will you design these pieces?

P: I will use wood.

R: Won't the pieces be too thick? Won't the thickness turn your field measurement into cubage? Could this lead to misconceptions among the students?

P: I didn't think of it. I think the students wouldn't pay attention to that.

Choosing the Right Supply

It was important in the development phase to design the material according to the requested specifications. But it is not possible to plan it exactly. One of the requested specifications was to use long-lasting and flexible material. After preparing the demonstration, the participants had to source for the correct supplies. In this research, the participants sometimes realized that preparing the material was very expensive or difficult; sometimes the supplies would be very heavy or couldn't be prepared to their exact measurements. In this case, either the design had to be changed or it had to be prepared again.

P: In my material, I wanted those lights to be turned on when this button was pressed. I checked with the electricians. An electric circuit is necessary to turn the lights on like this. Also the number of the led light emitting diodes which I will put in the spots in the coordinate axis is too many. That's why preparing the circuit and light part of the material costs … That is extremely expensive.

Materializing

The longest and toughest part of the production phase was materializing the idea in all aspects. In this phase, the participants consulted the experts to materialize their materials. But the fact that not knowing mathematics and preparing the supplies carelessly was an obstacle in materializing the idea. However, the participants held no clear knowledge about mathematics, so they prepared supplies carelessly. Also some instances could lead to some other problems, like the ability to produce the material, or some trouble in the working or misunderstanding how to produce certain materials. Give than, the materials were discarded and the participant returned to the "brainstorming" phase.

P1: I went to the carpenter for the preparation of the triangles. But I took a long time to make him understand that the edge length of the triangle has to be the same as the length of the other triangles. Additionally, he couldn't cut them to the exact size. As you can see there is a slight difference between them.

P2: I showed that this piece will be curved in this way. It will be put in here and will be like that. But he couldn't do that.

P3: This hemisphere will be made of mica (rigid plastic). They had to prepare a special moulding. He tried to make it look like a sphere two days after making the moulding and casting the mould. The first one cracked immediately. Then he made it again. A cylinder was somewhat easier to make.

The participants who completed the developing idea process thought that the idea could be materialized easily. Although most of the participants' ideas were ready at the end of the 7th or 8th week, they had to come up with other ideas for the demonstration phase or further phases. Only one of the participants submitted his material in the last week, while 14 participants were waiting for the materials to be completed by those constructing them. Three people were still in the demonstration phase. The remaining two participants did not have any developed ideas. During the last week there was a problem with the production of the material of one of the participants and he had to develop another idea. The participants who could not come up with an idea were helped by the researcher, then skipped the demonstration phase and began to make their material. After the last lesson, two extra weeks were given to the participants to complete completing their materials. Seven of the submitted materials were not appropriate for the requested specifications. These materials either would not work properly or did not reflect the requested idea exactly. Although these materials were appropriate in the idea phase, they were unusable when they were transformed to physical objects. Eight materials were relatively better, but they were not exactly appropriate for the requested specifications. Only five materials were prepared according to the requested specifications.

DISCUSSION AND SUMMARY

Many researchers have expressed that manipulative materials contribute to learning (Ball, 1992; Bellonio, 2012; Boggan, Harper & Whitmire, 2010; Brunner, 1966, as cited in Bayram, 2004; Dienes, 1960; Heddens, 1986; ; Kelly, 2006; Moyer, 2001; Ojese & Sexton, 2009; Piaget, 1973; Raphale & Wahlstrom, 1989; Ruzic & O'Connell, 2001; Sowell, 1989; Stacey at al, 2001; Swan & Marshall, 2010). Therefore, teachers are supposed to have the ability to use the materials as a support to learning. They are not only supposed to utilize them, but are also supposed to develop and adapt them to different occasions when necessary (NCTM, 2000; GDTT, 2010). Hence, it is crucial for teachers to know how to develop materials. Also practical knowledge forms are essential when educating school teachers (Verloop, 1992). For this reason, this research was intended to describe the material development process of prospective teachers.

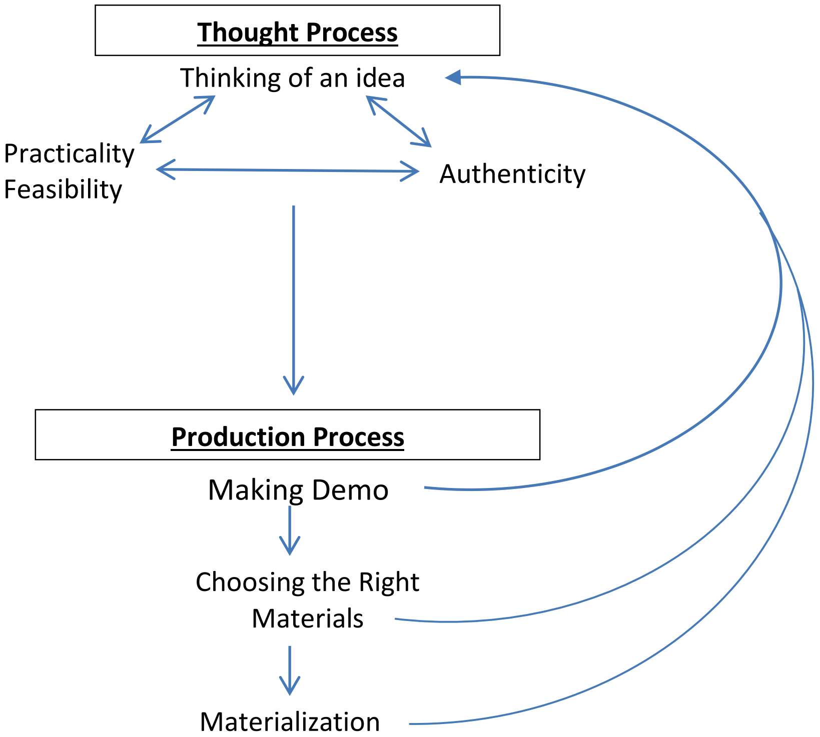

Figure 1 Material Building Processes

This process has been summarized in Figure 1 in light of the findings obtained in this study.

Teaching candidates began development process by straightforward ideas which were not elaborated according to the outlined criteria. It took some time for them to find an appropriate idea due to the fact that they had never developed the materials before. Their first ideas were failed due to the reasons such as not satisfying the criteria, being too irrational and particularly being not authentic. This failure was due to their lack of experience in developing a material and not checking beforehand whether their idea satisfies the criteria.

Furthermore, although the reasons of prospective teachers' failure in developing idea are not in the main interest of this study, from the observations and discussions with prospective teachers it can be argued that since they didn't have full knowledge about the curriculum, since they couldn't lower themselves to the students' level, or since they didn't know students' physical, psychological and cognitive evolution or simply since they didn't have sufficient teaching experience, their first ideas were very simplistic.

Since their ideas were not accepted by the researcher at the beginning, they were encouraged to research the problem. Thus, they examined the curriculum, and they researched particular student levels' difficulties, understandings and tried to produce more appropriate ideas. They used previously discussed material that were appropriate and found more authentic opinions. Also their materials not being realistic and achievable often stimulated their imagination. The candidates' development during this process is very important in terms of their development in their career (Cakiroglu & Tuncay, 2007). That is, prospective teachers are encouraged to research about students' understandings, difficulties, curriculum and other related areas of teaching while they are in the process of material development. A prospective teacher had thoughts of a class and had an imaginary teaching experience that he/she has wrote a story about using their own material. This is why developing materials may contribute to enhancing teaching skills. In this way prospective teachers were prepared according to NCTM (1991) and GDTT (2011) standards.

Preservice teachers whose ideas were well-developed were allowed to pass to the production phase. In this phase, firstly, they prepared demonstrations, by this way they examined how their imaginative scenarios work in real world without much expense, and they tried to realize their idea in the most appropriate way. When teacher candidates examined the demonstrations, they thought of how users may be mistaken and they widened their perspectives about students' misconceptions.

Similarly, the fact that they will continue to think about misconceptions throughout their career is a very important development in their teaching lives. Support for this claim comes from Sequin and Ambrosio (2002) who reported that teacher candidates' becoming aware of problems in a class would develop their pedagogical knowledge.

To sum up this research described the way prospective teachers develop concrete materials. In this description it is seen that prospective teachers' professional knowledge and teaching skills can be developed during this process. Future research can focus on prospective teachers' material development with different research approaches to give more detailed picture of this process.

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