

Investigating Seventh-Grade Students' Slope Preconceptions

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Abstract

This paper reports the results of research in which the objective was to explore the preconceptions of slope in seventh-grade students. Preconceptions are understood as students' knowledge prior to the formal teaching of a certain concept. For data collection, task-based interviews composed of ten tasks applied to 21 Mexican students were used. The data analysis was carried out using the Thematic Analysis method. Results indicate that the students have several preconceptions in which they consider the slope as any of the following: an intersection with the X or Y-axis, an arithmetic operation, a length, an object, a height, and *something to do*. These findings pose the challenge of achieving conceptual changes from these preconceptions. In this sense, science education has been the field most exploited in mathematics education; a collaboration between teachers and researchers from both fields could contribute to finding strategies to face this challenge.

Keywords: seventh-grade, slope preconceptions, task-based interviews, thematic analysis

INTRODUCTION

Several studies indicate that students' understanding of slope is limited (Greenes et al., 2007; Hoban, 2019, 2021) even when it has been considered a fundamental part of the mathematics curriculum (Dolores et al., 2020; Nagle & Moore-Russo, 2014; Stanton & Moore-Russo, 2012). In this regard, a great variety of misconceptions or confusions is known. For example, Barr (1981) identified that students have confusions: with the idea that the gradient can be considered as a ratio; whether the slope is "x over y" or "y over x" or when m and c are given in the general equation of a straight line of the form $y = mx + c$. Cho and Nagle (2017) detected similar mistakes: students do not change fractions to the simplest form; do not place the x variable after the slope in the equation; calculate a slope as run over rise instead of rise over run; calculate $\frac{y_2 - y_1}{x_1 - x_2}$, etc. Several misconceptions come from the interpretation of graphs of situations in science: students confuse slope with height when asked for speed (Bell & Janvier, 1981; Dolores et al., 2017; McDermott et al., 1987); students successfully find the slope of lines that pass through the origin, but have difficulty determining the slope of lines that do not go through zero (Beichner, 1994), if it passes through the origin they say that its slope is zero (Birgin, 2012; Dolores et al., 2017); Area/slope/height confusion; students often perform slope calculations or inappropriately use axis

values when area calculations are required (Beichner, 1994). Teuscher and Reys (2010), Planinic et al. (2012), and Dolores et al. (2019) reported that students consider slope and rate of change as disconnected concepts, thus showing poor understanding.

There is a considerable amount of information in the specialized literature on misconceptions, confusions, or errors made by students when performing tasks on the concept of slope. However, in this research, we do not focus our attention on these, but on the causes of this limited understanding, especially those related to the prior knowledge that students have before that concept is taught to them.

Barr (1981) and Crawford and Scott (2000) pointed out that the difficulties of the students that cannot understand slope as a rate of change lie in the fact that they understand it as a set of rules and procedures that must be memorized without understanding the relationship between both concepts. Planinic et al. (2012) argued that this occurs because students are not able to transfer knowledge from mathematics to physics or vice versa. Walter and Gerson (2007) considered that the difficulties may be related to the various meanings' teachers associate with the term slope such as inclination, steep, slant, steepness, etc. Similarly, Cho and Nagle (2017) assumed that a source of students' difficulties is in the vast assortment of forms that slope can be conceptualized; some of these ways are geometric

Contribution to the literature

- This study provides information on the preconceptions about the slope concept identified in Mexican students in the mathematics education context.
- Unlike various investigations that reveal misconceptions or confusion about slope after students have received instruction on it, our study explores their knowledge before they receive formal instruction on it.
- This study provides evidence that several of the misconceptions about slope reported since the nineties are present in students, but as preconceptions.

ratio, algebraic ratio, parametric coefficient, physical property, functional property, trigonometric conception, calculus conception, a real-life situation, a determining property, behavior indicator, and a linear constant (Moore-Russo et al., 2011; Stump, 1999, 2001). There is wide acceptance among researchers in science education (e.g., Campanario and Otero, 2000; Suparno, 2005; Yanik, 2011) that preconceptions can be the cause of misconceptions. We adhere to this acceptance. In fact, we assume that several of those misconceptions and confusions about slope already described above are actually part of the students' prior knowledge.

However, this prior knowledge is usually ignored when new concepts are introduced, and this prevents students from establishing connections between them (Fuson et al., 2005). Prior knowledge is one of the most important factors in influencing learning (Mahmud & Gutiérrez, 2010), therefore, it is necessary to identify and characterize it. The studies of conceptual change constitute a line of research in which the change from preconceptions to scientifically accepted knowledge is studied. It currently continues to occupy the interest of researchers (e. g., Lin et al., 2016; Nadelson et al., 2018) in order to improve learning.

LITERATURE REVIEW RELATED SLOPE PRECONCEPTIONS AND RESEARCH QUESTIONS

According to the literature consulted, little is known about preconceptions of slope, especially in mathematics education. We found two investigations in which preconceptions of slope were identified and another two in which they were considered as a way to advance in learning. Cheng and Sabinin (2008) explored the informal knowledge of slope in North American students from second to seventh grade in particular, to know which dimensions students attend and neglect when describing steepness. They showed that students most frequently refer to the vertical height of the ramp when explaining their conclusions about steepness and they have difficulties accurately describing how the different dimensions of the incline contribute to steepness. Rivera and Dolores (2021) expressly explored the preconceptions of slope in tenth-grade students who had already been taught the concept, however, they found preconceptions of slope such as: *length*, *object*, *physical property*, *x* or *y-intercept values*, etc. This may be

indicative of ineffective teaching or also resilience to conceptual change of those preconceptions.

Thacker (2019) found that most of the study participants (seventh and eighth grade) were able to make connections between their intuitive constructions of steepness and the mathematics of slope, as they were guided to attend to the steepness of the ramps, pay attention to its "rise and run" characteristics, compute slope, and comment on whether slope and steepness are related. An investigation with similar purposes but with engineering students was carried out by Vaara and Gomes (2019); they studied their preconceptions about the underlying slope in kinematic graphs, to evaluate preconceptions and learning gains. They found that the students had serious difficulties drawing and interpreting kinematic graphs, although the interpretation of graphs and understanding of speed concept and acceleration improved, the preconceptions were quite resilient. Dolores et al. (2017) also found resistance to change of the preconception height/slope widely reported in the eighties.

Unlike Cheng and Sabinin (2008), we are not only interested in the preconceptions associated with dimensions about the steepness, but we are also interested in studying broader conceptions as those associated with their Cartesian graphs or pictorial representations and those evoked by students using their own ideas. Unlike Rivera and Dolores (2021), we did not look for preconceptions in students who had already had a formal teaching of slope, but in those who had not received it; we assume that several of the ones they encountered are actually preconceptions that have not yet been changed by the formal teaching. Nor are we interested in this work to make a conceptual change, rather we are interested in identifying the students' prior knowledge through some representations used in the context of school mathematics. In summary, in this study we propose to find answers to the following research questions: What are slope's preconceptions that Mexican seventh-grade students have, and which ones are predominant?

CONCEPTUAL FRAMEWORK

One of the fundamental goals of mathematics education is to build mathematical concepts in students, and in this process the conceptions play an important role. According to the Oxford English Dictionary, the

concept is an idea or mental image which corresponds to some distinct entity or class of entities, or to its essential features, or determines the application of a term (especially a predicate). Concepts play a fundamental role in reasoning and human communication and therefore in the teaching and learning of science. Conceptions are a mental construct or representation of reality that contains beliefs, meanings, mental images, preferences, attitudes, etc. (Brown & Hirschfeld, 2007). The conceptions that students form can be a product of formal teaching or even from their own daily experiences. When students are instructed to learn new concepts, they have their own conceptions associated with those concepts. Studies such as Driver et al. (1994), and Thompson and Logue (2006) show that the students enter the classroom not with empty minds, but they bring with them prior knowledge about science which is developed from daily experiences.

When this prior knowledge, or student conceptions, enters into conflict with the accepted meanings in science, a number of cognitive phenomena appear which have been given several names, such as “misconceptions”, “systematic errors”, “alternative conceptions”, and “preconceptions”, to mention only the most common and of interest for this work. Fujii (2014) states that “misconception” implies a mistake or error; its connotation never implies mistakes from a child’s perspective since for them it is a reasonable and viable conception based on their experiences in different contexts or in their daily life activities. Systematic errors describe “incorrect features of students’ knowledge that are repeatable and explicit” (Leinhardt et al., 1990, p. 30). Alternative conceptions refer to a conception or idea that contradicts or is inconsistent with some aspects of the concept as per the negotiated or accepted scientific constructs (Chhabra & Baveja, 2012; Confrey, 1990; Narjaikaewa, 2013). Some researchers prefer to use the term “alternative conception”, “misunderstanding” or systematic errors to study these conflicts, we use the term preconceptions not to study these conflicts but to explore the students’ prior knowledge regardless of whether it is correct or incorrect.

The term preconception implies that the ideas being expressed by children do not have the status of generalized understanding characteristics and in a situation where children have developed autonomous frameworks or have conceptualized their experience of the physical world their ideas will be called ‘alternative frameworks’ whereas (Clement et al., 1989). Simons (1999) defines prior knowledge as “all the knowledge learners have available when entering a learning environment, that is potentially relevant for learning new knowledge” (p. 579); it can be formal or informal, implicit or explicit, and it can be partially correct or incorrect compared with the scientific standards. Johnston (2005), and Kambouri (2016) warned about the great danger of children reconstructing their initial

preconceived ideas if they do not receive an adequate orientation.

For the purposes of this study, the preconceptions term will be used to refer to the students’ knowledge about a concept (slope in our case) that they probably have developed autonomously through their own school or extracurricular experiences (Rivera & Dolores, 2021). Generally, these are misconceptions from a mathematical perspective and are developed prior to formal study. By formal study, it is understood as the intentional teaching of slope through its definition, properties, and habitual representations in mathematics education.

METHOD

This is qualitative research that used individual Task-based Interviews to collect data. According to Goldin (2000), the Task-based Interviews to study mathematical behavior, involve a minimum interaction between a subject (the problem solver) and an interviewer (the clinician) engaged in one or more tasks (questions, problems or activities) introduced by the clinician in a previously planned strategy. Goldin claims that the researcher can make inferences on the mathematical thinking, learning or problem solving from the analysis of the behavior of verbal or non-verbal interactions. In this regard, Assad (2015) highlights that task-based interviews not only give opportunities to evaluate the conceptual knowledge of the students but also widen this comprehension. According to Assad, the protocol of the interview may be structured with guidelines and previously planned answers by the interviewer, it can also be introduced as semi-structured interviews that allow the interviewer to judge the adequate response to the mathematical reasoning of the student.

Design of Task-based Interviews

Three criteria were used to design the Task-based interviews: first, it considers our assumption that the cause of various errors and misconceptions may come from preconceptions; second, that it considers the previous investigations that account for the errors and misconceptions; and third, that it promotes the manifestation of slope preconceptions.

The Task-based Interviews were designed in three sections: *Pictures, Graphics and Drawing or free descriptions*. Tasks 1, 2 and 3 correspond to the first category, *Pictures*, where they present ramps on which a car goes up or down. In the first, it is expected for them to know their own conceptions about “steeper” or “less steep” in relation to the angle of inclination or to the dimensions “high” or “long” of the right triangle, similar to that proposed by Cheng and Sabinin (2008). In tasks 2 and 3 we assume that the conceptions discovered by Rivera and Dolores (2021) about conceiving the slope as length, can emerge, although the arithmetic calculations

noted by Cheng and Sabinin (2008) are not ruled out. It is necessary to clarify that we use the term "slope" in formulation tasks in an intuitive sense and for communication purposes only, because it is a term familiar to students since they live in a mountainous relief region with various and different inclinations. So that sense or meaning was given to slope by them using their own ideas.

The second category, *Graphics*, includes tasks 4 to 8 in which straight line graphs are used on a Cartesian plane. In task 4 it is assumed that the concept of slope can appear as x, y intercept noted by Rivera and Dolores (2021). Tasks 5, 6, 7 and 8 were done following an idea similar to that reported by Bell and Janvier (1981), McDermott et al., (1987), and Dolores et al. (2002): enable the appearance of height/slope confusion, but as a preconception. The third category, *Drawings or free descriptions*, includes tasks 9 and 10 in which the students were asked to freely express their ideas about slope, being able to do so with drawings or verbal descriptions. We suppose that task 9 can allow the height/slope conception to emerge, but not in an interpretive manner, but as a representation made by the students themselves. Task 10 asks the students to express their own meaning about slope; we suppose they could use the synonyms of "slope" to explain their reasoning, using words such as: incline, steepness, level, flat, tilt, slant, angle, steep, pointing up. In this task, we also hope that they will be able to draw pictures to explain their own meanings.

To validate the instrument, three mathematics education experts reviewed and evaluated it, and it was applied in an experimental plan in a group of five students of primary and secondary education levels (fifth, sixth and seventh grades); these processes allowed us to reduce the number of tasks and select those that promote the emergence of the ideas of slope. A significant number of proposed tasks are related to the physical context, there are two reasons that support their validity. First, the Mexican mathematics curriculum (SEP, 2011a) indicates that mathematical training "should allow individuals to successfully face the problems of everyday life" (p. 67), thereby emphasizing the relationship of mathematics with the real world and, secondly, the textbooks used by Mexican primary school students (from first to sixth grade) and junior high school (from seventh to ninth grade) develop mathematical ideas from real-world situations. For example, in mathematical textbooks the ideas of proportionality, constant proportionality and even the same slope concept are used in physical situations that involve the variables time, distance, speed or rate of change. Where the use of graphic and pictorial representations is common. Therefore, students are no strangers to these representations in their math lectures.

Participants

According to the Mexican intended curriculum, the concept of slope should be taught in ninth grade from the study of physical phenomena that involve the rate of change (Dolores et al., 2020). However, the textbooks used by Mexican students address this concept beginning in eighth grade, by studying linear functions of the form $y = mx + b$ that model situations of physical variation. In ninth grade, textbooks continue their treatment by introducing the notion of "change" through the "increment" ($\Delta y, \Delta x$) and the ratio between them ($\frac{\Delta y}{\Delta x}$). Under these circumstances, we decided to conduct our research with seventh grade students, because with them we had a guarantee that they had not yet received instruction on the concept of slope.

Twenty-one students volunteered for this study, eleven boys and ten girls, between the ages of 12 to 13 years old, from three secondary public schools in the center of the state of Guerrero, Mexico. The secondary education level in Mexico ranges from seventh to ninth grade of schooling. The participants were the highest performing students in mathematics. They were selected by their respective teachers and asked if they wished to volunteer for this study. When the task-based interviews were carried out, the participating students had already been taught some topics preceding the slope concept, such as: reading information represented in bar and circle graphs, direct proportional variation; elementary equations of the form: $x+a = b$; $ax = b$; $ax+b = c$; linear variation and its graphic representation. In sixth grade they studied the following: reading of data contained in tables and circular graphs, comparison of ratios in simple cases, location of points or drawing of figures in the first quadrant of the Cartesian plane, ratios and equivalent ratios, proportions, percentages, and the constant proportionality factor.

Data Analysis

The interviews were videotaped and transcribed; the evidence of the written productions of the students were digitally scanned. The Thematic Analysis suggested by Braun and Clarke (2012) was used for the data analysis; its objective is to identify, organize, and systematize patterns of meanings (themes) using a set of data to answer the research question. According to Braun and Clarke (2012), a theme captures something important of the data related to the research question and represents some level of answer or meaning modeled within the data group. These patterns are identified through a rigorous process of familiarization and codification, development and review, and identification of themes.

Six phases structured this method: (1) familiarizing yourself with the data – this phase consists of repeatedly reading the transcriptions; (2) generating initial codes – the codes correspond with the phrases or words used to refer to the slope; (3) searching for themes – the initial

Table 1. Example of the synthetic process of analysis*

Task	Excerpts of the interviews	Initial code	Themes	Abb.*
T2	I: What is the slope in Figure 4? S1: It is five meters. I: Can you explain to me why? S1: <i>Because it is the measurement from here to the top</i>	Length of the straight line (hypotenuse).	Length of a segment	LT
T3	I: What is the slope in Figure 5? S4: It is 13 m. I: Can you explain to me why? S4: <i>Because the car will move thirteen meters.</i> I: What do you mean by that? S4: <i>This, it is what it will go down.</i>	Distance to travel		
T2	I: Tell me, what is the slope in Figure 4? S2: Well, <i>I am seeing it as a sum.</i> I: Would that be the slope? S2: Yes, for me it is $5+3+4=12$, therefore the slope is 12.	Sum of the sides of the triangle	Arithmetic operation	AR
T3	I: What is the slope in Figure 5? S3: Well, I multiply them, I think it is the answer. I: You multiply what by what else? S3: Look, this: $13 \times 12 \times 5$ E: But you wrote 30, how did you get this? S3: Oh! No! I did a sum: $13+12+5=30$ I: Can you explain to me why you did this? S3: Well, <i>it is the slope to me</i>	Sum of the data	Arithmetic operation	AR

*Italics represent phrases that allowed codification. T2 = Task 2. T3 = Task 3. I= Interviewer. S= Student interviewed.

Table 2. Preconceptions of the slope identified ($n = 21$)

Abb.*	Slope as ...	Subthemes	Tasks	Frequency	Total
CO	Intersection with the axis	y -intercept	4, 5, 6, 8	42	71
		x -intercept	4	2	
		x and y -intercept	4,5, 6	27	
LT	Length	Length of the line or distance	1, 2, 3, 5, 6, 9, 10	44	44
OB	Object	Slanted line	1, 2, 3, 4, 5, 6, 7, 10	18	34
		Go up or down a steepness	1, 2, 3,10	26	26
AL	Height	Height of a triangle	2, 3	4	33
		Greater height implies greater slope	7	10	
		Value of the height or the vertical	2, 3, 4, 5, 6	10	
		Height in the graph of the speed	9	9	
AR	Arithmetic	Sum of the sides of the triangle	2, 3	5	32
		Multiply or divide	2, 3, 4, 5, 6	27	
RD	Something to be done	Something to be done	10	7	12
		Segment to go	1, 2, 3, 10	5	

*Abb.= abbreviations

codes are compared and collated into a theme (in terms of the study, this is a preconception); (4) reviewing the themes, correspondence with the data; (5) defining and naming the themes (preconceptions of the slope); and (6) producing the report. During the first four phases, the researchers analyzed the data independently. Then, they compared and discussed their results using triangulation as suggested by Flick (2004) to verify the subjective opinions, balance the personal opinions, and eliminate biases associated with one researcher. In case of a disagreement, the data was analyzed collectively,

until consensus was reached. During the work sessions in phase five, the researchers discussed the results and reached conclusions. Table 1 shows the synthesis of the process of construction of two specific themes.

RESULTS

Eight preconceptions of slope and some subthemes were identified based on the responses of the questionnaire, the explanations, and the arguments given by the students during the interviews. Table 2

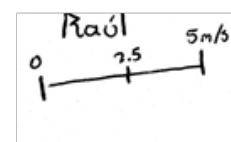
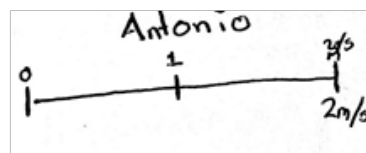
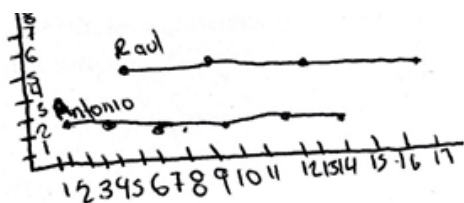


Figure 1. Speed as the length of segments of the straight line

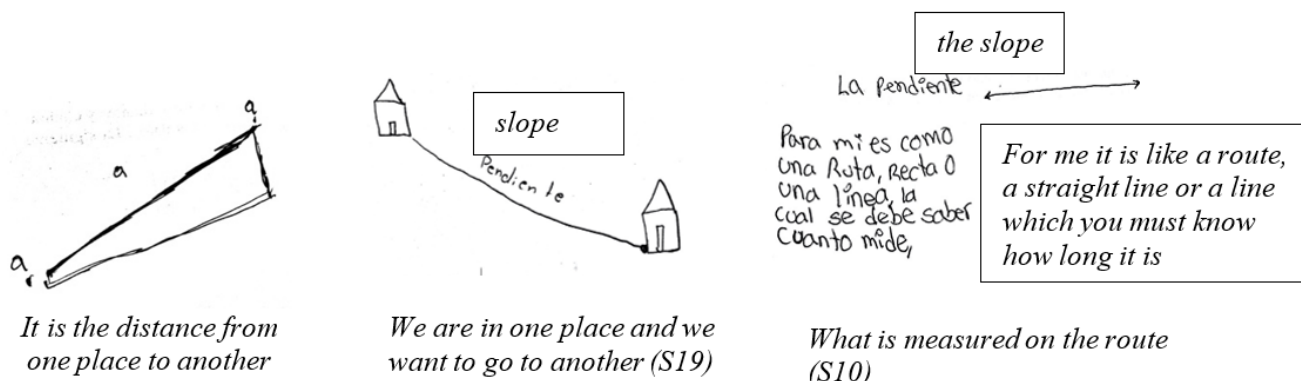


Figure 2. The slope as the length of a straight line or a trajectory

shows the set of preconceptions of slope identified in the seventh-grade students. The table also includes the abbreviations used, the frequency of the mentions, and the tasks from which they emerged.

The Slope as Intersection with the Axis

This idea emerges in the tasks that require the interpretation of the graphs of straight lines. Two subthemes were identified: the intercept in both axes and the intercept in some of them. For the first subtheme, the students associated the slope with the extreme points of the trajectory defined by the straight line justifying that those are the values through which the line passes. Other students only chose one of the two axes giving similar arguments; for example: “because it starts at 4” or “because it passes through there” (referring to the points where lines l or m pass through). Similarly, most of the students selected items a and c in task 8 which consisted in the identification of the graph of the speed; the characteristic of these items is the correspondence between the value of the speed and the initial numerical value of each graph, that is, the Y-intercept. Among the justifications that stand out: “because it starts on 50”, “because the graph represents fifty per hour”, “exactly at 50 km/h”, “because it goes straight forward, and let’s say that the car does not have difficulties”. In addition, those who chose one of the other items argued that “to reach 50 it has to start from 0 and then stay that way”. When asked about the slope of the line that was located in the second quadrant, two-thirds chose the two intersection points with the axes (task 4), when the lines were in the second quadrant (tasks 5 and 6); they were chosen by one-third of the students and when asked about speed (task 8), a little more than a third chose them. It seems that if the line is located in the second

quadrant, misconceptions increase, whereas when they are in the first quadrant, they decrease.

The Slope as a Length

This idea refers to the value of the slope as the length of a line or trajectory (the length of the hypotenuse of the triangles visualized on the ramps). Some of the arguments that evidenced this preconception were: “the slope is 5 because it is the distance to cover”, “because 5 meters is the measurement of the street that the car will travel”, “because it will travel for 13 meters”, “I think it is 13 because it is the measurement of the straight line”, or “the slope is the measurement of the line”. In task 9, several students represented the speed of the runners as segments of a line in the Cartesian plane or simply by drawing the segment, as shown in Figure 1.

This idea of the slope as the length of the line or trajectory was also identified in task 10 where students expressed what they think when they listen to the word “slope”. Below we transcribe the responses of several students in which they express these ideas:

S4: Mmm ... the distance from one place to another.

S19: It is like something intended... for example, when we are in one place, and we want to go to another place. This is the slope.

S18: It is a straight road and you want to know how long it is.

They were asked for a representation to infer the idea they intended to communicate, which is shown in Figure 2. Some of their ideas show that they appreciate slope as

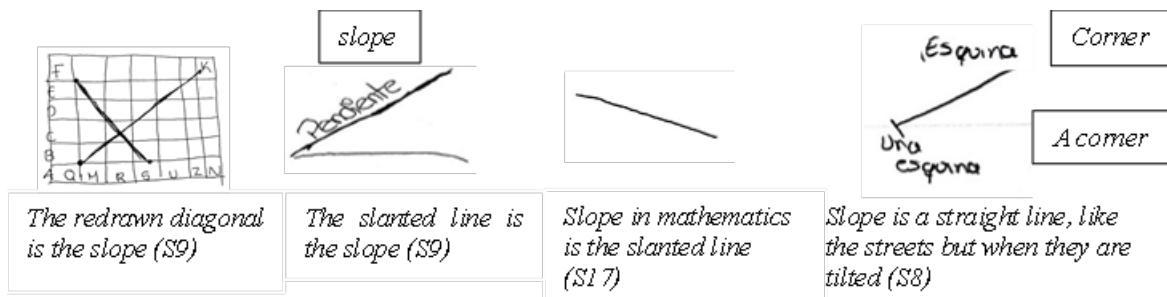


Figure 3. Examples of the subtheme slope as a slanted line

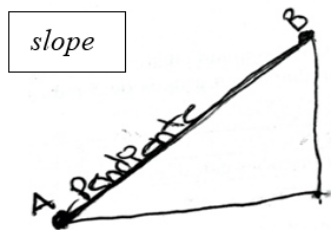


Figure 4. Drawing made by S3

something physical but they decided to attribute it as the length.

The Slope as an Object

This preconception appeared in all tasks. The idea of the slope as an object is identified when they referred to it as the straight line itself or as a physical situation like an ascent or a descent, the ramp, a stair, or a street under the condition that it is tilted. The following excerpt of the interview of S7 is shown as an example of the subtheme of slope as a slanted line: “I imagined that the slope was something that we were going to leave for later... but I can also say that the slope is a line that leans either way, up or down”. Some students were more expressive and showed drawings to talk about the slope, as shown in Figure 3.

The case of slope as an ascent or a descent was perceived as a physical situation and it was more frequent in the first three tasks. In this sense, the students showed expressions like “slope is the inclination on which the car goes”, they finally point to the same ramp or make reference to the ascent or descent as slope; they also gave the example “the stairs are a slope because it is tilted”. Care was taken to ensure that the terms “slope” or “inclination” did not appear in Task 1 to avoid influencing students’ ideas. However, two students involved the term “slope” to associate it with “the car

ride up” and thus describe the situation they could interpret in Appendix Figure 1. S3 also evidenced this idea of “ascent and descent” in task 10, as shown in the following excerpt of their interview:

I: What does slope mean to you?

S3: Something I left for later ... mmm... something to be done. It is also a hill, a slope that goes up or down.

I: How is it that the slope goes up?

S3: Like on a hill, you can see that it goes up or you can go down, that is a slope.

I: Can you represent what you said to me, please?

S3: (Draws a triangle as shown in Figure 4) There it is, it is where it goes up or down.

Slope as Height

This idea appeared in tasks 1, 2, 3, 4, 7, 8, and 9. The subthemes associated with these preconceptions are the height in a triangular form; greater height, greater slope; the value of the height in the plane; and the height of the graph associated with speed or velocity. For the first subtheme, students showed that slope is the height of the triangles that represented ramps and some of them traced an auxiliary triangle in the Cartesian plane (see Figure 5). The second subtheme is an inference made to describe the situations that involved different slopes; this is, “greater height, greater slope”. This same idea is used to compare the slopes of two given straight lines in the Cartesian plane but assigned to the value of the y coordinate, in this case, P and Q, and this gives place to the third subtheme. For the case of task 9, the students

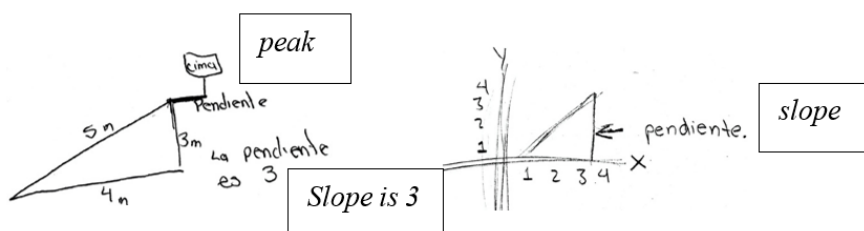


Figure 5. Slope as the height of a triangle

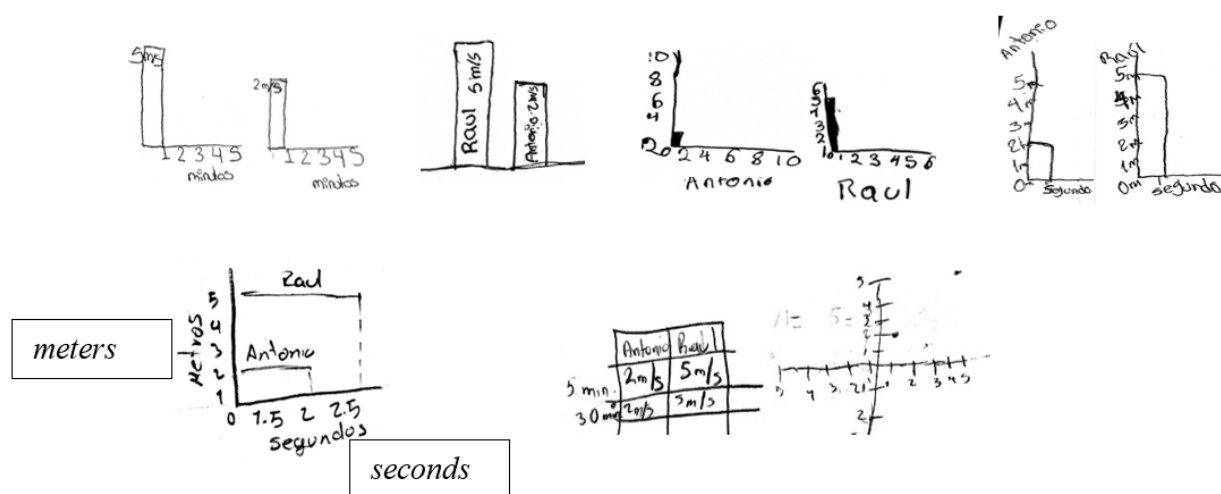


Figure 6. Slope as height in bars or points

used bars whose height is the requested speed or placed a point in the cartesian plane whose y coordinate is the value that represents the requested speed, as can be appreciated in Figure 6.

Slope as an Arithmetic Operation

This idea considers slope as an arithmetic operation, namely as the result of a combination of operations: addition, subtraction, multiplication, or division of the values obtained in the tasks, mainly in tasks 2 and 3. Five students in task 2 and eight in task 3 mentioned a sum of the values given in the tasks to find the slope of the ramp, they argued things like: "I find the slope by adding $5+4+3=12$ ", "I find it by adding ' $13m+12m+5m=30m$ ' or ' $12m+5m=17m$ and $17m-13m=4m$ ".

On the other hand, we found less frequent arguments that mentioned the use of multiplications and divisions or combinations of these operations; this happened mainly in those tasks where geometrical figures could be perceived. Therefore, it was natural for the students to associate their answers with the calculation of areas or the average of the given data. For example, "the slope is four because 5 plus 4 plus 3 is 12 , and 12 over 3 is 4 ", "it is 10 and I get it from 5 plus 12 plus 13 equals 30 but 30 over 3 is 10 ", "the slope is, multiply 5 by 12 that is 60 and 60 over 2 is 30 ". In the case of task 6 where the students were asked to find the slope of the line at a point, some of them found the value of the slope as the quotient (or multiplication) of the coordinates of the point.

Slope as "Something to be Done"

The Spanish word "*pendiente*" can be translated into English as "slope" but also as "pending"; so, the Spanish word is also used to express an activity that has been stopped and is left for the immediate future. The idea of slope as "something to be done" comes up when the students were asked to express what they think when they listen to the word "*pendiente*" or what does

"*pendiente*" mean to them, and they associated "*pendiente*" with pending. As a main idea, they declared "something that I stopped doing and left it for later", "something to be done, or when you say you have something pending to do, like doing your homework", "like something planned to be done", "something pending to be solved".

DISCUSSION

The results obtained in this investigation have allowed us to identify seven types of conceptions, slope as: *intersection with x or y -axis*, *a length*, *an object*, *a height*, *an arithmetic operation*, and "*something to be done*". The slope as *intersection with the x or y -axis* was the conception with the highest frequency obtained in the performance of the tasks. 20% of students pointed out that slope is a point where the straight line passes through, or the point where the line "starts" or "finishes" which are the values of the intersections with one of the axes. The same conception was found in students from the same region of Mexico: 48% of tenth-grade (Rivera & Dolores, 2021), and 33% of beginning university (Rivera et al., 2019), it was also found by Birgin (2012) in Turkish eighth-grade students when the concept is first taught. However, we found it in students who had not received instruction in this regard. The fact that it is still present in tenth-grade students and university beginners is indicative of how difficult it is to remove this preconception. Its permanence is notorious despite the fact that it is an object of teaching since ninth grade in Mexico. In some way, this is justified because graphing a linear function from the slope and the y -intercept is difficult for the students, as reported by Birgin (2012), and Hattikudur et al. (2012). In the same way, Moschkovich (1990) reports difficulties in the interpretation of the graphical representations of m and b in functions of the form $y = mx + b$ to the extent that, according to Knuth (2000), the students would rather move from algebraic representations to graphical representations. The points

of intersection with the axis are of great help to estimate the slope of lines from the graph; it is enough to identify the angle of inclination, and these intersections could determine the legs of a right triangle to use the rate $\frac{\Delta y}{\Delta x}$ correctly and determine it.

Slope as a length is mainly expressed in the tasks that visualize a segment of line or a trajectory, some students also expressed that slope is the distance. This finding has resulted in several investigations with American students from grades 2 to 7 (Cheng & Sabinin, 2008), Mexican tenth-grade students (Rivera & Dolores, 2021), university students (Rivera et al., 2019), and even with American mathematics teachers (Byerley & Thompson, 2017). This data suggests the resistance of this slope conception, or that teaching has done little to change or adapt it. Both (Rivera et al., 2019) and Byerley and Thompson (2017) have reported that some pre-university students and mathematics teachers can estimate slope correctly, but they associate the value with the length of the given straight line when interpreting it. Therefore, it is important to develop conceptual knowledge and not just procedural knowledge.

Rivera and Dolores (2021) included the preconception of *slope as height* in the category *the length of a segment* because some students expressed that the slope of a line is the length of the segment that goes from the top to the x -axis when the information of the task is given in a Cartesian plane. This conception appeared in this study when the students interpreted the pictures (tasks 1, 2, and 3), graphs (tasks 5, 6, and 7), or when they built graphical representations (tasks 9 and 10). This conception manifests when students associate slope with the height of a triangle that represents a type of ramp, when they represent the speed of two runners, or when they find the slope between two straight lines that intersect at a specific point in the Cartesian plane.

This result was already detected in American, English and Mexican students and reported long ago as a confusion between speed and height in the interpretation of kinematic graphs (e. g., Beichner, 1994; Bell & Janvier, 1981; Dolores et al., 2002, 2009; McDermott et al., 1987). In these investigations, Physics students associated "greater height with greater speed". In our case, we explicitly ask for the greatest slope given two lines, and also for the greatest speed in a moving situation. We find the same idea in both cases. Another difference with previous studies is that our results show that the idea of height/slope is conceived before students are instructed on the concept of slope. It is, therefore, actually a preconception. It is possibly caused by the strong association that, in the experiences of young students, they have developed between height and inclination reported by Cheng and Sabinin (2008).

The preconception of *slope as an object* is present in most of the proposed tasks, some students even use

"slope", "slanted line", and "inclination" as synonymous. This preconception was also found by Rivera and Dolores (2021) in grade 10 students when they were asked for the slope of the straight line; the most common answers found are: "the slope is a slanted line", "the slope is the line", "the slope is the staircase of the ladder", "the slope is the ascent", "the slope is a hill", "the slope is when you go in a straight line and suddenly you go up or down". Slope as a synonym of inclination was also reported by Cheng and Sabinin (2008) in American children from grades 2 to 7 when they had not yet received formal instruction on slope. However, this was expected because people start learning about slope through the inclination of daily-life objects like ramps, stairs, slides, streets, hills, roofs of houses, etc. (Cheng & Sabinin, 2008; Rivera et al., 2019). In this way, students can "visualize" the slope without relying on any mathematical ability like using a ratio or calculating a limit (Deniz & Kabaal, 2017; Nagle & Moore-Russo, 2013). However, the students' interpretation of slope as an *ascent or descent* should not be considered an irrelevant or mistaken idea, it should be kept in mind as a preconception when it becomes a teaching object and take it as a starting point to lead the students to more precise and rigorous ideas.

Slope as an *arithmetic operation* was expressed as the results of additions and subtractions, or as the result of a combination of additions, multiplications, and divisions. It appeared when students solved tasks associated with ramps (which included measurements) in which Cartesian graphs or equations appeared, and they were asked to find the numerical value of the slope. This trend of elementary school students performing arithmetic operations to solve problems is named operational strategies; in this sense, Alonso et al. (1988) highlights that reducing mathematics to numbers leads children to think that a problem is mostly a combination of numbers. The results show that 29% of the Spanish students in the sample did calculations even when they were not sure why, a similar result was found by Carpenter et al. (1980) in American students; Rizo and Campistrous (1999) named it irreflexive strategies. However, the students in our study could be invoking this type of operational knowledge to find the slope because it is widely privileged in elementary school.

The conception of *slope as something to be done* is part of the students' language or vocabulary used outside the mathematical context. "Slope" and "pending" are homographic words in Spanish; this word concerns the daily use that means "what is to be done". Other homographic words in Spanish are "pendant", "descent", "incline", "earring", etc. This result was expected because it is natural to acquire meanings during interactions in their social environment; therefore, the function of mathematics education is precisely to provide the scientific language. In this sense, Cobb et al. (1994) claim that the students do not use a

precise mathematical language automatically; teachers should guide them. The Mexican Curricular Standards of Mathematics (SEP, 2011b) to achieve that population can use mathematical knowledge (formulated in their vision) gives special attention to this transition; in this respect they suggest that students of basic levels of education should “transit from everyday language to mathematical language to explain procedures and results” (p. 15). Particularly, they recommend the use of the relationship between informal and conventional languages in solving problems or in their reconstruction in case of forgetfulness. On the other hand, The Principles and Standards (NCTM, 2000) also point out the importance of the use of familiar or everyday language by children of lower grades; since it provides them the basis on which to build connection with formal mathematical language “hence it is important to avoid a premature rush to impose formal mathematical language, students need to develop an appreciation of the need for precise definitions and the communicative power of conventional mathematical terms by first communicating in their own words” (p. 63).

CONCLUSIONS

We assumed at the beginning of this article that the causes of several misconceptions about slope came from students' prior knowledge or preconceptions. The results found in this investigation provide evidence that supports our assumption. Most of the found conceptions are erroneous from the point of view of mathematics. Considering that slope of a line is the value of intercept with the Cartesian axis is a misconception. And the mistake lies in considering only the value of the intersection points; the slope depends on magnitudes Δy and Δx and the ratio between these magnitudes, not on the value of the points. Nor is the slope obtained from an arbitrary arithmetic operation, this is also an error; Thompson (1990) specifies this ratio is obtained from a multiplicative comparison between two quantities through which it is possible to determine how many times one quantity is greater than another. On the other hand, the slope “as the line itself” or “as a length” are also misconceptions; it is not a material object and the line is just one of its geometric representations. Slope as a length or as a height comes from the experience of measuring the length of the lines or the length of the y-coordinate in the Cartesian plane. Slope is the measure of the steepness of lines or curves expressed as a ratio.

Chi (2008) states that learning in science classrooms can occur under three conditions of prior knowledge: it may be absent but have some related knowledge; it may be correct but incomplete, and it may be in the form of ideas that are “in conflict with”. All those described in the previous paragraph are of this last condition. However, we also found intuitive ideas related to slope. For example, the slope as the path that “goes up” or

“down” or the slope as the height of a ramp, is intuitive and incomplete because they emphasize a single dimension, but they can be starting points to complete the idea in the process of teaching as suggested by Carpenter and Fennema (1996) in their Cognitively Guided Instruction (CGI) proposal. On the contrary, conceptions that conflict with accepted mathematical knowledge must be subject to conceptual change.

The findings in this study pose formidable challenges for both researchers and teachers; these challenges are pinned on how to achieve conceptual changes under the basis of the students' prior knowledge. Conceptual change has developed more in science education than mathematics education; however, it is researchers such as Vosniadou and Verschaffel (2004), Merenluoto and Lehtinen (2002), and Prediger (2008), that argue that the conceptual change approach can be a fruitful one in mathematical learning and instruction.

Today there are at least two opposing educational paradigms regarding conceptual change: Piaget's constructivist and Vygotsky's sociocultural. In the first, the conceptual change is based on the “cognitive conflict” that drives change and the development of human cognition (Piaget, 1985), on the contrary, in the second, it is assumed that the change occurs because the child is exposed to the alternative speech of an expert and this speech is more sophisticated and more effective in describing the world (Sfard, 2008). Prediger (2008) advocates combining conceptual change approaches in the learning sciences with established categories from mathematics education research (such as ‘Grundvorstellungen’ and epistemological obstacles). We are also in favor of making this combination

On the other hand, Mathematics teachers need to develop their pedagogical content knowledge related to slope to improve their teaching and its effects on learning. According to Gess-Newsome (1999) this includes: subject matter knowledge, pedagogical knowledge and contextual knowledge. Subject matter knowledge should include the development of domain about slope conceptualizations discovered by Stump (1999), and Moore-Russo et al. (2011), and the development of a framework for slope as suggested by Nagle et al. (2019). It is required to develop their pedagogical knowledge that incorporates teaching strategies, to achieve connections with the previous intuitive knowledge or conceptual change of slope preconceptions; which could consider the CGI proposal, cognitive conflict and cooperative learning. The contextual knowledge where the practice of teaching the slope takes place could consider the sociocultural environment, didactic resources to be used (textbooks, technological resources, etc.) and the social interactions of the communities where the teaching takes place.

LIMITATIONS OF THE RESEARCH

This research focused on seventh-grade students and was conducted in the last month of the school year. This can be a limitation because it interviewed a specific population in a specific period. A broader exploration that includes from fourth to ninth grade and with other research instruments that include tasks close to the daily experiences of the students would allow to analyze the development of preconceptions and perhaps the discovery of others not reported in the literature. According to the intended curriculum (Dolores et al., 2020), the knowledge that precedes the slope is taught in these grades, which leads us to wonder if this knowledge would have some influence on their preconceptions.

The preconceptions that we found were analyzed with little interrelation between them. Several researchers argue (e.g., Clement et al., 1989; Kambouri et al., 2011) that preconceptions are alternative frameworks where some are related to others. In this sense, our work also was limited. Therefore, it would be necessary to carry out research that analyzes in greater depth the correlations between them in order to identify their alternative frameworks.

We use the term “slope” in the formulation of various tasks, it is possible that this has influenced their conceptions and limited their thinking. Formulating tasks where this term is not used and other less suggestive ones are used (or even omitting it) could give students the opportunity to use their own terms and their own conceptions associated with those terms. This could lead to the appearance of other preconceptions different from those reported in this document.

The tasks were formulated in the contexts of both mathematics and physics education mainly; this could be a limitation. Future research could consider parallel tasks, also considering other science teaching contexts such as biology and chemistry. One could even consider the context of real life, for example, analyze real ramps, real stairs, or flat and sloping floors. This would give students the opportunity to evoke their preconceptions in more familiar school and extracurricular situations.

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APPENDIX

Tasks Posed to the Students

1. Describe in your own words the situation shown in each of the figures.

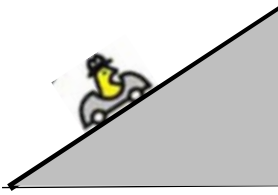


Figure 1



Figure 2

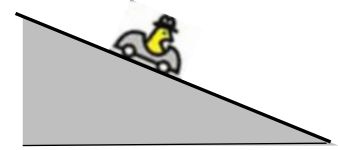


Figure 3

2. What is the slope of the ramp that the car should travel to reach the top of the hill?

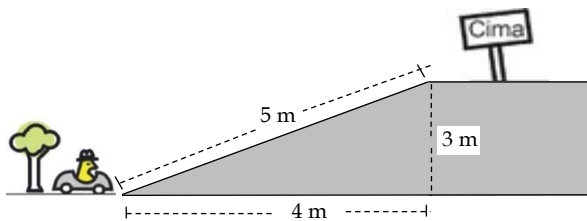


Figure 4

- a) 3 m b) 4 m c) 5 m d) $\frac{3}{4}$
 e) 10 m f) $\frac{5}{4}$ m g) $\frac{4}{3}$ h) $-\frac{5}{12}$
 Other: _____

3. What is the slope of the ramp that the car has to travel?

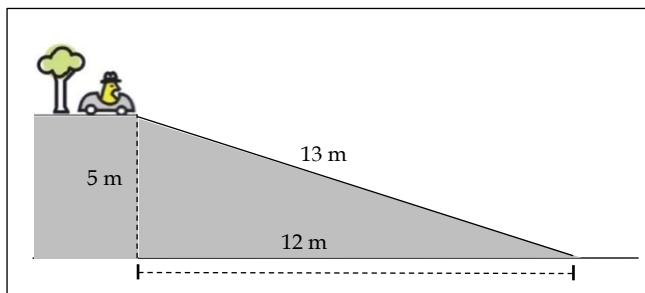
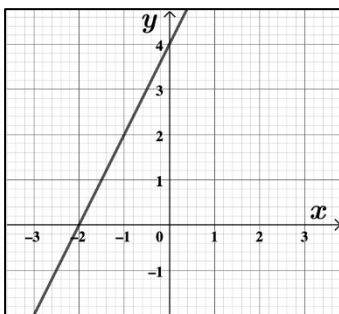


Figure 5

- a) -5 m b) 12 m c) -13 m d) $-\frac{12}{5}$
 e) 13 m f) $-\frac{5}{12}$ g) 5 m h) $\frac{5}{12}$
 Other: _____

4. What is the slope of the straight line shown in Figure 6? Explain how you find it.



← Figure 6

- a) -2 b) 4 c) $\frac{4}{2}$ d) -2 y 4 e) $\frac{1}{2}$
 f) 4x2 g) 2 Other: _____

Answer the following questions based on the information given in Figure 7:

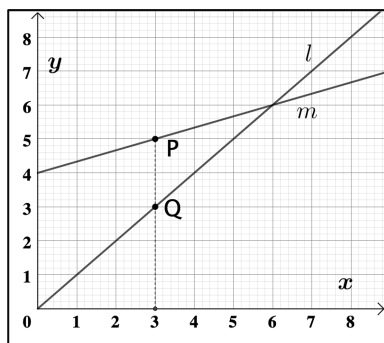


Figure 7

5. What is the slope of line m at point P?
 - a) 4 b) 5×3 c) 5 d) 3 e) $\frac{1}{3}$ f) 6 g) $\frac{5}{3}$ Other: _____
6. What is the slope of line l at point Q?
 - a) 2 b) 3×3 c) 0 d) $\frac{3}{3}$ e) 3 f) 6 Other: _____
7. What are the slopes like of lines l and m at points Q and P respectively?
 - a) Slope at P is greater than slope at Q
 - b) Slope at P is less than slope at Q
 - c) Slope at P equal the slope at Q
 - d) Other: _____

8. Which of the following graphs represent the speed of a bus that travels 50 km per hour?

Explain your answer.

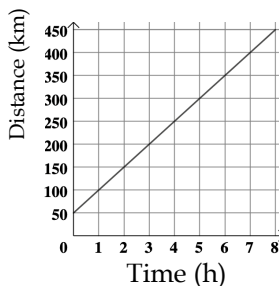


Figure 8

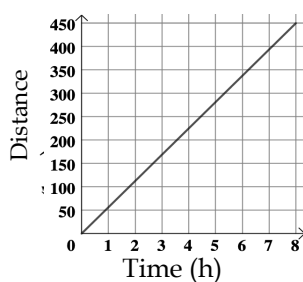


Figure 9

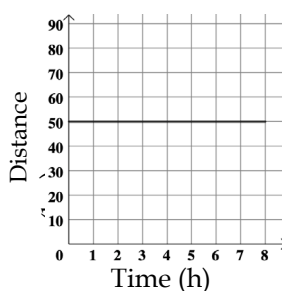


Figure 10

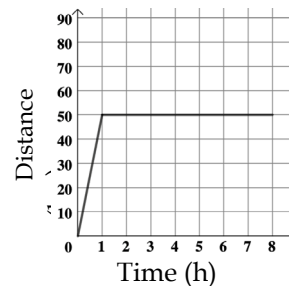


Figure 11

9. Antonio and Raúl start a race from the same starting point at an Olympic track. After 5 minutes, Antonio reaches a speed of 2 m/s and Raúl reaches 5 m/s, and they keep it that way for half an hour. Make the graphs that represent both situations, which goes faster? Why?

10. What does slope mean to you? What do you think when this word is mentioned? Make a representation associated with this idea.