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Learning to argue while arguing to learn: Students' emotional experiences during argumentation for graphing real-life functions

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Abstract
Argumentation in school mathematics is an important but demanding practice that supports important learning goals. Much of the research to date has focused on students' cognition but researchers are paying attention to students' affective experiences that influence their learning, and particularly in socio-cultural settings. In this qualitative case study, we drew on an emotion coding scheme to investigate students' emotions during a cyclic sequence of small-group argumentation tasks on graphing and critiquing graphs of real-life situations. Insights into the emotional experiences of argument construction, critique, and reaching agreement were gained by analyzing observations, written reflections, video data, and interviews of six 9th-grade students (aged 14-15 years). A wide range of emotions were observed and expressed throughout the sequence. We found examples of emotions, including tension and frustration, playing a productive role in the students' mathematics learning because of their experience of argumentation. The emotion of tension experienced by some students when receiving and giving critique of their mathematical ideas seemed to trigger productive attention to misconceptions. The emotion of frustration experienced by some students seemed to trigger productive co-construction of arguments and 'aha' moments of mathematical understanding. We also found examples of non-deliberative argumentation (disrespectful interactions, lack of final consensus) influencing emotions in the moment (distress) and in the longer term after the lesson sequence (disappointment). Implications and considerations for future research on argumentation for learning are discussed.

Keywords: argumentation, emotions, secondary mathematics, functions

INTRODUCTION

Extensive research in the field of education has highlighted the significance of argumentation activities in fostering deep reasoning and critical thinking abilities among students. These activities involve the exploration, confrontation, and evaluation of various viewpoints, as well as expressing support or objections, and providing justifications for different ideas and hypotheses (Evagorou et al., 2023; Francisco & Maher, 2005; Gravemeijer et al., 2017; Mueller et al., 2014). This view is reflected more generally in recent educational reform documents all over the world, highlighting argumentation as one of the most important goals for students (e.g., Common Core State Standards Initiative, 2010; Ministry of Education, 2021). Argumentation has a long tradition in both science and mathematics education, such as in the context of evaluating and making scientific claims (Evagorou et al., 2023) or formal geometric proof (e.g., Demiray et al., 2023). The focus has often been on students' individual reasoning and arguments, but increasingly researchers are paying attention to collective socio-cultural settings and opportunities for collective argumentation (e.g., Zhuang & Conner, 2022).

In order for argumentation to become an integral part of the class environment that enables effective learning, students need to take an active part in the process and show high levels of participation in building arguments and responsiveness to the contributions of their peers

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Contribution to the literature

- Students' collective argumentation on graphing real-life functions in school mathematics evoked a wide range of emotions, observed by researchers and described by students themselves.
- Experiences of negative emotions during argumentation, such as tension and frustration, may nonetheless play a productive role in learning, in helping students pay closer attention to particular mathematics concepts and to persevere with making sense of each other's arguments.
- It is important for teachers to monitor students' emotions and intervene if they are related to non-deliberative argumentation, such as disrespectful interaction or lack of eventual consensus, because of their potential to affect student motivation in the longer term.

(Webb et al., 2014). In particular, given the nature of learning to argue–to share ideas, hear questions and criticism, keep one's mind open, listen to and question peers–more research on the emotional dimension of students' argumentation–arguing to learn–is needed (Isohätälä et al., 2018; Plantin, 2004; Slakmon & Schwarz, 2019). However, there are few studies in the research literature that investigate students' emotions in their experience of argumentation, and particularly in mathematics learning contexts. The aim of this study is to fill the existing gap by investigating the emotional aspects experienced by students during a series of argumentation activities involving the construction and critique of graphs depicting real-life functional situations. Below we provide details on the context for the research and discusses theoretical perspectives from the research literature, which provided a framework for our work.

THEORETICAL BACKGROUND

Theoretical Perspectives on Argumentation and Learning Mathematics

Education researchers have drawn on the definition of collective argumentation proposed by van Eemeren and Grooendorst (2004). They defined argumentation as "a verbal, social, and rational activity aimed at convincing a reasonable critic of the acceptability of a standpoint by putting forward a constellation of propositions justifying or refuting the proposition expressed in the standpoint" (p. 1). This definition has been shown to be beneficial for investigating argumentation within the mathematics classroom (Ayalon & Hershkowitz, 2018). Mathematics students participating in argumentation are involved in building claims, providing evidence to support their claims, and evaluating evidence to judge the mathematical validity of the claims. When integrated into the discourse of the mathematics classroom by teachers, such experiences provide support for students to engage in reasoning with alternative ideas, engage in reflection, and collaboratively develop justifications (Tasdan et al., 2022; Zhuang & Conner, 2022). This view of argumentation forms a foundation for common descriptions of the type of argumentative discourse that is "fruitful" for learning (Asterhan & Schwarz, 2016, p. 167). In the literature, one such type is titled *deliberative argumentation* (Felton et al., 2009). Deliberative argumentation aligns with the view of mathematics as a social enterprise and mathematicians as communal members with established norms of argumentation for advancing mathematical knowledge (Davis & Hersch, 1981). In mathematics classrooms, this is demonstrated by students listening to and building on each other's ideas and critiquing ideas as the community moves toward consensus (Ball & Bass, 2003; Mueller et al., 2012). Studies in mathematics education highlight that engagement in such argumentative discourse promotes meaningful conceptual development and deep reasoning in mathematics (e.g., Mueller et al., 2014; Webb et al., 2014; Weber et al., 2008). In our study we have characterized deliberate argumentation as involving participants:

(1) giving and receiving critiques of alternative mathematical ideas,

(2) co-constructing arguments by building on each other's ideas,

(3) listening and responding respectfully to each other's ideas, and

(4) collaborating for the goal of reaching a consensus view (Felton et al., 2009; Schwarz & Baker, 2017).

Deliberative argumentation is contrasted with two other types: disputative argumentation and consensual co-construction. Argumentation that is disputative is characterized by participants disagreeing with others' ideas but without giving valid reasons and asserting their own ideas without the goal of reaching consensus collaboratively (Felton et al., 2009). With consensual co-construction, ideas may be expanded, elaborated, or explained, but they are not challenged or criticized, and so alternative ideas are not juxtaposed, restricting the opportunity for deep thinking.

Theoretical Perspectives on Emotions and Learning Mathematics

Several studies in the research literature dating back to the 1980s highlight the importance of attending to the mutual interaction between cognition and affect in learning mathematics (Hannula, 2012; Zan et al., 2006). Op't Eynde et al. (2006) argued that from a socio-constructivist perspective, affective processes are an integral part of problem solving and learning.

Within the literature on affect, there are differing theories about defining emotions and their mechanisms, yet it is commonly agreed that there are types of emotions universal to humans and distinctive in their physical expression, for example, anger, fear, joy, sadness, disgust, and surprise (Ekman, 1992). McLeod's (1992) affect framework has been widely used in mathematics education; it is comprised of three major aspects: beliefs, attitudes, and emotions. He viewed emotions as unstable compared to attitudes and beliefs. Hannula (2006) described emotions as being partially observable in a person's facial expressions and body language, but also partially inaccessible since they are experienced subjectively; they may even be inaccessible to the person experiencing them. DeBellis and Goldin (2006) provided a similar definition of emotions as "rapidly-changing states of feeling experienced consciously or occurring preconsciously or unconsciously during mathematical (or other) activity" (p. 135). According to this perspective, emotions can vary in intensity, be specific to certain situations, and be influenced by the surrounding context. However, Goldin (2002) and McLeod (1992) viewed emotional responses in the moment as having an impact on a student's long-term and overall attitude towards learning mathematics. Hannula (2006, 2012) also emphasized that automatic preconscious emotional reactions can be linked to past experiences. These understandings of emotions align with a neuropsychological standpoint, as exemplified by Immordino-Yang and Damasio's (2007) perspective on emotions as fundamental mechanisms for decision-making, enabling individuals to evaluate and react to various circumstances. Emotions guide a person's cognitive processes by incorporating relevant past experiences specific to the given context. A social-ontological viewpoint regarding emotions underscores the significance of attending to the emotions expressed by students within learning environments. Hannula (2006) emphasized the role of emotions in students' self-regulation of motivation and their connection to the students' goals, which stem from psychological needs such as social belonging, autonomy, and competence. These students' goals are shaped by their beliefs about learning mathematics, and themselves, as well as the norms within their classroom. Hannula (2012) conceptualized emotions as related to the biological human body and social systems. They have two social goals: affiliating, which involves building cooperative relationships with others, and social distancing because of social status or power (Fischer & Manstead, 2008). Therefore, the role of emotions in students' mathematics learning in collective socio-cultural contexts is important to research (Else-Quest et al., 2008). In our study, we used Else-Quest et al.'s (2008) definitions of specific emotion types for our analysis of the students' facial expressions and body language, as partial markers evidencing their emotions (Hannula, 2006), and also on students' self-reported emotional experiences, as partial evidence of their appraisal processes and actions (Goldin, 2002; Immordino-Yang & Damasio, 2007; McLeod, 1992).

Previous Research on Emotions During Argumentation

In researching participants' emotions in argumentative situations, it is important to consider their level of engagement (Plantin, 2004). Both personal and social aims of the participants are involved, and their actions have consequences for individuals and their relationships with others (Stein & Albro, 2001). Negative emotions are particularly salient when argumentative discourse involves perceptions of unfairness or confrontation (Muller Mirza et al., 2009). When people participate in argumentative scenarios, they assume a certain level of risk, since expressing their viewpoint may result in encountering opposition from other participants (Horn, 2008). Schreier et al. (1995) theorized a framework specifying two types of actions perceived as unfair during an argumentative episode:

(1) *violations of rationality*, such as making a faulty argument or refusing to justify an argument and

(2) *violations of cooperation*, such as discrediting someone, expressing hostility, hindering someone's participation and breaking off from discourse.

Mischo (2003) found that type 2 violations (noncooperative contributions) tend to be evaluated by group members more severely than type 1 (non-rational ones). According to Stein and Albro (2001), it is important to analyze the potential effects of various emotional reactions, such as unhappiness or anger, within the context of argumentation, particularly in relation to subsequent negotiation strategies and outcomes.

With regard to argumentation in educational contexts, Asterhan and Schwarz (2016) suggested that more research is needed into the nature of interpersonal and social interactions during different types of argumentation discourse (such as deliberative and disputative) to investigate how those experiences might affect the participants' engagement. They hypothesized that disputative argumentation would be more likely to lead to student disengagement from those actions that enable learning, i.e., lead away from deliberative argumentation. Students may even avoid engaging in argumentative interaction at all, as they may consider 'arguing' as something that people do when they are being impolite. Disagreement or challenge may be perceived as aggressive and antisocial (Lampert et al., 1996; Polo et al., 2016). Slakmon and Schwarz (2019) also called for researchers to examine the emotions of students when participating in deliberative argumentation.

Although research has extensively examined argumentation as a crucial aspect of mathematics education, there is a lack of studies specifically addressing students' emotions in relation to their participation in collective argumentation for the purpose of learning mathematics In this study, we investigated students' observed and self-reported emotions during their experience of small-group argumentation in three sequential cycles of interaction on different learning tasks about real-life graphs. We looked for themes that related their emotions to the characteristics of their argumentative discourse and mathematics learning. Our research question for this study was: *How do students' emotions, experienced during participation in small-group argumentation activities, relate to the characteristics of their argumentative discourse for learning about graphs of real-life functional situations?*

RESEARCH CONTEXT AND TASK DESIGN

Mathematics Context: Functional Situations in Real Life

The sequence of small-group argumentation activities involved both sketching graphs to match written non-linear "real-life" situations and critiquing the accuracy of some provided fictitious sample graphs for each real-life situation. The authors designed the tasks to elicit students' reasoning about functions and rates of change: identifying suitable variables for the situation, noticing contextual features of the situation, and forming a valid relation between them represented graphically. The tasks were deliberately designed to stimulate argumentative interaction by being both familiar yet slightly beyond students' likely prior learning experience (Schwarz & Baker, 2017)–they were about functional relationships in real-life situations, and the mathematics concepts were highly relevant to their prescribed secondary curriculum on graphing functions (Ministry of Education, 2021).

Although students' acquaintance with daily events can support them in learning to make sense of graphs (Goldenberg, 1987), dealing with graphs of real-life functional situations can challenge students (Leinhardt et al., 1990; Oehrtman et al., 2008). A widespread issue is students' "shape thinking"–that a graph is a static object with properties the student associates with learned facts, rather than a dynamic trace of quantities having co-varied (Moore & Thompson, 2015). Students have been found to apply such learned facts, such as 'gradient', to completing a specific task, but not to connect them to another task context (Wilkie & Ayalon, 2018). Other issues include not attending to both variables (covariation) in a functional situation, choosing only one relevant variable to graph an inappropriate relation, picking appropriate variables but graphing an inappropriate relation, and not noticing all of the contextual features that need to be graphed (Ayalon et al., 2018). It is important for students to learn connected ways of reasoning about functions, covariation, and rate of change, rather than memorize isolated procedures (Thompson & Carlson, 2017). We considered that opportunities to explore these difficult-to-learn functions concepts through argumentative discourse might provide students with the opportunity for deep mathematical reasoning needed for connecting such concepts. The cyclic sequence small-group activities on real-life functional situations were expected to stimulate students' questions, ideas, and disagreements, and therefore their active engagement in an argumentative process of learning (Schwarz & Baker, 2017).

Learning Task Sequence

The central component of the sequence consisted of three cycles of argumentation activities spanning several weeks. Each cycle, lasting 90 minutes and encompassing a double class period, centered around a distinct real-life situation, as depicted in **Figure 1**.

METHODOLOGY

In this study, we employed a case study design for qualitative in-depth exploration (Creswell, 2013) of student emotions during argumentative discourse for learning in a secondary mathematics classroom context. A small group of students comprised a case, and the students were studied individually and as a group to research their perceptions and experiences of argumentation. In the following two sub-sections we overview demographic information on the study's participants and data collection, and outline the data analysis process, including Else-Quest et al.'s (2008) emotions coding scheme that was adapted for this study.

Participants and Data Collection

The study was carried out in a regular 9th grade classroom. The study took place in a school with middle-to-high levels in mathematics achievement, to minimize the risk of students finding the tasks too difficult, while at the same time wanting them to be sufficiently unfamiliar with the tasks so as to stimulate deliberative argumentation (rather than consensual co-construction). The teacher who facilitated the sequence with the class, was informed about the research purpose and process, as well as the implementation of the classroom activities. She was also involved in choosing the students for the target group, with a preference for students she viewed as likely to be talkative.

The research centered on six students named Anna, Eva, Roni (girls), and Liam, Omer, and Tom (boys)

Cycle 1–"Watering a plant" situation:

Jack forgot to water his pea seedling. The seedling dried up and its growth decelerated for a while. Then, Jack remembered to water the plant, and its growth accelerated.

Cycle 2–"Filling a container" situation:

A tap is turned on hard, and water rushes into this container at a constant rate.

Cycle 3–"Pricing cakes" situation:

Karin, a cake shop owner, has to decide on the price at which to sell a new chocolate cake to customers. If the price is too low, she will lose money, but if the price is too high, fewer chocolate cakes will be sold, and she will lose money. She needs to choose a reasonable price so that she can sell enough cakes to make a profit.

Figure 1. Three cycles & real-life functional situations used in the learning task sequence (Ayalon et al., 2021)

(pseudonyms). We included these particular students in the study following the teacher's suggestion that some of them might occasionally be absent from class due to rehearsals for their end-of-middle-school celebration (which did not eventuate as all six students participated in the first two cycles and five in the third). The teacher told us that she focused her teaching on students developing a rich understanding of mathematics and that these students had previously experienced inquiry approaches and small-group work. She also indicated that the class was not used to participating in the activity of critiquing in a mathematics context.

The data collection comprised observations, written questionnaires, videoed student interactions within the group, audio-recorded interviews, student individual written work, and group written work.

The study primarily revolved around students engaging in argumentation within a small group setting, emphasizing collaborative learning and active participation in argumentative discussions. The researchers closely observed the actions of a specific group of students throughout the classroom sessions. For analyzing the students' actions during each of the three cycles, the researchers utilized various methods including:

(1) employing an observation proforma to systematically record student actions,

(2) maintaining a researcher's journal to document additional notes on significant events and reflections,

(3) conducting individual student interviews to gather insights from their perspectives, and

(4) engaging in a debrief session with the teacher to gain her perceptions regarding the students' actions during each cycle.

Four instances of written student reflections were gathered at various points during the study (**Appendix A**). These reflections served as additional data sources, offering insights into students' self-perceived emotions and behaviors within the classroom during the argumentation sequence. The purpose of collecting these reflections was to corroborate the researcher's observations in class, providing a triangulation of data for a more comprehensive understanding of the students' experiences. As part of the research process, each participant in the focus group underwent individual interviews. With these interviews we aimed to obtain a deeper comprehension of the students' viewpoints and allow them to expand on their experiences. More specifically, we asked the students to describe and clarify the emotions they experienced during significant moments that the researcher had observed and recorded in the classroom. Discussions with the teacher before and during the sequence provided information about the teacher's perceived usual classroom climate and teaching approaches; suggestions for group membership; the usual engagement of the chosen focus-group students; and about the teacher's experiences while facilitating the argumentation activities. A semi-structured post-sequence interview was also conducted with each student (**Appendix B**).

Data Analysis

In our examination of students' behaviors in the classroom, we focused on identifying signs of emotional experiences commonly understood within that particular subculture. We considered four types of indicators as identified by Evans et al. (2006): direct verbal expressions such as saying, "I feel anxious", the use of specific verbal metaphors, the emphasis placed on words, gestures, intonation, or repetition to convey strong or chronic feelings, and non-verbal cues such as body language, facial expressions, or blushing. While these indicators are seen as displays of emotions, it is important to note that students may not be consciously aware of them. Therefore, in presenting our findings, we aim to describe and share our observations in a clear and transparent manner, while acknowledging that they are based on the subjective interpretations of the researchers. For our analysis of video data, we utilized emotion coding scheme previously introduced by Else-Quest et al. (2008). We specifically looked for markers of emotional experiences and considered their possible interpretations, considering the specific context in which

Table 1. Emotion coding scheme* adapted for the study (adapted from Else-Quest et al., 2008 and Ayalon et al., 2021)

Category	Description	Markers in the study
Tension	Nervousness, anxiety, uncertainty, self-conscious, worry, & tension	Fidgeting, talking in tone of nervousness, & frowning
Distress/dismay	Distress, complaining, & disappointment	Raised inner eyebrows, leaning back, & sitting quietly (less intense facial expressions than sadness)
Frustration	Frustration, annoyance, & impatience	Outbursts, sharp movements, & talking in tone of irritation or frustration
Sadness	Sadness, withdrawal, & self-criticism	Leaning back, slouching, down-turned mouth, & raised inner eyebrows (more intense facial expressions than distress/dismay)
Boredom/apathy	Boredom, apathy, absence of all interest, & flat	Slouching, not listening attentively, & looking down/away
Contempt	Contempt, mocking, sarcasm, smugness, disrespect, scorn, & "brattiness"	Disrespectful interjections/interruptions & mocking tone of voice
Positive interest	Interest, engagement, positive attention, & eagerness	Leaning in towards task, focused on task, & engaging in discussion
Affection/caring	Encouragement, support, soothing, reassurance, trust, respect, & warmth	Smiling at person, leaning in towards person, listening attentively to person, verbally affirming, & soft tone of voice
Joy/pleasure	Happiness, excitement, pleased, having fun, & enjoyment	Smiling & exclamations ("Oh!")
Humor	Humor, joking, friendly teasing, & silliness	[Not seen with this focus group]
Pride	Pride, amazement, & focused on achievement or ability	Smiling when antecedent events is being praised/affirmed or revolving issue
Off-task	Group is not working on mathematics task	[Not seen with this focus group]

Note. Columns 1 & 2 are from Else-Quest et al.'s (2008) scheme; column 3 markers are from argumentation activities

they occurred, as outlined in Else-Quest et al.'s (2008) scheme. **Table 1** provides an illustration of how we applied Else-Quest et al.'s (2008) emotional categories in our analysis of the video data to interpret markers of students' physical and verbal actions. The activities within an individual cycle are presented in **Table 1**.

Anger is a category related to frustration but of greater intensity. At times the students appeared to be intense in their outbursts, but because they themselves used the term frustration to interpret their own emotions, we chose to leave out anger in our adapted framework.

When analyzing the students' written and verbal data, whether it was spontaneous or in response to a specific question about a particular class episode, we focused on identifying the connections that students made between their subjective emotional experiences and other aspects of their context, for example, argumentation processes, social goals, and social functioning within the group (Fischer & Manstead, 2008), or the mathematics in a task (graphing of real-life non-linear functions). Our interest lay in examining how students interpreted their emotions during a task in the immediate moment, as well as their subsequent long-term evaluation of their experiences and understanding of the mathematics functions involved. The data analysis process was comprised of two main stages. In the first stage we examined the data (videos, written reflections, interviews) for segments of the discourse that evidenced of or explicitly referred to each of the four characteristics of deliberative argumentation: co-construction of arguments, critique of alternative ideas, respectful interactions, and consensus.

We also sought to identify segments of the discourse that evidenced the opposite characteristic (such as disrespectful interaction or a standoff). In the second stage we looked for related indicators (Evans et al., 2006) or expressions of emotions: In the researcher's journal, video data, students' written reflections, and interview data, we actively sought explicit mention of specific emotions, as well as the use of emotive language or expressions that indirectly indicated some form of emotional response (Else-Quest et al., 2008). We used an analytical process among the researchers that was iterative and comparative. Decisions about interpretation were made collaboratively among the research team through check-coding and discussion to reach consensus and improve the reliability of the analysis (Miles & Huberman, 1994).

FINDINGS AND DISCUSSION

In the following four sub-sections we frame our findings according to themes on students' emotional experiences when participating in small-group argumentative discourse. They are structured according to the previously outlined four characteristics of deliberative argumentation to explore the role of students' emotions in achieving the learning goals of the

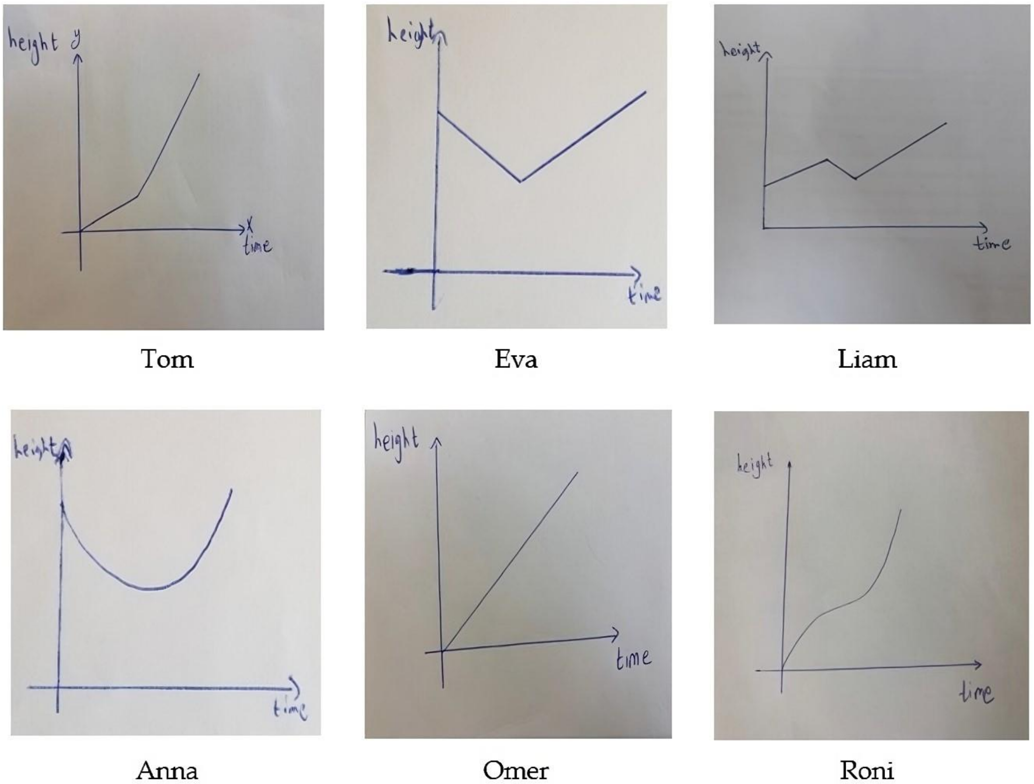

Figure 2. Constructed individual graphs for situation 1–'watering a plant' (for readability, drawings are facsimiles of the students' work with variables written in English) (Ayalon et al., 2021)

tasks. We draw on interpretations of emotions from the researchers' observations, verbal transcripts, and video data analysis as well as the students' own verbal and written expressions and interpretations of their emotional experiences and actions.

Potentially Productive Role of Tension: Receiving Critique

The experience of having one's mathematical ideas critiqued by peers was new to these students and often led to feelings of tension. Yet for some this was found to provoke their closer attention to their misconceptions and led to some 'aha' moments of worthwhile learning about graphs of functions. Some students also reported enjoying the pleasurable experience of having their ideas validated by a peer.

Across the three cycles of activities on different real-life situations, we found that the tasks did elicit differing mathematical ideas about functions from the students and discourse on them as hoped. Their ideas generated interactions, where the students gave and received critiques of their various ideas. The range of ideas were partly due to the possibility for more than one correct graph in each real-life situation, depending on the choice of variables. Unsurprisingly, some of the students evidenced and described experiencing tension when their graphs were disagreed with, even if the interaction itself was respectful and the focus was on the mathematics of their arguments. For example, in the first cycle ('watering a plant'), the students shared and discussed each member's graph (**Figure 2**), which were all different despite the students' selection of the same variables.

Eva initially disagreed with Omer's graph, saying it should not start from the origin. Anna supported Eva's argument, explaining that it did not make sense to start from the origin because it meant they did not start watering an actual plant–that there was no plant at all in the beginning. Omer reacted, "Obviously it starts from the origin" but then withdrew from further discussion for a time. He twitched his leg repeatedly up and down, suggestive of fidgeting, which is a marker for tension (Else-Quest et al., 2008). Later in his reflection, Omer reported that at that stage of the activity he felt anxious because it was unpleasant to be told that his graph was wrong. He experienced uncertainty (also a marker for tension) as he was no longer sure about his own mathematical reasoning.

Eva presented her graph, explaining that it starts from certain point, goes down and then goes up. Others then criticized her argument, as shown in **Table 2**.

Despite a number of self-reports of tension by the students when their ideas were criticized, some of the students reflected later on experiencing moments of pleasure when an idea of theirs had been critiqued and then supported by someone else, perhaps a validation of their mathematical thinking, or personal encouragement. For example, Tom reflected:

Table 2. Verbal excerpt from argumentative episode about situation 1 (translated into English)

Student	Verbal transcript	Student-reported emotions
Eva	[Presenting her graph] It starts from certain point, goes down and then goes up.	
Tom	Goes down and then goes up? How does it go down and go up?	
Eva	It is written here [points to the task]. He forgot to water his pea seedling, the seedling dried up and its growth decelerated for a while. For a while it went down and then it went up [showing with her hands].	
Tom	Not going down, decelerating.	
Roni	It did not go down [pointing at Eva's graph]. It should be a line a bit up [looking at her own graph].	
Anna	It's like, I think it is too sharp [pointing at Eva's graph]. It did not go down so sharp.	
Tom	The growth slowed down, so there was still growth, just slower [pause]. It does not follow that the graph should also go down.	
Eva	Maybe, I am not saying mine is right.	In her reflection, Eva wrote that she felt insecure & tense when she was criticized & realized that her graph was incorrect.
Anna	[Referring to Tom]: Why did it start from the origin?	
Liam	It doesn't start at zero because it is not written that he planted the seedling and immediately watered it.	
Anna	It does not make sense that it starts at zero because then it would mean that it was never watered, that there was no plant at all [pause]. If there is a plant, it means that it was watered sometime.	
Eva	Exactly.	
Tom	Zero is when Jack planted the seedling.	
Anna	But it is not written that he planted the seedling! It is the moment he began watering it. [pause] In the situation there is a seedling already.	Tom reported later that he felt tense when he was opposed repeatedly. Yet he was observed to remain engaged and persisted in trying to explain his arguments.

> "At the beginning of the activity first I had motivation, and then less motivation, and then more, decreasing and increasing according to how the students responded to my ideas, whether they accepted them or disagreed with them" (Tom, post-cycle-1 interview).

A few students also reported on appraising the experience of having an idea of theirs challenged in a positive light, describing it as an opportunity to learn a specific mathematics concept. For example, Omer said that after initially feeling tense, when evaluating their strengths and weaknesses, he experienced a sense of satisfaction. He added that even when he faced criticism, he felt positive as it promoted his growth and learning. It dawned on him, as he said, that receiving critique was an integral part of the process and that it is important to be an active participant in a group and engaging in open discussions.

He emphasized that he recognized the need to release stress and attentively listen to his peers, rather than becoming overwhelmed by every disagreement. Some students also reported experiencing pleasure when giving their critique to others and related it to being listened to by another student–their positive interest demonstrating emotional encouragement or support. For example, after the first cycle, Liam said,

> "When I showed Eva what was wrong with her graph, I felt good, I felt someone was listening to my opinion."

Felton et al. (2009) highlighted that for argumentative dialogue to be effective an individual needs both to seek to respond to questions and challenges with substantive answers and to be open to re-evaluating their claims. In this study, several students experienced tension from having their graphs disagreed with. In the mathematics classroom, it seems more likely that students when working in small groups are used to experiencing co-construction of knowledge rather than argumentative discourse per se. In ordinary conversations disagreements may be perceived as socially undesirable moves (Polo et al., 2016). Yet Omer was able to move beyond his experience of tension by re-appraising his experience, that "critique is part of the activity". This re-appraisal and shift in action from disengagement to re-engagement resonates with Immordino-Yang and Damasio's (2007) view of the processing of emotions for

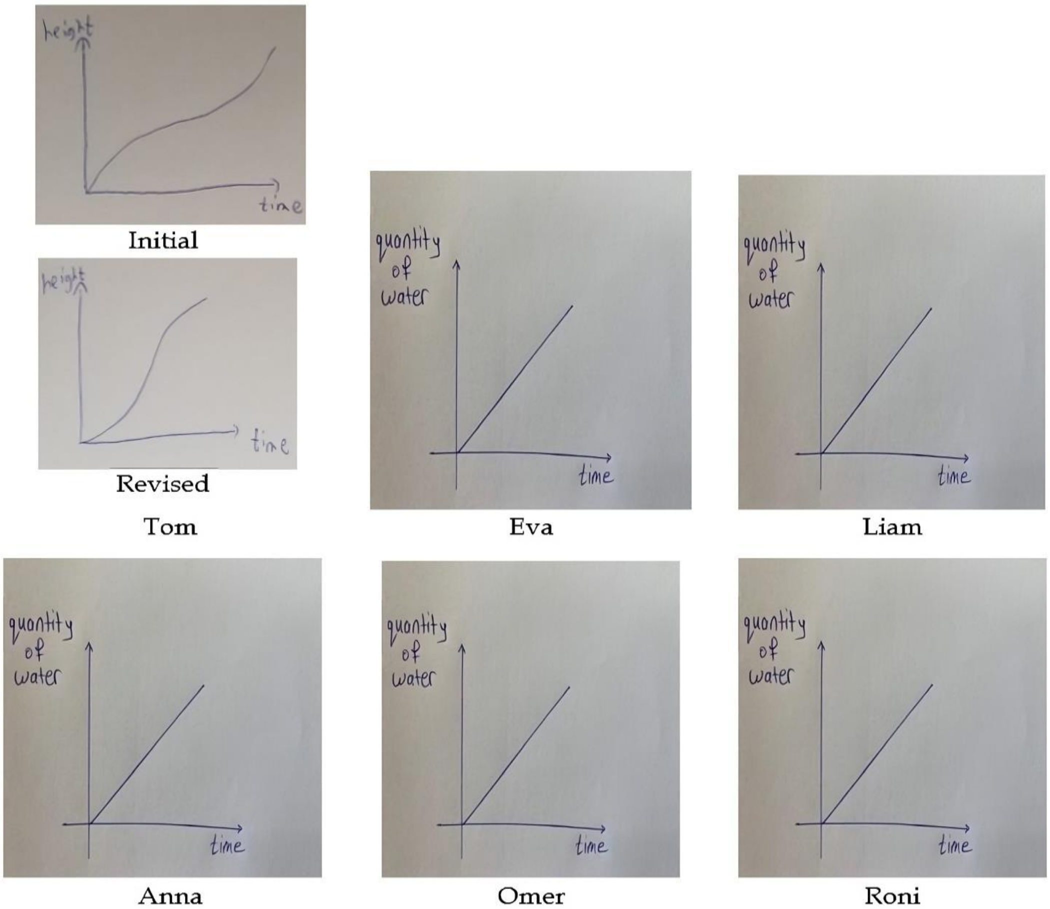

Figure 3. Constructed individual graphs for situation 2–'filling a container' (Ayalon et al., 2021)

subsequent decision-making. Omer was initially tense but reasoned about the usefulness of being critiqued and decided to re-visit his idea and ended up correcting a misconception. Overall, this suggests the value of students learning to give and receive critiques of mathematical ideas, and for teachers and students to be open to seeing feelings of tension that may accompany such critiquing, as potentially productive for learning.

Role of Frustration in Persevering (or Not): Co-Constructing Arguments

Across the three cycles during lengthy periods of trying to co-construct arguments to decide on a correct graph, various group members seemed to experience repeated waves of intense engagement, expressions of frustration, withdrawal from the discourse (while still following the discussion), and then re-engagement. Most seemed to harness their frustration for persisting with the difficulties of the mathematical ideas themselves as well as the difficulties of understanding each other's attempts to persuade clearly. One student's frustration seemed to play a role in his remaining disengaged, which had negative consequences for his learning.

The group seemed constantly to react to one member, Tom, and his arguments in particular. Several students reflected later on their experience of frustration and interpreted it as relating to Tom's incoherence when he was trying to persuade them about his ideas. One telling scenario occurred in second cycle ('filling a container') when Tom's graph was noticeably different from all the others, which were linear. As shown in **Figure 3**, Tom attended to the container's shape and chose the variables 'height' and time (although with an incorrect curve at this initial stage), whereas the other five students treated 'quantity of water' as being dependent on time.

Each of the students positioned his/her graph on **Table 3** to be visible to everyone. They noticed that Tom's graph was different from all the others and asked him to justify why, as shown in **Table 3**.

It seemed that a main source for the confusion was at least partly related to Tom's use of a different dependent variable (height, not quantity of water), which no one in the group had noticed yet. Liam claimed that Tom's graph was not justified because the 'constant rate' was explicitly described in the verbal situation, so it did not make sense for the slope in Tom's graph to be curved.

Tom did not give up and continued to explain his graph again and again. Each group member expressed their disagreement with Tom's graph, but he persisted in his confidence that it was correct. Eva eventually burst out and said in frustration,

"Ugh! How could it be that he insists like that?"

Table 3. Verbal excerpt from argumentative episode about situation 1 (translated into English)

Student	Verbal transcript
Roni	Let's draw this graph [pointing at her graph] since this is what most of us draw [No one responded to Roni].
Eva	[Turning to Tom] Why did you do it? How does the shape of the container relate to the graph? [gestures with her hand the shape of Tom's graph].
Tom	The height of the water in the container increases depending on the shape of the container.
Eva	The shape of the container is irrelevant.
Anna	You do not have to relate to the shape of the container, it is the amount of water. The shape of the container does not change the amount of water.
Liam	[Turning to Tom] They are right. You don't have to relate to the shape of the container. If we had a glass in the same situation instead of the container, how would you draw the graph then?
Anna	It does not matter whether it is a cup or any other container because regardless of the shape, there would still be a constant rate of change [in the quantity of water].
Tom	If it was a straight glass, I would have drawn your graph. But container is curved [pointing at shape of it].
Omer	I do not understand what you are trying to do.
Tom	Try to understand.
Omer	I am trying to understand.
Anna	I am trying to understand, but you referred to shape of container & it is not related to written situation.
Liam	Yes, I do not understand what connection is between quantity of water & the shape of the container.

Later Eva reflected:

> "I felt anger and frustration with Tom's behavior. He was so annoying and talked a lot. And it was so difficult to understand his explanations for his graph" (Eva, post-cycle-2 interview).

It seemed that the group were puzzled by Tom's graph in being a completely different shape to the others, and they repeatedly questioned him about it, but found his explanations unclear and unhelpful. It could be because of their perceived 'violation of rationality' in Tom using a type of faulty argument (Schreier et al. 1995) and also perhaps because since his graph, although curved, was actually incorrect at this point in the discourse. Schwarz and Baker (2017) emphasized that in order for a learning situation to provoke productive argumentation that is neither teacher-centered nor devoid of dialectic, *partial* rather than full or no knowledge of the focus of the discourse is necessary. In this study, the task was designed to be partially familiar to the students (filling containers with water from a tap) but mathematically challenging (by involving a non-constant rate of change of the height of a container) to stimulate dialectic. And Tom's graph did, as intended, stimulate argumentative discourse and co-constructing arguments for some members of the group, such as Eva. Tom continued to make repeated attempts to explain, while the others fired questions at him. For example,

> What is the difference between this container and a glass? (Liam).
>
> In the situation you have a constant rate of change, so what is this? (Omer, pointing to Tom's graph).
>
> Why did you draw a curve, you like refer to the shape of the container? It is not what the tasks asks us to do! (Anna).
>
> I do not understand, what are you trying to do? (Omer).
>
> Prove to us that you need to refer to the container (Anna).

He talked about the relationship between his chosen variables and gesturing repeatedly with his hands to demonstrate the curved shape of the container. Then he leaned back on his chair and said that he could not explain his ideas further. Then Anna had an "aha moment" when she observed that Tom had utilized a distinct variable (height instead of quantity) on the vertical axis. This discovery led her to understand that there could be multiple valid interpretations for the given situation. This revelation occurred after their previous lesson on 'Watering a plant', where they all selected the same variables of time and height. From then on, there was a clear change in the interactions of the group. Eva tried again to understand Tom's graph. Eva now joined Tom in trying to convince the others about the validity of his graph. She reflected later that "he is correct in his ideas even though he is irritating". Tom was apparently re-energized by Eva's being on his side: he leaned in towards the task, smiled, and focused again on his (incorrect) graph. Suddenly, he modified his graph, altering the curve so as to increase and then decrease (instead of decrease and then increase). During his renewed efforts to explain his graph to the group, he suddenly realized his mistake with the curve and corrected it. Tom then explained his newly corrected graph to the group.

For one student, Liam, Tom's revision of his own graph at this stage provoked further frustration. Liam conveyed his frustration towards Tom and questioned him why he had inverted his graph and abruptly determined that it was opposite. Liam subsequently leaned back, crossing his arms and raising his eyebrows.

Perhaps because of the lengthy amount of time he had already spent on trying to follow Tom's reasoning, in finding out that the graph was incorrect (and that he had wasted his time trying to understand it), Liam experienced anger towards Tom. Or it could be that the complexity of the real-life situation in involving constant *and* non-constant rates of change evoked his individual cognitive conflict and puzzlement (Chin & Osborne, 2010), expressed outwardly as frustration towards Tom. From that point Liam appeared to disengage from the group discourse. Liam said later in his interview that he had suffered from listening to Tom because it took him a lot of time to express himself. In particular, Tom's changing his graph frustrated him, and he felt desperate from not understanding him. Liam said that he felt so frustrated that he felt like wanting to punch someone.

Later Tom reported that he also felt frustrated from not being able to communicate his ideas persuasively enough. His self-confidence about his arguments appeared to alternate with distress and wanting to disengage when others in the group remained persistent in challenging his ideas. Once another group member finally understood him or was convinced by his arguments (which happened numerous times) and even joined him in explaining to the others, he reported that he experienced pride in achieving success but also weariness after expending so much energy.

In this study, we found evidence of different group members experiencing individual cognitive conflict (Chin & Osborne, 2010) while trying to co-construct arguments. They appeared to seek resolution through questioning Tom about his ideas, and reacting with frustration when they could not follow his arguments. Yet when Tom himself experienced cognitive conflict–when he realized his own graph was incorrect–and needed time to puzzle over and correct his ideas, some others in the group, like Liam, seemed to view the change as unacceptable. Noticeable verbal expressions of anger ensued. The process of co-constructing arguments, where the group members each had partial knowledge of the mathematics concepts and needed to resolve individual cognitive conflict in order to reach consensus, appeared to involve the emotion of frustration repeatedly. That provoked further persistence and learning for some, as with Eva, but also disengagement, as with Liam.

Painful Role of Distress: Disrespectful Interactions

Deliberative discourse is characterized by respectful interactions, even in the face of disagreement. The students in this study, while learning to argue, evidenced disputative rather than deliberative discourse at times. We found that a considerable issue was the difficulty of the students to explain their mathematical ideas coherently, and for their peers to follow their reasoning. The expectation of the students that one of them (Tom) ought to be able to explain clearly to them, perhaps because of their attributed mathematical authority to him, proved problematic, resulting in disrespectful interactions and distress.

Throughout the three cycles, there were several instances of respectful and disrespectful interactions that directly involved Tom. Several students reported that they had experienced irritation, even frustration, at their lack of comprehension when Tom was explaining his ideas. Observations of Tom's behavior throughout the cycles did evidence his intention to be understood–what we interpreted as a sincere rather than uncooperative stance. It seemed rather that the others' frustration related to their perception that Tom was making 'violations of rationality' (Schreier et al., 1995) in making incoherent mathematical arguments. For example, following the work on the 'watering a plant' situation, Liam said:

> "I felt desperate about Tom constantly changing his graph and helpless from not understanding him."

In turn, Tom's violations of rationality appeared to provoke 'violations of cooperation' in the others–verbal and facial expressions of hostility.

We found that expressions of frustration from various group members were mostly directed at Tom, less frequently at others. For example, while sharing and discussing each member's graph for the 'watering a plant' situation, Tom attempted unsuccessfully to communicate his ideas to the others. At some point Eva seemed to become exasperated with Tom, uttered an expletive, tilted her head up, held it in both hands and laughed. Liam and Eva joined her in laughing and Liam said to Tom, "time costs money" in a sarcastic voice. The group ended up choosing a graph different to Tom's and discovered that it was incorrect during the class discussion. Tom complained to his group members, "you did it without me. It's your problem that you did not listen to my opinions". Omer shouted at him, "shut up", and Anna also reacted to him by saying that *he* had not explained to them clearly enough and had not told them anything. Similarly, during the second cycle, when Tom suddenly changed his mind about his graph for the 'filling a container' situation and said he had to draw it the other way around, Liam made a face and became upset. He demanded that Tom explain his sudden change of idea. Anna had also banged on the table and shouted loudly at Tom that he was adding extra features to the written situation, which were unnecessary saying, "you are pathetic."

Although Mischo (2003) found that non-cooperative contributions are viewed more severely than non-rational ones in an argument, we were surprised by the emotional intensity of the group members' reactions to Tom's apparently sincere efforts to persuade them. Tom himself reflected on being on the receiving end of the others' hostility:

> They laughed at me for not being able to explain myself. It made me feel desperate … I was also told to shut up and they looked at me angrily … Sometimes I felt attacked and wanted to get up and leave the classroom (Tom, post-intervention interview).

The repeated instances of disrespectful interactions with Tom seemed to involve an additional dynamic related to their attribution of mathematical authority to him. We noticed that the group had given disproportionate attention to Tom's ideas throughout the cycles. For example, in situation 1 ('watering a plant') Roni's graph had been the only correctly curved shape (but started from the origin) and yet the group had not given their attention to it. It seemed that by default they turned to Tom rather than critiquing each member's graph in turn. It seemed that they may have expected (mistakenly) that his being perceived as 'good at maths' meant that he would also be 'good at explaining' mathematical ideas verbally. It was not clear if this attribution of authority had developed in the context of this learning sequence (from Tom's confident assertions about his ideas) or was pre-existent in the social dynamics of the class. The class teacher described her view that the group focused so much on Tom's responses to the tasks and deferred to his authority because of their prior perception that he was "smart at maths". Engle et al. (2014) studied the issue of undue influence and how some students end up having more influence than is explainable by the quality of their arguments. They hypothesized that an individual student's socially negotiated authority influences their access to opportunities to contribute (access to the 'conversational floor') and to be attended to spatially (looked at, actively listened to). Over time the merits of an individual's arguments also contribute to their level of influence, which can change over time. In the case of Tom, we found substantial evidence of frustration when his access to the conversational floor did not produce the looked-for arguments to help the others with the task (of reaching consensus on a graph).

Although we found disrespectful interactions with Tom during each cycle, they were of short duration, nonetheless, as was the temporary withdrawal of Liam. We did, however, find one exception with Roni, related not to an outburst of hostility of such but to repeated perceptions of being ignored or overlooked.

During the first cycle, while dealing with the 'watering a plant' scenario, a loud disagreement involving Anna, Eva, and Tom emerged. Roni expressed her annoyance at Tom by using a low tone, emphasizing that his graph should resemble hers (which was the only accurate curve among her peers). It was not clear whether the others had not heard Roni due to her low voice or if they were still recovering from the previous intense exchange. Roni positioned her graph in front of Anna, suggesting,

> "Perhaps this is how the graph should appear."

Anna glanced at the page without uttering a word and continued her ongoing discussion with Eva (**Table 3**). This behavior indicated a clear choice of Anna not to respond to Roni, nor allowing Roni to access the conversational floor (Engle et al., 2014). Roni subsequently refrained from further engagement for the remainder of that phase. Her two attempts to be heard had proven unsuccessful.

In the second cycle, following a lengthy discussion involving Tom's graph for the 'Filling a container' scenario, Roni patiently waited for the interaction to conclude. She had suggested that they select her graph, (incorrect in this cycle), arguing that most of them had suggested a similar graph, drawing on the notion of 'majority wins.' However, none of the others reacted to her comment. Roni demonstrated her continued attentiveness to the discussion but refrained from actively participating. She was observed leaning back in a thoughtful manner, with a downward-turned mouth and raised inner eyebrows. Subsequently, she expressed feeling sadness and fatigue, stating that she felt sad because her input was completely ignored. She said she felt tired and wanted to leave the classroom. Later in that phase, Roni voiced her objection to Tom's argument, expressing her disagreement with him. Surprisingly, Eva, not Tom, responded to Roni, attempting to clarify Tom's reasoning. However, Roni was still skeptical of Eva's explanation. The others seemed to disregard her. Roni placed her head on her arm, tilting her face downward, indicating her disengagement from participation.

In her interview, Roni said that the group had listened attentively to Tom, seemingly perceiving his words as unquestionably true while dismissing her contributions as incorrect. This situation proved challenging for her, resulting in a sense of exhaustion and a lack of energy toward anyone in the group. Their lack of attentive listening left her feeling unheard and invalidated. According to Stein and Albro (2001), it is important to analyze the potential effects of various emotional reactions, such as unhappiness or anger, within the context of argumentation, particularly in relation to subsequent negotiation strategies and outcomes. Roni was absent for the third cycle of activities, which may have been, we surmise, related to her distressing experiences of being overlooked in the first two cycles. It seemed that her level of influence remained low despite the merit of her mathematical ideas. Her attempts to take the floor were not successful and even though her ideas were correct in the first cycle, the group did not seem afford her increased access to the conversational floor in subsequent cycles. Roni herself did not admonish the group, as Tom had, for ignoring her correct ideas in the first cycle, and the group had not increased her authority during the second cycle. Unfortunately, the class teacher was not aware of, or did not intervene in, the situations with Roni, perhaps because of the presence of the video (and researcher). These disrespectful interactions point to the importance of closely monitoring student interactions in collective argumentation so that ideas that have merit can be aired, as with Roni's, and so that students are given room to be 'wrong' and to work through individual cognitive conflict emerging during the discourse, as with Tom. This finding highlights that learning to argue is an important but challenging focus for educators, so that arguing to learn can succeed.

The Concern of Not Reaching Consensus: Lingering Disappointment

In this theme, we found a differing relationship between emotion and argumentation that rather than negative emotions playing a productive role in the students' learning, the students' difficulties with reaching a consensus view contributed to lingering negative emotions, most notably disappointment. Throughout the cycles, there were multiple instances of individual members agreeing with the arguments of another member, and these were reported on as evoking pleasure or pride by the students. For example, Liam reported on "feeling good" and "enjoying" it when people agreed with each other. Tom said:

> "When we shared graphs for the 'filling a container' situation and the group members agreed with me, I felt great. I felt confident for succeeding in persuading the group with my graph" (Tom, post-intervention interview).

But these moments of consensus were different from the goal of reaching a consensus view of the best graph for the situation–an important characteristic of deliberative argumentation. Across the three cycles, the group did not experience reaching final consensus in which every single member was satisfied with the outcome, either cognitively or emotionally. Perhaps the task expectation of producing one final group graph at the end of the cycle for critique and discussion by the rest of the class contributed to their sense of anxiety in the moment. Although the group did submit a graph at the end of each cycle, it appeared to be the result of an unsatisfying 'standoff' or a deferral to authority rather than a 'win' or 'compromise' (Stein & Albro, 2001). Numerous students expressed a lingering sense of disappointment or frustration in their reflections after the learning sequence. There seemed to be notable issues at play that students reported in their post-cycle interviews: cognitive uncertainty about the correctness of their own graphs, too many alternative and even conflicting ideas about the real-life situations, time pressure, and a lack of process for what to do in the event of an ongoing stalemate.

In the first cycle ('watering a plant'), it appeared that Eva and Anna took the decisive role in pushing for group consensus on a final graph, since they had been scribing for the group. They chose to draw an incorrect final graph despite Tom's protest. During the class discussion (of all the groups' final graphs), their peers made the same criticisms of the graph that Tom had made previously. Later Tom said,

> "You did it without me. It's your problem that you did not listen to my opinions."

As mentioned previously, they had not evaluated the (correct) shape of the curve in Roni's graph.

In the second cycle ('filling a container'), the group again reached an impasse, and because of time pressure Eva and Anna deferred to Tom's opinion this time, perhaps because of what happened in the first cycle. Anna reflected later on her discomfort with having to draw a graph that the group did not agree with:

> "I was so nervous that it took us a long time, and nobody agreed with each other. And that we needed to present our graph to the class! What to do? We did not know what to do. I felt awful" (Anna, post-cycle-2 interview).

Unlike the first cycle class discussion, Tom joined Eva and Anna in explaining the group's graph to the class and how, using height and time as the variables, the curve of the line matched the shape of the container.

In the third cycle ('pricing cakes'), final consensus was again not reached, and the group seemed baffled (**Figure 4** for the graphs constructed by each student (except for Roni who missed this lesson) and the final group graph).

Several students reported experiencing negative emotions because of this lack of resolution. For example, Omer said in his interview:

> Ugh it's hard when there are so many opinions and there was no graph that convinced everyone. I felt confusion and tiredness, especially in the third task, everyone was already exhausted by not agreeing. I felt like giving up (Omer post-intervention interview).

Eventually they opted to defer to Tom's graph again for the class presentation. Unlike the previous two cycles, Anna chose not to present it to the class. Eva and Tom began, but when they encountered disagreement from classmates, Omer joined in and also tried to explain; he was also refuted by other peers. It eventuated that the whole class were divided, and in the last few

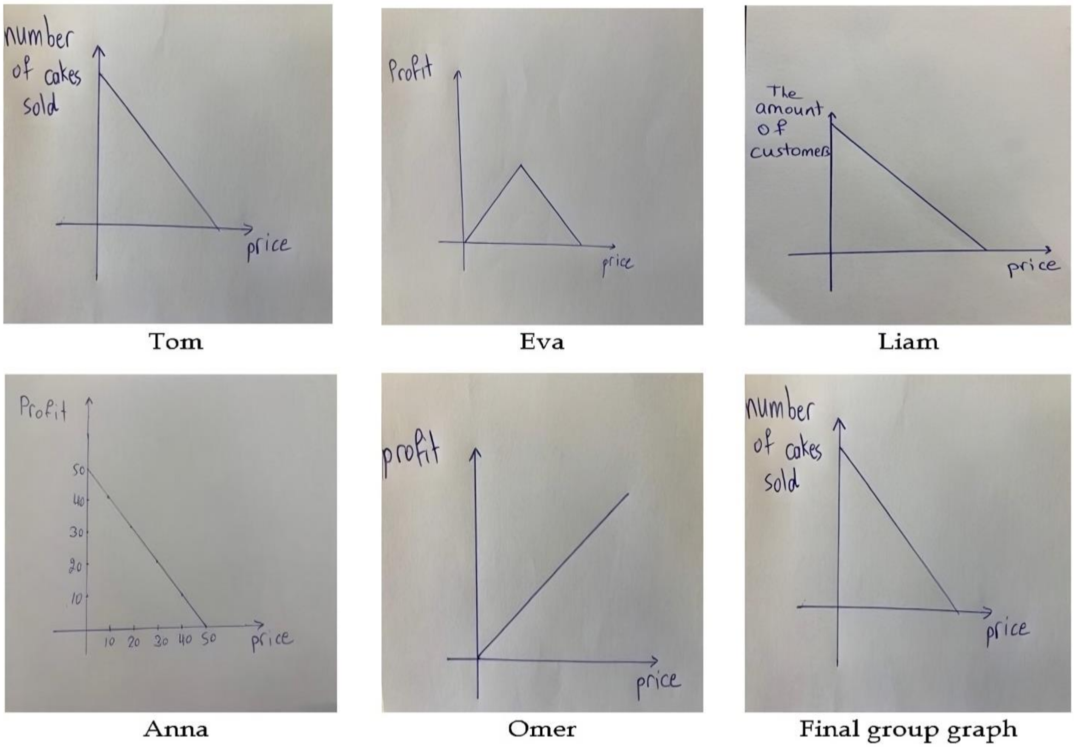

Figure 4. Constructed individual graphs for situation 3–'pricing cakes' (Source: Field study)

moments of the lesson, the teacher decided to intervene by suggesting the variables (price and profit) and directing another student to draw a correct graph.

Researchers have highlighted the emotional demands on teachers in facilitating collective argumentation activities in the classroom (Ayalon & Even, 2016; Staples, 2014). This suggests that teachers managing the 'conversational floor' (Engle et al., 2014) of the class is demanding both pedagogically and emotionally, particularly when students present ideas in public that prove to be wrong.

In this study, we found examples of emotions playing a productive role in the students' mathematics learning because of their experience of argumentation. The emotion of tension experienced by some students when receiving and giving critique of their mathematical ideas seemed to trigger productive attention to misconceptions. The emotion of frustration experienced by some students seemed to trigger productive co-construction of arguments and 'aha' moments of mathematical understanding. We also found examples of non-deliberative argumentation (disrespectful interactions, lack of final consensus) influencing emotions in the moment (distress) and in the longer term after the lesson sequence (disappointment). This relation has implications for their future learning because such negative emotions experienced in the past play a role in students' regulation of their motivation for future learning (Hannula, 2006). The intent is for students to want to experience argumentation for learning again in the future and not be dissuaded by recollections of distress when learning to argue was painful.

CONCLUSIONS

In this study, we sought to investigate how 9th grade students' emotions in small-group mathematics argumentation activities related to the four key characteristics of deliberative argumentation considered fruitful for learning: giving and receiving critiques of alternative ideas, co-constructing arguments, interacting respectfully, and seeking to reach consensus (Felton et al., 2009; Schwarz & Baker, 2017). The argumentation literature has highlighted the need for research on how emotional reactions relate to the process of argumentative discourse (Andriessen et al., 2013; Slakmon & Schwarz, 2019).

This study contributes to the literature in highlighting the productive role negative emotions can play in students benefiting with an argumentation learning task. The experience of having one's mathematical ideas critiqued by peers was new to these students and often led to feelings of tension, yet for some this provoked attention to their misconceptions and resulted in fruitful learning. Frustration from attempts to co-construct arguments was unsurprisingly a commonly experienced emotion, yet even when the discourse became disputative at times and even disrespectful rather than deliberative, some students harnessed their frustration to keep persisting with making sense of each other's arguments. Yet we found this was not the situation for each student. Some students' experience of frustration or distress seemed to lead to disengagement from their learning. In this study we also found a relation between distress from not reaching consensus over time and students' lingering disappointment, which has implications for their motivation for learning in the longer term (Hannula, 2006).

In researching the emotional dimension of argumentation in a mathematics classroom, we found these students willing to describe their emotions and they did so articulately. They identified likely sources of their feelings, which correlated with the researchers' and teacher's observations and interpretations (Fredricks & McColsley, 2012). Moreover, it seems that the argumentation tasks we designed, in being relevant to students' prescribed curriculum but only partially familiar mathematically (graphs of non-linear functions), did achieve our aim of eliciting differing mathematical ideas so that the students experienced argumentative discourse.

The full range of emotions from Else-Quest et al.'s (2008) framework were evidenced in this study. Various students reported on experiencing moments of pleasure when an idea of theirs had been critiqued and accepted by someone else, perhaps a validation of their mathematical thinking. Some students also reported encouragement when giving their critique to others, experiencing the sense of being listened to by others. In the argumentation literature, giving and receiving critique has been found to promote reasoning through students having to process information more deeply (e.g., Mueller et al., 2014; Weber et al., 2008). The students also described self-confidence when the others validated their ideas and a sense of being re-energized when others built on their ideas to co-construct arguments.

Tension and frustration were also expressed frequently, which appeared to relate to both cognitive and emotional issues. The intensity of emotion was at times surprising, since the context was mathematics rather than a controversial issue from the realm of politics, society or religion (Byford et al., 2009). Some of these self-reported emotions related to incidents of perceived unfairness in their social interactions, most often violations of cooperation rather than violations of rationality (Schreier et al. 1995). The literature suggests that unjust or disrespectful actions are likely to be viewed with more severity than incorrect thinking (Mischo, 2003) and our findings generally resonated with that view. We did find one exception, however: the students' pervasive frustration to one student's (Tom's) explanations of his ideas. We found students attributing undue mathematical authority to Tom and expressing irritation and frustration when he responded less coherently than they expected.

Engle et al. (2014) model of influence theorizes how some students end up having more influence than is explainable by the quality of their arguments. An individual student's socially negotiated authority influences their access to opportunities to contribute to the discourse (the 'conversational floor') and to be attended to in the 'interactional space' of the group. In the case of one student (Tom), we found substantial evidence of frustration from the rest of the group when his considerable access to the conversational floor did not produce the looked-for cognitive arguments to help reach consensus. At times the others withdrew from the discourse with him. Some withdrawals seemed to represent an offended stance but others more a time-out for cognitive processing. Most withdrawals were temporary; the students' re-engagement evidenced their willingness to persist in the argumentative process despite frustration. Another student (Roni) withdrew for longer periods of time. She had attempted to access with conversational floor a few times but without success (Engle et al., 2014). When her ideas (about the curve of the graph) were later validated in the class discussion, the group nevertheless continued to focus on Tom. So, we found patterns of undue influence (Tom) and lack of influence (Roni) affecting the group's cognitive processes and the students' emotional experiences. We think it is likely that the size of the group was a factor in making the conversational floor less accessible to Roni. It had been the class teacher's preference to include six students in that group as she was concerned about potential absence (for school rehearsals, which did not eventuate). Group size and grouping of particular students together are important considerations for teachers so that opportunities for deliberative argumentation are accessible to every student.

In this study, the relatively short duration of the argumentation sequence limited the opportunity to see if these positional patterns shift over time. To achieve deliberative argumentation the discourse needs to issue-driven rather than position-driven (Felton et al., 2009). Our study points to the need for further research on ways to design tasks and assign group members in mathematics classrooms for equitable access to the conversational floor and interactional spaces of argumentative discourse (Engle et al., 2014).

In an educational setting, where the argument is about mathematical ideas, rather than political or societal issues, there is more to understand about the influence of emotions and pre-existing social dynamics on reaching group consensus in mathematics contexts. In this study we found positive emotions expressed when the students experienced effective co-construction of arguments. However, the group did evidence repeated difficulty in reaching final consensus. Some of the students reported anxiety about having to produce one final group graph at the end of each cycle for presentation to the rest of the class. When the content being discussed involves new or partially understood concepts, the role of the teacher in guiding the conversation to uncover misunderstandings and misconceptions is both important and complex (Michaels et al., 2008). Our study additionally points to the importance of the teacher's expertise in addressing students' negative emotions (such as frustration or distress) that could attend such discussion. Further research on how to structure argumentation activities is warranted so that students' emotions support, not hinder, the learning process.

Overall, we found that arguing to learn is valuable for difficult mathematics concepts, like modeling real-life functions, but learning to argue is an important dimension that needs simultaneous pedagogical attention. Rogoff's (1995) conceptualization of participatory appropriation emphasizes that it is through participation that people change and prepare to engage in subsequent similar activities–that "participation is itself the process of appropriation" (p. 13). The challenge with mathematics argumentation is that we want students to learn to argue so that they can experience arguing to learn, but that process of doing so may involve painful emotions that can make it harder for some students to persevere with the necessary process. Research on incorporating argumentation in the mathematics classroom in ways that harness students' emotions to benefit their mathematics learning is a worthy endeavor.

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REFERENCES

Andriessen, J. E. B., Pardijs, M., & Baker, M. (2013). Getting on and getting along: Tension in the development of collaborations. In M. Baker, S. Järvelä, & J. Andriessen, (Eds), *Affective learning together* (pp. 205-228). Routledge.

Asterhan, C. S., & Schwarz, B. B. (2016). Argumentation for learning: Well-trodden paths and unexplored territories. *Educational Psychologist, 51*(2), 164-187. https://doi.org/10.1080/00461520.2016.1155458

Ayalon, M., & Even, R. (2016). Factors shaping students' opportunities to engage in argumentative activity. *International Journal of Science and Mathematics Education, 14*, 575-601. https://doi.org/10.1007/s10763-014-9584-3

Ayalon, M., & Hershkowitz, R. (2018). Mathematics teachers' attention to potential classroom situations of argumentation. *Journal of Mathematical Behavior, 49*, 163-173. https://doi.org/10.1016/j.jmathb.2017.11.010

Ayalon, M., Watson, A., & Lerman, S. (2018). Comparison of students' understanding of functions in classes following English and Israeli national curricula. *Educational Studies in Mathematics, 97(3)*, 255-272. https://doi.org/10.1007/s10649-017-9798-8

Ayalon, M., Wilkie, K. J., & Eid, K. H. (2021). Relating students' emotions during argumentative discourse to their learning of real-life functional situations. *Educational Studies in Mathematics.* https://doi.org/10.1007/s10649-021-10121-5

Ball, D. L., & Bass, H. (2003). Making mathematics reasonable in school. In J. Kilpatrick, W. G. Martin, & D. Shifter (Eds.), *A research companion to principles and standards for school mathematics* (pp. 27-44). National Council of Teachers of Mathematics.

Byford, J., Lennon, S., & Russell, W. B. (2009). Teaching conflictual issues in the social studies: A research study of high school teachers. *The Clearing House, 82*, 165-170. https://doi.org/10.3200/TCHS.82.4.165-170

Chin, C., & Osborne, J. (2010). Supporting argumentation through students' questions: Case studies in science classrooms. *The Journal of the Learning Sciences, 19*(2), 230-284. https://doi.org/10.1080/10508400903530036

Common Core State Standards Initiative. (2010). Common core state standards for mathematics. *National Governors Association Center for Best Practices and the Council of Chief State School Officers*. http://www.corestandards.org/the-standards

Creswell, J. W. (2013). *Qualitative inquiry & research design: Choosing among five approaches*. SAGE.

Davis, P. J., & Hersh, R. (1981). *The mathematical experience*. Birkhäuser.

DeBellis, V. A., & Goldin, G. A. (2006). Affect and meta-affect in mathematical problem solving: A representational perspective. *Educational Studies in Mathematics, 63*(2), 131-147. https://doi.org/10.1007/s10649-006-9026-4

Demiray, E., Isiksal-Bostan, M., & Saygi, E. (2023). How argumentation relates to formal proof process in geometry. *International Electronic Journal of Mathematics Education, 18*(3), em0741. https://doi.org/10.29333/iejme/13214

Ekman, P. (1992). An argument for basic emotions. *Cognition & Emotion, 6*(3-4), 169-200. https://doi.org/10.1080/02699939208411068

Else-Quest, N. M., Hyde, J. S., & Hejmadi, A. (2008). Mother and child emotions during mathematics homework. *Mathematical Thinking and Learning, 10*(1), 5-35. https://doi.org/10.1080/10986060701818644

Engle, R. A., Langer-Osuna, J. M., & McKinney de Royston, M. (2014). Toward a model of influence in persuasive discussions: Negotiating quality, authority, privilege, and access within a student-led argument. *Journal of the Learning Sciences, 23*(2), 245-268. https://doi.org/10.1080/10508406.2014.883979

Evagorou, M., Papanastasiou, E., & Vrikki, M. (2023). What do we really know about students' written arguments? Evaluating written argumentation skills. *European Journal of Science and Mathematics Education, 11*(4), 615-634. https://doi.org/10.30935/scimath/13284

Evans, J., Morgan, C., & Tsatsaroni, A. (2006). Discursive positioning and emotion in school mathematics practices. *Educational Studies in Mathematics, 63*(2), 209-226. https://doi.org/10.1007/s10649-006-9029-1

Felton, M., Garcia-Mila, M., & Gilabert, S. (2009). Deliberation versus dispute: The impact of argumentative discourse goals on learning and reasoning in the science classroom. *Informal Logic, 29*(4), 417-446. https://doi.org/10.22329/il.v29i4.2907

Fischer, A. H., & Manstead, A. S. R. (2008). Social functions of emotion. In M. Lewis, J. M. Haviland-Jones, & L. F. Barrett (Eds.), *Handbook of emotions* (pp. 456-468). The Guilford Press.

Francisco, J. M., & Maher, C. A. (2005). Conditions for promoting reasoning in problem solving: Insights from a longitudinal study. *The Journal of Mathematical Behavior, 24*(3-4), 361-372. https://doi.org/10.1007/s10649-006-9029-1

Fredricks, J. A., & McColskey, W. (2012). The measurement of student engagement: A comparative analysis of various methods and student self-report instruments. In S. L. Christenson, A. L. Reschly, & C. Wylie (Eds.), *Handbook of research on student engagement* (pp. 763-782). Springer. https://doi.org/10.1007/978-1-4614-2018-7_37

Goldenberg, E. P. (1987). Believing is seeing: How preconceptions influence the perceptions of graphs. In J. Bergeron, N. Herscovits, & C. Kieran (Eds.), *Proceedings of the 11th Conference of the International Group for the Psychology of Mathematics Education* (pp. 197-203).

Goldin, G. A. (2002). Affect, meta-affect, and mathematical belief structures. In G. C. Leder, E. Pehkonen, & G. Törner (Eds.), *Beliefs: A hidden variable in mathematics education?* (pp. 59-72). Kluwer. https://doi.org/10.1007/0-306-47958-3_4

Gravemeijer, K., Stephan, M., Julie, C., Lin, F. L., & Ohtani, M. (2017). What mathematics education may prepare students for the society of the future? *International Journal of Science and Mathematics Education, 15*, 105-123. https://doi.org/10.1007/s10763-017-9814-6

Hannula, M. S. (2006). Motivation in mathematics: Goals reflected in emotions. *Educational Studies in Mathematics, 63*(2), 165-178. https://doi.org/10.1007/s10649-005-9019-8

Hannula, M. S. (2012). Exploring new dimensions of mathematics-related affect: Embodied and social theories. *Research in Mathematics Education, 14*(2), 137-161. https://doi.org/10.1080/14794802.2012.694281

Horn, I. S. (2008). Accountable argumentation as a participation structure to support learning through disagreement. In A. J. Schoenfeld, & N. Pateman (Eds.), *A study of teaching: Multiple lenses, multiple views* (pp. 97-126). National Council of Teachers of Mathematics.

Immordino-Yang, M. H., & Damasio, A. (2007). We feel, therefore we learn: The relevance of affective and social neuroscience to education. *Mind, Brain, and Education, 1*(1), 3-10. https://doi.org/10.1111/j.1751-228X.2007.00004.x

Isohätälä, J., Näykki, P., Järvelä, S., & Baker, M. J. (2018). Striking a balance: Socio-emotional processes during argumentation in collaborative learning interaction. *Learning, Culture and Social Interaction, 16*, 1-19. https://doi.org/10.1016/j.lcsi.2017.09.003

Lampert, M., Rittenhouse, P., & Crumbaugh, C. (1996). Agreeing to disagree: Developing sociable mathematical discourse. In D. Olson, & N. Torrance (Eds.), *Handbook of education and human development* (pp. 731-764). Blackwell.

Leinhardt, G., Zaslavsky, O., & Stein, M. K. (1990). Functions, graphs, and graphing: Tasks, learning, and teaching. *Review of Educational Research, 60*(1), 1-64. https://doi.org/10.3102/00346543060001001

McLeod, D.B. (1992). Research on affect in mathematics education: A reconceptualization. In D. A. Grouws (Ed.), *Handbook of research on mathematics learning and teaching*, (pp. 575-596). MacMillan.

Michaels, S., O'Connor, C., & Resnick, L. B. (2008). Deliberative discourse idealized and realized: Accountable talk in the classroom and in civic life. *Studies in Philosophy and Education, 27*, 283-297. https://doi.org/10.1007/s11217-007-9071-1

Miles, M. B., & Huberman, A. M. (1994). *Qualitative data analysis: An expanded source book*. SAGE.

Ministry of Education. (2021). *Mathematics curriculum for grades 7-9*. https://edu.gov.il/special/Curriculum/Junior-High/Pages/Junior-High-List-of-subjects-per-grade.aspx

Mischo, C. (2003). Cognitive, emotional, and verbal responses in unfair everyday discourse. *Journal of Language and Social Psychology, 22*(1), 119-131. https://doi.org/10.1177/0261927X02250064

Moore, K. C., & Thompson, P. W. (2015). Shape thinking and students' graphing activity. In *Proceedings of the 18th meeting of the MAA special interest group on research in undergraduate mathematics education* (pp. 782-789). RUME.

Mueller, M., Yankelewitz, D., & Maher, C. (2012). A framework for analyzing the collaborative construction of arguments and its interplay with agency. *Educational Studies in Mathematics, 80*, 369-387. https://doi.org/10.1007/s10649-011-9354-x

Mueller, M., Yankelewitz, D., & Maher, C. (2014). Teachers promoting student mathematical reasoning. *Investigations in Mathematics Learning, 7*(2), 1-20. https://doi.org/10.1080/24727466.2014.11790339

Muller Mirza, N., Perret-Clermont, A.-N., Tartas, V., & Iannaccone, A. (2009). Psychosocial processes in argumentation. In N. Muller Mirza, & A. N. Perret Clermont (Eds.), *Argumentation and education* (pp. 67-90). Springer. https://doi.org/10.1007/978-0-387-98125-3_3

Oehrtman, M., Carlson, M. P., & Thompson, P. W. (2008). Foundational reasoning abilities that promote coherence in students' function understanding. In M. P. Carlson, & C. L. Rasmussen (Eds.), *Making the connection: Research and teaching in undergraduate mathematics education* (pp. 27-42). Mathematical Association of America. https://doi.org/10.5948/UPO9780883859759.004

Op't Eynde, P., De Corte, E., & Verschaffel, L. (2006). "Accepting emotional complexity": A socio-constructivist perspective on the role of emotions in the mathematics classroom. *Educational Studies in Mathematics, 63*(2), 193-207. https://doi.org/10.1007/s10649-006-9034-4

Plantin, C. (2004). On the inseparability of emotion and reason in argumentation. In E. Weigand (Ed.), *Emotion in dialogic interaction* (pp. 269-281). John Benjamins. https://doi.org/10.1075/cilt.248.18pla

Polo, C., Lund, K., Plantin, C., & Niccolai, G. (2016). Group emotions: The social and cognitive functions. *International Journal of Computer-Supported Collaborative Learning, 11*(2), 123-156. https://doi.org/10.1007/s11412-016-9232-8

Rogoff, B. (1995). Observing sociocultural activity on three planes: Participatory appropriation, guided participation, and apprenticeship. In J. V. Wertsch, P. del Rio, & A. Alvarez (Eds.), *Sociocultural studies of mind* (pp. 139-164). Cambridge University Press. https://doi.org/10.1017/CBO9781139174299.008

Schreier, M., Groeben, N., & Christmann, U. (1995). "That's not fair!" Argumentational integrity as an ethics of argumentative communication. *Argumentation, 9*(2), 267-289. https://doi.org/10.1007/BF00721962

Schwarz, B. B., & Baker, M. J. (2017). *Dialogue: Argumentation and education: History, theory and practice*. Cambridge University Press. https://doi.org/10.1017/9781316493960

Slakmon, B., & Schwarz, B. B. (2019). Deliberative emotional talk. *International Journal of Computer-Supported Collaborative Learning, 14*(2), 185-217. https://doi.org/10.1007/s11412-019-09304-3

Staples, M. (2014). *Supporting student justification in middle school mathematics classrooms: Teachers' work to create a context for justification*. CRME Publications.

Stein, N. L., & Albro, E. R. (2001). The origins and nature of arguments: Studies in conflict understanding, emotion, and negotiation. *Discourse Processes, 32*(2-3), 113-133. https://doi.org/10.1080/0163853X.2001.9651594

Tasdan, B. T., Dede, A. T., & Koyunkaya, M. Y. (2022). Examining pre-service mathematics teachers' argumentation-supported lesson plans and their noticing during planning. *International Journal of Mathematical Education in Science and Technology*. https://doi.org/10.1080/0020739X.2022.2054741

Thompson, P. W., & Carlson, M. (2017). Variation, covariation, and functions: Foundational ways of thinking mathematically. In J. Cai (Ed.), *Compendium for research in mathematics education* (pp. 421-456). National Council of Teachers of Mathematics.

van Eemeren, F. H., & Grootendorst, R. (2004). *A systematic theory of argumentation: The pragma-dialectical approach*. Press Syndicate. https://doi.org/10.1017/CBO9780511616389

Webb, N. M., Franke, M. L., Ing, M., Wong, J., Fernandez, C. H., Shin, N., & Turrou, A. C. (2014). Engaging with others' mathematical ideas: Interrelationships among student participation, teachers' instructional practices, and learning. *International Journal of Educational Research, 63*, 79-93. https://doi.org/10.1016/j.ijer.2013.02.001

Weber, K., Maher, C., Powell, A., & Lee, H. S. (2008). Learning opportunities from group discussions: Warrants become the objects of debate. *Educational Studies in Mathematics, 68*(3), 247-261. https://doi.org/10.1007/s10649-008-9114-8

Wilkie, K. J., & Ayalon, M. (2018). Investigating Years 7 to 12 students' knowledge of linear relationships through different contexts and representations. *Mathematics Education Research Journal, 30*(4), 499-523. https://doi.org/10.1007/s13394-018-0236-8

Zan, R., Brown, L., Evans, J., & Hannula, M. S. (2006). Affect in mathematics education: An introduction. *Educational Studies in Mathematics, 63*(2), 113-121. https://doi.org/10.1007/s10649-006-9028-2

Zhuang, Y. & Conner, A. (2022). Teachers' use of rational questioning strategies to promote student participation in collective argumentation. *Educational Studies in Mathematics, 111*, 345-365. https://doi.org/10.1007/s10649-022-10160-6

APPENDIX A

Table A1. Post-cycle and post-sequence individual student written reflection (Ayalon et al., 2021)

Activity	Emotions	Reason/s
Presenting my own graph of the situation to my group	I felt …	
Responding to peer's critiques of my graph (if it happened)	I felt …	
Sharing my opinion about my peers' graphs	I felt …	
Trying to reach agreement in choosing the best graph	I felt …	
Assessing the three fictitious students' graphs	I felt …	
Trying to reach agreement in keeping/revising the group graph	I felt …	
Assessing the different group graphs as a whole class	I felt …	
Other comments:		

APPENDIX B: POST-SEQUENCE STUDENT INTERVIEW PROTOCOL

a. [Using your written reflection] share how you felt when experiencing the different types of activities.
b. Choose and describe a situation you particularly remember from one of the lessons in the sequence where different people had different opinions and you needed to listen to others and explain or convince them about your own point of view.
c. What strategies did you use in that situation to try and convince others?
d. What strategies did you use in that lesson to evaluate and then respond to others' points of view?
e. How did you feel in this situation? Try to give a reason why you might have felt that way.
f. What do you think you learnt from this situation?

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