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Lexical ambiguities in statistics declared by in training and in-service teachers

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Abstract

This article aims to evaluate how teachers, in training and in-service, define the concepts of randomness, probability, chance and variability, fundamental terms in the teaching of statistics. To this end, a printed recording protocol of natural semantic networks was applied to 16 teachers in training and 58 in-service teachers, selected through non-probabilistic sampling. The results provide evidence that the concept with the lowest conceptual density in both groups is variability. Likewise, a greater presence of similar words was observed between randomness, chance and probability, despite being different concepts. Another finding is the association of the inducing concepts to words that are used within the basic lexicon in Chilean Spanish, whose dictionary is not specialized in statistics. It is concluded about the scarce use of technical language by the participants, which would eventually affect the teaching of statistics.

Keywords: terminology, statistical literacy, lexical ambiguities, natural semantic networks, teachers in training, in-service teachers

INTRODUCTION

Statistics, like any discipline, has a specialized language, which is used both to transmit ideas and to build knowledge. Learning this lexicon is key, because it constitutes a fundamental element of linguistic knowledge and is a cognitive tool that facilitates the learning of individuals (Crossley et al., 2017).

In particular, it means having a specific nomenclature for the communication of new ideas and statistical concepts, which helps students develop their understanding, allowing them to read and interpret information in everyday life (guidelines for assessment and instruction in statistics education [GAISE], 2016), skills that are found in close relationship with statistical literacy (Ben-Zvi & Garfield, 2004).

To achieve this goal, a higher level of precision than that of general communication is required, so a stable and unambiguous terminology is needed in which, theoretically, a single denomination corresponds to each concept (Pérez-Pascual, 2012).

Likewise, having conceptual clarity is key to responding to problem situations, which can help avoid errors potentially caused by the lack of an adequate language for decoding or interpretation (Nacarato & Grando, 2014). Thus, addressing this particularity of teaching is of vital importance (Kaplan et al., 2009).

However, as Leung (2005) mentions, the acquisition and learning of a language is not a trivial process; even more so when it comes to specialized terminology, where some of its key concepts show lexical ambiguities, with words that overlap or are used interchangeably to define different concepts (DC). This is highly possible, since ambiguity occurs constantly in languages (López-Cortés & Horno-Chéliz, 2021) and the same concept ends up receiving different names, some of which can be used also to designate very different ones (Pérez-Pascual, 2012). As can be seen from López-Cortés and Horno-Chéliz (2021), this phenomenon of use of everyday language is not isolated nor is it necessarily harmful to communication, since it validates the idea that language is based on use and on speakers' experience.

In addition, Schleppegrell (2007) states that the language of a new discipline is part of learning the discipline and that, in fact, language and learning cannot be separated, which reinforces the idea of addressing specialized lexicon as an object of study. In the case of statistics, an aggravating factor is the evidence that the students who enter the classrooms of this discipline, in

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Contribution to the literature

- The study shed light on the lexical ambiguity that the concepts of randomness and variability present to teachers in training and in-service teachers.
- The study demonstrates that both pre- and in-service secondary education mathematics teachers associated the concepts of chance and randomness with colloquially used words.
- This study provides evidence that participants (pre- and in-service teachers) use the words event and occurrence interchangeably. This study offers a description of the words used by pre- and in-service teachers to refer to randomness, chance, probability, and variability.

general, bring with them strongly rooted, but incorrect, intuitions that are highly resistant to change (Konold, 1995). This finding implies that to master a new topic it is necessary to understand and appropriately use the language of the discipline (Dunn et al., 2016).

Consequently, it is expected that teachers in training and in-service will have a specific lexicon to account for the mastery of technical concepts in the field of statistics and probability (from now on, only the term statistics will be used to refer to both themes) present in teaching at primary and secondary school and at university level; some of these concepts are randomness (aleatoriedad), chance (azar), variability (variabilidad) and probability (probabilidad).

In the Chilean context, there is a scarcity of studies that address the issue of specialized lexicon in statistics that is mastered by teachers in training and in-service. In general, these types of studies address lexical availability in mathematics by incorporating probabilities, data and chance as objects of interest (Ferreira et al., 2014). However, researchers in statistics education have highlighted that mathematics and statistics are sciences that have their own objects of study (e.g., Groth, 2007; Stohl, 2005), thus it is of interest to study the specialized language of statistics.

For these purposes, it is necessary to have word association techniques to approximate the mental representation of the concepts that the subjects possess. In particular, the prototype theory admits the distinction of central and peripheral elements through semantic networks (López-Cortés & Horno-Chéliz, 2021), as has already been studied in different fields of study (e.g., Ancer et al., 2013; Cantú-Martínez, 2023; Maldonado-Fuentes et al., 2019; Sánchez et al., 2011a; Vera-Noriega et al., 2005).

In accordance with the previous considerations, this research aims to evaluate how teachers, in training and in-service, define the concepts of randomness (aleatoriedad), probability (probabilidad), chance (azar) and variability (variabilidad), fundamental terms in the teaching of statistics. To do this, the semantic nodes related to these concepts are characterized; those mentioned in the teaching activities and in the Chilean school curriculum for teaching statistics and probability (Rodríguez-Alveal et al., 2022; Rodríguez-Alveal & Maldonado-Fuentes, 2023), as well as in GAISE (2016).

THEORETICAL FRAMEWORK

Literacy & Statistical Lexicon

Statistical literacy is understood as the ability to read, interpret and evaluate information from daily life using statistical language (Chance, 2002; Garfield, 2002). For their part, Ben-Zvi and Garfield (2004) include the understanding of symbols, vocabulary and statistical concepts. This knowledge will allow us to understand the messages behind the information circulating in the media, as well as the technical reports delivered by government agencies.

Among the fundamental concepts in statistics are randomness, probability, chance and variability. Of these, randomness and variability are among the most important (Watson et al., 2003). On the one hand, randomness is a polysemic concept of which students have informal ideas and prior judgments made in everyday life, such as its association with luck (suerte), that is, chance (azar) (Ramírez & Batalha, 2019). For its part, in math textbooks, it is linked to the calculation of probabilities (Ayer, 1974). There are also definitions that relate it to equiprobability (Zisimos & Tasos, 2021). This is how the concept of randomness has a complex character, hindering its learning in students (Ortiz et al., 2001).

On the other hand, the concept of variability has an essential role in solving statistical problems and making decisions, since its understanding and explanation is based on its quantification (Moore, 1997). Likewise, considering its importance in statistics, Wild and Pfannkuch (1999) placed it at the center of their model on statistical thinking. According to Franklin et al. (2005) this approach makes the difference between statistics and mathematics. Also, Ben-Zvi and Garfield (2004) concluded that variability is a concept that is complex to understand and learn for teachers in training.

Along these lines, Kaplan et al. (2010) mention that statistics books use the words variability (variabilidad) and dispersion (dispersión) or scale parameter (parámetro de escala) interchangeably. Furthermore, in the literature you can find studies that not only analyze knowledge but also teachers' attitudes towards statistics (e.g., Estrada et al., 2018). On the other hand, Ata (2014) and Kurt-Birel (2017) examined the conceptual levels of

knowledge about probability in pre-service and active teachers in Turkey, finding that the first group had mostly acquired procedural skills over conceptual ones.

In summary, teachers in training and in practice need to delve into the epistemological root of vocabulary so that students learn how these words construct statistical concepts (Schleppegrell, 2007) and use them appropriately in daily life to respond to problems, where statistics and probability intervene.

Statistical Vocabulary & International Guidelines

Learning new content, such as statistics, requires learning its language. In this sense, Rangecroft (2002) states that communication is at the heart of statistics and that it is necessary to be fluent in both the specialized language of the discipline as well as in everyday language. In this context, the statistical lexicon plays a crucial role in the classroom, the learning of which is not a trivial process, even more so when there are concepts such as randomness that resist a clear definition (Batanero & Serrano, 1995). That is to say, in statistics there are words that present lexical ambiguity, such as the notion of variability and randomness (Kaplan et al., 2010) from which its polysemic nature is evident (Durkin & Shire, 1991).

Along these lines, GAISE (2016) report states that having a good understanding of the concepts will make it easier for students to use the tools and procedures necessary to answer questions about a set of data. In summary, if students have adequate knowledge of statistical concepts, this will allow them to learn and understand statistical techniques to summarize and interpret data and communicate using statistical language.

However, Thibaut et al. (2018) have provided background information that suggests that the language used in textbooks presents difficulties for students, since it is considered complicated and far from reality. The latter is interpreted as the application of very technical language as opposed to colloquial use, an aspect that generates a dilemma for teachers, since it is known that people connect what they hear with what they have heard and experienced in the past (Kaplan et al. 2010). In the Chilean context, Rodríguez-Alveal et al. (2022), in their study of textbooks, report that activities related to the concept of randomness mention the use of physical devices and, on a small scale, the use of technological tools, without delving into the concept. Likewise, Rodríguez-Alveal et al. (2018), in a study carried out with teachers in training and in-service, found that participants put forward implausible arguments when explaining why a random phenomenon can or cannot be considered of this type. For its part, the concept of variability in Chilean textbooks is approached from a procedural perspective, restricting itself to the calculation of statistics, without delving into definitions about the notion of variability (Rodríguez-Alveal et al., 2021).

The evidence reported, therefore, supports the idea that students have a lexicon learned about statistics, which has a formal and informal origin, of a school and everyday type, with elements of semantic representation that adjust to or deviate from specialized knowledge of these concepts. In such circumstances, it is necessary to delve into empirical evidence that provides background information regarding which words are used for its definition, considering the participation of teachers in training and in-service.

METHODOLOGY

To respond to the study objective, the natural semantic networks technique was used (Hinojosa, 2008; Reyes, 1993; Vera-Noriega et al., 2005). In coherence, the scope of the study is descriptive, since it allows us to account for the different words attributed by the participants to the inducing terms.

Sample & Context

For the purposes of the study, a non-probabilistic sample was used by disposition (McMillan & Schumacher, 2011), made up of teachers in training and in-service. Specifically, the study considered 16 teachers in training in Mathematics Pedagogy who have completed and passed the subjects in the statistics line in their training itinerary. The second group was made up of 58 active teachers who participated in a summer workshop on statistics and probability at a university in south-central Chile.

Data Collection Instrument & Analysis Procedure

For data collection, a printed natural semantic networks questionnaire was used with four inducing concepts: randomness (aleatoriedad), probability (probabilidad), chance (azar) and variability (variabilidad). At first, participants spontaneously established and associated five words with the mentioned concepts. In a second moment, once the previous activity was carried out, they were asked to reorganize and prioritize these terms according to the importance they assign them to define the concept. In this way, a hierarchical list of terms was obtained, numbered from one (very important) to five (not very important).

From this record, a database was structured in the SemNet software version 3.22 (Sánchez et al., 2013), in order to carry out analysis at a global and specific level. Firstly, the corpus of different defining words was established for each center of interest (J value) and the density of the network was examined (G value), in order to study the semantic richness for each node or concept according to the data from each sample subgroup. In turn, DC, common concepts (CC), and free concepts (FC)

Table 1. Basic indexes of inducing terms according to groups studied

Inducing terms	Training teachers		In-service teachers	
	Value J	Value G	Value J	Value G
Randomness (aleatoriedad)	41	2.0	81	5.6
Variability (variabilidad)	39	1.6	111	4.8
Chance (azar)	41	2.3	83	6.2
Probability (probabilidad)	50	1.8	111	6.5
Data in total SAM Sets	Training teachers		In-service teachers	
Different concepts (DC)	35 (100%)		31 (100%)	
Common concepts (CC)	4 (11.4%)		9 (29.0%)	
Free concepts (FC)	31 (88.6%)		22 (71.0%)	

Note. DC: Total number of different words that appear in all SAM groups; CC: Total words that appear more than once in all SAM groups; & FC: Total words that appear only once in all SAM sets

were quantified in the total number of occurrences. In a second moment, an analysis was carried out for each key concept (inducing term), specifying the semantic weight (M value) of the defining words. From this, SAM set or central core of the semantic network was obtained, which corresponds to the 10 words with the greatest semantic weight or M value (Ancer et al., 2013; Cantú-Martínez, 2023; Reyes, 1993). Likewise, the semantic distance (FMG value) was calculated between defining words of SAM set, taking as the beginning the word with the highest M value, which represents 100% of the observations in relation to each stimulus. Finally, a comparison was carried out between the words that make up SAM sets, to identify the connectivity and relationship between the key concepts of statistics, proposed as terms with lexical ambiguity.

RESULTS & DISCUSSION

Indicators Associated with the Global Description of the Statistical Lexicon

Table 1 shows the corpus of different defining words that report the richness (J value) and density (G value) of the semantic network for each center of interest according to the sample.

It is seen that the greatest number of associations of the teachers in training was observed in the notion of probability (J value=50) and the least in variability (J value=39). On the other hand, in in-service teachers, probability and variability (J value=111) presented the greatest semantic richness and randomness the least (J value=81). It should be noted that both subgroups agree in associating a greater number of words for the same concept, which in global terms can be explained by the teaching of statistics. In the Chilean case, probability is introduced from primary education, making use of lexical combinations such as random experiment (experimento aleatorio), random games (juegos aleatorios) and probability calculation (cálculo de probabilidades) (Ministerio de Educación [MINEDUC], 2012).

On the other hand, the term with the lowest conceptual density was variability for both teachers in training (G value=1.6) and active teachers (G value= 4.8). A more compact and cohesive set of words is evident in both classes. Otherwise, the most dispersed term is chance (G value=2.3) in training teachers, and probability in in-service teachers (G value=6.5), with lower group consensus.

Regarding the total SAM sets, it should be noted that at least 11.0% of common words are observed in teachers in training and 29.0% in in-service teachers, although the centers of interest correspond to differentiated concepts in statistics. Consequently, the terms randomness, chance, probability and variability are notions that are carried over into classroom discourse, given that they are used in a related manner when quantifying uncertainty, a key aspect in the teaching of statistics.

Indicators of Semantic Network for Each Inducing Term

Next, the results are reported with a list of the 10 words from each of SAM sets. Likewise, ideas are provided that contribute to the reflection about the presence of lexical ambiguity of the inducing terms.

Randomness

In statistics books, such as Devore (2008) and Triola (2009), the word random is used as an adjective that modifies the nouns: phenomenon (fenómeno), event (evento), sample (muestra), data (datos), etc. In this sense, one of the definitions of randomness is the one given by Moore (2007), for whom a phenomenon is random if the individual results are uncertain. Likewise, Kaplan et al. (2010) note that randomness is a polysemous concept that also presents lexical ambiguity. **Table 2** shows that the semantic distance in both groups is numerically similar, in a range between 100% and 25.9% in training teachers and from 100% to 29.1% in in-service teachers.

Furthermore, it is observed that possibility is the defining word that has the greatest semantic weight in the group of teachers in training, which could be explained by the use of this word in the basic lexicon of Chilean Spanish, use value 100.87 (Castillo, 2021, p. 376).

Table 2. SAM sets in relation to inducing term randomness

No	In training mathematics teachers (n=16)				In-service mathematics teachers (n=58)			
	Defining word	Value M	FMG	Frequency	Defining word	Value M	FMG	Frequency
1	Possibility (posibilidad)	27	100	8	Chance (azar)	79	100	42
2	Luck (suerte)	16	59.3	5	Uncertain (incierto)	10	88.6	22
3	Unexpected (imprevisto)	14	51.9	3	Fortuitous (fortuito)	65	82.3	18
4	Causality (causalidad)	13	48.1	4	Unpredictable (impredecible)	44	55.7	17
5	Uncertain (incierto)	12	44.4	4	Luck (suerte)	38	48.1	12
6	Chance (azar)	12	44.4	10	Possibility (posibilidad)	37	46.8	11
7	Coincidence (coincidencia)	12	44.4	3	Casual (casual)	34	43.0	10
8	Opportunity (oportunidad)	10	37.0	2	Probability (probabilidad)	29	36.7	9
9	Spontaneous (espontáneo)	10	37.0	2	Uncertainty (incertidumbre)	27	34.2	9
10	Random (aleatorio)	7	25.9	2	Event (evento)	23	29.1	7

On the other hand, for in-service teachers it is chance, whose use value is considerably lower according to the same dictionary: use value 7.04 (Castillo, 2021, p. 146), followed by uncertain. Likewise, these associations could be explained from an epistemological point of view, given that the idea of randomness is usually represented in the school system with games of chance (gambling) (Huerta, 2020).

An example in the Chilean case is seen in the textbook by Merino et al. (2016), delivered by the Ministry of Education to public and subsidized schools, where it is mentioned that random (aleatorio) means by chance (azaroso), related to chance (azar), and that it refers to all uncertain knowledge. It comes from the Latin alias, which originally means dice and by extension, chance. A similar situation occurs in specialized statistics texts, where, for example, it is defined that a phenomenon is random (aleatorio) if the individual results are uncertain (Moore, 2007).

Other concepts that are part of SAM sets are luck (suerte), unpredictable (impredecible), event (evento), fortuitous (fortuito) and casual (casual). A plausible explanation in this regard is that these concepts are frequently used in everyday language, related to intuitive probability (Batanero & Serrano, 1995). Likewise, the term possibility (posibilidad) is present in training teachers, which is in line with what was expressed by Kaplan et al. (2010) regarding random sampling, defined as that in which all people in a population have the same possibility of being chosen to be part of the sample.

On the other hand, it stands out that the probability (probabilidad) is positioned with a greater semantic distance (FMG value=36.7) in in-service teachers. This result accounts for an emerging, unforeseen result that could be explained by the definition of Ayer (1974), who connects randomness with the calculation of probability. As can be seen from the data, the two groups of participants assign different words to the inducing term randomness; however, four of the ten words in SAM sets are common in both groups.

Variability

Variability is key in statistics and, therefore, it has been called the heart of it, being essential for its existence (Watson et al., 2003). Likewise, Reading and Shaughnessy (2004), when referring to this concept, declare that in the literature the terms variation and variability are used interchangeably. However, variation (variación) is a noun used to describe the act of varying or changing a condition, and variability (variabilidad) is a noun form of the adjective variable, meaning that something is apt or capable of varying or changing (Reading & Shaughnessy, 2004, p. 201). In line with the above, **Table 3** shows the words that make up each SAM set.

It is shown that training teachers refer, first, to the concept of variation (variación), followed by instability (inestabilidad). On the other hand, active teachers associate it first with change (cambio) and then with variation (variación). A plausible explanation for the presence of the words variation and change is that they have a high use value of 10.50 and 171.50, respectively, in the Spanish lexicon of Chile (Castillo, 2021). On the other hand, this is consistent with the study carried out by Sánchez et al. (2011b) who have explained that in the specialized literature variability (variabilidad) and variation (variación) are considered synonyms. Furthermore, the verb vary (variar) can be related to variable (variable) as an act of changing (Reading & Shaughnessy, 2004). However, Kaplan et al. (2009) have suggested avoiding the concept of dispersion to refer to the notion of variability, because it has different meanings. That is, it presents lexical ambiguity, which could eventually generate epistemological obstacles.

Other words mentioned only by active faculty are the statistics range (rango) and variance (varianza). That is, the sense of quantifying variability is recovered, an issue that suggests the strength of the use of algorithmic procedures in the teaching of statistics (Estrella et al., 2015). It is remarkable that both groups agree on five of the words mentioned in SAM (**Table 3**).

Table 3. SAM sets in relation to inducing term variability

No	In training mathematics teachers (n=16)				In-service mathematics teachers (n=58)			
	Defining word	Value M	FMG	Frequency	Defining word	Value M	FMG	Frequency
1	Variation (variación)	22	100	9	Change (cambio)	68	100	27
2	Instability (inestabilidad)	15	68.2	4	Variation (variación)	55	80.9	18
3	Variety (variedad)	14	63.6	4	Inestability (inestabilidad)	38	55.9	11
4	Change (cambio)	13	59.1	6	Dispersion (dispersión)	36	52.9	19
5	Diversity (diversidad)	11	50.0	4	Distribution (distribución)	32	47.1	11
6	Data (datos)	9	40.9	3	Variance (varianza)	30	44.1	9
7	Dispersion (dispersion)	8	36.4	3	Uncertainty (incertidumbre)	29	42.6	9
8	Difference (diferencia)	8	36.4	2	Variable (variable)	26	38.2	9
9	Fluctuation (fluctuación)	7	31.8	2	Range (rango)	20	29.4	8
10	Variable (variable)	6	27.3	3	Homogeneity (homogeneidad)	20	29.4	6

Table 4. SAM sets in relation to inducing term probability

No	In training mathematics teachers (n=16)				In-service mathematics teachers (n=58)			
	Defining word	Value M	FMG	Frequency	Defining word	Value M	FMG	Frequency
1	Possibility (posibilidad)	23	100	9	Possibility (posibilidad)	83	100	36
2	Occurrence (suceso)	15	65.2	6	Occurrence (suceso)	45	54.2	14
3	Chance (azar)	14	60.9	4	Incident (ocurrencia)	35	42.2	10
4	Eventuality (eventualidad)	12	52.2	3	Expectation (expectativa)	34	41.0	9
5	Event (evento)	9	39.1	4	Event (evento)	32	38.6	11
6	Option (opción)	9	39.1	3	Hypothesis (hipótesis)	26	31.3	10
7	Hypothesis (hipótesis)	8	34.8	2	Laplace's rule	23	27.7	8
8	Certainty (Certeza)	7	30.4	2	Calculation (cálculo)	20	24.1	5
9	Foreseeable (previsible)	5	21.7	1	Occurrences (sucesos)	19	22.9	7
10	Analysis (análisis)	5	21.7	2	Eventuality (eventualidad)	18	21.7	6

Probability

Another concept in the Chilean school curriculum is probability (MINEDUC, 2012), which is a polysemous concept. Among the most common meanings at the Primary and Secondary Education level are intuitive, Laplacian, frequent and subjective (Batanero, 2005). As seen in **Table 4**, the semantic distance of the words in SAM sets of both groups are equal from which it is inferred that the terms associated with the inducing concept in these nodes have a similar range.

Both training and in-service teachers relate, first of all, probability with the word possibility. This is a colloquial expression to quantify uncertain events and express the degree of belief in them (Batanero, 2005), which is usually associated with intuitive probability. Likewise, they refer to occurrence (suceso) and event (evento), both related to classical probability (Laplace's Rule). It should be noted that these words are used interchangeably in statistics books (Devore, 2008; Triola, 2009), as well as in primary and secondary education textbooks in Chile (Vásquez & Alsina, 2015). In addition, they are part of the basic lexicon of Chilean Spanish as documented in Castillo (2021).

In summary, the participants refer to words related to classical probability associated with the random experiment and the sample space. It is highlighted that in-service teachers mention Laplace's rule, a formula that allows quantifying probabilities, which demonstrates a procedural rather than a conceptual perspective. Likewise, within SAM set of teachers in training, the word chance appears in third place. A plausible explanation is the one provided by Huerta (2020), who states that one of the frequent actions in teaching probability is to make use of routine stochastic problems such as throwing dice or coins.

Chance

The evidence reported in the literature shows that the concept of chance (azar) is related to the notion of randomness and probability. This term derives from the Arabic zhar, which is related to dice. Next, **Table 5** reports the data for SAM sets in both groups of participants; highlighting that the semantic distance between the first two defining words (luck=suerte and coincidence=casualidad) is similar (FMG value 90.0 and 94.3), which is consistent with the definition given in the dictionary of the real lengua Española (RAE, 2014) that defines it as cause, which causal events or fortuitous cases are attributed to it. Furthermore, both words appear in the basic lexicon of Chilean Spanish with usage values of 40.56 and 7.80, respectively (Castillo, 2021). On the contrary, the rest of the words used move away from term with the greatest semantic weight by over 30.0%.

Also, two groups make common reference to four other words: random (aleatorio), destiny (destino), probability (probabilidad), and game (juego), whose representation can be explained by what was mentioned in relation to definition of randomness (Ayer, 1974).

Table 5. SAM sets before random inductor term

No	In training mathematics teachers (n=16)				In-service mathematics teachers (n=58)			
	Defining word	Value M	FMG	Frequency	Defining word	Value M	FMG	Frequency
1	Luck (suerte)	30	100	10	Luck (suerte)	87	100	28
2	Coincidence (casualidad)	27	90.0	9	Coincidence (casualidad)	82	94.3	27
3	Probability (probabilidad)	20	66.7	4	Random (aleatorio)	54	62.1	23
4	Unexpected (inesperado)	12	40.0	3	Destiny (destino)	41	47.1	11
5	Posibility (posibilidad)	10	33.3	4	Game (juego)	41	47.1	11
6	Destiny (destino)	10	33.3	3	Unpredictable (impredecible)	36	21.4	12
7	Random (aleatorio)	8	26.7	3	Eventuality (eventualidad)	34	39.1	15
8	Success (acierto)	7	23.3	2	Fortuitous (fortuito)	31	35.6	7
9	Accidental (accidental)	7	23.3	2	Probability (probabilidad)	31	35.6	10
10	Game (juego)	7	23.3	2	Uncertain (incierto)	25	28.7	7

Indicators Associated With Connectivity Between Concepts

As shown in the previous sections, there are several defining words that are associated indistinctly with the different key concepts. That is, these would be ambiguous concepts, since the same word is linked to multiple inducers, accounting for connections between them (Pérez-Pascual, 2012). This is the case of possibility, which teachers in training associated with three centers of interest: randomness, chance and probability.

To illustrate this set of concepts, **Table 6** shows the frequencies of similar words between SAM sets (FE value) and intergroups (IF value), proposed by Sánchez et al. (2011a) to account for the connectivity relationships between the different terms.

It is observed that randomness is the concept in which teachers in training and in-service teachers mention a greater number of words, four and eight, respectively, to refer to this concept among which those related to the concept of intuitive probability stand out: luck, coincidence, possibility, uncertain, fortuitous, unpredictable; as well as the center of interest chance. Of these words, possibility and luck are the ones that achieve the highest use value in the basic lexicon of Chilean Spanish. It should be noted that in this same dictionary the words uncertain, fortuitous, unpredictable are omitted, which could be interpreted as a type of word far removed from the common vocabulary of the population.

In summary, the concepts randomness, chance and probability, would be producing a lexical ambiguity for the participants of this study. However, it is remarkable that the concept variability was exclusively related to the word uncertainty by teachers in training. On the other hand, the main definers in the group of teachers in training were possibility with an IF of eight, four, and nine in the nodes: randomness, chance, and probability, respectively. With regards to in-service teachers, the word possibility was associated with the centers of interest randomness and probability with an IF of 11 and 36, respectively.

CONCLUSIONS

The objective of this research was to evaluate how teachers, in training and in-practice, define the concepts of randomness, probability, chance and variability, fundamental terms in the teaching of statistics. A key argument to consider is the fact that the language in statistics is not standardized, since several of its concept's present lexical ambiguities (Lavy & Mashiach-Eizenberg, 2009). This becomes a challenge for mathematics teachers in training and in-service, especially for those who must teach statistics in primary and secondary education.

In this regard, the present study reveals that participants associate words such as possibility, uncertainty, luck or probability, interchangeably with the centers of interest randomness and chance, that is, they relate them to colloquial concepts (Batanero, 2005), which have also been evidenced as words with a high presence in the basic lexicon of Chilean Spanish (Castillo, 2021). This encourages university professors to design and propose a greater number of activities so that future teachers acquire the lexicon associated with the concepts they must teach, which are reflected in the school curriculum. This focus on mastering statistical language is a relevant skill in statistical literacy (Garfield & Ben-Zvi, 2008) since communication is at the heart of statistics (Rangecroft, 2002).

On the other hand, something similar happens with the words occurrence (suceso) and event (evento), whose usefulness lies in their synonymy, which are also found in the basic lexicon of Chilean Spanish with usage values of 12.41 and 16.66, respectively (Castillo, 2021). When reviewing the temporality of these concepts in specialized books, it is observed that: Meyer (1973) refers to occurrence (suceso) (definition 1.5, p. 10), Canavos (1988) refers to event (evento) (definition 2.6, p. 33), Triola (2009) to occurrence (suceso) (definition 3.2, p. 120) and Devore (2008) to event (evento) (p. 48). A plausible explanation is that these references in both pre-service and in-service teachers would not be generational but rather relate to the selection of sources in teaching, which should be expected to align with the concepts present in textbooks.

Table 6. Connectivity between different terms according to teachers in training & in-service teachers according to FE value

	FE	Teachers in training	Basic lexicon use/dispersion	In-service teachers	Basic lexicon use/dispersion
Randomness (aleatoriedad)	2	Luck (suerte) (5) Coincidence (casualidad) (4) Chance (azar) (10)	40.56 (0.78) 7.80 (0.60) 7.04 (0.64)	Uncertain (incierto) (22) Fortuitous (fortuito) (18) Unpredictable (impredecible) (17) Luck (suerte) (12) Possibility (posibilidad) (11) Probability (probabilidad) (9) Uncertainty (incertidumbre) (10) Event (evento) (7)	40.56 (0.78) 100.87 (0.77) 3.57 (0.17) 14.49 (0.63) 16.66 (0.49)
	3	Possibility (posibilidad) (8)			
Chance (azar)	2	Luck (suerte) (10) Casualidad (9)	40.56 (0.78) 7.80 (0.60)	Uncertain (incierto) (7) Fortuitous (fortuito) (7) Unpredictable (impredecible) (12) Luck (suerte) (28) Probability (probabilidad) (9) Eventuality (eventualidad) (15)	40.56 (0.78) 3.57 (0.17)
	3	Possibility (posibilidad) (4)			
Probability (probabilidad)	2	Chance (azar) (4)	7.04 (0.64)	Possibility (posibilidad) (36) Event (evento) (11) Eventuality (eventualidad) (6)	100.87 (0.77) 16.66 (0.49)
	3	Possibility (posibilidad) (9)	100.87 (0.77)		
Variability (variabilidad)	2			Uncertainty (incertidumbre) (9)	14.49 (0.63)
	3				

Note. (Value IF): Intragroup frequency of concepts within each SAM set & value of use/dispersion according to basic lexicon of Chilean Spanish (Castillo, 2021)

This result invites us to stop at the lexicon used by teachers in training and in-service when defining statistical concepts, without losing sight of their epistemological basis, since the teaching of statistics and its communication involves addressing aspects such as those mentioned by Dunn et at. (2016):

(a) the understanding of the statistical content,

(b) the evaluation of the understanding of the questions to respond to them, and

(c) the interpretation of textbooks, government reports and making data speak according to a contextual situation, these latter skills inherent to statistical literacy (Ben-Zvi & Garfield, 2004).

This has been evidenced in textbooks when the concept of randomness is introduced (Rodríguez-Alveal et al., 2022). As a result of the above, teachers are required to observe and diagnose the relationship between ordinary language and formal statistical language (O'Halloran, 2000). Otherwise, the use of words in an indifferent manner could inhibit student learning by making incorrect connections between technical and colloquial meanings (Oliveira et al., 2023).

Likewise, we agree with Shaughnessy (1992), who attributes as one of the possible reasons why the conceptual knowledge of some notions such as randomness, variability, chance and probability has been left in second place, enhancing procedural knowledge, which would be influencing the acquisition of statistical literacy, as has been evidenced in research carried out by Rodríguez-Alveal et al. (2018) and Guven et al. (2021). That is, they are recipe-type or rule-based courses, which are based on performing calculations and memorizing content (Estrella et al., 2015; Rodríguez-Alveal et al., 2021). These types of practices would be insufficient to strengthen a deeper understanding of statistical concepts (Lavy & Mashiach-Eizenberg, 2009).

From this research emerges the need to study how the students at the school system define the concepts considered in this study, as well as the words they use for their semantic representation. Likewise, to increase the understanding of words that the specialized literature has defined as lexically ambiguous (e.g., randomness and variability) (Kaplan et al., 2009), it is necessary to constantly monitor the use given to statistical concepts in learning and evaluation proposed to students (e.g., tests, guides, assignments, workshops, projects, etc.), with the purpose of developing adequate statistical literacy (Garfield & Ben-Zvi, 2008).

Finally, there is a need to compare the findings of this research with those of teachers in training as well as active teachers graduated from other universities, in order to make visible regularities in the semantic networks of the terms analyzed.

Likewise, it would be of interest to incorporate trainers of trainers and authors of textbooks to investigate the lexicon they have about these concepts, since they are agents that participate in the communication of concepts to teachers.

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