



Light Source Matters – Students’ Explanations about the Behavior of Light When Different Light Sources are used in Task Assignments of Optics

Mikko Henri Petteri Kesonen
University of Eastern Finland, FINLAND

Mervi Anita Asikainen
University of Eastern Finland, FINLAND

Pekka Emil Hirvonen
University of Eastern Finland, FINLAND

Received 16 June 2016 • Revised 16 December 2016 • Accepted 10 March 2017

ABSTRACT

In the present article, the context-dependency of student reasoning is studied in a context of optics. We investigated introductory students’ explanations about the behavior of light when different light sources, namely a small light bulb and a laser, were used in otherwise identical task assignments. The data was gathered with the aid of pretest and post-test questions and semi-structured interviews. According to the results, the different light sources triggered different sets of students’ ideas about light and its behavior. The students’ ideas corresponded to the perceptible features of the light sources, and textbook presentations used at the earlier levels of education. The use of the ideas hindered students’ abilities to apply the scientific models of light in a coherent manner. Overall the present study suggests that students’ difficulties in understanding the behavior of light are partly caused by their ideas about light sources commonly labeled in the task assignments of optics.

Keywords: context-dependency of student reasoning, empirical study, students’ explanations about the behavior of light

INTRODUCTION

Students’ oral or written explanations have been a great source of information for researchers interested in students’ learning of science. For instance, they have revealed that experts and novices use different strategies in solving physics tasks (Hsu, Brewster, Foster, & Harper, 2004; Chi, Feltovich, & Glaser, 1981). Experts tend to recognize patterns that enable them to use the relevant subject matter of physics involving principles, laws, models and their idealizations. Novices, in turn, tend to pay attention to the surface features of a task, such as material objects mentioned in a task assignment. This has been recognized as one of the major sources of students’ misconceptions in various topics of physics (Reiner, Slotta, Chi, & Resnick, 2000).

© **Authors.** Terms and conditions of Creative Commons Attribution 4.0 International (CC BY 4.0) apply.

Correspondence: Mikko Henri Petteri Kesonen, *University of Eastern Finland, Finland.*

✉ mikko.kesonen@uef.fi

State of the literature

- Students unlikely develop sufficient understanding about the behavior of light at the introductory level of university studies.
- Students tend to face difficulties in applying the ray model and wave model of light.
- So far, it is unclear how light sources that are commonly labeled to the tasks assignments of optics impact on students' reasoning.

Contribution of this paper to the literature

- The role played by light sources on students' understanding of light is studied by investigating their explanations about the behavior of light in a simple optical setting.
- It was found that different light sources trigger different sets of students' ideas about light and its behavior.
- These ideas prevented students from applying the ray model and wave model of light in a coherent manner. Hence, they can be considered as potential source for introductory students' difficulties in understanding the behavior of light.

Besides students' misconceptions, paying attention to surface features often means that students' reasoning strongly depends on the situation or context (Bao & Redish, 2006; Palmer, 1997). Students' reasoning has been shown to get easily distracted by situational or contextual features that are irrelevant to physics perspective, such as wording or pictorial representations used in a task assignment. For example, in the context of mechanics, certain directions of the motion of an object have made students use "motion implies force" -misconception in their reasoning (Palmer, 1994). This kind of reasoning flaws have indicated that students' knowledge is not coherent, and therefore they are unable provide reasoning that is consistent with the subject matter of physics (Savinainen & Viiri, 2008; Sabella & Redish, 2007; McDermott, 1993).

Earlier studies regarding the context-dependency of student reasoning has mainly been implemented in the context of mechanics (Meltzer, 2005; Stewart, Griffin, & Stewart, 2007; Savinainen & Viiri, 2008; Palmer, 1997). The present study aims to widen this tradition by investigating the context-dependency of students' reasoning in the context of optics.

UNDERSTANDING THE RAY AND WAVE MODEL OF LIGHT

Light is a complex entity to be described thoroughly. In general terms, it can be seen as the propagation of electromagnetic energy that travels in space. Due to the complex nature of light, the subject matter of optics is typically simplified for the certain level of education, and the introductory level of university studies is not an exception. In the present study, we follow the simplifications presented in the textbook by Knight (2008a), where light is described in terms of the ray model and wave model of light¹.

The ray model and wave model of light

In the ray model, the light is described as infinitely thin lines, rays, that demonstrate the rectilinear propagation of light from its source. The light rays do not interact with each other, and hence they may overlap without causing any interference effects or increase the brightness of light. The ray model simply shows the route of light. The ray model is valid whenever details of an optical system are much larger than the wavelength of light. (Knight, 2008a)

In the wave model, light is described as waves that oscillate time-harmonic fashion. The wave model is needed whenever details of an optical system are comparable to the wavelength of light. For example, in the case of a small slit, each point of a slit can be considered as a source of time-harmonic elementary waves. When these waves overlap, they interfere with each other, which creates the diffraction pattern that is wider bright area than the ray model would predict. (Knight, 2008a).

Students' difficulties in understanding the ray model and wave model of light

Students tend to face difficulties in applying the ray model and the wave model of light (Sengören, 2010; Maurines, 2009; Colin & Viennot, 2001; Ambrose, Shaffer, Steinberg, & McDermott, 1999; Wosilait, Heron, Shaffer, & McDermott, 1999; Wosilait, 1996). For example, students commonly explain the formation of a single-slit diffraction pattern in terms of bending light rays rather than elementary waves. Earlier studies have explained that this type of students' difficulties is caused by their inadequate understanding of the subject matter of optics (Maurines, 2009; Ambrose et al., 1999).

In contrast to students' tendency to emphasize the ray properties of light, some studies have highlighted a contrasting opposite result: students have treated light as a wave in the circumstances where the ray model of light would have been valid (Ambrose et al., 1999). This has become evident when students have predicted that a 1 cm wide slit that was illuminated with a small bulb would produce a pattern of light dominated by the effects of diffraction (Wosilait, 1996).

As far as lecture-based instruction is concerned, students' tendencies to emphasize the inappropriate properties of light have been shown to be rather resistant to change (Maurines, 2009; Colin & Viennot, 2001; Ambrose et al., 1999; Wosilait et al., 1999; Wosilait, 1996). As a result of lecture-based instruction, students have typically learnt some desirable ideas associated to these light models, but failed to apply them properly. For instance, after instruction students have claimed that the central maximum of the single-slit diffraction pattern is caused solely by light that travels straight through the slit (Maurines, 2009; Ambrose et al., 1999; Wosilait, 1996).

According to Maurines (2009), the aforementioned difficulties indicate that students unlikely obtain sufficient understanding of light even at the introductory level of the university studies. To better understand what prevents students from developing their

understanding, we investigate the role played by light sources that are typically labeled in the task assignments of optics. The actual light sources used in the present study are a small light bulb and a laser. They were chosen because they are commonly used as light sources in everyday life and in optics instruction at the various levels of education. To understand the role of light source, students' explanations about the behavior of light are investigated before and after instruction. The research questions have been set as follows:

1. *How do introductory students explain the formation of a bright area created by light emitting from a small bulb or a laser based on their prior knowledge?*
2. *How does the teaching of the ray model and wave model of light and their validity ranges impact on introductory students' explanations about light emitted by a small bulb and a laser?*

In answering the research questions, students' ideas about light emitted by a small bulb or laser is compared with the ray model and wave model of light. Our aim is to understand whether students' ideas related to these light sources can explain the difficulties they tend to face in applying the ray model and wave model of light. It is also interesting to see how students explain the behavior of light after BPIV, where they were taught the ray model and wave model of light and their validity ranges in an explicit manner (as described in the following section).

CONTEXT, DATA GATHERING, AND ANALYSIS METHODS

Context

The present study was implemented in 2011-2013 as a part of a first-year physics course, Basic Physics IV (BPIV), at the University of Eastern Finland. The course covered the basics of waves, ray and wave optics, and modern physics. Each year, approximately 60 students actively participated in the course, typically majoring in physics, mathematics, chemistry, or computer science and aiming at graduate as teachers or scientists. On average, a total of 85% ($\pm 6\%$) of the students who participated in BPIV in 2011 and 2012 ($N=117$) had studied optics as part of their physics studies in the upper secondary school². Thus, the background knowledge on optics of most of the students was based on their upper secondary school physics.

The course instruction consisted of weekly lectures (36 h), weekly recitation sessions (20 h), and tutorial interventions (4 h) during a seven weeks period. During the lectures, a lecturer (the second author) presented the course content via PowerPoint slides, handwritten examples, and web-based applets, while engaging the students by using stop-to-think questions once or twice during a lecture hour. During the weekly recitation sessions, a course assistant (the first author) and students presented solutions to the weekly homework assignments to others. The tutorial interventions were based on the *Tutorials in Introductory Physics* curriculum (McDermott & Shaffer, 2010). The interventions were similar to the *interactive tutorial lectures* (Kryjevskaja, Boudreaux, & Heins, 2014), but excluded the whole-class discussions (For more information, see Kesonen, Asikainen, & Hirvonen, 2013).

The course followed the Knight's textbook (2008a) which explicitly introduces the ray model and wave model of light as two distinct descriptions of light that have different validity ranges³. The crossover point between the wave model and the ray model of light is addressed by simply comparing the size of an aperture to the wavelength of light as follows.

"If the spreading due to diffraction is less than the size of the opening (of an aperture), use the ray model and think of light as traveling in straight lines; if the spreading due to diffraction is greater than the size of the opening (of an aperture), use the wave model of light." (Knight, 2008a, pp. 686).

This principle was quantized in the context of a circular aperture, resulting in the equation $D_c = \sqrt{2.44\lambda L}$, where D_c is the crossover diameter between the validity ranges of the wave model and the ray model of light, L is the distance from the aperture to the screen, and λ refers to the wavelength of light. Finally, the principle was summarized by substituting the typical values for light ($\lambda=500$ nm, $L=1.0$ m). This then provided the 1 mm thumb-rule: *"When light passes through openings < 1 mm in size, the diffraction effects are usually important. Use the wave model of light. When light passes through openings > 1 mm in size, the diffraction effects are usually not important. Use the ray model of light."* (Knight, 2008a, pp. 687). According to Knight (2008b), the aforementioned thumb-rule is intended to assist students in recognizing the validity ranges of the ray model and wave model of light.

Data Gathering

Data was gathered with the aid of paper-and-pencil tasks at the beginning of the BPIV course (pretest) in 2011-2013, and at the end of BPIV (post-test) in 2013. The pretest was administered during the first BPIV lecture, and consisted of five conceptual tasks on the basics of the course content. One of the tasks was designed specifically for the present study, and it is presented in Appendix A. The post-test task was used as part of the BPIV final exam and is presented in Appendix B. The tasks were based on earlier studies reported by Ambrose et al. (1999) and Wosilait et al. (1999), and the alternatives used in the multiple-choice parts were designed to correspond to students' difficulties identified in those studies. In order to ensure the suitability of the designed tasks, two experts, the course lecturer (second author) and a professor who had had extensive (>10 years) experience in the teaching of optics at the university independently evaluated the tasks before they were used. The tasks were refined until all (first author, second author, and the professor) agreed that the task assignments were clear and suitable for the students participating in BPIV.

The pretest data-gathering settings varied slightly from year to year in 2011-2013. In 2011, 54 students answered the questions individually and the questions were posed without the diagram in the explanation part (see Appendix A). The first analysis of the data revealed that there were a number of vague or blank answers, and so in 2012 the diagram was introduced and 64 students answered the questions individually. In 2013, the questions were unrevised and the students were allowed to answer together with their peers, which provided us with a total of 35 answers. For the purpose of the present study, the pretest data sets have

been combined and the total of 152 answers has been treated as a single data set. More detailed justifications are given in the Results section (IV A).

In addition to the pre- and post-tests, in 2012 four student volunteers were individually interviewed two weeks after the pretest. The purpose of the interviews was to obtain a deeper understanding of the student reasoning underlying the most common selections in the pretest. The interviews were semi-structured (Kvale, 1996): the interviewer (first author) showed the students their pretest answers and asked the students to reflect aloud on how they had ended up with their answers. In addition, the students were asked to elaborate on how they would change their answers if some changes could now be made to the optical system described in the pretest task assignments. The interview protocol was prepared by first and second author, and it was not tested before hand since the protocol was rather straightforward. The interviews took about 25 minutes and were audio recorded with the students' permission.

Data Analysis

The data was analyzed in the following stages. First, the selection distributions of the pretest and post-test data were calculated using descriptive statistics. Second, the students' explanations for the most common students' responses were transcribed. These transcriptions were read several times in a row in order to gain a deep understanding of the student reasoning underlying their answers. Third, the interview data was analyzed by transcribing two out of four interviews. These two interviewees provided further information about the student reasoning involved in most common students' explanations. Unfortunately, other two interviewees mainly responded *I don't know or I can't remember*, and hence they were left out from the analysis. Finally, the textbooks that the students had presumably used at earlier stages in their education were reviewed in order to better understand the origins of the students' ideas frequently used in their explanations. The first author was responsible for all stages of the analysis.

RESULTS

The results obtained from the pretest and interviews are presented in subsection IVA and results from a post-test are presented in subsection IVB.

Pretest and interviews

The pretest asked the students to predict what they would see on the screen when a mask with a large rectangular shaped-aperture (1 × 3 cm) was lit up with (i) a small bulb and (ii) a laser whose beam was larger than the aperture (see Appendix A). One should note that we did not specify the exact size of the bulb or the shape of the laser beam, whether it was divergent, convergent, or maintaining its shape while traveling from the laser to the screen. This was intended to make students' ideas about light visible in the contexts of these light sources.

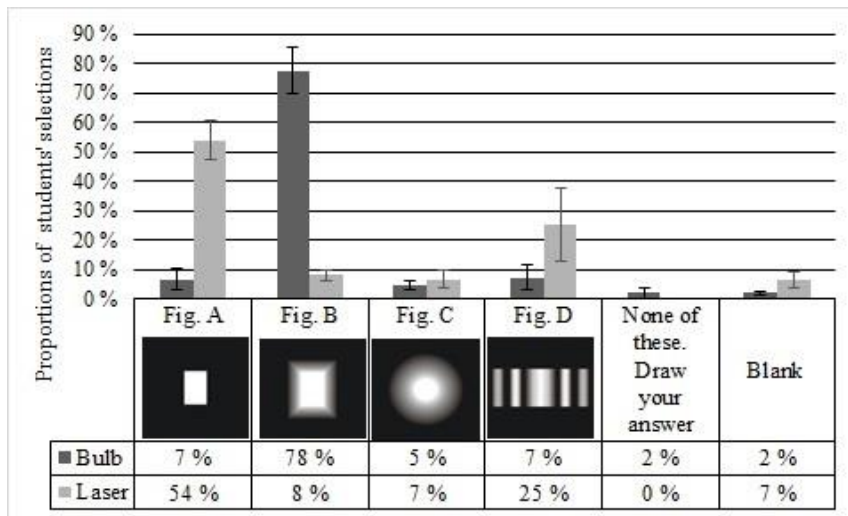





Figure 1. Proportions of students' pretest responses in a task where a rectangular slit was illuminated with a small bulb or a laser (N=152). The correct answer is underlined. The error bars show the standard deviations of responses that occurred in 2011-2013

The answer required for both pretest questions is that, seen on the screen, there would be an aperture-shaped bright area with sharp edges (see **Figure A, Appendix A**). In the case of the small bulb, the students were expected to idealize the bulb as a point source of light and apply the principle of rectilinear propagation of light. The point source idealization was expected to be known because it is typically covered in the lower and upper secondary school physics textbooks⁴⁵. In the case of the laser, the students were expected to apply the principle of rectilinear propagation of light to deduce that some of the laser beam stops at the mask while the rest of it creates an aperture-shaped bright area with sharp edges. The wave nature of light was expected to be ignored due to the large size of the aperture.

Figure 1 presents the students' responses to the pretest responses received during 2011-2013 in terms of the overall proportions (bars) and standard deviations of the yearly proportions (error bars). As can be seen in **Figure 1**, some variation occurred in the students' selections during 2011-2013. Here, we ignore the variation and focus on the obvious disparity between the students' responses when a small bulb or a laser was used as a light source in the pretest task assignment.

The results presented in **Figure 1** show that students' responses changed radically as a result of the light source being changed. When a bulb was used as the light source, 78% of the students responded with **Figure B**, which is the rectangular bright area with fuzzy edges. In the case of a laser, 54% of the students responded with **Figure A**, the rectangular bright area with sharp edges. In the case of the laser, the second most common selection (25%) was **Figure D**, the single-slit diffraction pattern.

Table 1. Students' explanations for the selections of Figure B, Figure A, and Figure D in the pretest task. Explanations labeled as A1-A3 are considered as acceptable explanations for the selections of Figure B and Figure A. Explanations labeled as B1-B8 are considered as inaccurate/inconsistent explanations for the selections of Figure B, Figure A, and Figure D

Small bulb		Laser		Laser	
A total of 119 students selected Figure B		A total of 82 students selected Figure A		A total of 38 students selected Figure D	
Acceptable explanations					
A1: A small bulb was treated as an extended light source that created penumbras around the bright area	19%	A2: The laser emits parallel rays of light	47%		
		A3: The laser's light spreads out in different directions	6%		
Inaccurate/inconsistent explanations					
B1: Light spreads out from a small bulb as an adequate criterion to account for the appearance the fuzzy edges	14%	B5: Overemphasizing the rectilinear propagation of light by claiming that the laser light does not scatter or diffract	10%	B7: The laser's light diffracts or bends or scatters when it passes through the aperture	53%
B2: Brighter light is travelling to the center of the bright area than to its edges (from the single point of a small bulb)	7%	B6: Referring to the properties of a laser, such as its high power and its monochromatic or coherent nature	9%	B8: The 1 x 3 cm aperture was treated as a diffraction grating	16%
B3: A small bulb is claimed to be too weak or too small a light source to create a sharp-edged bright area on the screen	8%	Vague explanations	6%	Vague explanations	6%
		Blank	22%	Blank	26%
B4: Diffraction and/or scattering is claimed to create visible fuzzy edges around the aperture-shaped bright area	15%				
Vague explanations	15%				
Blank	19%				

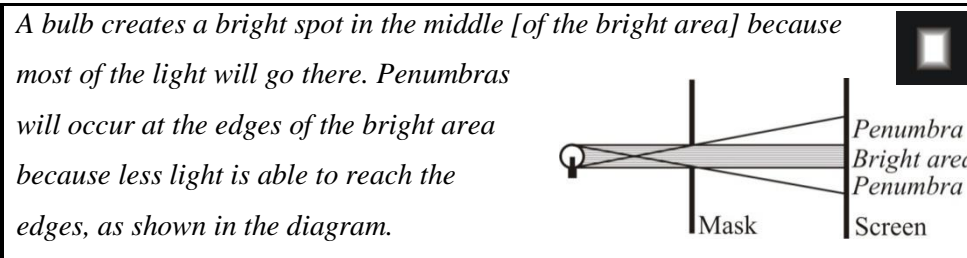


Figure 2. A student's answer in which a small bulb is treated as an extended light source creating penumbras around the bright area

Table 1 summarizes students' explanations for the selection of **Figure B**, and those of **Figure A** and **Figure D**. The explanations are categorized into four main categories: acceptable explanations, inaccurate/inconsistent explanations, vague explanations, and blanks. The acceptable and inaccurate/inconsistent explanations are further divided to subcategories that describe ideas that students used in their reasoning. These subcategories are abbreviated by using the following logic: A1-A3 correspond to acceptable explanations and B1-B8 correspond to inaccurate/inconsistent explanations. The contents of these subcategories are discussed in separate subsections.

A small bulb acts as an extended light source

In the case of a small bulb, a total of 19% of the answers contained an explanation in which the small bulb was assumed to act as an extended light source that created penumbras around the aperture-shaped bright area, as presented in **Figure 2** (A1 in **Table 1**). These students selected **Figure B**, the aperture shaped bright area with fuzzy edges, and explained the appearance of fuzzy edges by showing that less light is able to reach the edges of the bright area. Despite these students did not idealize a small bulb as a point source of light, they did not demonstrate any flaws in their reasoning, and hence their explanations were considered as acceptable.

Laser light travels rectilinearly

When a laser was used as a light source, most of the students who had selected **Figure A**, aperture-shaped bright area, idealized light as parallel light rays and provided a desirable explanation (A2 in **Table 1**). Some students described that the laser light spreads out in different directions, but in contrast to a small bulb, laser light formed sharp-edged bright area to the screen (A3 in **Table 1**). Both explanations were considered as acceptable since they did not violate the ray model or wave model of light.

Light spreads out from a small bulb and its brightness becomes fainter with distance

In the case of a small bulb, 14% of answers (N=119) contained an explanation according to which light from a bulb spreads out and creates the fuzzy edges around the aperture-shaped bright area (**Figure B** in the pretest). This explanation was considered as inadequate because

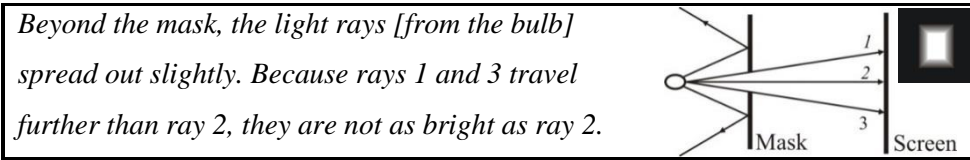


Figure 3. A student's answer in which the appearance of the fuzzy edges (Fig. B) around the aperture-shaped bright area is explained by assuming the brightness of a light ray decreases with distance

it did not specify the appearance of fuzzy edges (B1 in [Table 1](#)). 7% of the students (N=119) claimed that the fuzzy edges are seen because brighter light reaches the center of the bright area than its edges (B2 in [Table 1](#)). Most of these students did not explain why brighter light is emitted to reach the center of the bright area rather than its edges. One student, however, explained it by using an idea according to which the brightness of light rays becomes fainter with distance, as shown in [Figure 3](#). This idea might have also been used by the students who only referred to the direction of the light or claimed that brighter light is emitted to reach the center of the bright area, or at least their explanations were consistent with the idea. Using the idea that the brightness on a light ray becomes fainter with the distance shows that students were misunderstood the concept of light ray that only describes the route of light, and not its brightness.

In the course of the interviews, Student 1 suggested that fuzzy edges are caused by the small gaps between the light rays emitted by a bulb. This became evident when Student 1 was asked to elaborate whether the other bright areas presented in the pretest task assignment ([Figures A, C or D](#) in the [Appendix A](#)) could be seen by modifying the experimental setup. Surprisingly, he suggested that in the case of [Figure A](#), the aperture-shaped bright area with sharp-edges would be seen if the screen were moved closer to the mask.

Interviewer: Why would we see a sharp-edged bright area if the screen were moved closer to the mask?

Student 1: Because the light would not become so faint on its way to the screen.

Interviewer: Do you mean that the brightness of the light does not decrease because the light would travel a shorter distance, or...?

Student 1: Well... I have a common sense idea. Here's the bulb (see the circle in [Figure 4](#)), and it gives off these individual lines of light rather than this kind of uniform light (see [Figure 4](#)). Hence, there are always small gaps between these lines of light, and when the screen is moved closer to the bulb, these gaps get smaller and then we can see a sharper image on the screen.

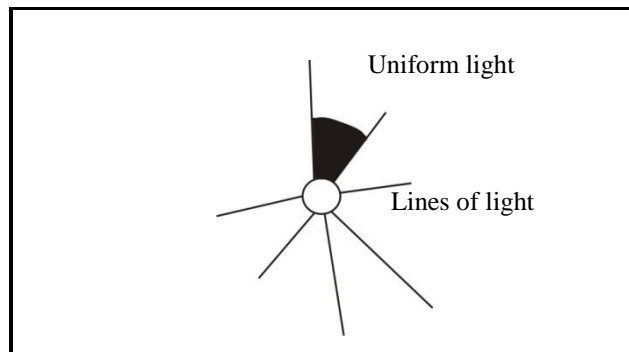
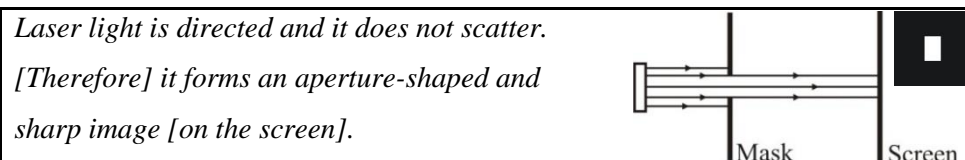


Figure 4. Student 1 had an idea that a bulb emits light as individual rays rather than as uniform light



*Laser light is directed and it does not scatter.
[Therefore] it forms an aperture-shaped and
sharp image [on the screen].*

Figure 5. A student's explanation in which the rectilinear propagation of light is overemphasized by assuming that a laser beam does not scatter or diffract while passing through the aperture

The idea by Student 1 according to that a small bulb emits lines of light is evidently inconsistent with the ray model of light.

Overemphasizing the rectilinear propagation of laser light

In the case of a laser being used as a light source 10% of the students who had selected **Figure A** overemphasized the rectilinear propagation of light by claiming that laser light does not diffract or scatter while it passes through the aperture (B5 in **Table 1**). This type of answer is illustrated in **Figure 5**. This particular student argued that because of the monochromatic nature of light the laser beam does not scatter, while others referred to the direction of the laser light, as illustrated in **Figure 5**. Interestingly, many of these students had referred to diffraction in the case of the bulb, but ignored it when the laser was used as the light source, as shown in **Figure 6**.

This type of disparity between a bulb and a laser became also evident during the interviews, where students were asked to give further explanations for their pretest responses. For example, Student 1 who had selected **Figure B** in the case of a bulb and **Figure A** in the case of a laser emphasized the direction of light when he explained his selections as shown below.

I guess, because it [the light from the laser] travels only in one direction and the laser beam stays narrow compared to the light of the bulb, which spreads out in all directions.

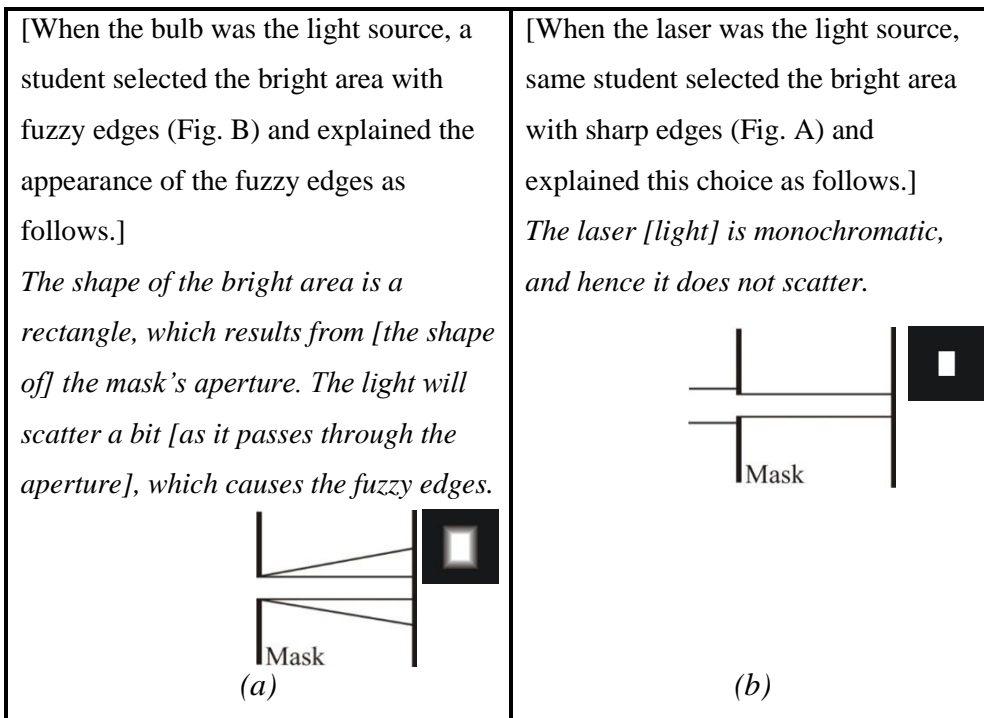


Figure 6. A student's answers that demonstrate how the diffraction or scattering is taken into account when a small bulb is used as the light source, but not when a laser is the light source

Another student, labeled here as Student 2, implied that light from a bulb bends (diffracts) from the edges of the aperture, argued that no bending would take place when a laser was used as a light source.

Student 2: I ended up with this [Figure A], because here there are clear edges and it [light] will pass straight [through the aperture] and it won't bend anywhere; it will keep traveling in the same direction and it will make this kind of clear [sharp-edged] picture on the screen.

Interviewer: Are you saying that, in the case of the laser, the bending does not happen around the mask?

Student 2: Yes, no bending.

These findings show that students treated the light emitted from a bulb and a laser differently. Besides this, it became evident that students tended to overemphasize the rectilinear propagation of laser light in their reasoning.

Bulb is weak light source, and laser has high power and other special properties

The disparity between a bulb and a laser also became evident in students' explanations in which the properties of a bulb and a laser were compared. 8% of the students (N=119)

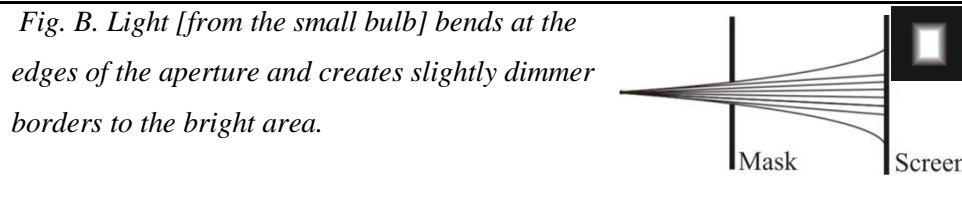


Figure 7. A student's answer in which the appearance of the fuzzy edges (Fig. B) is explained by assuming that light bends at the edges of the aperture

argued that the small bulb is too small or too weak light source to create a sharp-edged bright area on the screen, and hence fuzzy edges are seen around the bright area (B3 in [Table 1](#)). In the case of a laser, 9% of the students (N=82) claimed that the laser created a sharp-edged bright area due to its high power and its being monochromatic (B6 in [Table 1](#)). In addition, a minority of the students (< 5%, N=82) assumed that due to its special properties, the laser beam does not scatter or bend (diffract).

The diffraction of light is taken into account

In the contexts of both light sources, some students claimed that light diffracts, bends, or scatters as it passes through a 1 cm wide aperture. In the case of a bulb, 15% of the students (N=119) explained that the bending causes the fuzzy edges of an aperture-shaped bright area (B4 in [Table 1](#)), as illustrated in [Figure 7](#).

In the case of a laser, 25% of the students (38/152) selected [Figure D](#), the single-slit diffraction pattern. As shown in [Figure 8a](#), some of the students assumed that the laser beam automatically creates a diffraction pattern when it travels through an aperture that is smaller than the beam itself (B7 in [Table 1](#)). Others described the diffraction in terms of waves, as shown in [Figure 8b](#). In addition, 16% of the students (N=38) assumed that the aperture (1x3 cm in size) acts as a diffraction grating (B8 in [Table 1](#)). These students seemed to recall an experiment related to diffraction, as can be seen in the following sample response: There will be small [bright] lines separated from each other, by means of which we can calculate the grating period.

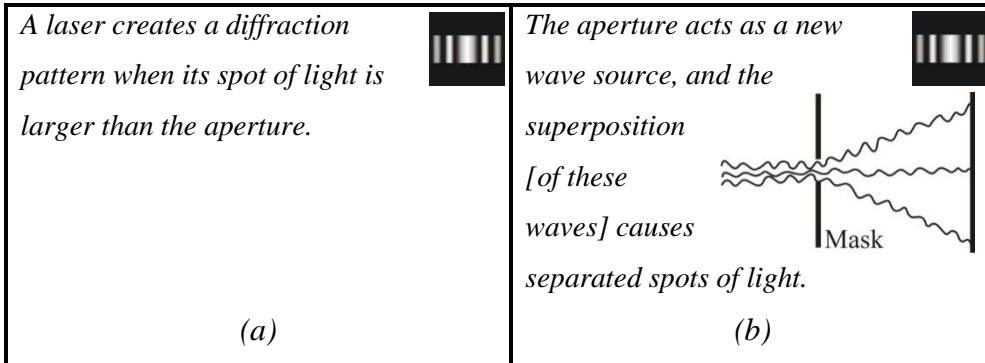



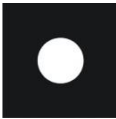
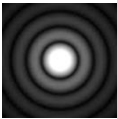

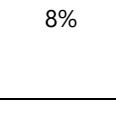


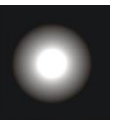

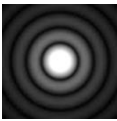


Figure 8. Students' explanations in the pretest for the selection of Fig. D, a single-slit diffraction pattern

The students, who thought that the diffraction pattern is seen on the screen, obviously did not recognize when the wave behavior of light needs to be taken into account.

Post-test assignment and post-test results

At the end of the course of BPIV, the students' explanations were investigated with the aid of an exam task, presented here in Appendix B. The task required students to predict what they would see on the screen if a mask with a circular aperture (diameter 1.0 cm) is illuminated with a small bulb or with a laser, and also to predict how large the bright area would be compared to the aperture. In addition, in the case of the laser, the students were required to predict what would happen if the diameter of the aperture were decreased to 0.015 mm (see Appendix B). In order to minimize any ambiguity on the exam task, the necessary idealizations were stated explicitly. The small bulb was identified as a point source of light, while the laser beam was said to be larger than the aperture, but keeping its diameter constant in travelling from the laser to the aperture.



Table 2. Students' answers to the post-test questions at the end of the BPIV course (N=54)

	Bulb (D=1.0 cm)	Laser (D=1.0 cm)	Laser (D=0.015 mm)
Correct bright area, correct size of the bright area, and acceptable explanations	48 % 	59% 	35% 
Correct bright area and correct size of the bright area, but with incomplete or vague explanation	4% 	8% 	7% 
Most common incorrect answers: either the bright area or the size of the bright or both were incorrect.	33% 	17% 	41% 
	11% 	15% 	11% 
Blank	2%	4%	2%

Central maximum is equal to or smaller than the aperture

In the case of the diameter of the aperture being 1.0 cm, the required response was that there would be a sharp-edged bright circle (Figure A in Appendix B). In the case of the bulb, the diameter of the bright circle would be greater than the diameter of the aperture, while in the case of the laser it would equal that of the aperture. When the diameter of the aperture was decreased to 0.015 mm, the students were supposed to notice that a diffraction pattern would appear on the screen. In addition, the students were required to deduce that the diameter of the bright area would be greater than that of the aperture, e.g., by applying the equation for the width of the central maxima $w=2.44 \lambda L/D$, where D is the diameter of the aperture. For the values presented in the task assignment, $w=10.3$ cm.

Table 3. Students’ explanations for their selection of Figs. B and C in the post-test

Bulb, D=1.0 cm, N=54, Students referred to		
C1: the direction of the light, e.g., the fuzzy edges are seen because the light from a bulb spreads out radially	15%	-
C2: the brightness of the bulb’s light, e.g., a bulb emits more light (rays) or brighter light to the center of the bright area than to its edges	6%	-
C3: the wave nature of light, e.g., the bulb’s light diffracts or scatters while passing through the aperture, thus creating the fuzzy edges/diffraction pattern on the screen	7%	9%
Vague	4%	2%
No explanation	2%	-
Total	33%	11%



A total of 54 students completed the task and **Table 2** presents the distribution of responses in terms of correct and incorrect answers. Roughly half of the students provided correct answers, while the rest provided incorrect responses. The explanations used to justify the incorrect responses contained similar students’ ideas that were observed in the pretest and interviews. The following subsection presents how these ideas interfered with students’ understanding of the validity ranges of the ray model and wave model of light at the end of BPIV. In the following subsections, students’ explanation categories are labeled as C1-C8.

A bulb emits light radially and creates fuzzy edges around the bright area

As shown in **Table 2**, a total of 33% of the students selected the aperture-shaped bright area with fuzzy edges, even though the point source idealization had been explicitly presented in the task assignment. **Table 3** summarizes the students’ explanations for this response, and indicates that a majority of the students (15%) explained the appearance of the fuzzy edges in terms of the direction of the light (C1 in **Table 3**). It is worth noting that all except one of these students realized that no diffraction pattern would be seen on the screen and they did so by referring to the size of the aperture or the 1 mm thumb-rule, as shown in the following sample extract. *Because diffraction takes place when the diameter of the aperture is less than 1 mm, in this case we won’t see any diffraction [pattern]. However, there will be fuzzy edges around the sharp image, because light spreads out radially from the bulb.* Despite understanding the applicability of diffraction, the students who answered as shown in this extract used the idea that the bulb would create fuzzy edges around the area of light because light spreads out radially from a bulb.

The appearance of fuzzy edges was described more comprehensively by students who claimed that the light emitted by a bulb is brighter in the center of the bright area and dimmer

Table 4. Students' explanations for their selections of Figs. B and C in the post-test (see Appendix B)

Laser D=1.0 cm, N=54 Students referred to		
C4: the appearance of the wide laser beam and how it might appear in real life. (B4)	14%	-
C5: the wave nature of light in terms of diffraction, scattering, bending, etc. (B5)	4%	15%
Total	18%	15%

on the perimeter. This idea was expressed explicitly by a total of 6% of the students (3), as shown in **Table 3** (C2). Two of the students stated explicitly that the intensity (~brightness) of the light of a bulb will decrease with distance, as illustrated in the following sample extract. **Figure C.** *From the point source of the light the light rays travel a shorter distance to the center of the bright area than to its edges. In consequence, the intensity of the light will decrease when moving toward the edges [of the bright area].* These students have misunderstood the concept of light ray that only describe the route of light rather than its brightness.

As shown in **Table 3** (C3), a total of 7% of the students explained the appearance of the fuzzy edges in terms of the wave nature of light. In addition, a total of 9% of the students who had selected the diffraction pattern referred to the wave nature of light in their explanations. Those students had not understood the validity ranges of the ray model and the wave model of light as they had been taught in BPIV.



Laser creates fuzzy edges or diffraction pattern due to its special properties

When the laser was used as light source in the case of the large aperture (diameter 1.0 cm), 18% of the students selected the bright area with the fuzzy edges, while 15% of them selected the diffraction pattern. **Table 4** presents the students' explanations for their responses.

Most of the students selecting **Figure C** (14%) explained the appearance of the fuzzy edges by claiming that the laser emits either more or brighter light toward the center of the bright area than toward its perimeter. These students explained the appearance of fuzzy edges by wondering how the wide laser beam might actually look like (C4 in **Table 4**).

Two students explained the appearance of the fuzzy edges by claiming that the diffraction or interference phenomenon would be visible even if the diameter of the aperture were 1.0 cm. In addition, those students who had selected the diffraction pattern referred to the diffraction and interference phenomena of light or special properties of laser (coherence etc.) in their explanations (C5 in **Table 4**). These students did not recognize the validity ranges of the ray model and wave model of light.

Table 5. Students' explanations in the post-test for the claim that the diameter of the light area is equal to or smaller than that of the aperture

Laser D=0.015 mm, N=54		
Students referred to		
	Central maximum is equal to or smaller than the aperture	
C6: the rectilinear propagation of light	22%	11%
C7: the diffraction and/or interference of light	9%	-
C8: equations such as $w = 2,44 \lambda L/D$, but they made mistakes when applying the equations	4%	-
Vague	6%	-
Total	41%	11%

Overemphasizing the rectilinear propagation of laser light

When a laser was used as the source of light in the context of a small aperture (D=0.015 mm), more than half of the students claimed that the diameter of the bright area would be either equal to or smaller than the diameter of the aperture (41% selected **Figure B** and 11% selected **Figure A**). **Table 5** presents the students' explanations for these responses, and it shows that the most of the students overemphasized the rectilinear propagation of light (C6 in **Table 5**). The overemphasis became evident when students claimed that the central maximum of the diffraction pattern was a geometrical image of the aperture, as presented in the following sample extract. *It is equal to [the diameter of the aperture, because the] light from the laser passes rectilinearly [through the aperture], so the bright spot [in the middle of the diffraction pattern] is equal in size to the aperture.* In addition, the overemphasis was apparent when students claimed that the distance between the first diffraction minima around the central maximum was equal to the diameter of the aperture, as shown in the following sample extract. *The bright spot [in the middle of the diffraction pattern] is smaller than the diameter of the aperture, because the first minima are formed at the width of the aperture.*

The overemphasis became even more evident amongst 11% of the students (N=54), who had responded that a bright circle would be seen on the screen. These students claimed that laser light travels in a straight line through the aperture and creates an aperture-shaped bright area on the screen, as shown in **Figure 9**. The student whose response is presented in **Figure 9** stated explicitly that the laser light does not diffract, but if the bulb had been used as the light source, then diffraction would have occurred. This type of disparity was also evident in the pretest and interview; its existence here highlights how greatly the type of light source labeled explicitly in the task assignment can impact on student reasoning.

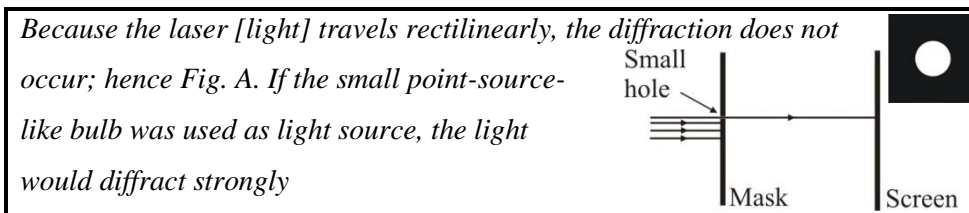


Figure 9. A student's explanation that demonstrates the overemphasis on the rectilinear propagation of light in the case of a laser

As shown in **Table 5**, 9% of the students (N=54) claimed that the central maximum of the diffraction pattern is smaller than the diameter of the aperture due to the diffraction or interference of light (C7 in **Table 5**). These students typically claimed that the *destructive interference restricts the size of the bright area seen on the screen*. These students may also have overemphasized the rectilinear propagation of light by expressing it in a far more implicit manner. Or they may have retained other ideas that we have not been able to distinguish in the present study.

4% of the students misapplied the equations derived from the wave model of light (C8 in **Table 5**). Consequently, these students deduced that the diameter of the central maximum of the diffraction pattern was smaller than the diameter of the aperture.

DISCUSSION

The results of the present study show that the introductory students used the different set of ideas about the behavior of light when different light sources were explicitly labeled to task assignments. In this section, we discuss how these ideas correspond to the subject matter of optics covered in lower and upper secondary school textbooks⁴⁵ used in Finland. Finally, we discuss how the instruction given in BPIV impacted on students' explanations of the behavior of light.

When the small bulb was used as a light source, students often argued that fuzzy edges are seen around an aperture-shaped bright area. To explain the appearance of these edges, some students assumed that the small bulb acts as an extended light source rather than a point source of light (see Section IV A1). The students have most likely learnt this idea from physics textbooks used at the lower and upper secondary schools. These textbooks⁴⁵ cover how extended light sources create shadows e.g. by presenting ray diagrams presented in **Figure 10a**. Interestingly, the textbooks also cover the point source idealization as presented in **Figure 10b**, but only a few of the students idealized the small bulb as a point source of light. The absence of point source idealization was surprising because the students who treated the small bulb as an extended light source had to apply more complex reasoning, e.g. in terms of the number of light rays, than would have been needed while using the point source idealization.

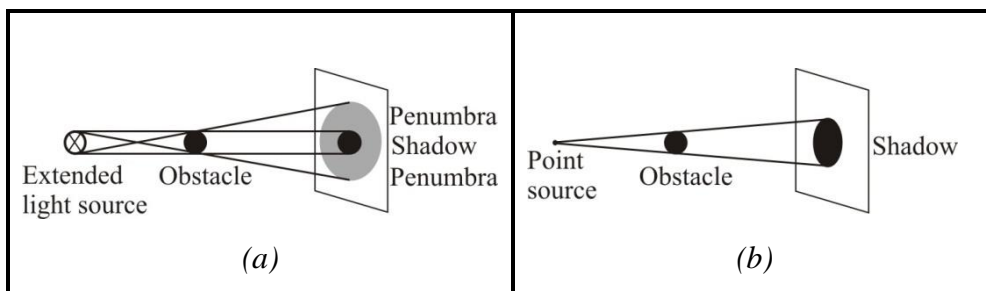


Figure 10. Commonly used textbook presentations that illustrate the rectilinear propagation of light in the context of shadows and penumbras

The majority of students might have treated the small bulb as an extended light source due to their everyday experience with flashlights, where small bulbs (or nowadays LEDs) are used as light sources. The flashlights typically create a spot of light that is brighter in the middle and appears dimmer at its edges, which corresponds to the bright area the most of the students chose when a small bulb was used as light source in the pretest. By treating the small bulb as an extended light source, the students were able to explain the appearance of dimmer edges around the aperture-shaped bright area. However, we found no explicit reference to flashlights in the students' explanations. Nevertheless, we suspect that experiences obtained with flashlights may have made students believe that a small bulb creates bright area with fuzzy edges, and treating the bulb as an extended light source offered a way to explain why these edges appear.

In the context of a small bulb, the students were found to use reasoning that was consistent with the idea the brightness of light rays becomes fainter with distance. This idea has potentially been learnt from lower and upper secondary level textbooks⁴⁵ that cover the basics of photometry. In those textbooks, the concept of the luminance (I) of an illuminated surface with respect to distance (r) has typically been presented by emphasizing the relation $I \propto 1/r^2$. This makes it possible to explain everyday phenomena such as why the brightness (luminance I) of the candle flame decreases with distance. However, photometry extends beyond the idealizations of the ray model of light (Ohta & Roberson, 2005). The lower and upper secondary school textbooks⁴⁵ do not emphasize it explicitly but rather present the ray model and the photometry as independent topics under the general rubric of optics. As a result of this type of instruction, the students may have identified brightness (luminance) as a relevant feature of a light ray, which violates the idealizations of the ray model of light. This may then explain why the students' reasoning was consistent with the idea that the brightness of light rays decreases with distance.

In the case of a laser, the students overemphasized the rectilinear propagation of light by ignoring the wave nature of light. Besides this, the overemphasis became evident whenever students referred to the wave nature of light but used reasoning that was more consistent with the rectilinear propagation of light than the wave model of light. The overemphasis most likely

originated from an everyday observation of a laser beam, that is, a narrow beam of light that travels rectilinearly. Besides the everyday observations, the textbooks⁴⁵ used in lower and upper secondary schools often visualize light rays by using laser beams while demonstrating the behavior of light in the context of lenses and mirrors. Demonstrations are also typically presented using Laser Ray Box Kits⁶, a detail that clearly shows the correspondence between special rays in ray diagrams and the behavior of real laser beams, implying at the same time that rays and beams are the same entity. In consequence, the use of such demonstrations may have facilitated the students' idea that the laser beam acts as a light ray that travels always rectilinearly.

Students were found to explain their incorrect answers by referring to the special properties of a bulb and a laser, such as coherence. In all probability, students have learnt these properties from the upper secondary school textbooks⁵ that briefly introduce them in the context of the diffraction of light. The following phrases recur frequently:

A bulb emits white light that contains various wavelengths and is incoherent; Laser sends light that is monochromatic and coherent; Laser light is monochromatic (it contains only a single wavelength) and coherent (the waves are in phase); With the aid of a laser, one can obtain coherent light in which all [light]waves are in phase. In addition, with the aid of a laser one can obtain light that has a single color, which means that it is monochromatic. As a consequence, the frequency of the laser [light] is constant. The intensity of the laser [light] is high, and the light waves created by the laser travel over long distances in a single direction (parallel).

These descriptions indeed summarize the properties of these light sources, but they may, as our data shows, also hinder students' abilities to apply the ray model and wave model of light in a coherent manner.

Some students believed that diffraction pattern appears on the screen whenever laser light passes through an aperture regardless of its size. These students obviously fail at recognizing the validity ranges of the ray model and wave model of light. This failure may be supported by upper secondary textbooks⁵ that typically cover the wave nature of light by presenting Young's double-slit experiment. The basic idea of this experiment is often illustrated with the diagram presented in **Figure 11**. The textbooks emphasize that the slits need to be comparable to the wavelength of light. However, as our results show, some students obviously overlooked this crucial piece of information.

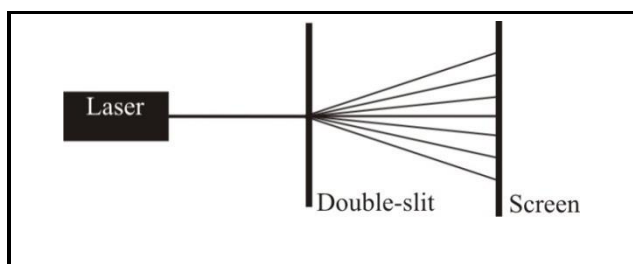


Figure 11. A typical textbook picture that illustrates the creation of a double-slit interference pattern with light

At the end of BPIV, approximately a half of the students was able to deduce the correct bright areas by recognizing the validity ranges of the ray model and wave model of light. (see Section IV B). This indicates that the instruction given at the course of BPIV supported students' understanding of the ray model and wave model of light. However, almost a half of the students failed at applying these light models, even if the validity ranges of the ray model and wave model of light were recognized in some cases by referring to 1 mm thumb-rule. Most often students provided incorrect answers due to their ideas about light emitted from a bulb or a laser. Overall this implies that the instructional solutions used in BPIV, such as the 1 mm thumb-rule, helped the students to deepen their understanding of the behavior of light, for instance, to determine when diffraction is relevant. However, it did not address the students' ideas regarding a small bulb and a laser that hindered their abilities to apply the ray model and wave model of light in a coherent manner.

CONCLUSION

Overall the present study indicates that students' understanding of light is not entirely independent on contextual features, namely light sources labelled in tasks assignments. Andersson & Kärrqvist (1983) have ended up similar conclusion when investigating young pupils' understanding of light. These pupils tended to view light as identical with its source (Reiner et al., 2000). Similarly, the findings of the present study show that students' ideas about light and its behavior strongly corresponded to perceivable features of the light sources and the subject matter covered at the earlier levels of education. The majority of students' ideas were, indeed, problematic because they evidently weaken students' abilities to apply the ray model and wave model of light in a coherent manner in the context of different light sources. The students' ideas were evident in the pretest, the interview, and the post-test, and hence they were not simply random errors but rather systematic and robust tendencies in the student reasoning.

Earlier studies conducted at the university level have concluded that students fail at developing sufficient understanding of light due to their difficulties to grasp the ray model and wave model of light (Sengören, 2010; Maurines, 2009; Colin & Viennot, 2001; Ambrose et al., 1999; Wosilait, 1996). We agree with this conclusion, and wish to sharpen it by suggesting

that students' difficulties are partly caused by their ideas about light sources that are commonly labelled in the task assignments of optics. The present study indicates that more attention should be paid to real light sources and their perceivable features in optics instruction given at the introductory level.

Finally, the present study yields implications for future investigations of students' understanding of optics. In the past, such investigations have often been conducted via paper-and-pencil test questions in which light sources have typically been labelled explicitly (e.g. Refs. Maurines, 2009; Colin & Viennot, 2001; Ambrose et al., 1999; Wosilait, 1996). Students' answers to these test questions have been analyzed without considering the impact of the light sources. The results of the present study indicate that explicitly labelled light sources should no longer be treated simply as neutral components of the test questions that need no attention. Instead, they are likely to trigger certain types of students' ideas about light and its behavior, and these ideas can determine what knowledge students bring to bear while answering test questions. Fortunately, the process of triggering has not been completely random. Instead, it has rather systematically biased the student reasoning toward the perceptible features of the light sources and textbook presentations used earlier. In order to better understand this bias, further studies will be needed to discover how they appear in other educational contexts where different instructional practices are employed, and how they can be tackled without expanding the optics instruction overwhelmingly.

REFERENCES

- Ambrose, B. S., Shaffer, P. S., Steinberg, R. N., & McDermott, L. C. (1999). An investigation of student understanding of single-slit diffraction and double-slit interference. *American Journal of Physics*, 67, 146-155.
- Andersson, B., & Kärrqvist, C. (1983). How Swedish pupils, aged 12-15 years, understand light and its properties. *European Journal of Science Education*, 5, 387-402.
- Bao, L., & Redish, E. F. (2006). Model analysis: Representing and assessing the dynamics of student learning. *Physical. Review Special Topics - Physics Education Research*, 2, (010103).
- Chi, M., Feltovich, P., & Glaser, R. (1981). Categorization and representation of physics problems by experts and novices. *Cognitive Science*, 5, 121-152.
- Colin, P., & Viennot, L. (2001). Using two models in optics: Students' difficulties and suggestions for teaching. *American Journal of Physics*, 69, S36-S44.
- Hsu, L., Brewster, E., Foster, T. M., & Harper, K. A. (2004). Resource letter RPS-1: Research in problem solving. *American Journal of Physics*, 72, 1147-1152.
- Kesonen, M. H. P., Asikainen, M. A., & Hirvonen, P. E. (2013). Assessing the impact of a tutorial intervention when teaching the ray model of light in introductory physics. *European Journal of Physics Education*, 34, 849-857.
- Knight, R. D. (2008a). *Physics for scientists and engineers with modern physics: A strategic approach*. San Francisco: Pearson: Addison Wesley.
- Knight, R. D. (2008b). *Physics for scientists and engineers (2nd edition) A strategic approach - Instructor Guide*. San Francisco: Pearson Addison-Wesley.

- Kryjevskaja, M., Boudreaux, A., & Heins, D. (2014). Assessing the flexibility of research-based instructional strategies: Implementing tutorials in introductory physics in the lecture environment. *American Journal of Physics*, 82, 238-250.
- Kvale, S. (1996). *InterViews: An introduction to qualitative research interviewing*. Thousand oaks: Sage Publications.
- Maurines, L. (2009). Geometrical Reasoning in Wave Situations: The case of light diffraction and coherent illumination optical imaging. *International Journal of Science Education*, 32, 1-32.
- McDermott, L. C. (1993). Guest Comment: How we teach and how students learn - A mismatch. *American Journal of Physics*, 61, 295-298.
- McDermott, L. C., & Shaffer, P. S. (2010). *Tutorials in Introductory Physics*. New Jersey: Pearson Education.
- Meltzer, D. E. (2005). Relation between students' problem-solving performance and representational format. *American Journal of Physics*, 73, 463-478.
- Ohta, N., & Roberson, A. R. (2005). *Colorimetry: Fundamentals and Applications*. Chichester: John Wiley & Sons Ltd.
- Palmer, D. (1994). The effect of the direction of motion on students' conceptions of force. *Research in Science Education*, 24, 253-260.
- Palmer, D. (1997). The effect of context on students' reasoning about forces. *International Journal of Science Education*, 19, 681-696.
- Reiner, M., Slotta, J. D., Chi, M. T., & Resnick, L. B. (2000). Naive physics reasoning: a commitment to substance-based conceptions. *Cognition and instruction*, 18, 1-34.
- Sabella, S., & Redish, F. E. (2007). Knowledge organization and activation in physics problem solving. *American Journal of Physics*, 75, 1017-1029.
- Savinainen, A., & Viiri, J. (2008). The force concept inventory as a measure of students conceptual coherence. *International Journal of Science and Mathematics Education*, 6, 719-740.
- Sengören, S. K. (2010). How do Turkish high school graduates use the wave theory of light explain optics phenomena? *Physics Education*, 45, 253-263.
- Stewart, J., Griffin, H., & Stewart, G. (2007). Context sensitivity in the force concept inventory. *Physical Review Special Topics - Physics Education Research*, 3, (010102).
- Wosilait, K. (1996). *Research as a guide for the development of tutorials to improve student understanding of geometrical physical optics*. Department of Physics, University of Washington: Ph.D. dissertation.
- Wosilait, K., Heron, P. R., Shaffer, P. S., & McDermott, L. C. (1999). Addressing student difficulties in applying a wave model to the interference and diffraction of light. *American Journal of Physics*, 67, S5-S15.

APPENDICES

Appendix A

THE PRETEST TASK

A mask with a rectangle aperture (1 cm x 3 cm) is placed between a small bulb and the screen, as shown on the left. The distance between the mask and the screen is a few meters. Which of the figures (A-E) best corresponds to the bright area seen on the screen?

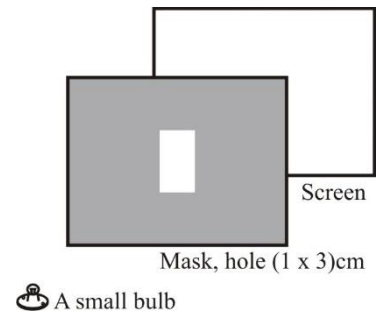


Fig. E:
None of these.
Draw your own
answer.

Explain your reasoning in words and sketch the path of the light on the diagram on the left.



The bulb is replaced with a laser whose beam is larger than the aperture in the mask. Which of the figures (A-E) best corresponds to the bright area seen on the screen?

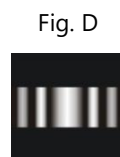


Fig. E:
None of these.
Draw your own
answer.

Explain your reasoning in words and sketch the path of the light on the diagram on the left.

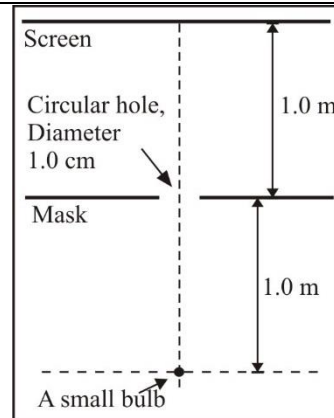


Appendix B

THE POST-TEST TASK

A small bulb illuminates a mask with a circular aperture, as shown on the right. The bulb is so small that it can be considered to be a point source of light. The diameter of the aperture is 1.0 cm.

- a. Which of the figures (A-D) below best corresponds to the bright area seen on the screen? Explain your choice.
- b. Is the diameter of the bright area a) *greater than* b) *equal to* c) *smaller than* the diameter of the aperture? If you selected figure B, consider the diameter of the central bright region. Explain your choice.



Top view (figure not to scale)

The small bulb is replaced with a HeNe-laser ($\lambda = 633 \text{ nm}$), and its beam is directed toward the aperture. The diameter of the beam is larger than the aperture. The diameter of the beam does not change as the beam travels from the laser to the mask. The diameter of the aperture is 1.0 cm.

- c. Which of the figures (A-D) below best corresponds to the bright area seen on the screen? Explain your choice.
- d. Is the diameter of the bright area a) *greater than* b) *equal to* c) *smaller than* the diameter of the aperture? If you selected figure B, consider the diameter of the central bright region. Explain your choice.

When the laser is used as source of light, the diameter of the aperture is decreased to 0.015 mm.

- e. Which of the figures (A-D) below corresponds to the bright area seen on the screen? Explain your choice.
- f. Is the diameter of the bright area a) *greater than* b) *equal to* c) *smaller than* the diameter of the aperture? If you selected figure B, consider the diameter of the central bright region. Explain your choice.

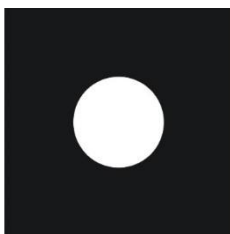


Figure A: Circle with sharp edges



Figure B: Circle with fuzzy border and rings

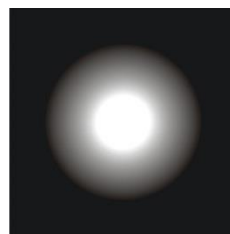


Figure C: Circle with fuzzy border

Figure D: None of these. Draw the bright area that will be seen on the screen

NOTES

- ¹ The particle (quantum) model of light is omitted in the present study, although it is also discussed in the textbook by Knight (2008a)
- ² No information was available regarding students' high school studies in 2013
- ³ In principle, the wave model of light could be used whenever the ray model of light is valid, but it would be terribly impractical.
- ⁴ Physics textbooks commonly used in Finnish lower secondary schools: Aine ja energia: fysiikan tietokirja (Aspholm S., Hirvonen H., Lavonen J., Penttilä A., Saari H. & Viiri J., 2000); Fysiikan avain 1 (Happonen J., Heinonen M., Muilu H. & Nyrhinen K.)
- ⁵ Physics textbooks commonly used in Finnish upper secondary schools: Physica 3 - aallot (Hatakka J., Saari H., Sirviö J., Viiri J. & Yrjänäinen S. 2005); Fysiikka 2: Lämpö ja aallot (Lehto H., Luoma T., Havukainen T. & Leskinen J. 2006); Fotoni 3 (Eskola S. M., Ketolainen P. & Sterman F., 2006, Otava).
- ⁶ See example of Laser Ray Box Kits, <http://www.arborsci.com/laser-ray-box-and-lenses> (valid 15.12.2016)

<http://iserjournals.com/journals/eurasia>