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Literature review on networking of theories developed in mathematics education context

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Abstract
The research aim is made a literature review on research focused on networking of theories developed in mathematics education field with several mathematical concepts. On the other hand, to illustrate what a networking of theories consists of, a synthesis of an articulation between the extended theory of connections (ETC) and the onto-semiotic approach (OSA) was presented using the study of the mathematical connections built by a university student on the derivative concept as an example. A qualitative study was developed in two stages: (1) Three phases were followed: search for information in various search engines and databases (ERIC, Google Scholar, etc.), organization and analysis of the documentation, finding works on articulation of theories focused on various mathematical concepts such as derivative. (2) A synthesis of the theoretical articulation ETC-OSA is presented, emphasizing the analysis of an episode over the derivative. It is concluded that the research reviewed on the articulation of theories about a certain phenomenon or teaching of content represents an important contribution to improving its understanding. In addition, this research provides a theoretical input or detailed panorama of background organized chronologically so that the community interested in this research line can use it for future studies.

Keywords: mathematical connections, networking of theories, literature review, onto-semiotic approach, derivative concept

INTRODUCTION

One of the community of researchers' fundamental characteristics in the mathematics education field is the wide diversity of theoretical views with principles, methods, and problematic questions. Each one of them tends to privilege some dimensions of the mathematics teaching and learning process over the others (Font et al., 2011), but there is one aspect in which many of these theories coincide, we mean that they consider that a characteristic of the mathematical objects that must be taught and learned is its complexity. Precisely, in our opinion, the complexity of mathematical objects, together with the complexity of their teaching and learning, is one of the reasons why there is a diversity of theoretical approach in mathematics education and that currently the need for dialogue and articulation of theories (Font, 2016; Prediger et al., 2008).

On the one hand, the existence of various theories to address didactic-mathematical problems can be a positive factor, given the complexity of such problems, if each theory addresses a partial aspect of them. On the other hand, when the same problem is approached with different theories, which frequently implies the use of different languages and assumptions, disparate and contradictory results can be obtained that can hinder the progress of mathematics education. Thus, the problem of comparing, coordinating, and integrating these theories in a framework that includes the necessary and sufficient tools to do the required work appears. Research related to the development of networks of theories aims to understand and interpret the complementarities between different theories and whether this can be successful in studying a certain phenomenon related to the teaching and learning processes of mathematics, respecting both the conceptual and methods of each theory (Kidron & Bikner-Ahsbahs, 2015; Prediger et al.,

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Contribution to the literature

- Review the literature on research focused on the networking of theories with different mathematical concepts, demonstrating the importance and functionality of pairs of strategies to create theoretical articulations. In addition, this research provides a theoretical input or detailed panorama of background organized chronologically so that the community interested in this line of research can use it for future studies.
- Show as an example of networking, a synthesis of one of the most recent networking of theories in the literature, the case of networking between ETC and the OSA. In this networking, the establishment of connections in the study of a derivative was used as a context for reflection, given that in the literature it is a concept that is important and in many cases is difficult for students and teachers to understand.

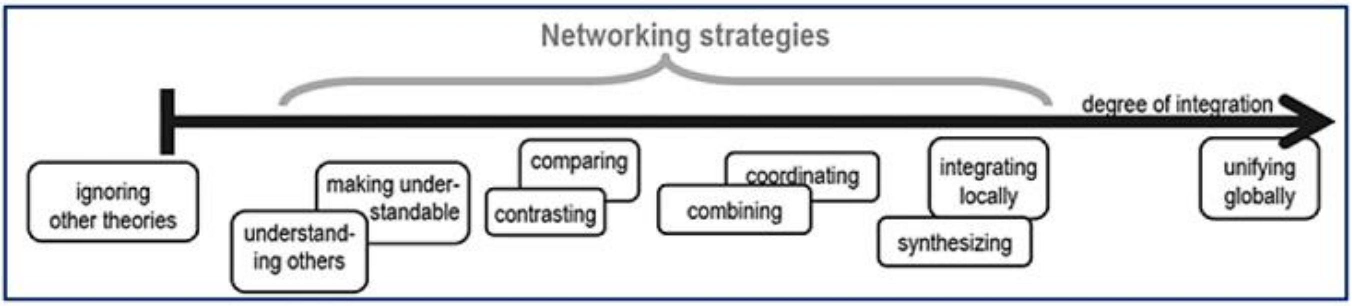

Figure 1. Strategies to articulate theoretical frameworks (Prediger et al., 2008, p. 170)

2008). This work on networks of theories has begun to provide concrete examples of articulations, pointing out the benefits and possible difficulties and limitations of a multi-theoretical approach (Bikner-Ahsbahs & Vohns, 2019).

A first aspect to consider is the characterization of what is meant by a theory and what are the theories developed in mathematics education, with the purpose of using it and establishing comparisons, complementarities, contrasts, and possible connections between them (Artigue & Mariotti, 2014; Niss, 2007; Radford, 2008; Tabach et al., 2020). The review of the literature shows various positions on the term "theory", however, in this research we will consider two of the most important that share aspects in common. Radford (2008) considers that a theory consists of the followings:

1. Basic principles, considered as a P system, which includes both implicit and explicit statements as the basis of its foundations.
2. A methodological process, M, consisting of data collection techniques as well as methods for analyzing data based on P.
3. A new set of research questions, Q, emerging from data analysis to provide a new interpretation of the study phenomenon and delve into the principles, if necessary, even expand them.

According to Kidron and Bikner-Ahsbahs (2015), a methodological approach to research, whether theoretical or empirical, where different theories are linked to deepen and broaden the understanding of a given problem, corresponds to a study of theoretical networks. There are different methods and strategies to achieve a study of this type. Prediger et al. (2008) report that the range of strategies and methods for articulating theories ranges from completely ignoring other theoretical frameworks, at one extreme, to globally unifying different approaches at the other extreme. As intermediate strategies, there is a need to: make one's perspective understandable to other theoretical perspectives, understand other theoretical perspectives, compare, and contrast different approaches, coordinate and combine perspectives, and achieve local integration and synthesis (**Figure 1**).

In detail, *ignoring the theories* refers to the fact that the author recognizes that there are several theories to research in mathematics education, but they are isolated except for one (s), which is the one that is (are) going to be used. The *understanding of theories* is a precondition or prerequisite before connecting them and it is that researchers must foster empathy in order to help colleagues understand the principles, methodologies or methods and paradigmatic questions of their theory and, their turn, understand that of the other theories. Next, *comparison and contrast* are a pair of strategies that do not differ substantially, for example, when promoting comparison, differences and similarities are sought in a general way to perceive theoretical components, while contrasting is the extraction of typical differences (Bikner-Ahsbahs & Prediger, 2010, 2014).

The pair of *coordination and combination strategies* refer to a deeper understanding of a research phenomenon or empirical episode (Bikner-Ahsbahs & Prediger, 2010). Particularly, the combining "is done when the theories are juxtaposed leading for example to complementary views. By coordinating, the connection between theories becomes tighter while common frameworks and methodologies for research can be built" (Kidron & Bikner-Ahsbahs, 2015, p. 225).

According to Bikner-Ahsbahs and Prediger (2014), the pair of local *integration and synthesis strategies* focus on structure a theoretical framework based on the coordination of other frameworks or set of small theoretical approaches. It should be noted that there is a degree of symmetry between the articulated theories, for example, when a new theory arises from two or more, it is called a synthesis because there was stability and deep relationship between its theoretical or conceptual and methodological tools.

However, in most of the networking of theories carried out in mathematics education, the degree of symmetry is not high because only some theoretical constructs of a theory A (specific) are integrated with the constructs of a more elaborated theory B (general) or drawn from another theoretical articulation. This integration process is not a total unification; therefore, it is understood as a local integration of two or more theories.

Just as research has been carried out on networking or theoretical connections, there have also been works focused on mathematical connections, which is a relevant topic in mathematics education because the establishment of connections contributes to the understanding of mathematical concepts (De Gamboa et al., 2020, 2021). Most of the research work on establishing connections has been carried out on key calculus concepts such as the derivative (Rodríguez-Nieto et al., 2021a, 2021b, 2021c, 2022), the derivative and its relationship with the integral (García-García & Dolores-Flores, 2019, 2021), the rate of change (Dolores-Flores et al., 2019); and on functions, such as exponential and logarithmic function (Campo-Meneses & García-García, 2020; Campo-Meneses et al., 2021), quadratic function (Businskas, 2008). In addition, among others, connections between geometric concepts (Caviedes et al., 2019) and ethnomathematical connections in daily practices (Pabón-Navarro et al., 2022; Rodríguez-Nieto, 2021). Among other results of these investigations, it is worth noting, in the case of the derivative, that both students, pre-service and in-service teachers have difficulty understanding it because they have difficulties establishing multiple connections between its partial meanings and between its representations of this concept (Badillo et al., 2011; Borji et al., 2018; Fuentealba et al., 2018a, 2018b; Yavuz-Mumcu, 2018).

In summary, the reviewed research on "networking" shows the diversity of networks between different theoretical approaches to analyze phenomena of interest for the teaching and learning of mathematics, using different specific mathematical objects such as the derivative and its representations, functions, the rate of change in application problems such as reflection context or topics such as Cartesian graphs, among others (Font et al., 2016; Pino-Fan et al., 2017). In particular, the Theory of mathematical connections (García-García & Dolores-Flores, 2019, 2021; Rodríguez-Nieto et al., 2022) recognizes that there is a need for this theory to be complemented to better understand the connections, especially about the derivative. In this sense, after analyzing the different theories involved in theoretical networks, we identify that the onto-semiotic approach (OSA) is a theoretical approach that articulates theories and, in addition, considers the notion of mathematical connection to be important, for which it would be adequate to complement each other and coordinate with the theory of connections.

Therefore, the research aim is, first, to review the literature on research focused on the articulation of theories with different mathematical concepts, and second, illustrate what a networking of theories refers to, presenting a synthesis of a networking between the extended theory of connections (ETC) and the OSA was deepened using the study of the mathematical connections made by a university student on the derivative concept as a context for reflection.

Now, why is this research important? Because the works interested in the review of the literature focus on: understanding the concept of a fraction (Arenas-Peñaloza & Rodríguez-Vásquez, 2021), how the COVID-19 pandemic has influenced (favorable or not) the educational practices and adaptations of science teachers (Lucena Rodgríguez et al., 2021), bibliographic reviews of the bibliometric type on the state of research on a topic investigated in mathematics education (Julius et al., 2021), the evaluation of the mathematical modeling of pre-service mathematics teachers (Hidayat et al., 2022), importance of sustainable development to improve people's quality of life (Husamah et al., 2022). Likewise, the interest of Sibgatullin et al. (2021) for reviewing research on algebraic thinking given that teachers need to develop this thinking with non-routine tasks favoring multiple representations. Another literature review focused on the "*inverted classroom*" as a diversified methodological strategy that has yielded effective results in the teaching and learning processes of mathematics (Fung et al., 2021).

In the study by Ukobizaba et al. (2021) reviewed the literature about problem-solving and main directions in mathematics education field that assume it as fundamental. In turn, these authors reveal that little is known about the implemented assessment strategies that have a fundamental contribution to the development of problem-solving mathematical skills and competencies in children from kindergarten to middle school. However, on networking of theories (according to what is recognized from the literature) no research has been carried out where the current state of this research agenda is evidenced and where, in addition, the trend of networking between general approaches to the analysis of mathematical activity and specific approaches that only analyze one aspect of mathematical activity of students or teachers.

METHODOLOGY

This research was carried out under a qualitative methodology focused on a bibliographic review and treatment of information on scientific topics as proposed in Gómez-Luna et al. (2014), which allowed the search, organization and analysis of the documentation related to the networking of theories developed in mathematics education and the case of mathematical connections, facilitating the acquisition of information available in different databases (articles, books, book chapters, etc.), the identification of the most relevant authors, the quantity of publications per year, the most relevant areas of work and future trends in the topic of interest.

The methodology demands considering the definition of a research problem and topic, since this shows clarity about the needs of the researcher on the bibliographic search, in such a way that the scenario for said activity is broadened. Subsequently, the phases must be considered:

1. search for information,
2. organization of information, and
3. analysis of information.

Specifically, the problem from which this research is based is the need to understand the principles, methodologies or methods and paradigmatic questions between two or more theories to enrich the analysis of a certain aspect of the instruction and learning process (connections in mathematics). Hence, the theme for the bibliographic review has been defined as the analysis of the articulation of theories developed in mathematics education field with respect to mathematical connections. Next, the three phases for the bibliographic review are described (Gómez-Luna et al., 2014).

Phase 1: Search for Information

This phase consists of a structured and professional search for bibliography that is directly related to the topic to be investigated. To do this, reliable documents were selected based on norms or standards at the national or international level, that is, obtaining informative material such as books, articles, theses, among others, in accordance with the subject and belonging to databases such as *Conricyt, Scopus, Eric, Elsevier, Google Scholar*, etc. (**Figures 2** and **3**).

In addition, search equations (expressions consisting of keywords such as mathematical connections, understanding, derivative concept difficulties, and networking of theories) were used to not only define the research domain but also study semantic patterns and citation, and identify the cognitive structure that determines the main research advances developed in the world regarding the subject of study.

In this first phase, 17 scientific articles, two books, 15 book chapters and four proceedings documents were identified. 17 investigations on mathematical connections were recognized, eight from the onto-semiotic approach and 20 that address the importance and problems about the derivative.

This phase is linked to phase 2 of the method (Gómez-Luna et al., 2014), to select the documents used

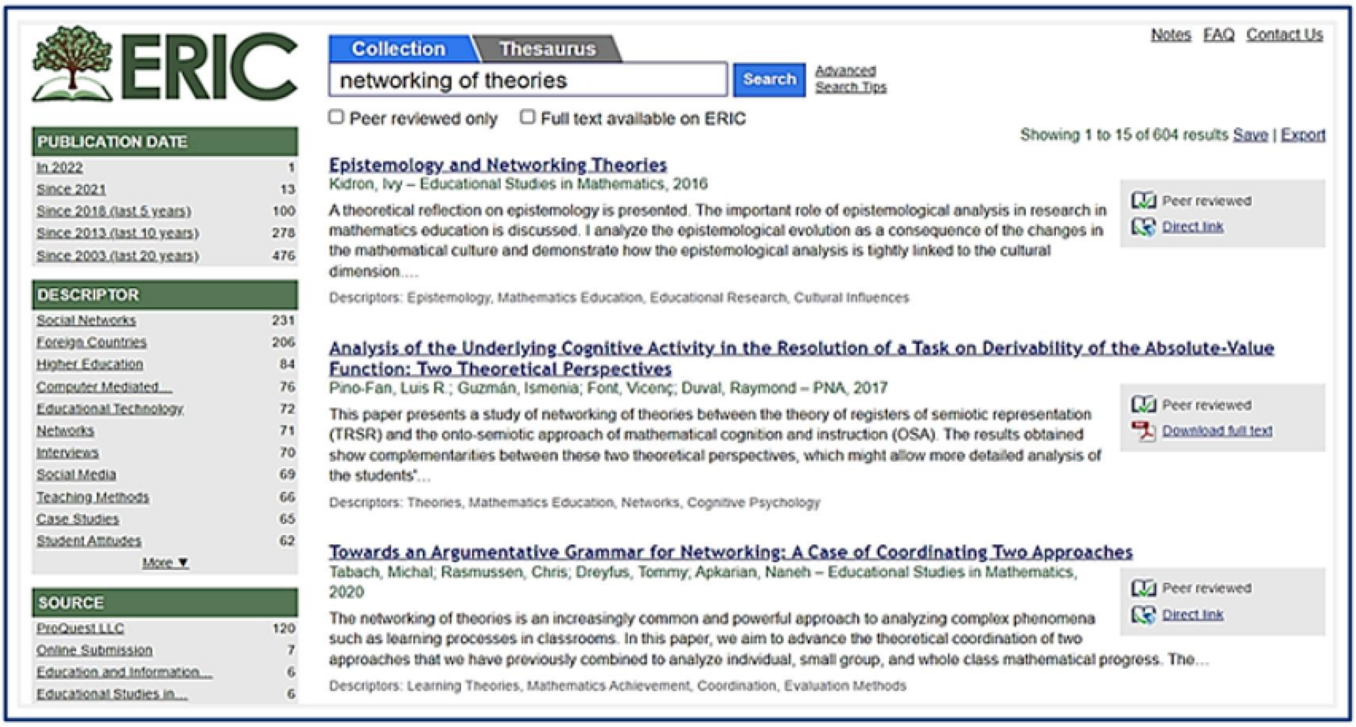

Figure 2. Search for research on networking (Own elaboration based on information from ERIC)

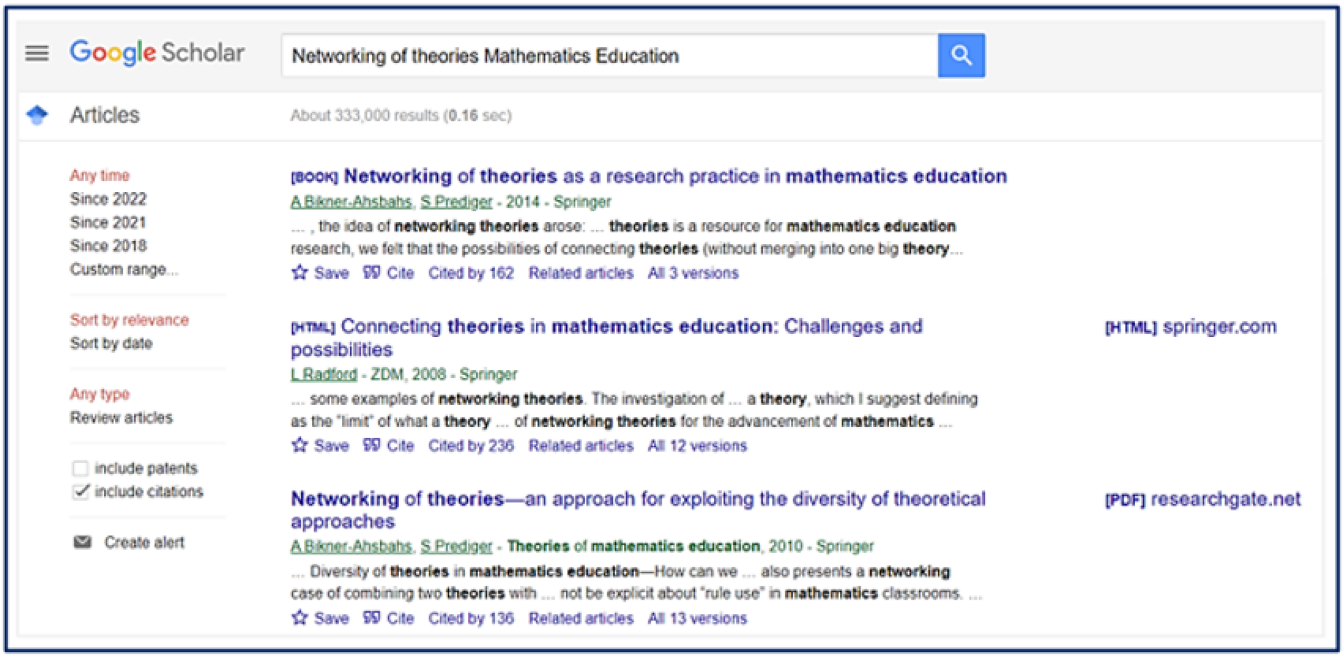

Figure 3. Search for research on networking (Own elaboration based on information from Google Scholar)

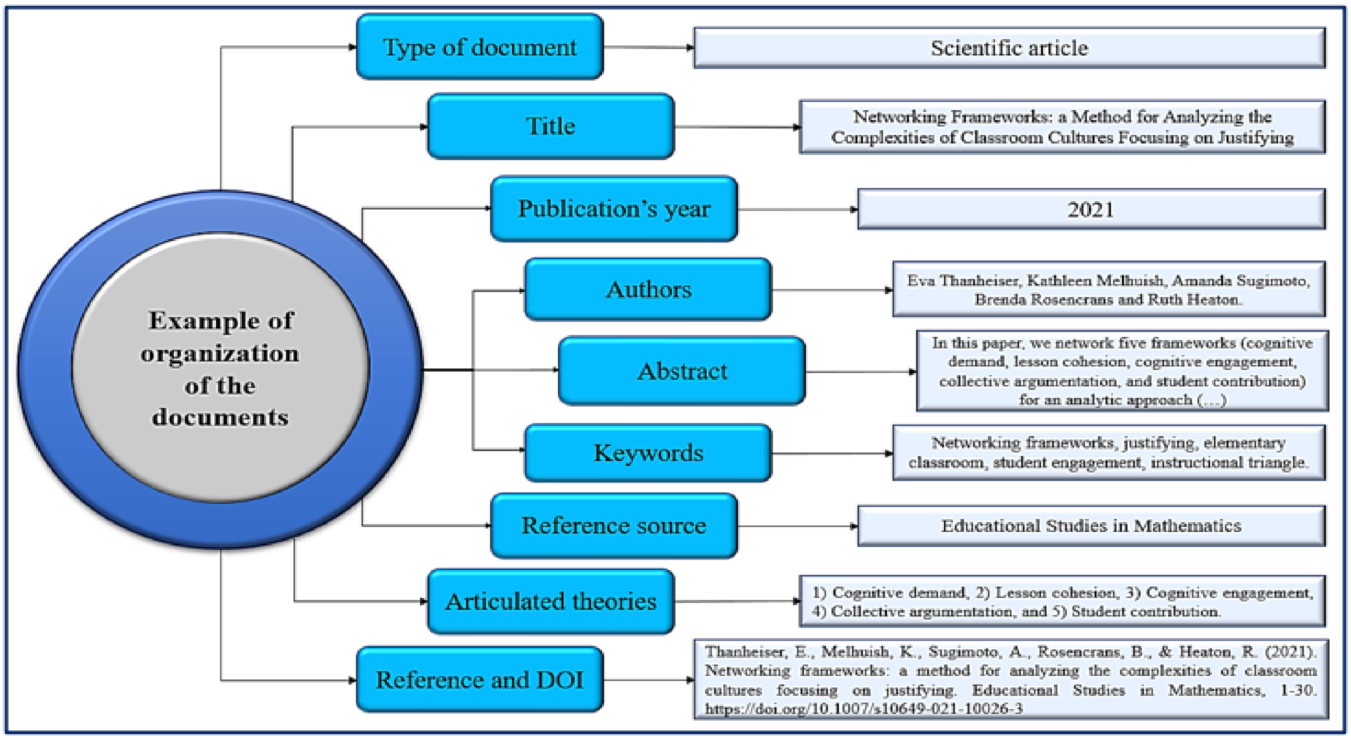

Figure 4. Example of organization of the documents found in phase 1 (Own elaboration)

in the research, based on establishing selection criteria and analysis categories among the documents, as shown below.

Phase 2: Organization of Information

This phase consists of systematically organizing the documentation found in the first phase. Therefore, a detailed organization was carried out, based on tables using Word and diagrams, in which data such as the name of the document, title, type of article, year of publication, authors, abstract, keywords, etc. (**Figure 4**).

In this phase, a timeline on the development of research related to the articulation of theories was identified, and for phase 3, the criteria for selecting scientific articles related to:

1. articulation of theories with different mathematical concepts was established and
2. articulation between the ETC and the OSA.

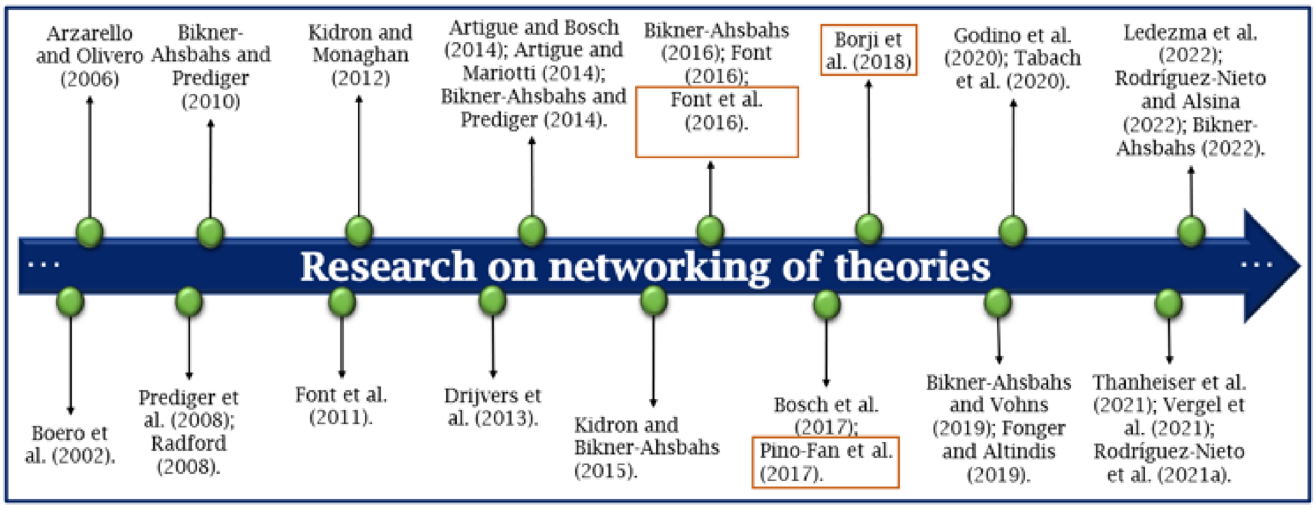

Figure 5. Scheme of the timeline about the networking of theories (Own elaboration)

Phase 3: Data Analysis

This phase was carried out in parallel with the previous phases since it was a constant and iterative process. The key aspects of the research were read, starting with the abstract and conclusions. This made it possible to identify the link between the frameworks they used and their impact on mathematics education.

ANALYSIS AND RESULTS

Current Systematic Review of Research on Networking of Theories

Figure 5 shows some of the research on the articulation of theories carried out with different contexts of reflection or mathematical concepts.

In mathematics education research field, interest in the integration processes of different theories is due to two complexities, one is that of mathematical objects and the other is their teaching and learning (Boero et al., 2002). Another motivation to articulate theories is because the data collected in an investigation becomes difficult to analyze with a single theory (Arzarello & Olivero, 2006; Font, 2016).

Radford (2008) proposed that theoretical networks consist of differentiation and complementarity based on a tripartite vision referring to theoretical principles, methodologies, and research questions, which have emerged and are necessary to strengthen collaborations between researchers at an international level and generate knowledge structures both educational and social, political, cultural, among others. At the same time, the potential of the theoretical connections that arise from the limitations that a theory A can present and the complementarity that a theory B can give it and vice versa, or concordances between the two theories A and B to analyze phenomena, is manifested.

Prediger et al. (2008) show a discussion on the variety of theories in mathematics education, systematizing the strategies and methods for the creation of theoretical networks, which, in fact, propose that networking strategies seek to reduce the number of unconnected theories while respecting their specificity, principles and methods. In addition, he proposed four pairs of strategies to articulate frameworks (understanding other theories-understanding the theories to use; contrast/compare; coordinate/combine; synthesize/local integration) contained between two poles, the non-relationship of ignoring other theoretical approach and the unification of theories globally (see **Figure 1**).

In this context, Bikner-Ahsbahs and Prediger (2010) characterized each pair of strategies as follows: the *first pair* of strategies refer to the understanding of the theories and interpretation and deep reading is very common when researchers are beginners, otherwise it is faster to find connections. The *second pair* of strategies invites comparison between theories from their principles to paradigmatic questions. The *third pair* of strategies allows us to search for what is most common among the theories that leads to coordination or complementarity. Finally, the *fourth pair* of strategies focuses on the balance between theories, reduction of tools (parts of the theory) to consolidate those that are articulated and work consistently. In fact, if it is desired to integrate theories locally, researchers must carefully recognize the theories to be used in order to apply the strategies and make decisions that lead to integration.

Font et al. (2011) linked the OSA, the APOS, and cognitive science of mathematics (CSM) for a better use of the notion of object since these theories assume similar elements about said notion. In addition, they concluded that the APOS and CSM theories only highlight partial aspects of the complexity and emergence of mathematical objects in mathematical practices, while from the OSA perspective, mathematical objects allow explaining two aspects: the first is the emergence of mathematical objects primary mathematicians of the configurations built through practices and, the second is the sample of the realistic vision of mathematics in the classroom. Especially in this research, the added value of the OSA is highlighted when it is articulated with the other theories, since they have focused on the fact that a solid understanding of mathematical objects from a theoretical perspective should be considered an important part of any research on the teaching and learning of mathematics.

For their part, Kidron and Monaghan (2012) worked on the complexity of the dialogue between theories, focusing on the difficulties and benefits, showing that networks of theories develop theories more through dialogues between researchers with visions based on different scientific cultures. Drijvers et al. (2013) analyzed an episode on the use of computer algebra for learning the concept of parameter with the theory of instrumental genesis (TIG) and the OSA. They found that the instrumental and onto-semiotic network approaches complement each other and one of the main results is that in TIG the artifact concept can be similarly worked from the OSA view as a primary mathematical object contained in mathematical practice connected to other objects. They concluded that theoretical advances can indeed benefit as a network of theoretical activities. In addition, these two investigations share the vision that networking of theories offers the possibility of identifying limitations and possibilities for the development of theories and how one can affect the other. In fact, the researchers who connected OSA with TIG are from different cultures and research schools (in order, the Netherlands, Spain, and France).

Next, in the development of the ReMath project, Artigue and Mariotti (2014) after clarifying the different theoretical perspectives and networking, presented their methodological constructs based on the project, who managed to identify connections between theories and when they complement each other, likewise, managed to identify the limits of the theoretical constructs between different theories and progressively build a theoretical framework on semiotic representations with an emphasis on interpreting an investigation of praxeologies. It is important to mention that the research was the product of a team of researchers from different cultures who shared the same cultures in deep dialogue.

In this context, Bikner-Ahsbahs and Prediger (2014) published a book referred to Networking of theories as a research practice developed in mathematics education field, where the starting points are shared to address the diversity of theories, among which stand out: the approach of action, communication and production (Arzarello & Sabena, 2014), the theory of didactic situations (Artigue et al., 2014), the anthropological theory of the didactic (Bosch & Gascón, 2014), abstraction and context (Dreyfus & Kidron, 2014), the theory of dense situations of interests (Bikner-Ahsbahs & Halverscheid, 2014). Also, case studies on theoretical networks were developed (Bikner-Ahsbahs et al., 2014; Dreyfus et al., 2014; Kidron et al., 2014; Sabena et al., 2014; Prediger & Bikner-Ahsbahs, 2014). In particular, Artigue and Bosch (2014) extended the notion of praxeology to research practices, which, from the beginning, had been introduced to model mathematical and didactic activities.

The investigations reported in the previous paragraph have their origins in the book by Bikner-Ahsbahs and Prediger (2014), which was motivated by the research question: How can we deal with the diversity of theories developed in mathematics education? Thus, generating multiple perspectives for the interaction between theories, such as understanding, comparing-contrasting, coordinating, and integrating locally between them. In addition, we highlight that the authors have stated that connecting theories is a challenging but fruitful research process, especially, they considered the dialogues with five theories finding that these works are a good resource for mathematics education that generate questions at a theoretical, methodological, and practical level. However, it is important to extend the theoretical networks since there are more approaches that require attention for their own and articulated development to carry out detailed analyzes of mathematical activity.

With the aim of promoting research through theoretical interconnections, Kidron and Bikner-Ahsbahs (2015) mentioned that in the work on networking of theories, the group of researchers must be supportive and empathic when understanding the theories, for example, if there are two researchers and they will articulate two theories and each one dominates one of the theories, researcher A must communicate their ideas about their theory to researcher B and vice versa, in order to help the colleague to understand the ideas, principles, methodologies and paradigmatic questions of their theory. Likewise, they emphasized the evolution of the networks of theories that has been a point of discussion in different editions of the CERME congress. In addition, they proposed the strategies to connect theories and the different cases:

1. networking between two theories,
2. articulation of 3 theories and complementarities,
3. clarify the role of various concepts in theories through the construction of networks, and
4. ReMath example of a multi-layout project.

Bikner-Ahsbahs (2016) recognizes that networking of theory is strengthened from the different theories, in addition to interpreting not only the practical aspect under a specific approach but also taking advantage of the interpretation made of empirical situations. For these authors, networking means building theoretical relationships to better address the complexity of the phenomena and not making simplistic interpretations.

Font (2016) states that in research on didactics of mathematics there is a need to use theoretical frameworks because there is a consensus that a research work should follow, through some key steps such as

1. the formulation of the research question,
2. the selection of a framework with the possibilities of coordinating the question with the selected framework and coherence with the methodological decisions,
3. operation of the theoretical framework, and
4. application of research techniques.

Font (2016) affirms that the diversity of frameworks can be convenient since each theory develops a partial aspect, but the results of the investigations that address the same research problem with different theories, most of the time are disparate, due to the complexity different languages and fundamentals. For this reason, it is necessary to compare, coordinate and combine theories for a comprehensive framework with theoretical and methodological tools to address a research problem.

Font et al. (2016) articulated the APOS theory with the OSA to contrast and compare the conceptualization of the notion of object from both theoretical perspectives, that is, first proposed a genetic decomposition of the concept of derivative and then analyzed it considering the principles both theoretical and methodological of OSA. Likewise, they made confrontations between the notions of action and practice, processes and procedures, encapsulation and primary object, cognitive configuration, and scheme, thematization and second level of emergence of a mathematical object. One of the authors' conclusions is that basic local coordination is provided through the articulation of the theories, to understand and interpret difficulties related to learning concepts, especially the derivative. It should be noted that this research is a clear example that these theories complement each other to analyze a particular mathematical object. Bosch et al. (2017) established dialogues between the anthropological theory of the didactic and the APOS theory to analyze the notion of praxeology.

Pino-Fan et al. (2017) articulated the theory of registers of semiotic representation (TRSR) with the OSA to analyze the mathematical activity associated with the resolution of problems related to derivability of the absolute value function. The authors conclude that the notions of the TRSR are important to understand the cognitive activity necessary to solve a mathematical task, and the OSA allows to analyze the cognitive activity of the subject that shows mathematical objects that intervene in the processes of treatments and conversions between the representation's registers. In addition, the analysis with the OSA "complements the analysis carried out using the TRSR tools, since with the configuration tools of objects and processes and semiotic function (SF), the contents of the representations become explicit and are used as part of said cognitive activity" (Pino-Fan et al., 2017, p. 121). It is reflected that, just as the TRSR is a semiotic approach, the OSA also assumes fundamental the use of various representations (verbal, symbolic, graphic, algebraic, numerical, etc.), however, unlike the cognitive approach (TRSR) that considers representations basically from a representational point of view (something for something), the OSA emphasizes its instrumental perspective (what can be done with them) (Font, 2016).

Bikner-Ahsbahs and Vohns (2019) present the articulation between the theory of the semiotic vision of mathematics, which, based on the use of signs and a semiotic game, explains the dynamics of doing mathematics to analyze schematic reasoning; and the theory of learning activity applied to mathematics to develop students' skills in doing mathematics considering learning objectives to be achieved. It is concluded that the theoretical networks of German precursors present a meta-research program that emphasizes how it improves problem-solving in the field of mathematics. Fonger and Altindis (2019) integrated the theory of quantitative reasoning and the theory of multiple representations (constructivist and semiotic lenses on cognition) with the aim of conducting empirical research on the study of meaningful understanding of function. The authors affirm that they were based on these theories because they allow students to learn from the creation, interpretation, and connection between different mathematical representations from a covariational reasoning perspective to favor students' understanding of functions. Borji et al. (2019) combined APOS and OSA theory to study students' mathematical understanding of polar coordinates.

Godino et al. (2020) were motivated to explore the affinity and complementarity that exists between the Objectivation Theory and the OSA, where they found that both theories share theoretical and sociocultural principles about mathematics in the learning and teaching processes. In addition, they analyzed the dimensions: epistemological, ontological, and semiotic-cognitive with their respective implications in teaching, taking as a context of reflection a report of a Cartesian graph.

It is worth mentioning that, in the research by Bikner-Ahsbahs and Vohns (2019), it is pointed out that theoretical articulations are necessary because the research focuses on the description and interpretation of the teaching and learning processes of mathematics (pointing out in the problems-solving) or find results in diverse cultural groups, which is consistent with multi-theoretical lenses because "they can be much more useful to understand the complex nature of environments in the field" (p. 193). For this reason, in the studies by Borji et al. (2019), Fonger and Altindis (2019), and Godino et al. (2020), the theoretical articulations are used as a previously established theoretical foundation and that its applicability helps to analyze in more detail problematic situations such as the understanding of functions and polar coordinates. In fact, in Borji et al. (2019), the use of OSA and APOS is given with confidence because in two previous studies (Borji et al., 2018; Font et al., 2016) this articulated framework has allowed analyzing the notion of mathematical object and the derivative concept' understanding.

Tabach et al. (2020) consider that articulating theories is a powerful approach to know about the possibility of new knowledge and links between different theoretical approaches. These authors coordinated two theoretical approaches. After achieving the articulation, they used it to analyze and reflect on the argumentative grammar that reveals the implicit structure of levels of analysis of combination and coordination of theories. Vergel et al. (2021) compared the perspectives of the theory of objectivation and the OSA to theoretically analyze and reflect on the nature of algebra and the development of algebraic thinking, considering that from the first educational levels it is an important theme in mathematics education. It should be noted that this study shares theoretical and methodological views with the research by Godino et al. (2020), but they analyze different mathematical objects according to the problem identified.

Following the chronological order of research on networking, Thanheiser et al. (2021) coordinated five theoretical approaches (cognitive task demand, lesson cohesion, student contribution types, collective argumentation, and student cognitive engagement activity), with the purpose of presenting a holistic perspective of classroom culture engage students around the rationale and illustrate observable relationships between three important elements: teacher, students, and content (see articulation in **Figure 6**). This study (Thanheiser et al., 2021) is a powerful and important case of networking because its content looks in detail at how researchers made high-level efforts to connect five theoretical approaches. This could mean that multi-theoretical analyzes are relevant (Bikner-Ahsbahs & Vohns, 2019), but also one or more theories could help refine or improve conceptual and methodological tools of other theories (e.g., Bikner-Ahsbahs & Prediger, 2014; Kidron & Bikner-Ahsbahs, 2015; Ledezma et al., 2022; Rodríguez-Nieto et al., 2021a).

A triangle model for the coordination of the five frames and the connections between student, teacher, and content is presented in **Figure 6**.

According to Thanheiser et al. (2021), the analysis depends on the operation of the triangle centered on:

1. the relationship between the student and the content through types of student contribution,
2. the teacher and the content through the cohesion of the lesson,
3. the way in which teachers shape the content of students through cognitive demand,
4. students-students and teacher through collective argumentation, and
5. student-student and content through student cognitive engagement activity (p. 14).

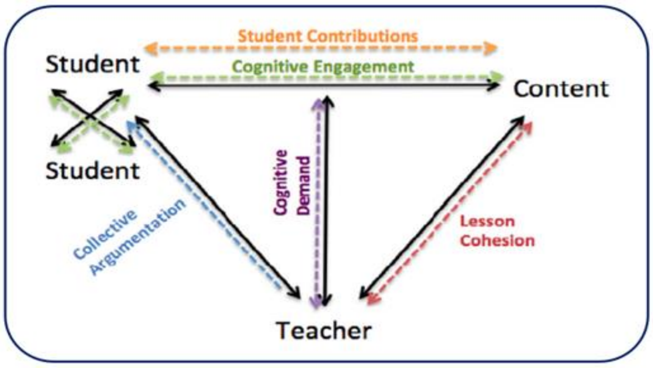

Figure 6. Networks between five frameworks to analyze high-quality math classrooms focused on motivating students to justify or argue (Thanheiser et al., 2021)

Currently, the theoretical articulation between the ETC and the OSA has been proposed (Rodríguez-Nieto et al., 2021a) with emphasis on the derivative, because despite the fact that in some investigations they articulated theories using the derivative as a reflection context (Borji et al., 2018; Pino-Fan et al., 2017), in their results they strongly state that students and some in-service teachers have difficulty understanding the derivative because they do not connect meanings or definitions and several representations of the derivative concept.

Ledezma et al. (2022) can be identified the network between OSA and the modeling cycle from a cognitive perspective. The first gives tools for mathematical activity analysis and the second gives an appreciation of the mathematical activity of modeling. Applied a model of a problem to pre-service mathematics teachers, which was analyzed with both OSA and modeling cycle from a cognitive perspective, in specific, with both analyzed the mathematical activity relative to the modeling process. They concluded that both frameworks (specific and general) complement each other to deeply analyze the mathematical modeling processes of a person. In fact, with the OSA tools it is possible to reveal the phases or the way in which the modeling cycle moves as a set of mathematical practices, primary processes/objects activated in the mathematical activity.

Rodríguez-Nieto and Alsina (2022) articulated STEAM, globalized approach and ethnomathematics the for the analysis of mathematical connections and found the following:

1. interdisciplinary connections that allow establishing relationships between knowledge from different subjects that feed each other,

2. intradisciplinary connections where mathematics is presented in its entirety, and not as separate knowledge, and
3. connections between the mathematics used by various cultural groups with the institutionalized mathematics found in the curricula and, simultaneously, globalized because they relate mathematics to the sociocultural context (an ethnomathematical view).

Bikner-Ahsbahs (2022) addressed the empirical research phenomenon of adaptive instruction through a case study on covariational reasoning emphasizing the importance of collective argumentation. In fact, this research shares a family resemblance with the articulation carried out in Tabach et al. (2020) and Thanheiser et al. (2021) for the indispensable use of collective argumentation promoted in the processes of participation in the learning and teaching of mathematics. We suggest that, in future research, other theories could be connected with argumentation, communication, connection, problem-solving, since these approaches permeate most of the experiences promoted in the classroom.

The main conclusion that we want to highlight from this literature review is the evolution that goes from networking between general theories towards networking between general theories and more specific theories (Ledezma et al., 2022; Vergel et al., 2021). In the next section we will delve into this type of networking, describing the articulation between the ETC and the OSA, taking as a context of reflection the establishment of connections in the study of the derivative. In particular, a synthesis of the networking between ETC and OSA carried out in Rodríguez-Nieto et al. (2021a). Furthermore, with the reported investigations, a great theoretical, methodological, and practical approach was visualized that invites and motivates other theoretical approaches to be used to analyze data, but not only individually, but to look at what is inside each theory, the possibilities that there is to make interconnections to improve the analyzes avoiding partiality, locality, and exercising the plurality of correct interpretations offered by a networking of theories.

Research Problems About Derivative's Understanding

Studies on the concept of derivative in Rodríguez-Nieto et al. (2021a) reveal that some pre-service teachers have difficulties in finding the equation of the tangent line, as a consequence of the inadequate meaning they have of the derivative. In other investigations whose main focus is not to investigate the establishment of mathematical connections on the derivative, but its understanding, it has been reported that many teachers and students have difficulty connecting various representations (graphic, verbal, numerical, symbolic) of the derivative and use formulas in a mechanized way (Amaya, 2020; Borji et al., 2018; De la Fuente & Deulofeu, 2022; Dolores-Flores & Ibáñez-Dolores, 2020; Font, 2000; Font et al., 2011; Fuentealba et al., 2018a, 2018b; Galindo-Illanes et al., 2022; Pino-Fan et al., 2017, 2018; Sánchez-Matamoros et al., 2015); and, also, connect derivative concept' meanings (Borji et al., 2018; Fuentealba et al., 2015, 2018a; Vargas et al., 2020).

In addition, university students have difficulties relating the increase and decrease of f and the sign of f' and f'' and they work more with graphical information and the derivative' punctual-analytic meaning and have difficulty drawing e interpret the graphs (Fuentealba et al., 2018a, 2018b; Ikram et al., 2020; Nemirovsky & Rubin, 1992). García-García and Dolores-Flores (2019) showed that pre-university students need the algebraic expression of the function to make a graphic representation of itself and its derivative, otherwise they do not graph. Nurwahyu et al. (2020) found that students have difficulties in problems-solving about derivatives because the answers are not directly related to a formula and proceed in a memorized way, which implies the disconnection between the meanings, the definition, and the formula. Feudel and Biehler (2020) argue that students have difficulty understanding the derivative in different application contexts because they have a little understanding of the rate concept.

In another view of understanding, Rodríguez-Nieto et al. (2021b) state that quality level 0 mathematical connections (with mathematical errors or inconsistencies) are the causes for which students have difficulty understanding the derivative concept and quality level 2 mathematical connections (based on mathematically consistent arguments) are a guarantee of understanding by students or teachers. Haghjoo and Reyhani (2021) concluded that university students have inadequate understanding of the derivative concept in contexts (numerical, physical, verbal and graphic), for example, they state that the inconsistency between interpretations of slope in mathematic classes (involving difference quotient limit, velocity, rate of change, etc.) and interpretations of rate of change in engineering classes may cause some problems for students in understanding the derivative, additionally, they share similar results with previous research in two directions:

1. As mentioned by Borji et al. (2018), Pino-Fan et al. (2015, 2017, 2018), and Rodríguez-Nieto et al. (2021d) that students did not establish relationships or connections between different derivative representations or had insufficient relationships, which means that they had little conceptual understanding and more instrumental understanding.
2. Coincide with Artigue (1995), Fuentealba et al. (2015), Nurwahyu et al. (2020), and Oehrtman et al. (2008), because they converge in that students use formulas in a memorized way in problems-

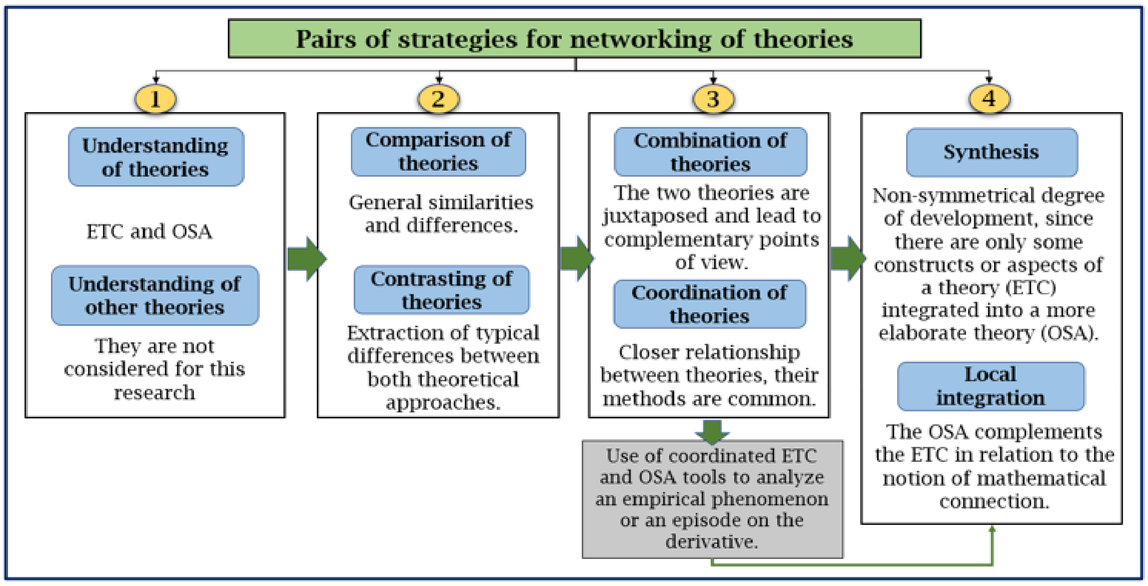

Figure 7. Path followed for the theoretical articulation (Own elaboration based on Bikner-Ahsbahs and Prediger (2010) and Prediger et al. (2008))

solving, ignoring the mathematical connections between the partial meanings of the derivative.

Networking Between ETC and OSA as an Example

In Rodríguez-Nieto et al. (2021a), the four pairs of strategies were followed to articulate theories (Bikner-Ahsbahs & Prediger, 2010; Radford, 2008), which, among other aspects, allows a better understanding of the cases or phenomena investigated (**Figure 1-Figure 7**).

Understanding of theories

Extended theory of mathematical connections: In the ETC, a mathematical connection is defined from a cognitive process perspective where a person establishes links or relationships between mathematical objects such as meanings, concepts, characteristics, representations, procedures with each other and with real life situations that may be framed in other scientific disciplines (García-García & Dolores-Flores, 2018). Mathematical connections have been classified into two groups (**Figure 8**): intra-mathematical connections that are made between representations, concepts, definitions, arguments, theorems among themselves (Dolores-Flores & García-García, 2017), and extra-mathematical connections are established by relating a mathematical concept (or model) with a contextualized situation or not mathematics or vice versa (Dolores-Flores & García-García, 2017).

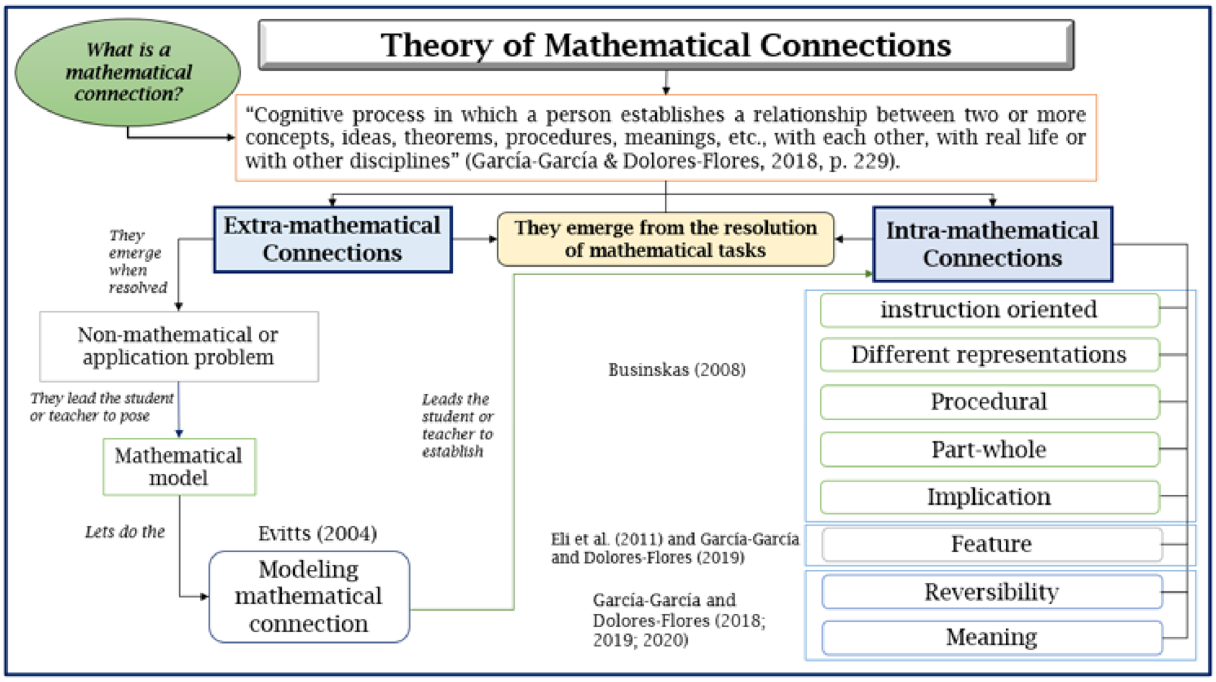

Figure 8. Schematization of the ETC (information adopted from García-García and Dolores-Flores (2019))

De Gamboa et al. (2020) state that extra-mathematical connections are based on intra-mathematical connections and are important for students and in-service teachers in problems-solving in the classroom. Mathematical connections of the ETC is described below:

1. *Modeling*: They are relationships between mathematics and real life and are evidenced when the subject solves non-mathematical or application problems where he has to pose a mathematical model or expression (Evitts, 2004). For example, translating the information expressed in the statement of a problem (which requires finding the measure of the side of a box so that it has the greatest volume) to an initial mathematical expression that models the volume, $v = length * width * height$.
2. *Instruction-oriented*: Refers to the understanding and use of a mathematical concept D from two (or more) previous concepts B and C (which are related), required to be understood by a person. These connections type can be recognized in two forms:
   a. the relationship of a new topic with previous knowledge and
   b. the mathematical concepts, representations and procedures connected to each other are considered fundamental prerequisites that people must have to develop new content (Businskas, 2008). For example, when the in-service teacher tells the students that, to work on the derivative of a function, they must first remember the concepts of function and slope of a line.
3. *Different representations*: they are identified when the person represents mathematical objects using representations (equivalent and alternate). Equivalents refer to transformations of representations of the same register ($P(x) = x^3 - 3x^2 - 4x + 12$ is equivalent to $P(x) = (x-3)(x-2)(x+2)$ in the algebraic register). Alternates refer to representations of the same mathematical object where the register in which they were formed (initially) is changed (graphical-algebraic) (Businskas, 2008).
4. *Procedural*: they are identified when a student o teacher uses algorithms, rules, or formulas to mathematical problem-solving. This type of connection is of the form: C is a procedure to work with a concept D (Businskas, 2008). For example, when the person uses the general formula $\left(x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}\right)$ to find solution of a quadratic equation of the form: $ax^2 + bx + c = 0$.
5. *Part-whole*: it occurs when the logical relationships are made in the following two ways:
   a. The generalization relationship is of the form B and is a generalization of C and C is a particular case of B (Businskas, 2008) or
   b. The inclusion relation is given when a concept is contained in another. For example, the function $g(x) = x^3 - 4x^2 - 17x + 60$ is a particular case of the general mathematical expression $f(x) = ax^3 + bx^2 + cx + d$.
6. *Implication*: They are recognized when a concept *P* leads to another concept *Q* through a logical-relationship (*P*→*Q*) (Businskas, 2008). Let f be a function whose second derivative exists on an interval (a, b). a) If $f''(x) > 0$ on all (a, b) then $f(x)$ is concave up on (a, b). b) If $f''(x) < 0$ on all (a, b) then $f(x)$ is concave down on (a, b).
7. *Feature*: Is recognized when the person expresses characteristics of the mathematical concepts or identifies their properties in function of other concepts (that help define it) that make them similar or different to the other concepts (Eli et al., 2011). For example, when the person says that a right triangle is distinguished because it has an angle of 90°.
8. *Reversibility*: They occur when a person (student or teacher) starts from a concept B to obtain a concept C and reverse the process, starting from C until returning to B (García-García & Dolores-Flores, 2019). For example, the reversibility relationship that exists between the logarithmic and exponential functions, between the derivative and the integral, or when addition and subtraction operations are used.
9. *Meaning*: It is recognized when a person attributes a meaning to a concept, as well as the inclusion of those cases in which a person gives a definition that he has constructed through his experience (in the field of mathematics) for these concepts (García-García & Dolores-Flores, 2019). This connection is identified when the person relates different meanings attributed to a concept to problems-solving. For example, the derivative $f'(a)$ is the instantaneous rate of change of $y = f(x)$ with respect to x when $x = a$ (Stewart, 1999).
10. *Metaphorical*: It is identified when a person establishes a relationship or projection of the properties, characteristics, etc. starting from a known domain (related to a person's bodily experiences) to form another domain less known (in mathematical terms) (Rodríguez-Nieto et al., 2022). For example, this connection is activated when the person (student or teacher) mentions metaphorical expressions such as *a function is continuous if the graph can be sketched in a single stroke without lifting the pencil from the paper, that is, it makes use of a metaphorical type of connection and*

comes to light the conceptual metaphor: the graphical representation of a function is a path.

Onto-semiotic approach: It is an inclusive theoretical approach that emphasizes a person's mathematical knowledge and how mathematical instruction is carried out, which arose from the need to clarify, connect, and improve theoretical and methodological notions of several theories used in mathematics education field. From this approach, it is essential to describe mathematical activity from an institutional or personal perspective, which is modeled in terms of practices and configuration of primary objects and processes that are activated in said practices (Drijvers et al., 2013). For Godino and Batanero (1994) mathematical practice is understood as "any situation or expression (…) carried out by someone to solve mathematical problems, communicate the solution obtained to others, validate it or generalize it to other contexts and problems" (p. 334). And they include objects used in a broad sense to refer to any entity that, in some way, is involved in mathematical practice and can be identified as a unit (Font et al., 2013). Six primary objects are considered:

1. problem situations,
2. representations,
3. definitions,
4. propositions,
5. procedures, and
6. arguments.

These interconnected objects form the configuration of primary objects (Godino et al., 2019).

The primary objects that emerge in mathematical practice can do so in different ways, which are the result of the different ways of seeing, speaking, operating, etc., on the primary objects, which allows us to speak of primary personal or institutional objects, ostensive or non-ostensive, unitary or systemic, intensive or extensive, and of content or expression (Godino et al., 2007). Now, a configuration is a heterogeneous set or system of interrelated objects, which can be institutional (*epistemic*) or personal (*cognitive*) (Godino et al., 2019).

According to Godino et al. (2007), the set of primary objects emerges in mathematical activity through the activation of primary mathematical processes (communication, problem posing, definition, enunciation, procedures (algorithms) and argumentation) derived of the application of the process-product perspective to said primary objects, these precise ones occur together with those derived from applying the process-product duality to the five dualities mentioned above (institutional/personal, ostensive/non-ostensive, expression/content, extensive/intensive, and unitary/systemic): personalization-institutionalization; materialization-idealization; representation-meaning; synthesis-analysis; generalization-particularization (Font et al., 2013; Godino et al., 2007) (**Figure 9**).

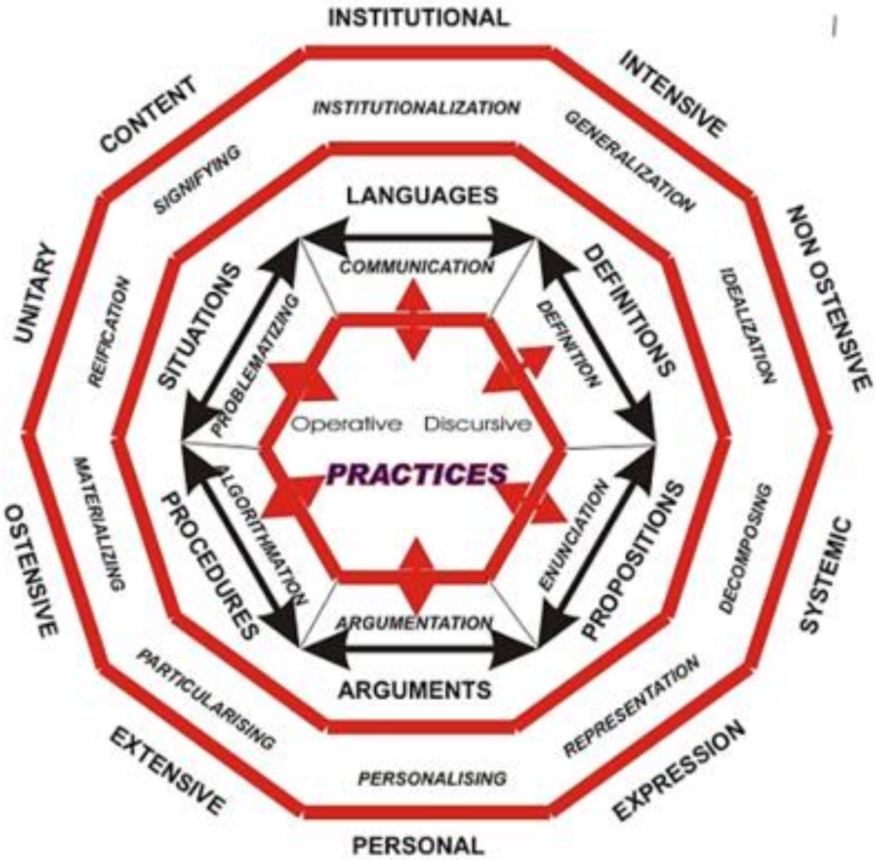

Figure 9. Schematization of mathematical knowledge from an onto-semiotic view (Adopted from Font and Contreras (2008))

Table 1. Comparison of theories

Aspects to compare	ETC	OSA
Principles	Connections are important for understanding (NCTM, 2000). Make connections as a fundamental principle of the teaching and learning of mathematics (ETC).	It assumes the connections in components and indicators of epistemic suitability criteria (richness of processes (modeling, argumentation, problem solving, connections)) and in the representativeness of complexity. Ecological suitability (interdisciplinary connections).
Mathematical connection vs. SF	Cognitive process of a person to relate meanings, concepts, definitions to each other and to everyday life.	Relationship between an antecedent (object 1) and a consequent (object 2), supported by a correspondence code.
Mathematical understanding	Mental process that is the result of having a well-connected network of representations, concepts, definitions to solve problems.	Competent use of mathematical objects competently in different practices. Activate a configuration of primary objects and establish appropriate SFs.
Methods	Thematic analysis	Analysis of mathematical activity
Research questions	What connections are made when studying a particular mathematical object? How do we generate a new typology of mathematical connections? It is necessary to build and validate a reference framework to study mathematical understanding based on connections.	Epistemological problem: How does mathematics arise and develop? Ontological problem: What is a mathematical object? Semiotic-cognitive problem: What does a mathematical object mean? Educational-instructional research problem: What is teaching? Learning optimization problem-Didactic suitability criteria.

Note. Own elaboration based on Rodríguez-Nieto et al. (2021a)

Another important component in OSA is the notion of SF that allows practices to be associated with the processes and objects activated and allows the construction of an operational notion of knowledge, mathematical understanding, meaning, and competence (Godino et al., 2007). An SF is a triadic relationship between an antecedent (expression/object) and a consequent (content/object) made by a person (person or institution) according to a certain criterion or correspondence code (Font, 2007). Semiotic functions (SFs) are inferred when mathematical activity is viewed from the expression/content duality. In Rodríguez-Nieto et al. (2021a) it is stated that the notion of SF (OSA) is more general than the notion of mathematical connection (ETC) since the connections are considered particular cases of SFs of a personal or institutional nature. In the ETC, the mathematical connection may or may not be true, revealing from the perspective of OSA that, when a subject makes a correct connection, it coincides with the institutional one, and when it is incorrect, it is of a personal nature.

Comparing and contrasting theories

The comparison was oriented towards the differences and similarities of each theory (**Table 1**).

The contrast was aimed at highlighting different elements, for example, the OSA emphasizes the suitability criteria where connections are considered, but not all of them are considered in the ETC, since it only considers the criterion of making and promoting mathematical connections in the students.

Coordination and combination of theories

The mathematical connection and the SF can be considered similar, since the ETC defines the mathematical connection as an SF. The notion of SF is more general than the notion of mathematical connection as conceptualized in ETC since connections are understood as particular cases of SFs (institutional or personal).

Now, in the thematic analysis of the ETC, carried out in accordance with the proposal of Braun and Clarke (2006), a typology of mathematical connections made a priori is used and they are called themes. While the analysis developed with the OSA uses various tools (practices, processes, mathematical objects, SFs, etc.) (Godino et al., 2007), although the level of detail of the analysis methods (thematic analysis and mathematical activity analysis) is different, both are complementary, and their functionality has allowed for a more detailed exploration of mathematical connections (**Table 2**).

The research questions formulated in the ETC can be located in the problem areas of the OSA or in the intersection of some of them, especially the cognitive-semiotic. Mathematical understanding is the cognitive process that emerges in mathematical activity through which a subject X in his mathematical practice relates primary objects O (*problem situations, concepts/definitions, propositions/properties, representations, arguments, procedures*) among themselves, with other subjects or with everyday life, relationships established through SFs (*grouped in mathematical connections of the ETC*).

Table 2. Analysis method based on the articulation between ETC and OSA

Phases	Description
1 Transcript of the interviews	It is transcribed so that the researcher becomes familiar with the data collected. In addition, the written productions are reviewed, which, in fact, is the first phase of thematic analysis of ETC.
2 Temporal narrative	It is explained mathematically, what the subject does when solving the task. In it are the practices carried out by the student or the teacher and some primary objects that play the role of the protagonists of the temporal narrative.
3 Mathematical practices	Based on the narrative, mathematical practices are enunciated, understood as sequenced actions that a person performs when a mathematical problem-solving (Godino et al., 2007).
4 Cognitive configuration	Set of primary objects that a person (teacher or student) activates in the mathematical practices you develop in a specific problem-solving.
5 Semiotic functions	They are established among the primary objects of the cognitive configuration.

Note. Own elaboration based on Rodríguez-Nieto et al. (2021a)

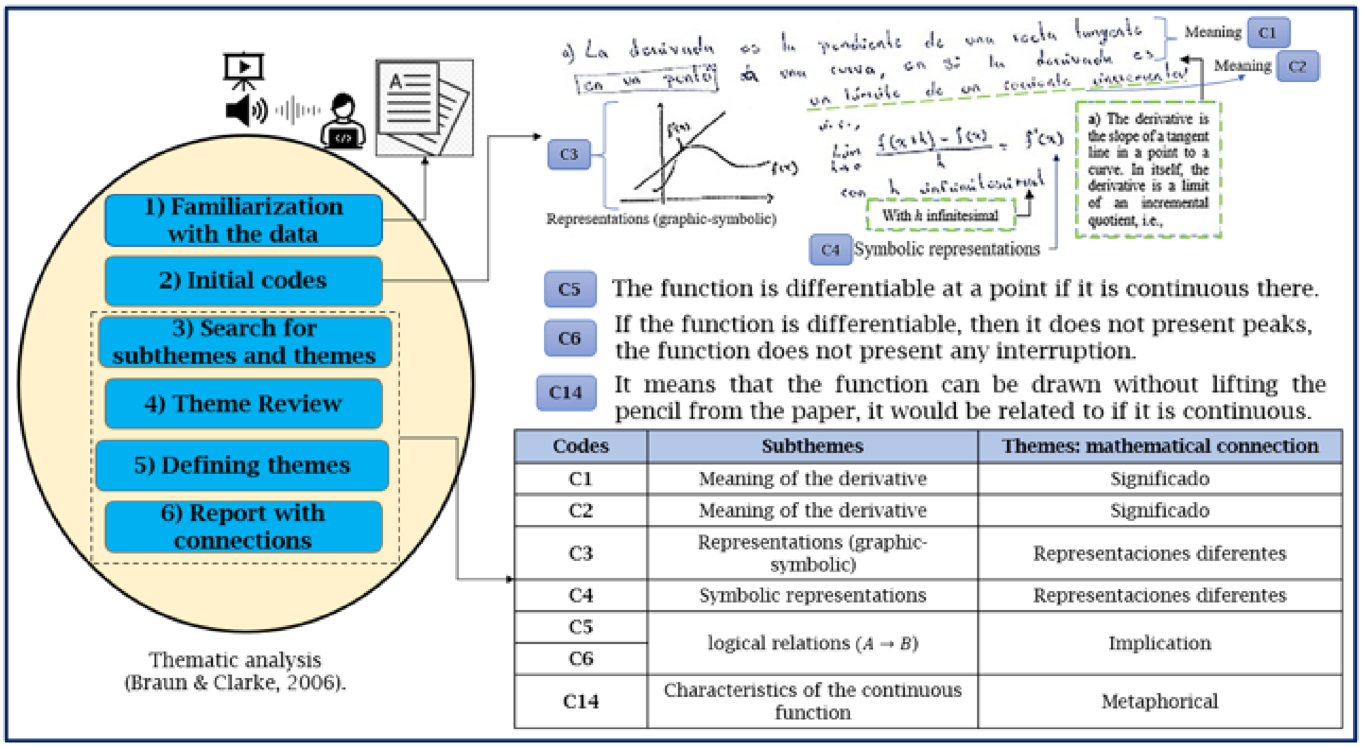

Figure 10. Summary of five phases of thematic analysis (Own elaboration based on Rodríguez-Nieto et al. (2021a))

Furthermore, understanding a mathematical concept allows the subject to use it competently in problem-solving.

In relation to the comparison, it is stated that the theories are juxtaposed and lead to complementary points of view of analysis. While the coordination is characterized by the relationship between both theoretical approaches allows to build a common theoretical-methodological framework for research on mathematical connections. In addition, the perspective of Bikner-Ahsbahs and Prediger (2010) is assumed when they mention that, by coordinating and combining theories, the articulation of complete theories should not necessarily be directed, but rather some analytical tools of both coordinated theoretical frameworks can be used for the analysis of empirical phenomena. Therefore, an analysis of the connections that a mathematics undergraduate student makes when solving a task is shown, for example:

1. What does the derivative mean? Explain your answer and if possible, give examples.
2. When is a function differentiable at a point? Argue your answer.

Summary of the thematic analysis: Figure 10 presents the first five phases of the thematic analysis used in the ETC, beginning with the transcription of the interviews organized with the written productions.

Subsequently, Figure 11 shows all the mathematical connections established by the university student.

Summary of the analysis with the OSA tools: After presenting the analysis of the connections in a summarized way from the ETC, the use of the OSA tools to analyze connections will be presented (Figure 12).

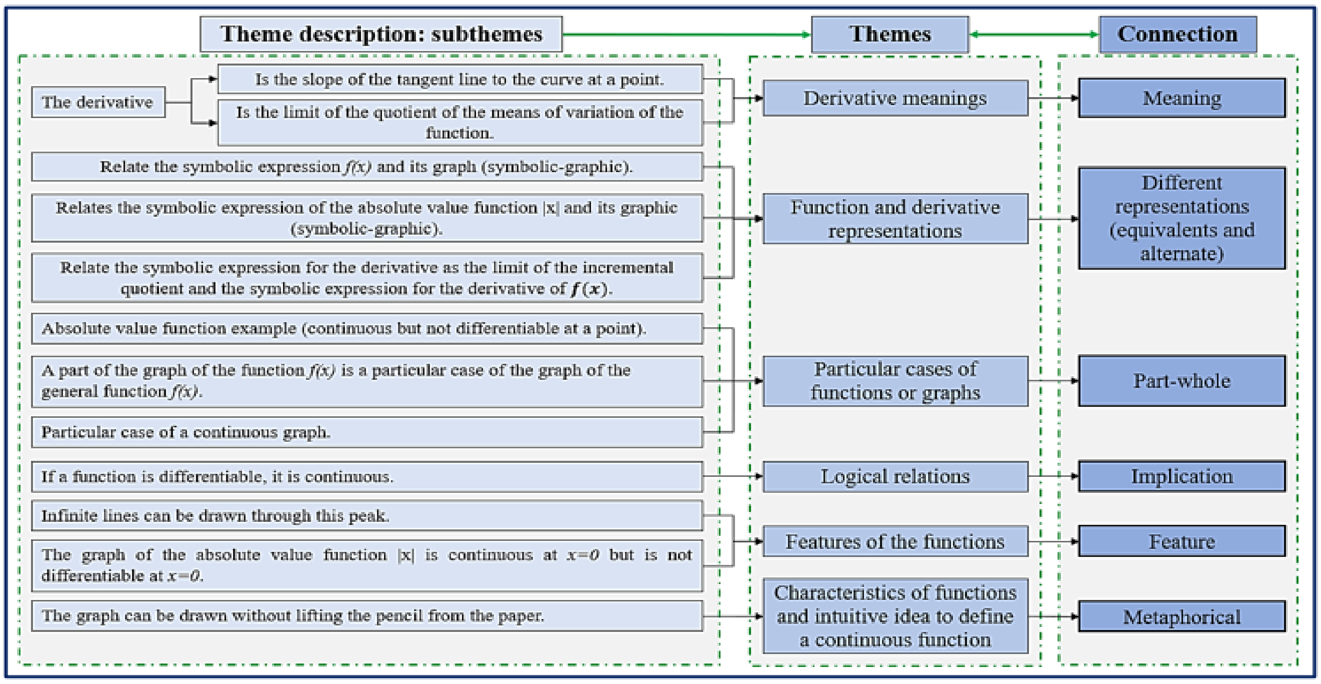

Figure 11. Mathematical connections from the ETC (Own elaboration based on Rodríguez-Nieto et al. (2021a))

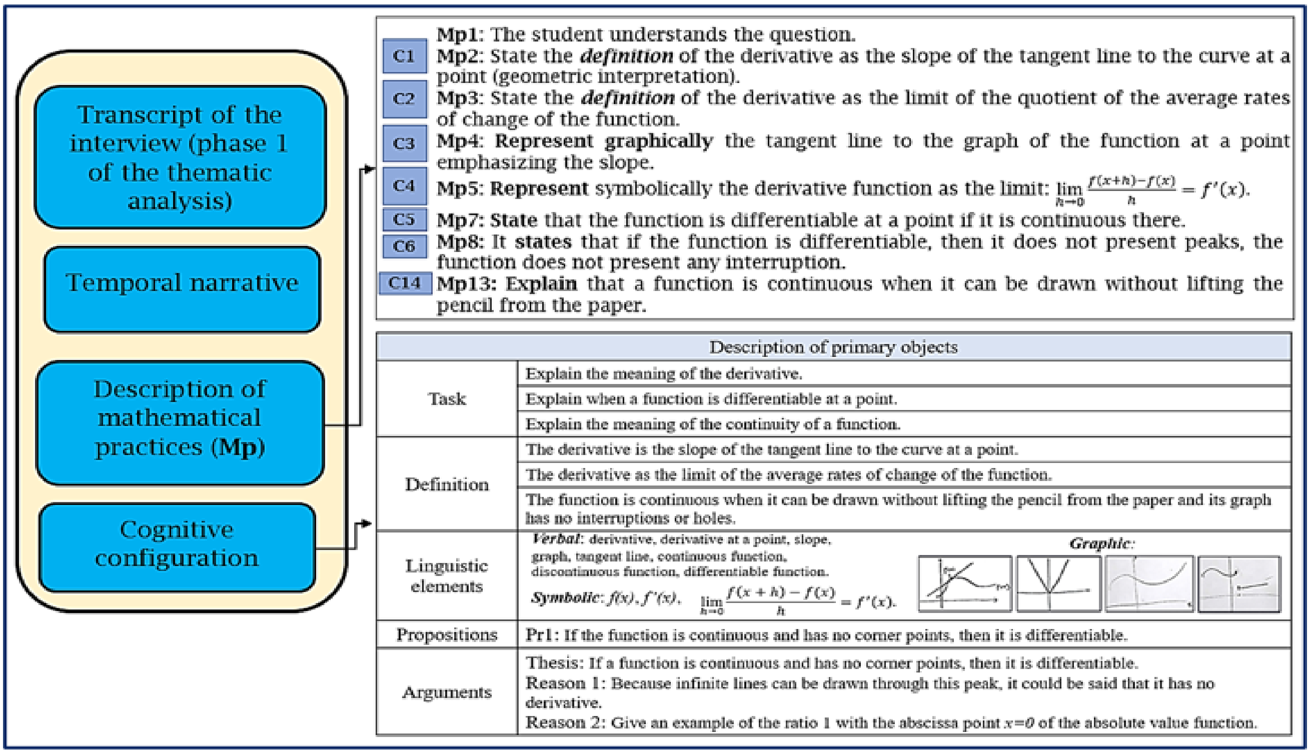

Figure 12. Some mathematical practices and cognitive configuration of objects (Own elaboration based on Rodríguez-Nieto et al. (2021a))

Finally, in this pair of strategies the SFs are presented (based on the information in **Figure 11** and **12**) that relate to the primary objects (**Figure 13**), which appear as the student was solving the task.

Synthesis and local integration of theories

Based on the perspective of Rodríguez-Nieto et al. (2021a) in **Table 3** the synthesis and local integration between the theories is presented and a portion of the analysis of the mathematical activity developed by the university student with OSA tools that refer to the first four columns is presented. Likewise, we state that the fifth column summarizes the analysis following the ETC, carried out before.

In addition, **Table 3** shows that both methods complement each other and in an articulated way allow a more detailed and in-depth analysis using the OSA tools, and in this way the mathematical connection is

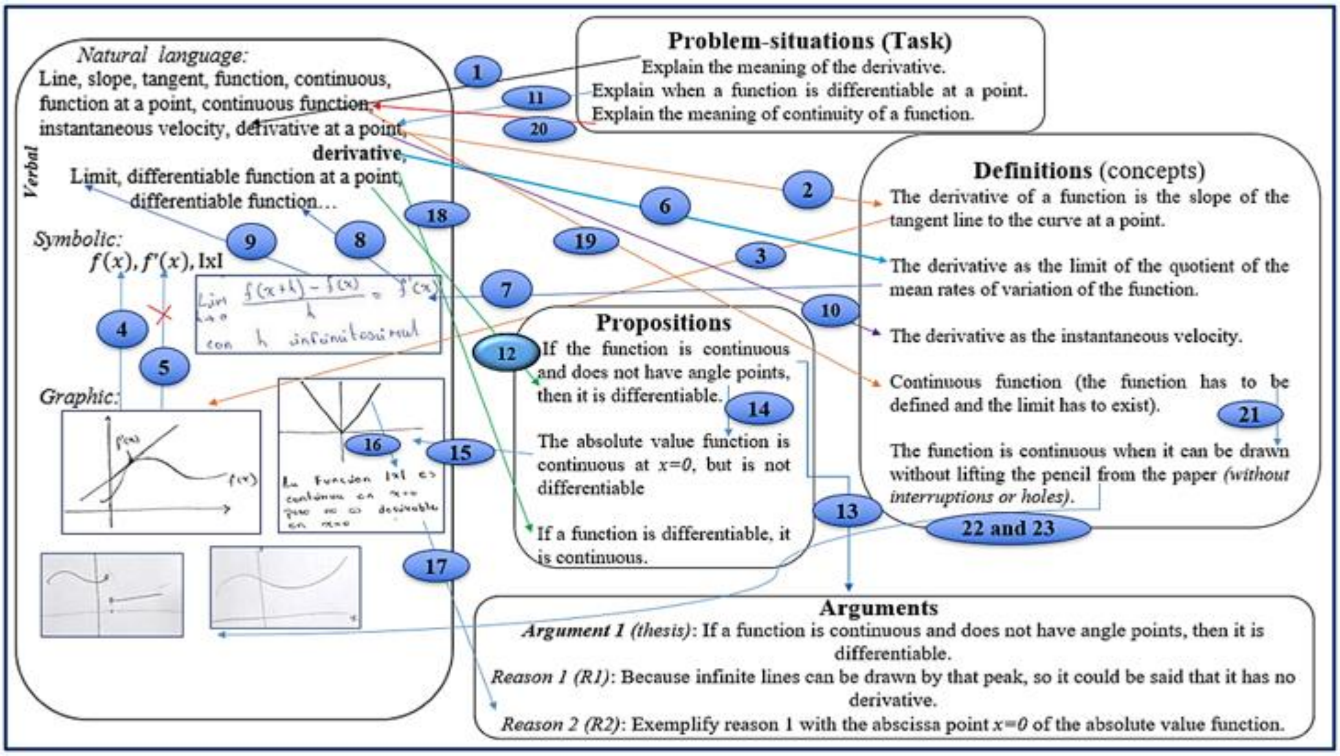

Figure 13. SFs' system (Adopted from Rodríguez-Nieto et al. (2021a))

Table 3. Detailed analysis of mathematical activity through local integration between ETC and OSA

Practices	Processes	Primary objects	SF	Mathematical connections
Mp1	-Signification/understanding (the act or process of signifying by signs or other symbolic means) -Problematization (take on the task as a challenge to be solved)	T1: Explain the meaning of the derivative concept.	SF1	
Mp2	-Problem-solving (the act of identifying, prioritizing, and selecting alternatives for a solution; and implementing a solution) -Enunciation (the person expresses some mathematical idea)	D1: The derivative (*geometrically defined*) is the slope of the tangent line to the curve at a point.	SF2	Meaning
Mp3	-Problem-solving -Graphic and symbolic representations	Graphic Symbolic	SF3, SF4, & SF5	Different representations
Mp4	-Problem-solving -Enunciation	D2: The derivative is the limit of the incremental quotient.	SF6	Meaning
Mp5	-Representation	Symbolic representation 3	SF7, SF8, & SF9	Different representations
Mp13	-Problem-solving -Enunciation	D5: The function is continuous when you can draw without lifting the pencil from the paper.	SF21	Metaphorical

Note. Some information was taken from Rodríguez-Nieto et al. (2021a)

understood metaphorically as the tip of an iceberg constituted by a set of practices, processes, primary objects recognized in the mathematical activity of a student or teacher when solving a task and SFs that play the role of relating said objects, processes, etc.; Finally, the ETC categorizes a segment of a person's mathematical activity as a type of mathematical connection (Rodríguez-Nieto et al., 2021a).

Figure 14 shows another detailed example of the mathematical connections in terms of the articulation between ETC and OSA, as is the case of the metaphorical connection based on the information in **Figure 12** and **Figure 13** and **Table 3**. Finally, **Figure 15** presents the ETC from a vision articulated with the OSA.

FINAL REFLECTIONS

In this research, a review of the literature on studies referring to the articulation of theories was carried out, highlighting the tendency to compare general models of mathematical activity (such as the OSA) with specific models of mathematical activity such as the ETC. In this case, the OSA tools made it possible to establish a deep and detailed method of analysis that details, defines the

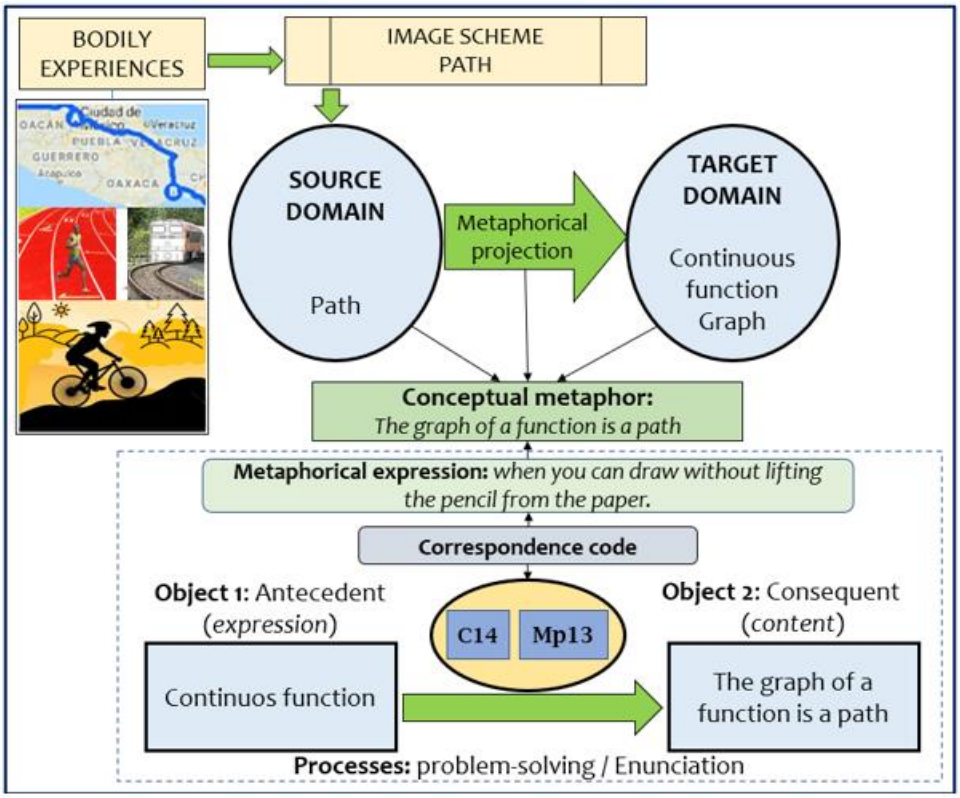

Figure 14. Functioning of the metaphorical connection articulating the ETC and the OSA (Own elaboration)

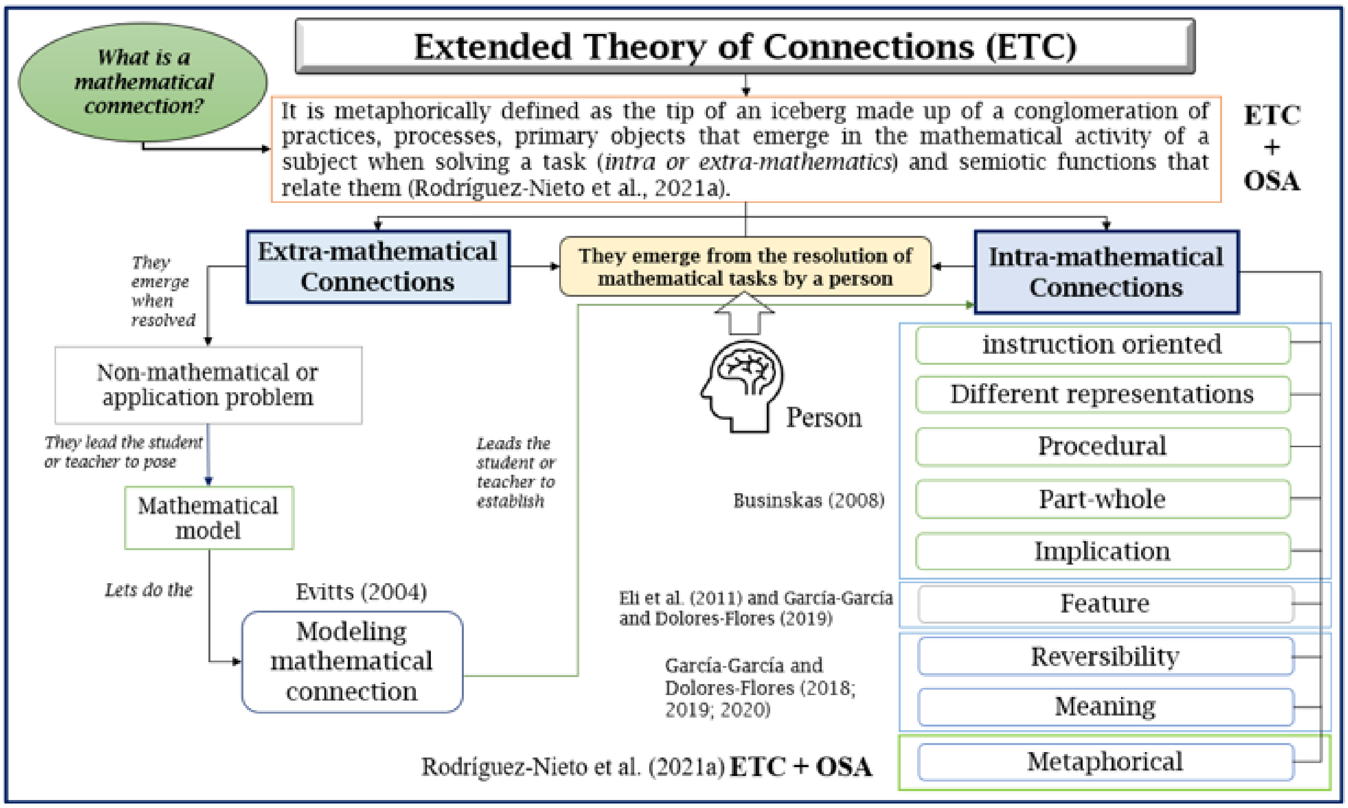

Figure 15. ETC articulated with OSA (Information adopted from García-García and Dolores-Flores (2019) and Rodríguez-Nieto et al. (2021a))

structure and functioning of the mathematical connection as the tip of an *iceberg* made up of a conglomeration of practices, objects/processes, and SFs that relate it. This networking between the ETC and the OSA also makes it possible to identify the difficulties of the students, which are caused by the key or necessary connections that cannot be made in the mathematical activity.

This research is relevant because, although many reviews of the literature have been carried out on specific topics (Arenas-Peñaloza & Rodríguez-Vásquez, 2021; Fung et al., 2021), for example, Julius et al. (2021) highlighted the importance and contributions of research in mathematics education field with topics related to problem-solving, testing, professional development of teachers and mathematical content of algebra, calculus, technology, geometry, and modeling.

However, no relationship was identified with the networking of theories, which is an important line of research that has given good results for the analysis of mathematical activity (Bikner-Ahsbahs, 2022; Font et al., 2016; Rodríguez-Nieto et al., 2021a). In fact, various authors (Bikner-Ahsbahs, 2022; Bikner-Ahsbahs & Prediger, 2010, 2014; Boero et al., 2002; Ledezma et al., 2022; Radford, 2008; Rodríguez-Nieto et al., 2021a; Tabach et al., 2020; Thanheiser et al., 2021) have investigated networking of theories, but not to report a detailed study that demonstrates the current state of research on this topic. Therefore, this research becomes relevant.

This type of networking of theories between general and specific theories increases the possibilities of interpretation and understanding of parts of the mathematical activity and can be applied, in addition to the connection and modeling processes, to other mathematical processes for which a specific theory has been developed. In this research, it was decided to exemplify the networking between the ETC and OSA (Rodríguez-Nieto et al., 2021a) because a significant and complementary contribution between both theories is recognized, especially from the OSA to the ETC where the connection was detailed from an onto-semiotic view. In fact, it is reported in the literature that university students have difficulties connecting derivative concept's several representations and partial meanings (Borji et al., 2018; De la Fuente & Deulofeu, 2022; García-García & Dolores-Flores, 2019; Galindo-Illanes et al., 2022; Pino-Fan et al., 2017, 2018; Sánchez-Matamoros et al., 2015), but in the example presented it is evident that the student makes connections to answer the proposed task, it gives examples and especially connects various derivative's meanings such as the slope of the tangent line to a curve at a point and the average rates of variation of the function, highlighting its verbal, graphic and symbolic representations.

Finally, for future research we suggest that the theories (two or more) continue to connect as it is a powerful approach, respecting their principles and methods, in order to find concordance and complementarities (following the pairs of strategies) and qualify the theoretical concepts and methods for a better analysis of the mathematical activity of any phenomenon or episode that is chosen as an object of study. In addition, the result of the networking of theories can be used by researchers to analyze without having to implement the pairs of strategies again, but rather as a combined use of theoretical frameworks.

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Declaration of interest: No conflict of interest is declared by authors.

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