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MAKING CONNECTIONS: SCIENCE EXPERIMENTS FOR ALGEBRA USING TI TECHNOLOGY

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ABSTRACT. Using science experiments in life science, chemistry, and physics, helps ground students' understanding of abstract algebra concepts in real-world applications. Hands-on activities connect mathematics with science in a way that is accessible to teachers and students alike. Each activity explores a scientific phenomenon, connecting it to algebra concepts such as quadratic functions and trigonometry. Students understand abstract algebra concepts by experiencing how scientists solve problems and use mathematical models to design experiments. They apply a variety of techniques to verify their experimental results and develop conjectures. These activities: use the Calculator Based Laboratory, CBL2™, with different probes, common science equipment, and basic tools, in addition to calculators. The experiments can be used as hands-on activities or demonstrations.

KEYWORDS. Algebra, Science, Technology, Quadratic, Trigonometry.

INTRODUCTION

'Why do we need to know this?" Sounds familiar? I had to answer this question to my students over and over again. I was facing a challenge - how can I make mathematics meaningful and real for most if not all my students?

We know that real-life applications, especially, visual and hands-on demonstrations enhance students' learning of the material, meet needs of kids with different learning styles, and create additional motivation for learning a discipline. The use of experiments allows students to create visual image and practical understanding of abstract mathematics concepts and relationships. Experimental demonstrations and lab activities in the course of mathematics make mathematics more interesting and appealing to students. Real experimentation with mathematical concept adds students' emotional component to the learning process. Coincidence of the experimental and theoretical results is equivalent to the Archimedes' "Eureka!".

In this article I would like to share with you two of several science experiments that I designed for algebra class. In the first experiment students develop understanding of the graph of cosine function by using fan cart, in the second one students learned properties of parabolas by using projectile launcher. Suggested experiments are designed for students taking different

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courses of algebra. Use of hands-on activities within rigorous mathematics content provides additional opportunities for students to make connections, and to master algebra concepts and skills. Another important part of this approach is use of technology and different measuring equipment in mathematics classes. Real-life problems do not provide us with “nice” numbers. Students educated on sets of standard problems get accustomed to the fact that only “nice” numbers, usually integers, can be correct answers to the problem. Real practical problems give students an understanding that any number that are not very “nice” can be a correct answer to the problem.

I also felt that from this experience students realized that mathematics plays an important role in every aspect of our lives, and especially in science applications. Mathematics is needed at each step of scientific investigation. In high school science curriculum students are usually exposed to only one role of mathematics – use of mathematical technique for computations of parameters from the experimental results or verification of experimental and theoretical data. In students’ minds this approach reduces the role of mathematics to a basic computational tool. By using science experiments in algebra class students are exposed to different roles of mathematics in science:

* use different mathematical technique to verify results of experiments with theoretical predictions

* use set of mathematical models and methods that allow them to describe some real-life situation, and to design an experiment for this situation.

* develop conjectures based on the results of the experiment that go beyond the scope of the experiment and only mathematics allows verification of these conjectures for general case.

Cosine Graph. Thrust Force of Fan Cart.

In a *Die Another Day* James Bond is in a fast-paced hovercraft chase. Hovercraft is a ground or water-effect vehicle. There is very little friction between the craft and the surface. Like the hovercraft, the fan cart that students will be using in this experiment is powered by the airflow created by the fan mounted on top of the cart (Fig. 1). The airflow produced by the fan creates a force F acting on the cart in the direction opposite to the airflow that causes the cart to move. Since the fan cart is designed to move in one direction only, the thrust force of the fan cart is the component of the force F parallel to the wheels of the cart. This component can be defined as $F_x = F \cos \theta$, where θ is the angle between the cart’s direction of motion and the direction of the force F (Fig. 2) By turning the plane of the fan students can change the direction of airflow and observe the effect of the angle θ on the thrust force F_x. In this experiment students investigate the graph of the cosine function by measuring the thrust force as a function of the angle. In a sense this experiment is a hands-on technology based version of unit circle analysis.

Figure 1

Before we start the experiment, I ask students to predict when the thrust force will be maximal and minimal. Most of kids can easily predict that if thrust force is parallel to the wheel axes of the fan cart, it should go fastest, and of course they can easily check that by turning the switch on and let the cart go. It is not as obvious for them what happens if we turn the fan perpendicular to the wheel axes. Immediate check demonstrates that cart does not go anywhere, since the thrust force pushes perpendicular to the direction it can go. Now, with the use of Vernier Dual Force Sensor (Vernier Software and Technology), data interface (CBL2 or EasyLink also available from Vernier Software and Technology), and graphing calculator we can collect data for all different positions of the fan dial. The force sensor allows us to measure an average force for each dial position and present data graphically. Sample data for this experiment are shown on Fig. 3. Students can now analyze the graph and answer set of questions about graph properties, for example:

1. At what angle(s) the magnitude of the thrust force is zero? Why?

2. At what angles does the thrust force reach its maximum possible magnitude (equal to the air flow force F)? Why?

3. What is the function that describes the ratio F_x/F? Does it depend on actual values of F_x and F?

4. What is the domain and range of this function? What are x and y – intercepts?

5. What will happen to this function if we keep turning the fan dial through several revolutions?

Figure 2

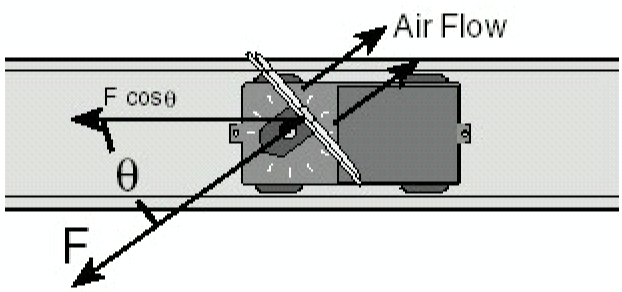
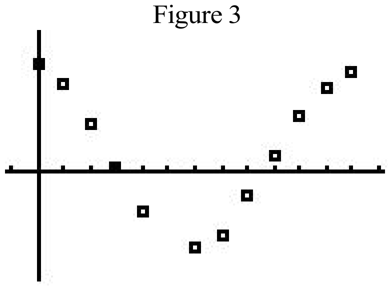


Figure 3



One of the advantages of this experiment is the fact that the dual force sensor can measure both, pull and push. When the airflow is directed backwards, the force sensor measures negative force. I always ask students "What is the meaning of the negative part of the graph and what does it tell us about magnitude and direction of the thrust force?" This question creates a very rich discussion in class and helps students to make connections between hands-on experience of how fan cart moves and increasing and decreasing behavior of cosine graph.

The activity also opens several opportunities for further explorations, for example, you may ask students to convert degrees into radians and find sine regression. For the sample data presented on Figure 3, the regression equation is $y = 0.83 \sin(.97x + 1.74)$. Ask them to plot sine regression equation along with the function $y = 0.83 \cos(0.97x)$. Do these functions describe the same graphs? Is it possible for the same graph to have different equations? Ask students to explain their statements using properties of right triangle and definitions of sine and cosine. Or you may want to ask them the meaning of the horizontal shift of 1.74 in the expression for the sine regression and help them to realize that this just represents the co-function identity: $\sin(x + \pi/2) = \cos x$.

In this experiment we combine fun and engaging activity for students with rigorous mathematics. We help them to make connection between "making sense" real-life situation, how does airflow affect the motion of a fan cart, and properties of an abstract mathematical object, cosine graph. I learned that after my students did this experiment, they never had problem with recalling that cosine is a decreasing function in the 1st quarter period, that it has maximum at zero degrees and zero value at 90^0. All they needed was to think of a motion of a fan cart and position of the fan. Can we explore properties of the sine graph using fan cart – absolutely yes! The fan cart also comes with the sail, an attachment that you place on top of the cart. Students can measure the angle of the sail instead of the angle of the thrust force, and they will get sine graph instead of cosine graph. And the best part is this comment from one of my student: "Can we play with FUN cart again?"

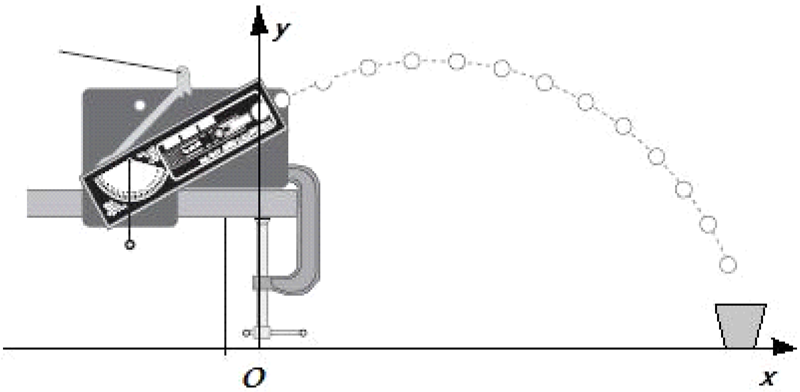
Equation of Parabola. Catch the Ball.

"The Russian Space Agency guided Mir back to Earth on March 23rd, 2001. Fragments of the huge spacecraft splashed down in the South Pacific Ocean just as ground controllers had planned - it was a flawless re-entry. No one was hurt. On the contrary, onlookers who saw Mir's blazing fragments described it as the experience of a lifetime!" (The End is Mir 2001). The scientists had to calculate details of the Mir's trajectory before it entered Earth's atmosphere and plan control of the station that would allow the Mir's pieces to fall into the ocean.

In this experiment students will perform a similar task at a much smaller scale. They will need to find equation of the trajectory of a ball launched off the table under the angle in order to predict the ball's height at any position and catch it in a cup. This is a classic physics experiment, but when used in an algebra class along with right goals and questions it provides an excellent opportunity for students to explore properties of parabolas.

In order to complete this experiment you will need a projectile launcher (PASCO) or any type of toy that would shoot a ball at a constant initial speed, three ring stands, meter stick and a plastic cup. The setup of the experiment is shown on Fig. 4. The first task that I ask of students is to determine the least number of different points on the trajectory (path) of the ball they need to know in order to predict location of the ball at any point of the path. Mathematically we are asking students to determine experimentally how many points is sufficient and necessary in order to define a parabola uniquely. Students start with measuring coordinates of the launching point. The question they have to answer first " Can you predict where will the ball land on the floor if you know coordinates of the launching point?" I usually ask students to come up with the equation of parabola through the launching point and using that equation to predict the position of the ball on the floor. At this stage of the activity some of my students used $y = a - x^2$ to start with and found a by plugging in launching point. They check if their equation works by trying to place a cup at the predicted position, and find out that this is insufficient information. After they find the landing point by trials and errors, they record coordinates of the second point. The question is still open: "Now that you have coordinates of two points on the part of the ball, do you have enough information to predict at what height above the floor will the ball go through a ring mounted on a stand located at a specific position?" Students repeat prediction process, trying to model equation of parabola with 2 points and use it for prediction. Students may create an equation by trials and errors or start with familiar form of quadratic, like $y = ax^2 + b$, or by using quadratic fit option on the calculator. They are again learning that having coordinates of 2 points is insufficient for determining unique equation of parabola and are forced to use experimental trial and error approach until they determine coordinates of the 3rd point.

Figure 4



At the next step of the investigation, they determine equation of parabola based on coordinates of three points they found from an experiment. They realized at this time that they can only come up with one equation of parabola, but not all of them are sure why this is the case. So, experimental check is necessary. We keep the cup on the floor and 1st ring stand in positions they found in previous trials. The 2nd ring stand is placed in a new position and students calculate the height of the ring in order for the ball to go through the ring. With the small ball, PASCO launcher, and careful calculations, the ball goes through both rings and is caught in the cup. Loud cheering and clapping usually express the students' joy. So, what have we accomplished so far? Students determined that you must have 3 points in order to determine parabola uniquely. Now we can have several mathematics tasks for them: find equation of parabola based on 3 points by different methods:

* Substitution of x and y values into a general form of quadratic function, $y = ax^2 + bx + c$ and solving for a, b, and c. By the way, it is also a great place to make a connection to this form of quadratic polynomial, 2nd degree polynomial has 3 constants that need to be determined; linear polynomial requires 2 constants, cubic polynomial requires 4 constants, etc. Students can determine this pattern on their own.

* Using matrices to solve system of equations for a, b, and c.

* Using quadratic fit to find equation of parabola passing through three given points. In this case students will need to enter three pairs of points into Lists and use quadratic regression option on the calculator.

Here are other tasks and questions that I ask students in this experiment:

1. It is known that motion of any objects is directed along the tangent line to the path of the motion. Confirm that for your equation at the initial point:

    a. Measure the launching angle and determine its tangent

    b. Use calculator and graph the equation of trajectory that you found

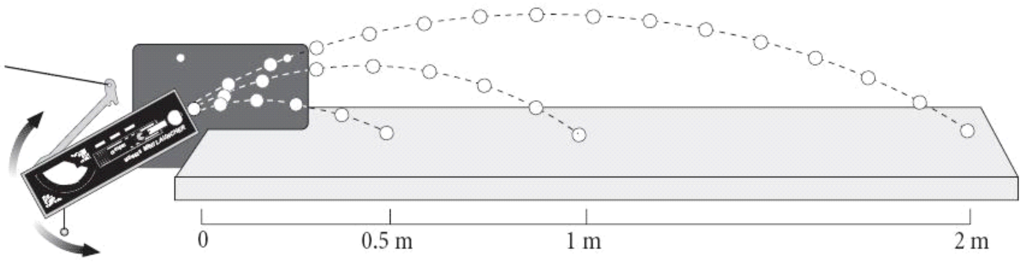
    c. Zoom in and calculate slope of the tangent line at the launching point

    d. Compare calculated and measured values of tangent

2. Use parametric equations for the projectile motion, measure initial speed of the ball with Vernier Photo Gate, and confirm that two parametric equations for the motion of the ball produce the same parabolic trajectory as the equation you found earlier.

This experiment can also be used to study symmetry of parabola, but adjusting launcher to shoot at the level (Fig. 5) and asking students to explore the distances from the vertex position to the launching and landing points.

Figure 5



ASSESSMENT

One of the most important goals of assessment is to make it a learning tool for the students. If students know how to prepare for the laboratory experiment, what they need to know before they come to class on the day of the experiment, and how their lab reports will be assessed, they will do much better job in class and on the written report. Whenever these activities are used as laboratory experiments, it is recommended that students will write laboratory reports to present their data, calculations, and analysis.

I have developed assessment tools to reduce amount of time that teachers will have to spend for grading of lab reports and at the same time to help students to learn how to write laboratory reports. The assessment of the experiment includes two parts. The first part is pre-lab performance based assessment (*see Performance Based Assessment form*). It includes set of questions that students should be able to answer **before** they start an experiment and scoring rubrics. In many cases, that also means that students are expected to complete necessary calculations prior to the data collection. The performance based assessment form is provided to students at the time when laboratory experiment is assigned. Teacher has an option of using this form for students' self-evaluation, peer evaluation, or for interviewing students before or during the experiment, and assessing students' preparation with or without the grade. In my classes each group of students go through the questions offered in the *Performance Based Assessment* form together before I assess their knowledge of the experiment they are about to do. When I work with the group of students, I ask each student in the group randomly 3-4 questions, so that all students in the group cover all questions on the form. Each student in the group is assessed individually. These interviews could occur a day before the experiment, at the beginning of the experiment, or during the experiment.

The second part of the assessment is written laboratory report (*see Assessment of Laboratory Report form*). All students should know requirements for the laboratory reports before they turn them in. The Assessment of Laboratory Report form is designed to provide students with the checklist/criteria that they can use when preparing written reports after completion of the lab. Students use this form for self-evaluation and peer evaluation, and it becomes a learning tool for them. I expect that each student check his/her laboratory report against this checklist. Then, I require students to have their lab report evaluated by their peers. Usually, students have their lab partners to evaluate the lab reports. This evaluation does not include grading by the peers or evaluation of the contribution made by each member of the team. The purpose of the peer evaluation is to allow someone else to go through the lab report and check it against the criteria, make comments and suggestions for the author of the lab report to revise and perfect their work before it is turned in to the teacher. Each student (or group) is asked to turn in an original draft of the lab report with comments and markings made by the peers, checklist from the peers and final revised copy. I assess the final copy of the lab report using the same form. This three-step evaluation allows me to teach student to check their work before turning it in, to learn from each other and to succeed in lab report writing. At the same time, standardized expectations force students to develop a uniform structure of the lab reports; self and peer evaluation reduces amount of careless mistakes and omissions in the lab report, and all that facilitates teacher's grading and reduces time necessary for grading.

One more concern of assessment of laboratory experiments (or any group projects) is how to assess individuals within a group. There are different approaches to the group assessment. Due to the need to ensure that all students have a clear understanding of the material covered within each laboratory/project and to ensure a level of equity in the distribution of work, the laboratory assessment options are offered to students. My approach is to allow students to take responsibility on themselves and choose laboratory assessment option (*see Laboratory/Project Assessment Options form*) that better fits their needs and ability to work within a group. Students are expected to make the choice of an option before they turn in lab reports. My experience shows that about 70% of students usually choose 1st option, working together and submitting one lab report per group, while 30% of students choose 2nd option, working individually on the lab report and limit group work to experimentation only. There are a lot of factors that could affect students' choice of 1st or 2nd option. These factors could include day schedule that may or may not allow students to work together out of class, established personal relations between the students, small or large range of students' abilities and skills within the group, reputation of being responsible or irresponsible person, etc. The main advantage of allowing students to choose an assessment option by themselves is to shift the decision making process from the teacher to the students in forming groups and sharing the group work, and to provide students with the opportunity to develop responsibility for the shared work.

All assessment forms are provided here as an aid to the teacher when assessing students' work. Teachers may use these forms as it is or modify them to better fit the needs of their students.

CONCLUSION

These experiments are intended as supplementary activities. Any activity can be used as a teacher's demonstration, class exercise, or a laboratory assignment. Using an experiment as a demonstration allows a teacher to talk about real-life applications of mathematics without necessity to have multiple sets of equipment for the students groups. When activities are used as class exercise or laboratory assignment, students have an opportunity for teamwork and interaction with each other as well as learning skills of using measuring devices; however, any group work is more time consuming and usually requires at least one class period for completion of the experiment and additional time for pre-lab calculations and/or post-lab analysis. Some experiments may be divided up in parts and completed within two or three lessons. Teacher may use one part of the experiment as a class demonstration and another part as a lab exercise.

In teaching a particular topic, teacher has an opportunity to introduce the experimental activities in different place within the topic. Labs could be great exploration type introduction to a new topic that would be followed by the teacher's instructions and explanations. The experiments could also be used as review exercise. In some cases experiments allow for more engaging way to exercise algebraic skills necessary for successful learning of mathematics. Most commonplace of lab experiments is at the end of studied topic when students are expected to use what they learned for applications and problem solving.

Whatever place the experiments are used within the context, they can enhance students' learning of the mathematics, allow students to see real-life applications and allow the teacher to have performance-based assessment of students' understanding of learned material.

Please contact the author for a copy of the student lab handouts described in this article.

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APPENDIX

Student Name(s): ______________________

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ASSESSMENT OF STUDENT PERFORMANCE

Laboratory and Technology Use

na = not assessed

nr = no response

4, 3, 2, 1 = see attached scoring rubric

1. Can the student clearly state the problem that is being investigated?	nr 1 2 3 4 na
2. Can the student make predictions as to the outcome of the experiment(s) or analysis?	nr 1 2 3 4 na
3. Can the student defend his/her predictions based on background information gained through preliminary research?	nr 1 2 3 4 na
4. Can the student thoroughly describe the experimental procedure conducted or technology used?	nr 1 2 3 4 na
5. Can the student explain the relevance of the experimentation in terms of real-world applications?	nr 1 2 3 4 na
6. Can the student describe the quantitative or qualitative aspects to be explored during the laboratory?	nr 1 2 3 4 na
7. Can the student describe how the measurements will be taken?	nr 1 2 3 4 na
8. Can the student describe the laboratory or technology set up?	nr 1 2 3 4 na
9. Can the student demonstrate the ability to use the equipment/technology properly?	nr 1 2 3 4 na
10. Can the student identify possible sources of error?	nr 1 2 3 4 na

_________/_________ = _________%

Student Total / Total Possible Percentage

Comments:

Scoring Rubric

Level 4

- The student demonstrates a clear understanding of the problem and investigations performed.
- The student’s observations are valid and demonstrate attention to detail.
- The quality of the data analysis reflects his/her ability to utilize computer software in the data analysis.
- The design of experiments suggested by the student are well thought-out and scientifically accurate.
- All data are presented in an organized fashion, utilizing appropriately designed tables, charts, and graphs.
- Answers to questions are well thought out and supported by experimental data.
- In general, there are no false assumptions or misleading statements made by the student.
- The student recognizes the need for additional testing and provides appropriate suggestions related to the problem.
- The conclusions reflect the student’s ability to effectively analyze experimental data and draw appropriate conclusions.
- The student shows a deep understanding of the technology being used.
- The student successfully proposes an explanation which clearly shows a relationship between his/her data and conclusions.

Level 3

- The student demonstrates an understanding of the problem and investigations performed.
- The student’s observations are valid and demonstrate some attention to detail.
- The quality of the data collected generally reflects his/her success at performing each experiment.
- The experiments suggested by the student are well thought-out, but may have flaws in scientific design.
- Most data are presented in an organized fashion, utilizing appropriately designed tables and charts.
- Answers to most questions are well thought-out and supported by experimental data.
- In general there are few false assumptions or misleading statements.
- The student recognizes the need for additional tests, but is unable to provide appropriate suggestions related to the problem.
- The conclusions reflect the student’s ability to analyze experimental data and draw conclusions.
- The student shows a moderate understanding of the technology being used.
- The student successfully proposes an explanation which generally shows a relationship between his/her data and conclusions.

Level 2

- The student demonstrates some understanding of the problem and investigations performed.
- The student’s observations are vague and lack attention to detail.
- The quality of the data collected reflects limited success at performing the tasks.
- The experiments suggested by the student are unclear or riddled with flaws in scientific design.
- The data are recorded in a disorganized fashion, with tables, charts and graphs poorly designed or missing.
- Answers to questions are not well thought-out or supported by experimental data.
- In general, the student makes both false assumptions and misleading statements.
- The student fails to recognize the need for additional tests.
- The student has difficulty analyzing experimental data and drawing conclusions.
- The student shows little understanding of the technology being used.
- The student fails to propose an explanation which shows a relationship between his/her data and conclusions.

Level 1

- The student demonstrates little or no understanding of the problem or investigations performed.
- The student’s observations are poor or missing and show no attention to detail.
- The quality of the data collected reflects little or no success at performing the tasks.
- The experiments suggested by the student are difficult to follow , or missing.
- The data collected is haphazardly recorded, or missing.
- Answers to questions are implausible and not related to the experimental data.
- In general, the student makes many false assumptions and misleading statements.
- The student fails to recognize the need for additional tests.
- The student is unable to analyze experimental data and draw conclusions.
- The student shows lno understanding of the technology being used.
- The conclusions are unrelated to the experiments performed.

NAME OF STUDENT:______________________________

ASSESSMENT OF LABORATORY REPORT

Check each item present and circle the total for each category. na = not assessed, nr = no response

A. FORM OF THE REPORT (5 pts)	nr 1 2 3 4 5 na
_____ Title, objective(s), names of group members are included	
_____ Background information is provided and thorough	
_____ Hypotheses/predictions are stated	
_____ Diagram of set up with necessary labels is shown	
_____ Procedure is thorough and sequential, materials are listed	
B. QUALITY OF THE OBSERVATIONS/DATA (4 pts)	nr 1 2 3 4 na
_____ Accurate measurements/observations	
_____ Complete data table/list and qualitative observation	
_____ Correct units	
_____ Data consistent with the event	
C. GRAPHS (6 pts)	nr 1 2 3 4 5 6 na
_____ Appropriate title	
_____ Curve appropriate to data trend	
_____ Data points plotted accurately/shown	
_____ Appropriate scale with units is shown	
_____ Axes labeled with correct variables and units	
_____ Legend if more than one set of data included	
D. QUALITY OF CALCULATIONS (8 pts)	nr 1 2 3 4 5 6 na
_____ Mathematical relationship/formula stated	
_____ Necessary formula(e) derived	
_____ All steps are mathematically correct	
_____ Selected and substituted proper data into relationship	
_____ Calculated correctly	
_____ Units stated and used correctly in the relationship	
_____ Error calculation(s)	
_____ Results	
E. CONCLUSION (7 pts)	nr 1 2 3 4 5 6 7 na
_____ Consistent with scientific and mathematics principles	
_____ Consistent with objectives and hypotheses	
_____ Consistent with data	
_____ Relationship among variables stated	
_____ Sources of possible error identified	
_____ Specific questions are answered	
_____ References/Citations are provided	
	_______ / 30 = ______%
	Total Pts.

Comments:

LABORATORY ASSESSMENT OPTIONS

Due to the need to ensure that all students have a clear understanding of the material covered within each laboratory and have an adequate understanding of how all of the laboratories and projects relate to each other; and to ensure a level of equity in the distribution of work, the following options have been developed:

Option 1. Three Heads Are Better Than One

1. Each lab/project group will consist of no more than three students.
2. All students are required to participate equally in the performance of each lab/project.
3. Group members will share the research, development, analysis, and writing of the report equally.
4. The group will submit one laboratory/project report.
5. All group members should use the laboratory report/project checklist to ensure that they have included all the necessary components into their report.
6. A peer critique along with the first draft should be attached to the final corrected report. A peer critique must be performed using lab report/writing project assessment.
7. If the group chooses this option, the group members should be aware that every member of the group would receive the same grade for their efforts.

Option 2. Then I'll Do It Myself!

1. Each lab/project group will consist of n more than three students.
2. All students are required to participate equally in the performance of each lab/project.
3. Group members will share data acquired during lab experiment. No sharing is allowed for the writing projects. Each individual student will research, develop, and analyze **all** portions of the lab/project.
4. Each group member must submit individual lab/project report.
5. All group members should use the laboratory report/project checklist to ensure that they have included all the necessary components into their report.
6. A peer critique along with the first draft should be attached to the final corrected report. A peer critique must be performed using lab report/writing project assessment.
7. Each member of the group will be graded individually.

Lab/project groups cannot change the assessment option after laboratory report/writing project has been turned in for a grade. All group members should know what assessment is chosen before starting the lab/project. In case of conflicts/problems within the group, group members should seek an advise from the teacher as early in the work as possible.