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Mathematical reasoning of prospective mathematics teachers in solving problems based on working memory capacity differences

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Abstract

This study aims to investigate algorithmic reasoning and creative reasoning of prospective mathematics teachers in solving problems. This research is a qualitative research. The research subjects are prospective mathematics teachers with high working memory capacity (HWMC) and low working memory capacity (LWMC). Based on the results on algorithmic reasoning, it is known that for subjects with HWMC using algorithms to solve the given problems. While subjects with LWMC using sum and multiplication rules to solve the given problems. On creative mathematical reasoning it is known that for subjects with HWMC using a novel strategy, providing predictive and verification reasons for the selection of strategies, and providing convincing reasons accordance with the intrinsic mathematical properties. While subjects with low memory capacity had difficulty providing a verification reason and the reason that the way of solving was in accordance with the intrinsic mathematical properties.

Keywords: mathematical reasoning, open ended, problem-solving, working memory capacity

INTRODUCTION

Reasoning is the key to learning mathematics. Because solving mathematical problems requires a lot of reasoning. Solving problems is a means to learn mathematics (de Ron et al., 2022; Dröse et al., 2021; Fatmanissa et al., 2020; Habtamu et al., 2022; Hokor et al., 2022; Julius, 2022; Portaankorva-Koivisto et al., 2021; Ramdani et al., 2019). Learning aims to understand the concept being studied (Langi et al., 2021). The process of thinking used to obtain logically based conclusions is called reasoning (Aineamani, 2018; Sumpter, 2015). Mathematical reasoning itself is reasoning on mathematical objects or topics (Lithner, 2008). Mathematical reasoning consists of two types, namely creative and imitative mathematical reasoning (Bergqvist, 2007). The characteristics of a thinking process carried out in the learning process are imitative and creative mathematical reasoning (Lithner, 2015).

Solving mathematics using reasoning or processes found or used by others, or in other words using reasoning that was already available before, is called imitative reasoning (Sidenvall et al., 2014). That is the reason why imitative reasoning is used to solve routine problems. This reasoning becomes the main choice when the task given to students is a task that requires the ability to calculate or something that is solved by remembering the algorithm (Muzaini et al., 2019). Imitative reasoning consists of two types: memorized and algorithmic reasoning (Hershkowitz et al., 2016). In Memorized reasoning, solving problems is done by remembering answers, specifically by writing down what is remembered on the answer sheet (Boesen et al., 2010). Boesen et al. (2010) provide the conditions for memorized reasoning:

(i) how to solve the problems is based on what is remembered and the answers and

(ii) writing is the only way to solve the problems.

One can describe an object of a part without being affected by the preceding elements. This type of reasoning can be used as a solution only on relatively easier tasks with a small scope (Lithner, 2008).

Another imitative reasoning is algorithmic reasoning (Lithner, 2015). Algorithms are a collection of instructions that can be used to find definite results for a problem (Sidenvall et al., 2014). Algorithmic reasoning is

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Contribution to the literature

- This study adds to the literature for educators on how students of prospective mathematics teachers solve problems using mathematical reasoning.
- This study examines the creative mathematical reasoning and Algorithmic reasoning of prospective mathematics teacher students based on working memory capacity.
- The study examined student teacher candidates with working memory capacity to solve problems following a reasoning structure.

more dominant in using sought or remembered algorithms that are considered suitable for solving problems (Lithner, 2008). Algorithmic reasoning is used if a given task is completed with counting. Applying a previously learned algorithm to be recalled is an effort used for algorithmic reasoning (Norqvist et al., 2019). All pre-defined procedures are scopes for algorithms (Lithner, 2015). Therefore, using an algorithm means one should determine the previous algorithm. This is because the purpose of using algorithms is not to discover new things or information, make new decisions, or interpret things (Øystein, 2011). Therefore, the use of an algorithm can provide a reliable and very fast process used to find answers (Fan & Bokhove, 2014), so that the advantage of algorithmic reasoning lies in its purpose to get answers from a task. If the goal is to know the solving process and not the final answer, then it is not recommended to use algorithmic reasoning (Jonsson et al., 2014). Something that is difficult to do conceptually can be done with algorithms so that students play a really easy role (Jonsson et al., 2016; Lithner, 2015).

According to Lithner (2008), algorithmic reasoning meets two conditions:

(i) the method chosen uses available or existing algorithms and

(ii) sloppiness will result in incorrect answers since the completion step is very easy by following the algorithm (Norqvist, 2017).

The mathematical reasoning process will occur if the learning process provides opportunities for students to solve problems according to their own abilities. Creative mathematical reasoning meets three criteria (Norqvist, 2019), namely creative (able to use new reasoning), plausibility (able to provide arguments that can explain why the implementation of the procedure is right or reasonable), and anchoring (able to offer ideas based on mathematical properties (Aineamani, 2018). Lithner (2008) states that students should have opportunities to learn conceptual aspects and solve non-routine problems. Creative mathematical reasoning can provide such thing. Based on this, the problem given to students in this study is open-ended. Open-ended problems are problems that are open to being solved with not only one solution.

In an effort to solve problems with mathematical reasoning, it can be seen that the process of reasoning requires the ability to sort and use a lot of information into information relevant to the problem. The ability to use the intrinsic properties of relevant mathematics is needed to prove the truth. It is common knowledge that every experience leaves an indelible mark on our memory. Unlike computers, the average human brain never reaches the point where new experiences can no longer be processed in memory; the brain cannot be full (Baddeley, 2000). But at the same time, people can be overwhelmed in processing new information that seems too difficult to understand, confusing, or complicated to store in memory. Feelings of being overwhelmed by a lot of new information can occur because of a special type of memory that is usually called working memory. It refers to a relatively small amount of information that can be remembered, noticed, or maintained, technically, in a rapidly accessible condition at a time. Based on research, students' creative mathematical reasoning in solving problems is not optimal. Students have difficulty connecting information about the intrinsic mathematical properties relevant to the task. In that case, working memory capacity is needed.

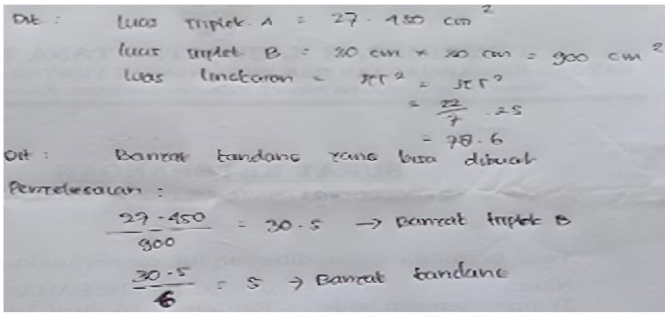
Working memory capacity is important because it helps select the relevant information used to resolve the problem (Anjariah et al., 2022). In an effort to solve problems with creative mathematical reasoning, students need relevant information. For this reason, working memory capacity helps individuals to only direct their attention to relevant information (de Fockert et al., 2001; Kane et al., 2001). Working memory capacity can reduce information retrieval errors that are not needed in problem-solving (Wiley & Jarosz, 2012), so Fyfe et al. (2019) state that working memory capacity is involved in problem-solving.

The learning process of mathematics nowadays opens opportunities for students to solve open problems in their own way. The same is true for college students who are prospective mathematics teachers. As prospective teachers, they must be proficient in solving open problems well. Eventually, they will already be familiar with those kinds of problems later when they become mathematics teachers. Therefore, research that investigates how mathematical reasoning is applied by students as prospective teachers in the mathematics education department is needed. In addition, like the previous explanation about the importance of working memory capacity, it is necessary to pay attention also to the process of solving the problems of prospective students based on working memory capacity. So far, there has been no research investigating the mathematical reasoning of prospective teachers in solving problems based on working memory capacity. Therefore, this research becomes something very important to do.

This research is underpinned by previous research, which explains that mathematical reasoning is not as expected. This is in accordance with research conducted by Norqvist (2019), which states that some students who practice with creative mathematical tasks tend to use algorithmic reasoning, which seems to impact their underperformance in both practice and exam situations. One thing that could happen is that individuals with lower memory capacities have more difficulty sorting out much information (Palengka et al., 2019, 2021). Relevant information and very large tasks must be processed on-line, which essentially requires great attention, and thus the selection of information relevant to the task is highly expected. This inhibits cognitively weaker students, whose search process is easier, and potentially causes them to use information that is not in accordance with the problem. Furthermore, it is known that there has been no research on how the mathematical reasoning process is carried out by students who are prospective mathematics teachers to solve mathematical problems. Based on the background above, the purpose of this study is to determine the mathematical reasoning of students who are prospective mathematics teachers based on working memory capacity.

MATERIALS AND METHODS

This study is a qualitative research. The research subjects were students in semester three who are prospective teachers in the mathematics education study program, Toraja Indonesian Christian University. Specifically, two subjects from 35 subjects with high working memory capacity (HWMC) and two from 42 subjects with low working memory capacity (LWMC) were selected. The subjects were selected using the OSpan test. This instrument was used to determine the working memory capacity of the subjects by recalling the numbers displayed. This test is a computer-based test using automatic time settings. There are some numbers to remember, and the math operation is used as a distraction in the time frame of four seconds. Students are asked to write down the number they remember within 10 seconds. For more details, an example of o-span test can be seen in (Juniati & Budayasa, 2020). After that, one subject with HWMC and one with LWMC were selected. The two subjects were chosen based on the same gender and level of mathematical abilities. Both subjects were male students with equivalent mathematical skills. Both subjects were given a reasoning test in the form of an open-ended assignment. There were two questions given to each subject. The first test was about making a square-shaped swift bird cage. They were requested to specify how many boxes can be made if plywood with an area of 27,450 cm² is provided. Furthermore, it was known that 21 sheets of 30 cm × 30 cm plywood are required to form four boxes. In each box, there is a hole for the entry and exit of the bird in the shape of a circle with a diameter of 10 cm. The second test was about dividing inheritance in the form of 100 buffaloes, which will be divided into four children. They were requested to determine the number of buffaloes received by the fourth child if there is a condition that the first child gets twice as much as the second child, the third child gets more than the first and second child, and the fourth child gets less than the second child. After given the test, an interview was conducted with each subject. The results of the reasoning test and the interview results were then analyzed to obtain the conclusion. The aspects examined in imitative reasoning, namely algorithmic reasoning, are the process of remembering and using algorithms in solving problems. The aspects to be examined for creative mathematical reasoning are novelty, plausibility, and anchoring.



Condition: area of plywood A = 27. 450 cm²
Area of plywood B = 30 cm x 30 cm =900 cm²
Area of circle = πr^2
$= \frac{22}{7} \times 25$
= 78, 6
Question amount of cages that can be made
Solution: $\frac{27.450}{900}$ = 30, 5 amount of plywood B
$\frac{30.5}{6}$ = 5 amount of cages

Figure 1. LWMC answer to the first problem (Source: Authors' own elaboration)

RESULTS AND DISCUSSIONS

The results of mathematical reasoning analysis in problem-solving that are reviewed based on the working memory capacity of both subjects based on mathematical reasoning tests and interviews are described as follows.

Description of Mathematical Reasoning of Subject with Low Working Memory Capacity

Algorithmic reasoning

The subject with LWMC solved the problem by dividing the size of the plywood provided by the box area to make the cages and then dividing the area by 4 to get the number of cages made. These results can be seen in **Figure 1**.

From the subject's answer in **Figure 1**, it can be seen that the subject with LWMC solves problems involving algorithmic reasoning. This can be seen from the process of solving problems conducted by LWMC. Subject with LWMC determined the square area using the square formula. After that, he resolved the problem using the division process. Subject with LWMC divided the known plywood area by the square area used to make the cages. Then, the result of the division was divided by the number of square plywood used to make the cages. This is in accordance with the results of the interview with LWMC, as follows: Subject with LWMC interview transcript:

R: How did you solve the problem?

LWMC: I determined the area of the plywood first, ma'am.

R: How did you do that?

LWMC: Using the square formula, ma'am.

P: So?

LWMC: I divided the known area of the plywood by 900.

R: Why did you divide it by 900?

LWMC: That is the area, ma'am.

R: Then, what is next?

LWMC: I divide the result of dividing the area of the plywood and the area of the square by six.

R: Why should it be divided by six?

LWMC: Because the number of square plywood made for one box is six plywood.

R: So, how many cages can be made?

LWMC: Five cages.

R: Are you sure that your answer is correct?

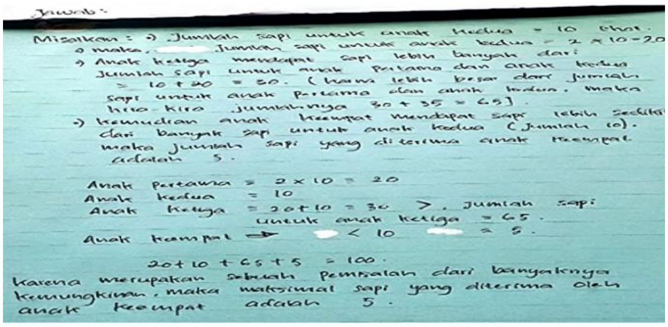
LWMC: Yes, ma'am.

R: Why are you so sure?

LWMC: Because making one box uses six plywood.

R: If the boxes are arranged side by side, does it also need six sides?

LWMC: Yes, ma'am.



Answer:
If:

- The amount of cows for the second child is 10 cows
  Then the amount of cows for the second child = 2 x 10 = 20
- The third child gets more cows than the first and second children combined = 10 + 20 + 30 (because it should be more than the first and second children, then the approximate amount is 30 + 35 = 65)
- Then the fourth children gets fewer cows than the second child (10 cows) then the fourth child gets 5 cows.

Fist child = 2 x 10 = 20
Second child = 10
Third child = 20 +10 = 30, the amount of cows for the third child = 65
Four child = <10 = 5
20+10+65+5 = 100
Because of the assumption from the various possibilities, then the maximum amount of cows that the fourth child can get is 5

Figure 2. Subject with LWMC answer to the 2nd problem (Source: Authors' own elaboration)

From these results, it is revealed that the reasoning process carried out by LWMC is algorithmic. The subject solved the problem by using a pre-existing algorithm using the square area formula. After using the algorithm, the following solution was to use the division process only. This can be seen in the completion process in **Figure 1**. It can be seen that the subject divided the area of the plywood by six. According to him, six plywood must be used to make a cage. Meanwhile, the cages are arranged side by side so that each cage does not have to consist of six sides.

In different problems, the algorithmic reasoning the subject with LWMC is, as follows: For the issue of inheritance distribution, the answer given by the subject is in **Figure 2**.

It can be seen in **Figure 2** that subject with LWMC used multiplication to determine the number of buffaloes obtained by the first child, summation for buffaloes obtained by the third child, and "more than" to determine the number of buffaloes obtained by the fourth child. From these results, it can be seen that the subject with LWMC solved the second problem by using summation, multiplication, and "more than" rules. Thus, it is revealed that the subject used algorithmic reasoning in solving problems in the second problem.

Creative mathematical reasoning

The mathematical reasoning flow of the subject with LWMC for problem number one, in addition to using the algorithmic reasoning flow, also used creative mathematical reasoning. Creative mathematical reasoning appears in the step of solving done by the subject using the reasoning flow found by the subject himself after understanding the given problem. The subject symbolized the known plywood area with A and

B. The subject also divided the known plywood with the square plywood area to obtain the number of square plywood.

Thus, it was revealed that the subject used a new line of reasoning or what is called novelty in solving problems. The novelty here is not something completely new, but new to the subject himself. This flow of reasoning may not be new to other teachers or students but is something new for the subject. This is shown in the following interview transcript:

> R: Is that a new problem or a problem that has been done before?
>
> LWMC: It is new, ma'am.
>
> R: Why is it new?
>
> LWMC: It is just new, ma'am.
>
> R: What I mean is what the new thing you are talking about? Is it the difficulty level or the way you solve it?
>
> LWMC: It is the way I solve it because it is not the same as the examples in the book, so, I find the answer with my own way.
>
> R: So, do you think the solution is your own?
>
> LWMC: Yes.
>
> R: Why?
>
> LWMC: Because no examples of the solution are given.

The subject also explained that selecting strategies can solve problems, often called predictive arguments. The subject stated that the number of square plywood must be divided by six to find out the number of boxes to be made because each cage consists of six sides. The subject also argued verification that the given answer is a correct and reasonable solution. The subject stated that he is sure that the answer given was the correct answer because to make one box, it takes six square sides. However, the subject had difficulty explaining that the process he used to solve the problem was based on intrinsic mathematical properties.

For problem number two, it can be seen in **Figure 2** that the subject with LWMC solved the problem by giving assumption. The subject first determined the number of buffaloes to be given to the second child, then from its result, the subject could determine the number of buffaloes to be given to the other three children. This is supported by the subject's statement seen in the following interview transcript:

> R: How did you solve the inheritance division problem?
>
> LWMC: First of all, I determined the amount that will be accepted by the second child, ma'am.
>
> R: How did you decide?
>
> LWMC: I determined the amount myself, ma'am.
>
> R: Why did you determine the value to the second child?
>
> LWMC: Because if the amount obtained by the second child is already known, then the others can be determined.
>
> R: Well, what did you do after that?
>
> LWMC: I determined the number of children obtained by multiplying 10 by two according to the question, so the result is 20.
>
> R: Then?
>
> LWMC: I determined the number that the third child gets by adding what the first and second children got.
>
> R: How many did the third child get?
>
> LWMC: 65, ma'am.
>
> R: How did you get that number?
>
> LWMC: I added what the first and the second child got to get 30, but the condition said that it has to be more than the second and the third child. So, I added another 35, so it is 65.
>
> R: Why adding by 35?
>
> LWMC: So, it has more than the second and third children, ma'am.
>
> R: Is there a reason why you chose 35?
>
> LWMC: No, ma'am, I just decided it.
>
> R: So, how many did the fourth child get?
>
> LWMC: Five.
>
> R: How do you get the answer?
>
> LWMC: 10+20+65=95. So, we have to add five to be 100. So, what the fourth child gets is five, ma'am.
>
> R: Are you sure the completion steps you used can give a correct and reasonable answer?

LWMC: Yes.

R: What is the reason?

LWMC: (Smiling).

Based on how to solve given problems, it is revealed that LWMC subject also uses mathematical reasoning in solving the problems. This can be seen in the step of solving the second problem. The subject used creative mathematical reasoning in which the subject determined the number of buffaloes that will be given to the second child through a repeated attempt. This meets the criterion of novelty, which is to use a new line of reasoning. The next step was to determine the number of buffaloes obtained by the first child by multiplying the value determined previously by two. Similarly, to get the number of buffaloes received by the third child, the subject summed up the number of buffaloes received by the first and second children. To determine the number of buffaloes obtained by the fourth child, the subject summed up the number of buffaloes obtained by the first, second and third children, and then the remaining number of buffaloes were given to the fourth child under the condition that that the number of buffaloes received by the fourth child should not exceed the number received by the second child. It appears that the subject solved the problem completely based on the information mentioned in the given problem and did not use other ways that were possible to solve the problem. And, according to the subject there was no other way that he can solve the problem than the way he uses it.

LWMC subject could also provide reasons related to the completion strategy that was used correctly and sensibly or what we know as plausibility. The subject could provide a predictive reason that the chosen strategy can solve the problem (Swanson, 2017). This appears in the interview transcript, where the subject determined the value for the second child and stated that because if the amount obtained by the second child is already known, then the others can be determined. Here it appears that the subject determined for himself the number of buffaloes obtained by the second child on the grounds that if the number received by the second child is known, the number obtained by the first, third and fourth children can also be known. However, the subject could not provide a verification reason that the strategy used is a correct and sensible strategy. The subject agreed that the strategy used to solve the problem can provide a correct and reasonable answer, but the subject cannot provide a reason for it. The subject was also unable to give reasons that the steps taken are in accordance with the intrinsic nature of mathematics. Thus, anchoring cannot be investigated more deeply in the case of the subject with LWMC.

From the discussion above, it was revealed that the subject with LWMC uses creative mathematical reasoning and algorithmic reasoning in solving the given problems. From the results of the discussion, it can be seen that the subject used the rules of summation, multiplication, and the methods that have been done before to solve the problem. Thus, it can be said that the subject tended to use algorithmic reasoning to solve problems. Nevertheless, the subject with LWMC also used creative mathematical reasoning although not optimally. This can be seen in the reasoning flow used, where the subject used a new reasoning flow (Yackel & Hanna, 2003). The subject could also provide predictive reasons (plausibility) although the subject has difficulty to provide verification reasons (plausibility), and the reason that the way of solving is in accordance with the intrinsic mathematical properties (anchoring).

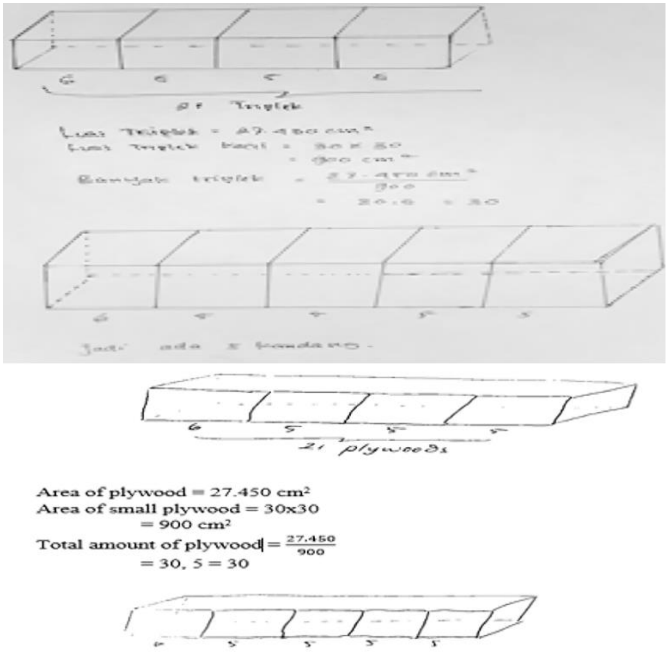


Figure 3. Subject with HWMC answer to the 1st problem (Source: Authors' own elaboration)

Description of Subject Mathematical Reasoning Based on High Working Memory Capacity

Algorithmic reasoning

The subject with HWMC used algorithmic reasoning in solving the given problems. This can be seen in the process of solving the first problem as seen in **Figure 3**, where the subject uses a square area formula to obtain the area of small plywood. Likewise, to get the many square-shaped plywood available, the subject also used division rules. The reasoning process carried out by the subject with HWMC subject can be seen in **Figure 3**.

The subject with HWMC used algorithms in solving the first question. For the case of making swift bird cages, the subject used a square area formula, namely the side × side formula to solve the problem. The subject also used division to determine the total amount of plywood, which is to divide the known plywood area with small plywood area. For the second question, the subject with

HWMC also used algorithmic reasoning. This appears in the process of solving the problem by considering the number of buffaloes received by each child with variable x as well as in determining the number of buffaloes that will be obtained by the fourth child in which the subject uses linear inequalities to get answers.

Thus, it appears that the subject with HWMC used imitative mathematical reasoning, which in this case the subject uses algorithmic reasoning in solving the problems (Øystein, 2011). Algorithmic reasoning flow applied is by using algorithms to solve the problems.

Creative mathematical reasoning

Based on the results of the study, it can be seen that the subject with HWMC solved the problems by using the methods that the subject finds himself. The subject firstly drew four boxes with 21 sides then determines how many squares are needed to make the box. The subject then determined the area of the square box using the square area formula. After that, the subject divided the area of the plywood by the area of the box. Lastly, the subject calculated how many boxes can be made by using the drawing method.

The data are obtained through the interview below:

P: So, how did you solve the problem?

HWMC: I drew it first, ma'am.

P: What did you draw?

HWMC: Cage with 21 known sides in question

P: What did you draw the cages for?

HWMC: So that I could know how to get 21 sides to make the cage, ma'am.

P: Then what was the next step?

HWMC: I determined the area of small plywood used to make the cage.

P: How did you determine the area?

HWMC: With the square area formula, ma'am.

P: And then?

HWMC: I divided the area of big plywood to the area of small plywood.

P: What for?

HWMC: To find out how many small plywood that could be made into cages, ma'am.

P: Then what else did you do?

HWMC: I drew again the cage until the known small plywood ran out, ma'am.

From the interview above, it is revealed that the subject with HWMC solved the first mathematical problem by using creative mathematical reasoning. This can be revealed from the process of solving the problem by using a new line of reasoning (Lithner, 2015), namely knowing the number of plywood used to make cages by drawing. It was also done in the same way for the subject to be able to determine the number of cages that can be made with the 30 available small plywood. Thus, the subject with HWMC met the criteria of novelty in the reasoning process in solving the problem. Furthermore, the subject with HWMC was able to give the reason that his chosen method is the correct or reasonable way, and the used method can solve the given problem. This is revealed in the following interview transcript:

P: Are you sure that the way you use to finish is correct?

HWMC: Yes, ma'am.

P: What makes you so sure of that?

HWMC: Because by drawing, then the shape can be clearly seen.

P: Are you sure that the answer you get is the right one?

HWMC: Yes, ma'am.

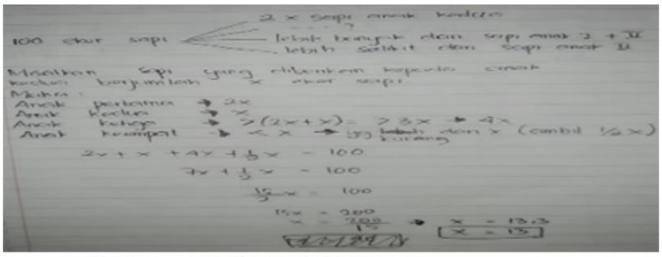
P: Why?

HWMC: Because based on the drawing the cage could be formed, ma'am.

P: Were not there 30 plywood given in the problem? Why did you only use 26 plywood?

HWMC: Because after making five cages, it took five more small plywood and the remaining plywood are only four; so, it is not enough ma'am.

The interview reveals that the subject with HWMC believed that the method used to solve the problem is correct because the drawing method produces a lucid idea of the cage. In accordance with the answers obtained by the subject with HWMC, he believed that the answer is correct because the image that he made shows the correct result. The subject with HWMC also stated that there are only five side-by-side cages that can be formed from the 30 available plywood. Although there were still pieces of plywood with a side of 30 cm left, but according to the subject with HWMC, it was not enough to make one more cage. Thus, it can be concluded that the subject with HWMC met the plausibility criteria in the process of solving the problem



100 cows : 2x of second child's cows
: ……?
: More than the first child, second child's cow
: Less than the second child's cows
If the amount of cows given to the second child is x then:
First child: 2x
Second child: x
Third child : >(2x+x) =>3x = 4x
Fourth child: <x = less than x (take $\frac{1}{2}$ x)

$2x+x+4x+\frac{1}{2}x=100$

$7x+\frac{1}{2}x=100$

$\frac{15}{2}x=100$

$15x=200$

$X=\frac{200}{15}$

X= 13, 3 =13

Figure 4. Subject with HWMC answer to the 2nd problem (Source: Authors' own elaboration)

(Schoenfeld, 2014). It is also revealed that the subject with HWMC used intrinsic mathematical properties to solve the problem (Schoenfeld, 2014). One of the properties used was a square area formula to solve the problem that is the concept that the area of a square is equal to the result of the multiplication of its sides. This means that the subject met the anchoring criteria.

For the second problem, namely in the case of dividing the inheritance of buffaloes to four children, the subject with HWMC used the concept of linear inequality as shown in **Figure 4**. From **Figure 4**, it can be seen that the subject first interpreted the terms of distributing buffaloes into mathematical forms using the concepts of linear equations and inequalities. After that, the subject solved the problem by using the summation of linear equations. The same thing was also stated by the subject at the time of the interview as in the following interview transcript:

P: How did you solve that?

HWMC: First of all, I identified the condition of dividing 100 buffaloes as the inheritance in accordance with the existing conditions in the question, ma'am.

P: Then what did you do next?

HWMC: Quantify the buffalo as x, ma'am

P: And then?

HWMC: Write all the conditions into a mathematical sentence.

P: Which mathematical sentence do you mean?

HWMC: Writing it into the form of linear equation in one variable, ma'am.

P: How was the notation?

HWMC: Which was x, 2x, greater than 2x+x and less than x?

P: And then?

HWMC: I added up all the initials and it is equal to 100.

P: What is that supposed to mean?

HWMC: I added up everything that's 2x+x+4x+½x=100.

P: How did you get 4x and ½x?

HWMC: That is the condition for the third child's buffalo, ma'am, because it has to be more than the first and second child buffalo, so it's more than 2x+x=3x, because it's more than 3x, so I take 4x ma'am.

P: Then what about the ½x?

HWMC: Because the fourth child's buffalo must be less than the second child and the second child's buffalo is as much as x then I took it as ½x, ma'am.

P: All right, so how did you get the number of buffaloes that the fourth child got?

HWMC: I summed it all, ma'am, then I got the x value, and the sum was 13.3.

P: So, the number of buffaloes the fourth child got is 13.3?

HWMC: No, ma'am, 13.

P: Why?

HWMC: Because there are no buffaloes in the form of a fraction.

From the result of problem solving and interview above, it can be revealed that the subject with HWMC solved the second problem by using creative mathematical reasoning. The subject with HWMC used a new flow of reasoning by using variable x for buffalo and using supposition of a variable to write the conditions specified in the question to solve the problem. The subject also determined the number of buffaloes received by the third child by summing up the number of buffaloes received by the first and second children and determining the number of buffaloes received by the fourth child, namely ½x. Thus, the subject's reasoning met the criterion of novelty. In addition, the subject gave the reason that the strategy used is in accordance with the intrinsic nature of mathematics. To determine the number of buffaloes obtained by the third child, the subject used 4x with the reason that 4x is more than 3x. Likewise to determine the number of buffaloes owned by the first and second children, the subject used the rule of summing one variable, namely by summing 2x with x. Similar to determine the number of buffaloes received by the fourth child, the subject took ½x with the reason that ½x is less than x. The subject with HWMC also gave the reason that the selected strategy is a correct and sensible strategy (Lester & Cai, 2016). This is revealed through the following interview transcript:

> P: Are you sure that the strategy you use can solve the problem?
>
> HWMC: Yes, ma'am.
>
> P: Why?
>
> HWMC: Because the number of buffaloes that will be divided is 100 buffaloes, so the condition used to determine the number that each child receives is that I sum them all up equal to 100.
>
> P: What is that supposed to mean?
>
> HWMC: I would say that the first child is 2x, the second child is x, the third child is 4x and the fourth child is ½x. So, I sum it all up to 2x+x+4x+½x=100.
>
> P: Are you sure that the strategy you use can solve the problem?
>
> HWMC: Yes, ma'am, I'm sure.
>
> P: What makes you so sure?
>
> HWMC: Because I have used all the conditions in the problem, and I can also determine the value of x to determine the number of buffaloes

From the interview transcript above, it is revealed that the subject HWMC met the criteria of plausibility, namely that the subject could provide predictive reasons that the selected strategy is a correct and sensible strategy (Wirebring et al., 2015). The subject explained that he firstly forms a linear equation of the terms of the distribution of inheritance in the given problem, and then applies the sum of one variable by summing the number of received by the first, second, third and fourth children, which are all equal to 100 to solve the problem. The subject could also provide a verification reason that the application of the chosen strategy can provide the correct answer. The subject stated that all the requirements proposed in the question have been used to solve the problem so that the application of the selected strategy is believed to provide the right answer.

The subject with HWMC also gave the reason that the selection of strategies is based on the intrinsic mathematics properties. This is stated in the following interview transcript:

> P: Are you sure that the completion steps you use correspond to the intrinsic properties of mathematics?
>
> HWMC: What kind of properties are that ma'am?
>
> P: Basic properties of mathematics that you use to solve problems.
>
> HWMC: Oh, do you mean about what am I going to do with the same number of variables, like that, ma'am?
>
> P: What is that supposed to mean?
>
> HWMC: This is 2x+x+4x+½x=100. I call the buffalo x, and the fourth child gets 100, so I sum up everything the first, second, third, and fourth child gets is equal to 100.
>
> P: Why is it denoted by x?
>
> HWMC: Because all that can be summed is the one with same variable, ma'am.

From the interview transcript above, it appears that the subject with HWMC knew that the summing variable method is eligible for the same variable only. Variable sum rules are used to solve the problem (Fan & Bokhove, 2014). Thus, the subject with HWMC met the anchoring criteria in solving the second problem. Thus, it can be seen that the subject with HWMC solved the second problem by using creative mathematical reasoning that meets three criteria, namely novelty, plausibility, and anchoring.

Based on the analysis of the data above, it is revealed that the subject with HWMC used mathematical reasoning in which the subject used algorithmic reasoning in solving problems. The algorithmic reasoning flow applied is by using algorithms to solve problems (Jonsson et al., 2016). In addition, the subject also used creative mathematical reasoning that meets three criteria, namely novelty, plausibility, and anchoring. Thus, it appears that students with HWMC have a more optimal performance in solving problems.

CONCLUSION

The subject with LWMC used creative mathematical reasoning and algorithmic reasoning in solving the given problems. On the other hand, the subject with LWMC used algorithmic reasoning by using sum and multiplication rules and using methods that have been done before to solve the given problems. Nevertheless, the subject with LWMC also used creative mathematical reasoning but the use of this reasoning was not optimal. This can be seen in the reasoning flow used, where the subject used a new reasoning flow (novelty) and was able to provide a predictive reason (plausibility), but the subject had difficulty providing a verification reason (plausibility) and the reason that the way of solving was in accordance with the intrinsic mathematical properties (anchoring).

The subject with HWMC on the other hand used algorithmic mathematical reasoning and creative mathematical reasoning. The algorithmic reasoning flow applied was by using algorithms to solve the given problems. In addition, the subject with HWMC also applied creative mathematical reasoning by using a novel strategy, providing predictive and verification reasons for the selection of strategies (plausibility), and providing convincing reasons that the completion steps used are in accordance with the intrinsic mathematical properties (anchoring).

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REFERENCES

Aineamani, B. (2018). How learners communicate their mathematics reasoning in mathematics discourse. In J. Moschkovich, D. Wagner, A. Bose, J. Rodrigues Mendes, & M. Schütte (Eds.), *Language and communication in mathematics education* (pp. 65-74). Springer. https://doi.org/10.1007/978-3-319-75055-2_6

Anjariyah, D., Juniati, D., & Eko, T. Y. (2022). How does working memory capacity affect students' mathematical problem solving? *European Journal of Educational Research, 11*(3), 1427-1439. https://doi.org/10.12973/eu-jer.11.3.1427

Baddeley, A. (2000). The episodic buffer: A new component of working memory? *Trends in Cognitive Sciences, 4*(11), 417-423. https://doi.org/10.1016/s1364-6613(00)01538-2

Bergqvist, E. (2007). Types of reasoning required in university exams in mathematics. *The Journal of Mathematical Behavior, 26*(4), 348-370. https://doi.org/10.1016/j.jmathb.2007.11.001

Boesen, J., Lithner, J., & Palm, T. (2010). The relation between types of assessment tasks and the mathematical reasoning students use. *Educational Studies in Mathematics, 75*(1), 89-105. https://doi.org/10.1007/s10649-010-9242-9

de Fockert, J. W., Rees, G., Frith, C. D., & Lavie, N. (2001). The role of working memory in visual selective attention. *Science, 291*(5509), 1803-1806. https://doi.org/10.1126/science.1056496

de Ron, A., Christiansen, I., & Skog, K. (2022). Configuring the landscape of research on problem-solving in mathematics teacher education. *International Electronic Journal of Mathematics Education, 17*(4), em0712. https://doi.org/10.29333/iejme/12457

Dröse, J., Prediger, S., Neugebauer, P., Delucchi Danhier, R., & Mertins, B. (2021). Investigating students' processes of noticing and interpreting syntactic language features in word problem solving through eye-tracking. *International Electronic Journal of Mathematics Education, 16*(1), em0625. https://doi.org/10.29333/iejme/9674

Fan, L., & Bokhove, C. (2014). Rethinking the role of algorithms in school mathematics: A conceptual model with focus on cognitive development. *ZDM, 46*(3), 481-492. https://doi.org/10.1007/s11858-014-0590-2

Fatmanissa, N., Kusnandi, K., & Usdiyana, D. (2020). Effect of schema use in solving word problems: Emphasis on linguistic difficulties. *Contemporary Mathematics and Science Education, 1*(2), ep20006. https://doi.org/10.30935/conmaths/8495

Fyfe, E. R., Matz, L. E., Hunt, K. M., & Alibali, M. W. (2019). Mathematical thinking in children with developmental language disorder: The roles of pattern skills and verbal working memory. *Journal of Communication Disorders, 77*, 17-30. https://doi.org/10.1016/j.jcomdis.2018.11.001

Habtamu, S. B., Mulugeta, A. A., & Mulugeta, W. G. (2022). The effect of cooperative problem-solving method on students' motivation towards learning algebra. *Pedagogical Research, 7*(2), em0123. https://doi.org/10.29333/pr/11906

Hershkowitz, R., Tabach, M., & Dreyfus, T. (2016). Creative reasoning and shifts of knowledge in the mathematics classroom. *ZDM, 49*(1), 25-36. https://doi.org/10.1007/s11858-016-0816-6

Hokor, E. K., Apawu, J., Owusu-Ansah, N. A., & Agormor, S. (2022). Preservice teachers' misconceptions in solving probabilistic problems.

Pedagogical Research, 7(1), em0112. https://doi.org/10.29333/pr/11441

Jonsson, B., Kulaksiz, Y. C., & Lithner, J. (2016). Creative and algorithmic mathematical reasoning: Effects of transfer-appropriate processing and effortful struggle. *International Journal of Mathematical Education in Science and Technology, 47*(8), 1206-1225. https://doi.org/10.1080/0020739x.2016.1192232

Jonsson, B., Norqvist, M., Liljekvist, Y., & Lithner, J. (2014). Learning mathematics through algorithmic and creative reasoning. *The Journal of Mathematical Behavior, 36*, 20-32. https://doi.org/10.1016/j.jmathb.2014.08.003

Julius, E. (2022). The relationship between self-concept and problem-solving skills on students' attitude towards solving algebraic problems. *Contemporary Mathematics and Science Education, 3*(2), ep22020. https://doi.org/10.30935/conmaths/12509

Juniati, D., & Budayasa, I. K. (2020). Working memory capacity and mathematics anxiety of mathematics students and its effect on mathematics achievement. *Journal for the Education of Gifted Young Scientists, 8*(1), 279-291. https://doi.org/10.17478/jegys.653518

Kane, M. J., Bleckley, M. K., Conway, A. R., & Engle, R. W. (2001). A controlled-attention view of working-memory capacity. *Journal of Experimental Psychology: General, 130*(2), 169-183. https://doi.org/10.1037/0096-3445.130.2.169

Langi, E. L., Juniati, D., & Abadi. (2021). Understanding definite integral concepts of prospective teachers through actions and processes based on gender difference. *Journal of Physics: Conference Series, 1747*(1), 012026. https://doi.org/10.1088/1742-6596/1747/1/012026

Lester, F. K., & Cai, J. (2016). Can mathematical problem solving be taught? Preliminary answers from 30 years of research. In P. Felmer, E. Pehkonen, & J. Kilpatrick (Eds.), *Posing and solving mathematical problems* (pp. 117-135). Springer. https://doi.org/10.1007/978-3-319-28023-3_8

Lithner, J. (2008). A research framework for creative and imitative reasoning. *Educational Studies in Mathematics, 67*(3), 255-276. https://doi.org/10.1007/s10649-007-9104-2

Lithner, J. (2015). Learning mathematics by creative or imitative reasoning. In S. J. Cho (Ed.), *Selected regular lectures from the 12th International Congress on Mathematical Education* (pp. 487-506). Springer. https://doi.org/10.1007/978-3-319-17187-6_28

Muzaini, M., Juniati, D., & Siswono, T. Y. (2019). Exploration of student's quantitative reasoning in solving mathematical problem: Case study of field-dependent cognitive style. *Journal of Physics: Conference Series, 1157*, 032093. https://doi.org/10.1088/1742-6596/1157/3/032093

Norqvist, M. (2017). The effect of explanations on mathematical reasoning tasks. *International Journal of Mathematical Education in Science and Technology, 49*(1), 15-30. https://doi.org/10.1080/0020739x.2017.1340679

Norqvist, M., Jonsson, B., Lithner, J., Qwillbard, T., & Holm, L. (2019). Investigating algorithmic and creative reasoning strategies by eye tracking. *The Journal of Mathematical Behavior, 55*, 100701. https://doi.org/10.1016/j.jmathb.2019.03.008

Øystein, H. P. (2011). What characterizes high achieving students' mathematical reasoning? In B. Sriraman, & K. H. Lee (Eds.), *The elements of creativity and giftedness in mathematics* (pp. 193-215). Springer. https://doi.org/10.1007/978-94-6091-439-3_13

Palengka, I., Juniati, D., & Abadi. (2019). Creative mathematical reasoning of prospective teachers in solving problems reviewed based on working memory capacity. *Journal of Physics: Conference Series, 1417*(1), 012055. https://doi.org/10.1088/1742-6596/1417/1/012055

Palengka, I., Juniati, D., & Abadi. (2021). Mathematical reasoning structure of junior high school students in solving problems based on their working memory capacity. *Journal of Physics: Conference Series, 1747*(1), 012023. https://doi.org/10.1088/1742-6596/1747/1/012023

Portaankorva-Koivisto, P. M., Laine, A. T., & Ahtee, M. (2021). Two primary teachers developing their teaching problem-solving during three-year in-service training. *International Electronic Journal of Mathematics Education, 16*(1), em0624. https://doi.org/10.29333/iejme/9617

Ramdani, R., Syamsuddin, A., & Sirajuddin, S. (2019). Development of mathematical module-problem solving approach to train student's reflective thinking. *Pedagogical Research, 4*(4), em0040. https://doi.org/10.29333/pr/5861

Schoenfeld, A. H. (2014). What makes for powerful classrooms, and how can we support teachers in creating them? A story of research and practice productively intertwined. *Educational Researcher, 43*(8), 404-412. https://doi.org/10.3102/0013189x14554450

Sidenvall, J., Lithner, J., & Jäder, J. (2014). Students' reasoning in mathematics textbook task-solving. *International Journal of Mathematical Education in Science and Technology, 46*(4), 533-552. https://doi.org/10.1080/0020739x.2014.992986

Sumpter, L. (2015). Investigating upper secondary school teachers' conceptions: Is mathematical reasoning considered gendered? *International Journal of Science and Mathematics Education, 14*(S2), 347-362. https://doi.org/10.1007/s10763-015-9634-5

Swanson, H. L. (2017). Verbal and visual-spatial working memory: What develops over a life span? *Developmental Psychology, 53*(5), 971-995. https://doi.org/10.1037/dev0000291

Wiley, J., & Jarosz, A. F. (2012). How working memory capacity affects problem solving. In B. H. Ross (Ed.), *Psychology of learning and motivation* (pp. 185-227). Academic Press. https://doi.org/10.1016/b978-0-12-394393-4.00006-6

Wirebring, L. K., Lithner, J., Jonsson, B., Liljekvist, Y., Norqvist, M., & Nyberg, L. (2015). Learning mathematics without a suggested solution method: Durable effects on performance and brain activity. *Trends in Neuroscience and Education, 4*(1-2), 6-14. https://doi.org/10.1016/j.tine.2015.03.002