Mathematics Enrichment for All – Noticing and Enhancing Mathematical Potentials of Underprivileged Students as An Issue of Equity

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ABSTRACT

Whereas equity issues are mainly discussed with respect to students at risk, this article focuses on mathematical potentials of under-privileged students and therefore elaborates a wide, dynamic and participatory conceptualization of (sometimes still hidden) mathematical potentials. An extended research review theoretically and empirically grounds the presented approach for uncovering and enhancing (possibly underprivileged) students in design principles for the instructional design and the ways in which teachers notice students' and situations' potentials. Dual design research methodology on students' and teachers' level is adopted to develop whole class enrichment settings with rich tasks and empirically study the initiated processes. The empirical investigation of the classroom processes show that the chosen design principles can enhance the intended enrichment processes on the student side, but need to be strongly supported by teachers' expertise in noticing and fostering students. An important outcome is the perspective model for teachers' necessary diagnostic perspectives for noticing and enhancing the potentials. Consequences are formulated for professional development programs.

Keywords: Mathematical Potentials, Task Design, Teacher Noticing

INTRODUCTION

For a long time, the mathematics education discourse about equity and the discourse about mathematics enrichment for students of high potential were conducted rather separately: On the one hand, the discourse on equity and access mainly referred to offering access to a minimum of basic mathematics, without systematically taking into account that there might be high potentials among the underprivileged students. Only recently, the underprivileged access to tertiary education came in view (Pateman & Lim, 2013).

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State of the literature

- Mathematical potentials for gifted students is characterized by multiple facets and in a dynamic way, well underpinned with empirical research, but only for a small top group of students
- Different design principles for enrichment settings are well established, although mainly in extra-curricular settings
- Equity issues are mainly discussed with respect to students at risk, not yet to mathematical potentials, although underprivileged students are repeatedly shown to be underrepresented in gifted programs

Contribution of this paper to the literature

- A wide, dynamic and participatory conceptualization of (sometimes still hidden) mathematical potentials is extended to a wider group of students, with a specific focus on underprivileged students
- Seven design principles for heterogeneous whole class enrichment settings are specified and investigated in a dual design research project with respect to the possibilities for noticing and enhancing students' hidden potentials

On the other hand, Leikin's (2011) insightful overview on giftedness and creativity mentions issues of equity only as contradicting the idea of ability grouping for gifted students (p. 179). This gap is slowly overcome by newer research and development that also focuses on potentials among underprivileged students (Suh & Fulginiti, 2011).

In this article, we plead for intensifying the connection between these two discourses and contribute to the search for ways of mathematics enrichment for all students in order to uncover and foster potentials also among underprivileged students and those who are not immediately identified as high potentials. We present design principles for enrichment settings allowing to activate, notice and enhance mathematical potentials and describe requirements for diagnostic perspectives of teachers. For this, we rely on data of a design research project in German grade 8 classrooms dealing with context-relevant and mathematically rich problem situations.

We first present the theoretical background (Section 1) and the design principles for fostering underprivileged students with mathematical potential (Section 2). We present the dual design research methodology of the project (Section 3) from which we draw some empirical insights into our work (Section 4).

THEORETICAL BACKGROUND: EQUITY FOR STUDENTS WITH MATHEMATICAL POTENTIALS

Extending equity concerns to higher education access

Due to the social disparities found in school success rates and large scale assessments, equity is an increasingly discussed issue (DIME, 2007; Secada, 1992; Pateman & Lim, 2013)

which has recently also reached Germany. Pateman and Lim define: "For mathematics learning to be equitable and accessible, all students, regardless of social and cultural background, gender, religious beliefs, ethnicity, geographical location, and family financial circumstances, should have the same 'opportunity to learn'" (Pateman & Lim, 2013, p. 244).

In many countries, equity and the necessary opportunities to learn are mainly discussed with respect to low achieving underprivileged students and their chances to get some access to basic mathematics (DIME, 2007). Only in the last years, an extension has been reached by considering also the participation rates of different socio-economic and ethnic groups in advanced secondary mathematics courses as an additional "measure of equity and access" (Pateman & Lim, 2013, p. 244). This is of particular importance due to the role of mathematics as a gate keeper for higher (secondary and tertiary) education, especially in the field of science, technology, engineering, and mathematics (STEM) for which the job prospects are good in many countries.

For the European societies, this shift in equity concerns from basic access to access to higher STEM-education is not only inspired by democratic ideas, but also by economical requirements raised by the huge economic need of STEM academics in the technical civilization (Taskinen, 2010). Whereas the small number of STEM academics needed in former times could be recruited among privileged students alone, the increased need of STEM professionals also calls for opening up the social groups of students to look for. This calls for a wider conceptualization of mathematical potentials.

Conceptualizing 'mathematical potential'

The construct mathematical potential is often referred to students "who can achieve a high level of mathematical performance when their potential is realized to the greatest extent" (Leikin, 2009a, p. 388) and characterized as follows: "In sum, the mathematical potential of a student includes abilities (analytical and creative), affective factors (…), commitment and other personal characteristics, and multiple opportunities" (Leikin 2009a, p. 390; similarly in Sheffield, 2003 and many other sources). This conceptualization can be transferred from the top 2 % to a wider group. With Suh & Fulginiti (2011), we assume that mathematical potential

- has a "dynamic nature" (Leikin, 2009a, p. 388) so that it can be fostered in students, whereas talent tends to have a flavor of being "gifted", i.e. given by nature (cf. Sheffield, 2003 for this difference)

- might not yet be seen in constantly good performances but appear situationally (Sheffield, 2003) – thus we call them 'hidden' potentials

- refers to about 20 % of all students, thus are less exclusive than usual (the terms "talented" or "gifted" are often referred to only the top 2 % or 1 %).

While Leikin (2011) relates the construct mathematical potential to learning situations, not to students, we use it for both, the students and the learning situation, because we see both connected: Only if the situation has a potential of becoming mathematically rich, then the student can show some potentials. And in longer-term perspectives: if the student experiences his or her mathematical potential in a mathematically rich learning situation, then the potential can become a stable characteristic of the student in the long run.

Equity issues in the discourse about students with high potentials

The discourse about equity issues with respect to gifted students had three phases: In the 1980s and 1990s, the equity principle was "(mis-)interpreted as a recommendation to provide all students with identical instruction" (Leikin, 2011, p. 179), without room for fostering gifted students. But equity also entails adaptivity of learning opportunities, so it was later interpreted as fitting to individual potentials, including the possibility of selective ability grouping for students identified as gifted (Leikin, 2011; Sheffield, 2003).

Only in the last decade, students' diverse backgrounds received more attention, when teachers and researchers became increasingly aware of racial, social and gender gaps: girls and students from underprivileged racial or social background are often underrepresented in programs for gifted students (cf. overview on empirical results in Suh & Fulginiti, 2011; DIME, 2007 for the general participation gap). The delayed shift of attention is astonishing because already in 1990, Lubinski and Humphreys pointed out that even though mathematical talent is in principle spread across all socio-economic states (SES), students with higher SES are more likely to have successful academic careers than those with low SES background. They warned "a search for talent must cast a wide net. And this fact is not emphasized nearly enough in contemporary investigations" (Lubinski & Humphreys, 1990, p. 340). In the last decade, their early call for interventions directed at students with lower SES is increasingly heard and also extended to the background factor gender: For the German context, Benölken (2011, 2015) shows the massive underrepresentation of girls in talent programs and calls for specific approaches.

In order to reduce the underrepresentation of students with underprivileged social background and of girls, it is necessary to identify the obstacles for these groups to unfold their potential. Interestingly, Benölken (2011, 2015) and Suh and Fulginiti (2001) identified similar obstacles for both groups:

- biased assessment measures (Suh & Fulginiti, 2001, p. 67: e.g. language bias for migrant students, rarely for girls);

- low-self-esteem and expectation set by the individual or others (Suh & Fulginiti, 2001, p. 6; Benölken, 2011; for girls an often described phenomenon, cf. Leder, 1992);

- the lack of strong advocacy or referral from parents and teachers (Suh & Fulginiti, 2001, p. 6; Benölken, 2011);

- missing interest among the adolescents (Benölken, 2015; similarly Piggott, 2004).

Thus, potentials can be sometimes hidden to students and their parents. The approaches for fostering these underprivileged groups should therefore be situated within whole-class settings rather than extra-curricular activities (Sheffield, 2003) in order to identify the potentials without the help of parents or formal tests (cf. next section). Furthermore, it must be connected to fostering self-esteem and interest.

Interest in mathematics has repeatedly been identified as a critical factor, not only for unfolding mathematical potentials (Benölken, 2015; Kruteskii, 1976, p. 347), but also for mathematical success in the long run (Leder, 1992), and for later choosing a mathematics-related STEM-study program (Taskinen, 2010). Theories of interest refer to both, affective and cognitive factors in situational or personal dimensions (Krapp, 2005). Interventions for raising interest can successfully be grounded on the self-determination theory according to which interest development can be enhanced when three basic psychological needs are taken into account: experiences of (1) competence, (2) autonomy in learning situations and (3) social relatedness (Deci & Ryan, 2002; Krapp, 2005). For situational interest to emerge in classroom interactions, Bikner-Ahsbahs (2014) has specified conditions of so-called interest-dense situations: situations can become interest-dense when students become intensively involved in a task; if they progress in deep constructions of knowledge; and if they value the activity highly. This is specifically the case when teachers and learners engage in discussions about students' thinking (Bikner-Ahsbahs, 2014). Krapp (2005) shows that repeated experience of situational interest can enhance the stabilization into personal interest as a more stable disposition.

Lessons to be learnt from mathematics enrichment programs and research on high potentials

Although our target group is widened compared to most gifted programs, there are some lessons to be learnt from mathematics enrichment programs and research on mathematically gifted students. They concern (1) facets to identify potentials, (2) general approaches of fostering students with potential, and (3) design features for enrichment settings.

Facets of mathematical potential

The research on gifted students has a long tradition in specifying characteristics for identifying talents, starting with Krutetskii (1976). Today, researchers emphasize the need to take into account many different facets rather than relying only on psychometric measures of intelligence or mathematics tests, because students with potentials are characterized by the interplay of several attributes (Käpnick, 1998; Sheffield, 2003; overview in Singer, Sheffield, Freiman, & Brandl, 2016) However, not all facets have to be equally developed; some researchers found students with very specific giftedness, excelling only in certain attributes (e.g. Käpnick, 1998, p. 66ff.). As potential is conceptualized to appear situationally (last

section), the facets and related constructs are identified within specific situations rather than as stable dispositions per se. **Table 1** outlines a state-of-the-art overview of facets which were identified in a literature research.

Table 1. Five facets of mathematical potential – research overview

Facets	Selection of related constructs	Selected relevant literature
Cognitive facet	• Conceptual understanding • Procedural fluency • Strategic competence • Adaptive (logical) reasoning • Generalizing, • Recognizing and making use of patterns and mathematical structures, • Dealing with and changing different representations, • Problem solving, hypothesizing or finding related problems. • • …	(Bauersfeld, 1993; Käpnick, 1998; Krutetskii, 1976; Kilpatrick, Swafford, & Findell, 2001; Sheffield, 1994)
Meta-cognitive facet	• Planning • Monitoring • Evaluating • …	(Cheng, 1993, Schoenfeld, 1992)
Personal & affective facet	• Mathematical self-concept, self-efficacy & self-confidence • Interest in mathematics • Commitment & persistence • Creativity • …	(Bauersfeld, 2002; Benölken, 2011, 2015; Goldin, 2009; Leikin, 2009a; Leikin, Berman, & Koichu, 2009; Rinn et al., 2010; Vygotsky, 1930/1984)
Communicative & linguistic facet	• Complex argumentation • Deep discursive involvement • …	(Bauersfeld, 1993, 2002; Vygotsky, 1996)
Social facet	• Cooperative skills • Social involvement • …	(Diezmann & Watters, 2001; Leikin, Berman, & Koichu, 2009; Swan, 2008)

Krutetskii's (1976) foundations for identifying specific cognitive characteristics of mathematically gifted students have constantly been refined (e.g. Bauersfeld, 1993; Sheffield, 1994; Käpnick, 1998). Among these characteristics in the cognitive facet are for example an affinity for generalizing, recognizing and making use of patterns and mathematical structures, dealing with and changing different representations, problem solving, hypothesizing or finding related problems.

With respect to abilities in the meta-cognitive facet, empirical studies show how excelling in problem solving is intertwined with competences in reflecting the own cognition, in conscious regulations and control of working process (e.g. Schoenfeld, 1992).

Certain general attributes of a personal facet such as persistence, commitment or intellectual curiosity are also regarded as possible characteristics for mathematical potential (Krutetskii, 1976; Käpnick, 1998; Bauersfeld, 2002). Especially creativity has been given a lot of attention as one characteristic deeply intertwined with mathematical potential (e.g. Vygotsky, 1930/1984; Leikin, 2009a; Leikin, 2011).

Additionally, researchers emphasize the influence of affective factors (Leikin, Berman, & Koichu, 2009; Leikin, 2009a; Goldin, 2009; Benölken, 2015), such as intrinsic and extrinsic motivation, beliefs and attitude towards mathematics. The mathematical self-esteem also belongs to this facet; it is especially important as low self-efficacy and low self-concepts in mathematics tend to correlate with lower performances (e.g. Pajares, 1996).

As mathematical thinking is always intertwined with language aspects (Vygotsky, 1996), a communicative and linguistic facet must also be considered. Mathematical potential thus can become apparent e.g. in students' exceptional verbal or written expressions of mathematical relations and the depth of their discourse activities. Although Bauersfeld (1993) points out the specialty of language in the thinking of (outstandingly) gifted students, he also describes cases where students' lack the competences to express their highly developed mathematical thoughts (Bauersfeld, 2002).

Lastly in this (probably not complete) list of facets is the social facet, which includes constructs such as cooperative skills or social involvement. As Diezmann & Watters (2001) point out, gifted students tend to start fruitful collaborations when working on mathematically challenging tasks, which includes shared cognition and critical thinking together. But this is also a typical example of a facet which does not apply to all students (Käpnick, 1998).

General modes of fostering students with potentials

The international discourse about fostering students with potentials distinguishes two general modes, both for heterogeneous whole class settings as well as for more homogeneous ability groups in and out of school (cf. overview in Leikin, 2011; Singer et al., 2016): Acceleration versus enrichment. Acceleration means learning mathematics in accelerated pace (mainly by taking special courses ahead of the normally scheduled year). Enrichment means to expand students' experiences and skills by exposing them to rich learning processes. It has been realized as enrichment by broadening (i.e. learning additional mathematical topics or subjects, mainly in special courses out of school) or enrichment by deepening (i.e. staying with the same topics, but enhancing the depth and complexity of the treated aspects and practices).

We chose the mode of enrichment by deepening as it fits best with regular classrooms (Piggott, 2004), when all students are engaged in open tasks and practices, and those with potentials are supposed to expand their experiences and skills and provide them with fulfillment. In contrast to broadening approaches in extracurricular activities, the enriched topics and tasks for deepening are chosen mostly in-line with the normal curriculum (Sheffield, 2003).

Design features for enrichment settings

Most researchers agree that the key design feature for enrichment settings is a mathematical challenge. It can appear in different forms: "proof tasks in which solvers must find a proof, defining tasks in which learners are required to define concepts, inquiry-based tasks, and multiple-solution tasks.[… It comprises] conceptual density, mathematical connections, the building of logical relationships, or the balance between known and unknown elements" (Leikin, 2011, p. 180, more details in Sheffield, 2003). Thus, mathematical challenges go beyond the classical problem-solving tasks. Like Leikin (2011), other authors also plead for placing substantial mathematical ideas at the core of different task formats to allow for "enriching concepts (new types of tasks, new mathematical topics), extending the concepts' possible connections (by applications and analogies …)" (Bauersfeld, 1993, p. 262, translated by the authors). However, deepening mathematical concepts and substantial activities can be achieved either by working on context-free mathematical problems (e.g. proofing geometrical relations) or real-world problems. The latter are called 'model-eliciting tasks' (Lesh & Doerr, 2003), in which students make mathematical descriptions of meaningful real-world situations, e.g. by "quantifying, dimensionalizing, coordinatizing, categorizing, algebratizing and systematizing relevant objects, relationships, action, patterns and regularities" (Lesh & Doerr, 2003, p. 5). Overall, these rich tasks should allow for a variety of explorations and discoveries on different levels of mathematical expertise so that they can be used in whole classroom lessons. This resonates with the general discourse on substantial learning environments (Wittmann, 1995) or rich tasks (Flewelling & Higginson, 2003).

Additionally, Swan (2008) emphasizes the importance of student collaboration for rich tasks, discussing multiple solution pathways and drawing on collective knowledge. He stresses the importance of using multiple representations, focusing on (rather than avoiding) conceptual obstacles, building on the knowledge that students already have and creating tension and cognitive conflict to be resolved through collaborative discussion.

Leikin (2009b) has shown in several studies how multiple solution tasks can foster the richness of mathematical potential. Additionally, multiple-solution problems have been shown to foster interest by allowing experiences of autonomy and competence (Schukajlow & Krug, 2014). Furthermore, researchers (Silver, 1994, 1997; Sheffield, 2006; Leikin & Lev, 2007) emphasize that besides finding (multiple) solutions, problem posing is an especially fruitful activity that fosters creative mathematical thinking. Not only is the generation of new

or further questions "central to the discipline of mathematics and the nature of mathematical thinking" (Silver, 1994, p. 22), but it can also serve as a means to improve students' problem posing skills, help noticing their conceptual understanding and improve their attitude towards perceiving mathematics as a creative activity (Silver, 1994, 1997).

PRINCIPLES FOR UNCOVERING AND FOSTERING UNDERPRIVILEGED STUDENTS WITH MATHEMATICAL POTENTIALS

The summary of the literature allows to characterize our approach for uncovering and fostering (possibly underprivileged) students with mathematical potentials by principles for the instructional design and the teaching in situ, i.e. the ways in which teachers notice students' and situations' potentials and in which they interact with the students. From this we derive design challenges and research questions with respect also to teachers' professionalization.

Fostering by the task design and pedagogy – process-oriented mathematical enrichment settings in whole classrooms

Based on general concerns on equity, a wide dynamic and participatory conceptualization of students with mathematical potential and the empirical findings on underprivileged students being underrepresented in enrichment programs, we justified the need for uncovering and fostering them. From the literature on fostering potentials and possible obstacles for underprivileged students, we derive seven strongly intertwined design principles as resumed in **Table 2**.

The design principles (P1), (P2), and (P4) have been theoretically and empirically grounded in the previous sections. The differentiation principle (P3) is a direct consequence of the whole class settings: As the setting does address students with and without potentials, the problem must be accessible for all students (which also avoids an untimely in-class-selection). At the same time, it must provide "ramps" (Hengartner et al., 2006) for students to unfold their potentials (the idea of "low-entry–high-ceiling" has already been formulated by Shade, 1991). These ramps lead to natural extensions into the depth of rich mathematics (Foster, 2015). The differentiation principle might be realized by a level structure of the worksheet, but more profoundly by self-differentiating rich problems (Foster, 2015).

The dynamic and sometimes fragile nature of (hidden) potentials calls for teachers' high sensitivity on the quality of students' processes (P6) and the ability to spontaneously extend the tasks by enriching prompts (Sheffield, 2003). The processes are supposed to be cognitively demanding on different levels (P3) and thereby to allow experiences of autonomy and competence (P5). The principle of raising positive engagement is intertwined with the need for social relatedness (P7) and with creating diagnostically valid situations for identifying mathematical potentials. It is realized in cooperative and communicative settings (e.g. Swan, 2008).

Table 2. Seven design principles in instructional designs for fostering mathematical potentials

Context conditions and challenging aspects		Design principles
Potentials of underprivileged students are often hidden, thus no prior selection possible	→	(P1) Place enrichment settings in heterogeneous whole class settings (Sheffield, 2003)
Whole class must stick to curriculum without wasting time	→	(P2) Deepen topics close to the standard curriculum (Piggott, 2004)
All students should be involved, with and without potentials	→	(P3) Find self-differentiating problems with low entrance and high ceiling (Foster, 2015; Shade, 1991)
Potentials need to be challenged	→	(P4) Create mathematical challenges by rich mathematical problems (e.g. by complex open problems or by problem posing) (Leikin, 2009a/b; 2011; Suh & Fulginiti, 2011; Silver, 1994)
Interest and self-esteem of those students with potentials are possibly still fragile	→	(P5) Establish experiences of autonomy and competence by openness of problems, adaptivity and feedback (Deci & Ryan, 2002; Krapp, 2005)
(Hidden) potentials have a dynamic nature	→	(P6) Focus on cognitively demanding processes rather than perfect products (Leikin, 2009a; Bauersfeld, 1993) with teachers' spontaneous prompts to extend the mathematical richness (Sheffield, 2003)
Interest-dense situations are most appropriate for identifying potentials	→	(P7) Raise positive engagements in cooperative discussions of students' thinking (Bikner-Ahsbahs, 2014; Swan, 2008)

Whereas the principles (P1) and (P2) refer to the overall organization of the enrichment setting, the principles (P3-P5) are mainly addressed by the design and preparation of the lessons. In contrast, the principles (P6) and (P7) are of highest importance during the lesson itself. This calls for considering fostering in the interaction.

Fostering in the interaction – Teachers' role in noticing and fostering students

The main aim of the designed enrichment setting is to engage the whole class in rich activities and then foster those students with hidden mathematical potentials. This puts the teacher in an important role for the classroom interaction: Already for everyday classroom settings with rich problems, teacher's facilitation of small group and whole group discussions is crucial for reaching mathematical depth and for connecting multiple ideas (Swan, 2008). For the intended enrichment, the teacher is additionally supposed to identify situational potentials even if they only occur occasionally or in just partly correct contributions of the students. Thus, noticing and moderating are essential skills for teachers to make the most of an enrichment setting.

Especially positive engagement and experiences of autonomy and competence (principles P5 and P7) cannot be enhanced by the task design alone, as it mainly depends on

the teacher's facilitation in the classroom, e.g. for giving suitable feedback (Benölken, 2011) and to allow for early successes, which contribute to learners' motivation (Foster, 2015). However, this is not at all a trivial task: "We need to know how to recognize creativity and giftedness in a person before he or she has a chance to demonstrate outstanding mathematical achievements" (Leikin, Berman, & Koichu, 2009, p. vii). This is not only true for a stable outstanding dispositions, but even more for identifying mathematical potential during the process of working on a challenge before (or sometimes without) reaching completely correct solutions by paying attention to half-baked ideas or discontinued approaches.

But what do teachers need for meeting these expectations? It is part of our dual design research project to specify or refine the professional demands for noticing and moderating in these enrichment settings.

Design challenges and research questions

Leikin (2011) repeatedly emphasizes the need for empirical research on talent and creativity. This need for empirical research applies even more to enrichment by deepening in heterogeneous whole class settings for underprivileged students with hidden mathematical potentials. Summing up the state of research, there is not only a necessity to develop enrichment settings for underprivileged students, but also a need for further research. This research must not only focus on the students, but also on the professional demands for the teachers (as emphasized also by Leikin, 2009a). We therefore pursue two research questions, with a focus on Q1 in this article:

Q1 on Students. What kind of students' processes does an enrichment setting initiate, and which facets of students' mathematical potentials can become apparent?

Q2 on Teachers. What can teachers do in order to support the processes and to foster the potentials?

DUAL DESIGN RESEARCH METHODOLOGY

Methodological framework of dual design research

The described design challenges and research questions call for a research methodology in which design and research are systematically combined, with respect to the students' processes as well as the professionalization necessities for teachers. That is why we work in a design research methodology (Prediger, Gravemeijer, & Confrey, 2015), here specifically as Dual Design Research (Gravemeijer & van Eerde, 2009) by which a knowledge base can be built for both layers, students and teachers. In iterative design research cycles, the processes of students and teachers in the designed enrichment settings as well as in professional development sessions for the teachers are investigated in order to optimize the enrichment setting and to gain insights into typical challenges, learning processes and

necessities for teachers to support those processes (cf. Prediger, Schnell, & Rösike, 2016, for more details on the framework for teachers' professional development).

Methods of data gathering

In our on-going dual design research project *DoMath,* the two authors (as researchers) worked so far with two groups of teachers from urban schools in underprivileged areas (in the first year with 5 teachers, in the second year with 20 new teachers). During one year, the group of teachers and researchers jointly develop enrichment settings for uncovering and fostering potentials, which are then tested within the teachers' own classrooms. The teaching learning processes are videotaped, and selected video scenes are discussed in the mode of video clubs (Sherin, 2007) in meetings for professional development. Iterativity of design experiment cycles is realized in three ways: (1) as mini cycles for the same enrichment setting, when one teacher learns from the other, (2) as cycles between different enrichment settings, and (3) between the two groups of teachers in the first and second year.

In the following section, we report on one enrichment setting called "flexible percentages in soccer journalism" realized in five classes (grade 6-9) with the first group of five teachers, altogether with approximately 150 students. The five teachers each dedicated 90 to 240 min for implementing the enrichment setting in their classrooms. The lessons were completely video-taped, with a focus on the student groups' work with several cameras. Intermediate results were discussed with the teachers during 2 hours of video club meeting. The data corpus comprised roughly 20 hours of video material from classrooms and 2 hours from teacher video club meeting as well as all students' notes.

Methods of data analysis

The qualitative data analysis focused on

(1) the facets of students' potentials which become apparent in their working processes,

(2) effects of the design elements and how they correlate with their intentions,

(3) teachers' perspectives and enacted strategies in noticing and fostering their students, as well as

(4) other teachers' perspective and uttered strategies while watching video clips from the classrooms.

For focus (1) and (2), the categories derived from literature research (**Table 1** and **Table 2**, see above) served as foundation for the transcript analysis which was iteratively refined and optimized for reaching interrater agreement. Focus (3) and (4) required an explorative approach by more in-depth reconstructions of teachers' professional vision in order to generate categories from the transcripts (Sherin, 2007 for methodological concerns, and Prediger, Schnell, & Rösike 2016 for details). The findings were compared and contrasted with similar and different scenes and thus further elaborated – this process is at the moment

still on-going. For consensual validity, all analyses were conducted and/or discussed among a group of researchers.

INSIGHTS INTO FIRST DESIGN AND RESEARCH OUTCOMES ON FOSTERING UNDERPRIVILEGED STUDENTS

Exemplary design outcome:
Enrichment setting "Flexible percentages in soccer journalism"

One example for the realization of the design principles is the enrichment setting "Flexible percentages in soccer journalism", developed for whole classes (design principle P1) of grade 7-9 (students of age 13-15). The curricular goal (design principle P2) is to deepen and flexibilize students' conceptual understanding of percentages (for varying referent wholes, cf. Parker & Leinhardt, 1995). The problems arise in the real-world context of statistics from soccer world cups.

In this setting, students are first confronted with a worksheet containing statements (of a fictional, not very reliable newspaper) that are partly incorrect or make no sense in the context (cf. **Figure 1**). Students are asked to evaluate the statements by means of large data tables (from which a short extract is also printed in **Figure 1**). The students' core activity then is to create statements themselves, which are correct, incorrectly calculated, or correctly calculated but meaningless in the context. Errors and meaningless statements must be hidden well to 'fool' other students. Within the classroom, the best headlines are then collected and forwarded to other groups of students, who analyze and evaluate them.

Incorrect or useless statements about soccer performance
In the weeks before the World Soccer Cup 2014, people are often exposed to percentage information about soccer results.
Many journalists use data to extract percentages and use them for evaluating teams. Many of the used percentages are weird….

Germany is especially fair!
In 2010, less than 15 % of all German fouls were punished with a yellow card.

Why didn't we win?
Germany scores in the World Cup 2010 50 % more goals than the winner Spain!

(Correct but meaningless, as most other countries had lower percentages and the absolute number of German yellow cards is quite high)

Germany won more than 70 % of all games played at the World Cup 2010.

(Incorrectly calculated, Germany had 16 goals and Spain 8.)

(Correct, 5 out of 7 games were won.)

Results of the the FIFA World Cup 2010

final ranking	team	played	wins	draws	losses	goals for	goals against	goal difference	points	fouls	yellow cards	2nd yellow card	red cards
1	Spain	7	6	0	1	8	2	6	18	81	8	0	0
2	Netherlands	7	6	0	1	12	6	6	18	126	22	1	0
3	Germany	7	5	0	2	16	5	11	15	75	11	1	0
4	Uruguay	7	3	2	2	11	8	3	11	99	9	1	1
5	Argentina	5	4	0	1	10	6	4	12	65	7	0	0

Figure 1. Introduction of task, selected headlines and extract of data table (translated and evaluations added)

The enrichment setting reflects the formulated design principles as it creates mathematical challenges and room for creativity (principle P4) and initiates cognitively demanding processes (P6). Rather than just calculating percentages, the students have to analyze, explain and evaluate, which values can be used for (meaningful or not meaningful) statements and which errors could be hard to find, for example as they are common for other students to make (such as mixing base and amount up). The demand for 'cleverly hidden' mistakes is highly self-differentiating (P3) as it can be interpreted in a basic way (low entrance) and can encourage students with mathematical potential to use their deeper conceptual knowledge of percentages (high ceiling). This enrichment setting was created and tested in school a few weeks before the World Cup 2014, so that soccer was an appealing context for nearly all students. Beyond the context, positive engagement was intended to be enhanced by cooperative work in small groups of three or four (P7) on the creative activity of problem posing (P4). The experience "I can manipulate with mathematics and can identify manipulations in the newspaper" offered opportunities for experiencing autonomy and competence (P5).

Empirical snapshot into students' processes and potentials in the enrichment setting

In this section, two scenes provide empirical insights into how students deal with the "flexible percentages in soccer journalism" enrichment setting. Scene 1 shows that the setting bears opportunities for deep mathematical reflections beyond the authentic context.

Scene 1. 100 % more yellow-red cards

Olaf, Gina and Lara (grade 9, about 14 years old) evaluate a statement (written by another group of students) comparing the numbers of second yellow cards in the data table (0 yellow cards for Portugal and 1 for Germany): "Germany was given 100 % more second yellow cards than Portugal at the World Cup 2010":

1 Olaf   That's correct.

2 Gina   No, zero times zero is zero

3 Olaf   Yeah, but this is 100 % more second yellow cards

4 Gina   Then it's infinitely more cards

5 Lara   The base is zero. You can't divide one by zero, it's impossible.

While creating the statement, the other group deliberately included the mistake '1 is 100 % more than 0'. And indeed, Olaf agrees with the statement, in line 3. Gina and Lara argue against it and show a deep discursive involvement (communicative facet). The girls draw on their conceptual knowledge of percentages (cognitive facet). In turn 2, Gina tries to convince Olaf by referring to the multiplicative relation within percentages (base multiplied by rate equals amount). Then she and Lara switch to an argument about the division by zero.

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Gina shows an interesting mathematical potential by relating calculation arguments to relational arguments about a forbidden referent whole for percentages (cognitive and communicative facet). It is typical for students with high potential to find these deep mathematical issues (e.g. Leikin, 2011).

Scene 2. Discussing the referent whole

The group of four students (grade 8, about 13 years old) in Scene 2 shows much more fragile potentials: While creating correct statements, they intend to compare the 22 yellow cards of the Netherlands with Portugal's 8 yellow cards. As Leon and Oliver disagree in their drafts for statements, they have an intense discussion.

1	Oliver	What do you want to calculate?
2	Leon	How much more percent of yellow cards did the Netherlands get than Portugal?

…

10	Oliver	(…) In total, there are 30. A[mount] divided by B[ase], A divided by B, A by B, A by B. 22 divided by 30 that is [*uses calculator*]
11	Leon	[*uses calculator*] is 73 percent, then? No
12	Oliver	Yes
13	Leon	No
14	Oliver	Yes
15	Leon	Uh-uh, it is wrong for this reason *(looks at data table)* here, where is it. See, it is already more than the double. And 73 – that is the error in thinking – it is not half of it more, because 50 percent is only the half
16	Oliver	I don't say that, though
17	Viola	How many statements do we have to [create]?
18	Leon	But it is the double and the double is 100 percent, not 50

...

25	Oliver	[*uses calculator*] 26.6 period. And then period minus period [*mumbles incomprehensibly*] *(10 sec)*
26	Leon	[*mumbles*] I'll make a second one with the half now
27	Oliver	47.3
28	Leon	Eh Portugal [*writes*]

…

30	Oliver	47.3 percent more
31	Leon	47? Or 74? [Looks at calculator in Oliver's hand]
32	Oliver	47

…

34	Leon	[looks at his paper, puts pen on paper to write, stops] Yeah it- No. Yours, not mine. Not the – not this [points to the data table and shows it to Oliver]. 2 times 8 is already 16 and that's more than double, so more than 100 percent. 200
35	Oliver	No, you distribute the B completely wrong
36	Leon	Oliver, here [looks at data table], see, you can- with Portugal – see, the Netherlands already have more than twice of the
37	Oliver	No, you distribute the B completely differently. In total, there are 30 yellow cards, because 22 plus 8
38	Leon	Yes yes, in total
39	Oliver	That is 30. That is your B. B equals 30
40	Leon	And A divided by B is then-

41	Oliver	Yes, 22 divided by 30 and 8 divided by 30. You then have 73.3 percent and 26.6 percent. 73.3 minus 26.6 is 47.3 percent. I'll have a look now [raises hand to get teacher's attention]
42	Leon	I still don't get it. It cannot be possible that it's only 47 percent. It is more than twice. I can see it like that, I can see it.
…		[Oliver calls the teacher and both students present their ideas. Then the teacher appraises Oliver's solution without commenting further on yellow cards than Portugal, that's already more than 100 percent or 200. Leon's idea:]
44	Teacher	So, you talk about the yellow cards, which both teams got in total. Then this [points to Oliver's calculation] is correct. Then you only have to- if you want to have this as a correct statement, then you have to write it so that you talk about the yellow cards both teams got in total and exactly only these two teams. Then, in relation to the in-total 30 cards, the Netherlands has 40 percentages more than Portugal.
45	Leon	Oh [incomprehensible] that was my error in thinking. Has 40 percent more, not just 40 percent. But 40 percent more. That's the reason, oh gosh.

While Leon and Oliver agree on creating a statement about comparing the percentages of yellow cards between the two countries, they differ in their approach (**Figure 2**). Oliver is very explicit in communicating what he is calculating also by referring to the rule he learned for determining percentages ("A[mount] divided by B[ase] equals R[ate]", turn 10). He is able to identify that Leon's approach differs in the base rate and thus explains his calculation repeatedly (turn 35, 37, 39, 41). Leon, in contrast, is not making a concrete calculation or explicitly stating his approach as 16/8, but validates Oliver's result by drawing on his conceptual knowledge of percentages. In line 15, he possibly assumes that Oliver miscalculated the rate as roughly "half of it more", for example by wrongly believing that 22 in comparison to 8 is approximately 50 % more rather than more than 100 %, which is a common mistake for students dealing with percentages (Parker & Leinhardt, 1995).

Oliver's calculation	Leon's reasoning
Base rate: 22 + 8 = 30 yellow cards	(Base rate: 8, not made explicit)
Rate of the Netherlands: $\frac{22}{30}$ = 73.3 %	Comparison of NL and PRT: $\frac{22}{8}$ > 200 %
Rate of Portugal : $\frac{8}{30}$ = 26.6 %	
Difference: 73.3 % - 26.6 % = 47.3 % (miscalculated)	NL has more than 100 % more yellow cards than PRT

Figure 2. Reconstruction of Oliver's and Leon's approaches

Analyzed with respect to **Table 1**, the situation can be interpreted as containing several facets of mathematical potentials: Regarding the cognitive facet, the students' approaches show a conceptual understanding of percentages as well as a procedural fluency in determining them. Oliver miscalculates the difference of decimals and wrongly calls the result 'percentage' rather than 'percentage points', so that the result is not perfect. But the process nevertheless shows potential which is worth being fostered since the contingent choice of the base is a core idea of percentages. In the personal and affective facet, Oliver and Leon show deep engagement and persistence (over 5 minutes) to resolve their conflict in this

situation. This resonates with the social facet in which their cooperative skills become apparent in trying to identify each other's approach. Thus, the boys' process can be considered to show creativity in creating correct and correct but meaningless statements. Concerning the communicative and linguistic facet, Oliver is quite precise in describing his approach, although using only abbreviations and adopting a procedural stand. In contrast, Leon has difficulties to express his thoughts which is a typical limit of German underprivileged students' discursive competence. Viola contributes very little in the scene (possibly also due to her low mathematical self-concept mentioned by herself in unprinted lines of the transcript), but she shows some metacognitive skills in monitoring the group working process and keeping the group focused on the overall goal. However, unfolding this potential would require much more focused support.

Summing up, the overall group work process shows some flaws such as insecurities with calculations or that the students did not cope with the problem themselves but called the teacher for a resolution. This certainly identifies the students as not gifted in the narrow sense but as having some mathematical potentials which would be worth to be fostered. Even though Leon states he understood the reason for his differing result in the end (turn 45), it remains unclear if he just follows teacher's authority (as the teacher is not making a corresponding statement about the "percent more"). Thus, there is some room for improvement, especially in the outcome of the group work. That is exactly why this scene is inspiring for a video club discussion in the professional development sessions.

The very brief analysis of the two scenes shows the design principles at work (**Table 2**): The self-differentiation works for the high ceiling (principle P3), and the problem turns out to be mathematically rich (P4). The students experience autonomy and competence in creating their own statements (especially later in the lesson when the statements are discussed and positively evaluated in the whole classroom) (P5), even if not yet by the feedback. The created situations are interest-dense, raising positive engagement (P7). However, the fragile processes in the second scene underline the necessity of teachers' process orientation rather than focusing on perfect products (P6). The limitations in the teacher-student interaction call for a deeper look at demands for teachers.

Refining the demands for teachers: Adaptive support requires multiple perspectives in noticing potentials

The presented scenes above illustrate how subtle the hidden potentials can be, especially when the result of a working process is not excellent. The aim of a participatory approach to potentials of all students requires the variety of different facets as well as its dynamic and situational conceptualization, but this raises the demand for teachers to be specifically sensitized. Looking at the outcome of Scene 2, the limited result for the given problem and teacher's dominant role in resolving their issue of differing base rates, one could say the scene does not contain much potential. As the in-depth analysis of the PD

meetings show, this product perspective is often uttered by in-service teachers when they evaluate the students' progress towards a certain goal (cf. Schnell, 2016).
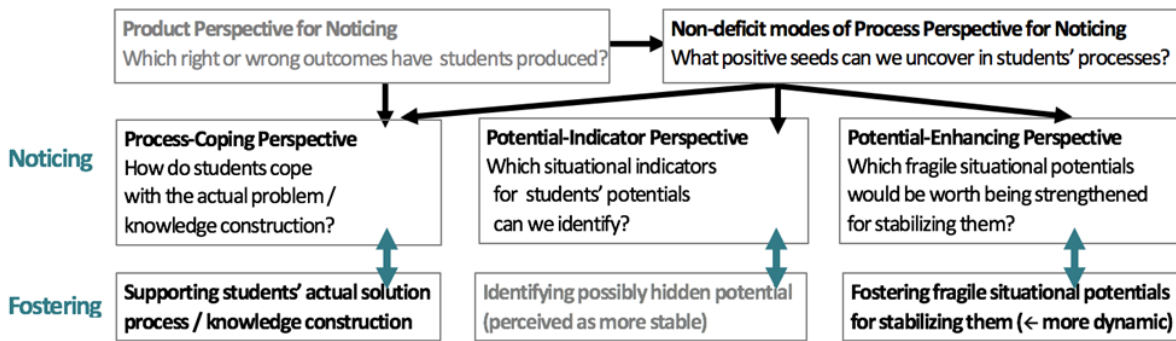

Figure 3. Perspective model for noticing and fostering potentials – Result from analyzing teachers' perspectives

While this is a justified perspective for teachers especially when grading students' work, the identification and adaptive fostering of situational potential requires further perspectives on noticing potentials. Although we cannot account for the research process on teachers' perspectives here (research question 2), we present its condensed result, the perspective model for noticing and fostering potentials (cf. **Figure 3** from Prediger, Schnell, & Rösike, 2016).

Well known in the literature (e.g. Leikin, 2009a and **Table 2**) is the required shift from the product perspective to the process perspective, and we add in a non-deficit mode. What our research added to the existing literature is a distinction of three different process perspectives, the process-coping perspective, the potential indicator perspective and the potential-enhancing perspective, which are all linked to certain focuses in teachers' moderation for fostering students in these processes:

The main part of the literature on noticing students' processes adopts the potential-indicator perspective, seeking to identify (preexisting, but more or less hidden) potentials that can be expressed in different facets (cf. **Table 1**). The analysis in the last section is conducted in this perspective, seeking to identify indicators for these facets. For example, in Scene 2, Leon's approach might be identified to show a deeper conceptual understanding of percentages than Oliver's more procedural approach, but Oliver has the higher capabilities to communicate his ideas appropriately. For the teacher to be able to identify these potentials, he could have listened longer before intervening (line 44). During the PD program, we experienced that teachers get increasingly used to identify potentials in different facets by discussing the videos.

However, the process-coping perspective is more familiar to most teachers, as they usually aim at supporting students' actual solution or knowledge construction process. Rather than looking beyond the concrete situation, teachers often show a powerful and

persisting routine in identifying actual obstacles and help students overcome them. In Scene 2, the teacher adopts this perspective and helps the students with their conflict (in line 44). Also in the video clubs, the teachers mainly asked how students coped with the actual situation rather than looking beyond the situation. This often turns out to hinder the full development of the situational potential of the enrichment setting. For example, teachers discussing Scene 2 mainly focus on the students' difficulties to communicate ideas rather than noticing their engagement and the mathematically interesting contingency in choosing the base for the percentage comparison.

During the video analysis, only a very small group of teachers focused their attention on identifying situational potentials worth to be strengthened in the situation for stabilizing them in the long run. However, this potential-enhancing perspective is required for the dynamic and participatory conceptualization of potentials: only by stabilizing the first seeds, the hidden and fragile potentials can grow into a more stable disposition. In the case of Oliver and Leon, this could mean for example to give feedback to Leon how he is persistent in the pursuit of clarifying the issue (personal and affective facet) and shows a good understanding of the concepts of percentages (cognitive facet). If he would show these aspects more often, he could become a very good mathematician. Furthermore, the conflict about contingent choice of the base for percentage comparison would be worth discussing with the whole class as it touches a core idea of percentages (Parker & Leinhardt, 1995). In this way, noticing in a potential-enhancing perspective can contribute to prepare for enhancing all students' conceptual understanding. This very brief insight into the analysis of teachers' obstacles show that enrichment settings do not only require rich tasks and cooperative teaching methods, but also fine-tuned and subtle noticing and fostering by the teachers.

DISCUSSION & LIMITATIONS

In this article, we elaborated our wide dynamic and participatory conceptualization of mathematical potentials, especially for socially underprivileged students. Seven design principles for heterogeneous whole class enrichment settings with rich tasks were adapted from the literature for the specific whole class settings (**Table 2**), and the analysis of initiated classroom processes show these principles at work (following Leikin's 2011 advice for future research needs). For identifying potentials, five facets of mathematical potential could be specified based on the literature (**Table 1**). The five facets are discussed by teachers in video clubs quite naturally, as the enrichment settings invite not to focus too narrowly on the cognitive facet alone. These results of the design research projects raise optimism for fostering equity also among the best students (Suh & Fulginiti, 2011).

Beyond evaluating materials and teaching approaches, the design research project also allows deep insights into the chances and obstacles of the micro processes in classrooms and PD sessions. Of course, the results should be interpreted in respect to their methodological limitations. As the presented study is exploring the rather under-researched topic of equity

in fostering underprivileged students' potentials, the approach is necessarily qualitative and aims at uncovering and systemizing important phenomena and relations such as the perspective model for noticing potentials. However, the data corpus especially concerning the meetings of 25 teachers in total is too small to conduct quantitative analyses. As the research is still on-going, generalizability of the findings is not yet achieved. Nevertheless, by triangulating analysis along categories identified from literature research with category-generating analysis, findings are promising for further research which can hopefully end also in quantitative proofs of effectiveness in the long run.

Although this article could only give very brief and limited insights into the students' and teachers' processes, it already showed an important future design challenge for professional development of teachers: The success of the enrichment setting to enhance students' fragile potentials heavily depends on the teachers' expertise to notice and foster them. Thus, an important result of the project is the perspective model for noticing and fostering (**Figure 3**). Whereas the principle shift from product to process perspectives is mastered by all participating teachers, a distinction between three different process perspectives became necessary and allows to explain obstacles for some teachers to exploit the situational potentials for a longer-term stabilization (for more details see Prediger, Schnell, & Rösike, 2016).

This has important consequences for professional development programs for teachers: For carrying a dynamic and participatory conceptualization of mathematical potentials into mathematics classrooms, teachers need to be sensitized for the potential-indicator perspective and especially for the potential enhancing perspective. Hence, the perspective model can inspire further research and design activities for sharpening PD programs which prepare teachers for fostering underprivileged students' in their dynamic and still hidden potentials (Suh & Fulginiti, 2011). So this article can contribute on an empirical base to filling the design and research gap about fostering potentials as claimed by Singer et al. (2016).

ACKNOWLEDGEMENT

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