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Mathematics Student Teachers' Epistemological Beliefs about the Nature of Mathematics and the Goals of Mathematics Teaching and Learning in the Beginning of Their Studies

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This article examines Finnish mathematics student teachers' epistemological beliefs concerning the nature of mathematics and the goals of mathematics teaching and learning solely in the beginning of their studies at university. A total of 18 students participated in a study consisting of a short questionnaire and interviews. The data was analyzed by categorizing the views presented in terms of four mathematics-related orientations. According to the results, student teachers often regarded mathematics mainly as a static system, but their beliefs about the goals of mathematics teaching and learning consisted of features that derive from all of the four orientations. Future teachers' beliefs can be affected by teacher education and hence the results are important for teacher educators to be recognized.

Keywords: beliefs, nature of mathematics, student teacher, teacher education

INTRODUCTION

Learning about mathematics and mathematical problem-solving are processes that are strongly affected by *mathematics-related beliefs*. Beliefs about mathematics determine, for example, how an individual chooses to approach a problem, and which techniques and strategies she will use (Op't Eynde, de Corte & Verschaffel, 2003). Beliefs also affect "motivational decisions" (Op't Eynde et al., 2003, p. 15) in the learning and problem-solving process.

Beliefs about the structure, certainty, and source of knowledge are termed *epistemological beliefs*. In this study we intend to focus on Finnish mathematics student teachers' epistemological beliefs concerning mathematics solely as they exist at the start of their studies. We determine which goals in mathematics teaching and learning the students consider important and also the nature of the epistemological beliefs that they possess concerning the nature of mathematics. We categorize the features presented by the students in terms of four different orientations: formalism-related, scheme-related, process-related, and application-related. These four orientations were originally put forward by Grigutsch, Ratz and Törner (1998)1, but they will undergo some modifications for the present study.

Mathematics student teachers confront two challenges with respect to their forthcoming mathematics teacher studies. The first challenge is that they are now entering tertiary-level mathematics. In Finland, most of the mathematics courses in the teacher education program are taught by mathematicians, and hence a general mathematical way of thinking is required. The student teachers generally possess no

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State of the literature

- Several studies about the impact of learners' beliefs on learning have been published.
- Several studies about the impact of teachers' beliefs on teaching practices have been published.
- Some categorizations for the nature of mathematics have been presented in the literature.

Contribution of this paper to the literature

- In this paper the framework concerning mathematical orientations is modified so that it is more relevant for studies dealing with beliefs about the nature of mathematics and the goals of mathematics teaching and learning.
- This paper reveals what kind of beliefs student teachers have about the nature of mathematics and mathematics teaching and learning just in the beginning of their university studies.
- According to the results of this study, the student teachers often regarded mathematics mainly as a static system, but their beliefs about the goals of mathematics teaching and learning were more multifaceted in the beginning of their studies.

previous experience of tertiary-level mathematics and hence their mathematics-related beliefs have mainly been affected by sources that exist outside the community of mathematicians. In general they have probably been influenced by mathematics teaching at school. Since mathematics teacher students have chosen mathematics as their major subject, it can safely be assumed that they are interested in mathematics, and hence they have probably thought about the nature of mathematics and its learning to a greater extent than the average student at upper secondary level.

The second challenge faced by mathematics student teachers is that they should mature into teachers, a demanding process that often requires reflection on one's previous experience of the teaching and learning of mathematics. Learning to teach mathematics can be regarded as an ongoing and lifelong process originating from experiences gained as a mathematics learner at school (Johnson & Golombek, 2002; Llinarres & Krainer, 2006).

Teachers'beliefs have an important impact on teaching practices in the classroom and also on student learning outcomes, and a change in beliefs is considered to be a perquisite for changes in teaching practices (Ernest, 1989). Because student teachers have years of experience of mathematics teaching during their time as students, their beliefs are difficult to change (Llinares & Krainer, 2006). In consequence, the development of beliefs should be taken into account throughout the process of teacher education. For this reason, it is important that teacher educators are aware what of the beliefs that their teacher students possess at the beginning of their studies and how those beliefs can develop in the course of the teacher education program. Based on this kind of understanding, teacher educators can offer their teacher students opportunities to reflect on their beliefs critically during different phases of the training program.

In the present study, however, we focus on the mathematics-related epistemological beliefs of Finnish student teachers at the start of their studies. Our attention is focused on the following themes:

- *Theme 1: Beliefs about the nature of mathematics: What kind of science is mathematics? What are the essential features of mathematics and mathematical knowledge?*
- *Theme 2: Beliefs about the goals of teaching and learning of mathematics: Why is mathematics taught? What is essential or important in the learning of mathematics?*

A Finnish mathematics teacher education program provides the qualification of a secondary mathematics teacher. The students taking the program are majoring in mathematics, although not all of the students will necessarily have decided on their study program when they start their studies; in reality, it is possible to select the final program at a later date since the initial courses are usually common to all programs. It is likely to prove interesting to discover the range of beliefs that first-year university students possess with regard to these two themes when they start their mathematics teacher studies immediately after leaving upper secondary school. It appears that insufficient attention has been paid thus far to this aspect of the majority of mathematics education research, and thus it is hoped that this study will at least partly fill this gap.

The concept of belief in mathematics education

Several studies focusing on beliefs have been published in the field of mathematics education literature in the course of the past two decades. No unified definition of the concept of belief has, however, been agreed on, but researchers have generally presented their own definitions or they have even left it undefined (Furinghetti & Pehkonen, 2003). Beliefs have usually been considered primarily in terms of cognitive elements, but they are also closely connected with affective elements. In theoretical discussions the relationships between the concept of belief and concepts such as knowledge, conception, affect, attitude, or emotion have remained vague.

Knowledge has been defined traditionally as a justified true belief (Sartwell, 1992). This definition implies that knowledge is a sub-class of belief. An alternative approach has been presented by Furinghetti and Pehkonen (2003), who separate objective/official knowledge from subjective/personal knowledge. The first category means knowledge that is accepted by a community, whereas the latter refers to knowledge "that is not necessarily subject to an outsider's evaluation" (p. 43). Beliefs belong to the latter category, and hence beliefs cannot be considered to be absolutely valid, but rather that they are exposed to doubts and disputes. Furinghetti and Pehkonen (ibid.) also discuss the relationship between the concepts of belief and conception, which has tended to receive mixed treatment in the literature. By referring to the following references, they claim that some researchers have regarded beliefs as a sub-class or as elements of conceptions (Thompson, 1992; Lloyd & Wilson, 1998), whereas others have considered conceptions to be conscious beliefs (Saari, 1983; Pehkonen, 1994). In contrast, however, Ponte (1994, quoted in Furinghetti & Pehkonen, 2003) regards these two terms as referring to totally different spheres: beliefs are statements, whereas conceptions are cognitive constructs.

Hannula (2011) illustrates the inconsistencies between the concepts of belief, attitude, affect, and emotion by comparing McLeod's (1992) framework for affect and Hart's (1989) framework for attitude. McLeod considers belief to be an element of affect alongside attitude and emotion, whereas Hart sees belief as an element of attitude alongside emotion and behavior. Other definitions have, however, also been provided, such as the concept of attitude. Hence, in practice it is impossible to combine the various different frameworks concerning these concepts.

In general, beliefs have been considered to be stable objects that do not change rapidly. Hannula (ibid.), however, also considers that beliefs have a state aspect: thus, for example, in the course of a problem-solving process instantaneous beliefs concerning the problem in question may also arise.

A number of different classifications of mathematics-related beliefs have been presented in the literature. Op't Eynde, de Corte and Verschaffel (2003) review some of the categorizations of mathematics-related beliefs presented in the literature in the 1980s and 1990s. These categorizations mainly consist of similar elements, but they are grouped differently. Op't Eynde et al. attempt to integrate the main components of the different categorizations, and so they divide students' mathematics-related beliefs into three main categories: *beliefs about mathematics education*, *beliefs about self*, and *beliefs about the social context*. The first category includes beliefs about mathematics as a subject, beliefs about mathematical learning and problem-solving, and beliefs about mathematics teaching. A newer, modified version of this model is presented in Op't Eynde, de Corte & Verschaffel (2006).

In the present study we have restricted our attention to beliefs about learning mathematics and about the nature of mathematical knowledge, both of which have been noted to be connected with each other (Crawford, Gordon, Nicholas & Prosser 1998). In consequence, this study is restricted to the first main category in Op't Eynde's classification, whereas beliefs about mathematical problem-solving and beliefs about mathematics teaching are not explicitly focused on here. Both are, however, quite closely connected to beliefs about the nature of mathematics and beliefs about mathematical learning: it can, for example, be assumed that beliefs about the nature of mathematics influence beliefs concerning mathematical problem-solving or vice versa, and that beliefs concerning the learning of mathematics also imply beliefs about the teaching of mathematics.

Beliefs about the nature of mathematics and about its teaching and learning

According to Op't Eynde, de Corte & Verschaffel (2006), it is reasonable to assume that epistemological beliefs are dependent on the domain that they concern. Their domain specificity can be seen from the different problem-solving activities in different disciplines, as well as from the individual contents and methods in school teaching of a certain domain. This means that epistemological beliefs in mathematics can be discussed separately from the general epistemological beliefs.

Ernest (1989) presents three different philosophical views for the nature of mathematics. According to the *instrumental view*, mathematics is a collection of facts, rules and skills that are needed in the pursuance of external ends. Mathematics is also seen as "a set of unrelated but utilitarian rules and facts" (p. 250). The *Platonist view* refers to platonic philosophy, and means that mathematics is seen as a static but unified body. According to this view, mathematical objects are seen to be real and to exist independently of human beings (Brown, 2005). Mathematical statements are considered to be objectively true or false, and their truth value is also seen to be independent from the human. Consequently, mathematical knowledge is not *created* but *discovered*. In addition, mathematical knowledge is seen to be non-empirical: it is not based on sensory experiences. The third view is the *problem-solving view*, which means that mathematics is seen as "a dynamic, continually expanding field of human creation and invention, a cultural project" (Ernest, 1989, p. 250). According to the problem-solving view, mathematical results and findings are always open to revision.

The view of mathematics has an impact on how the role of teacher and the nature of teaching and learning of mathematics are seen. According to Ernest (1989), the teacher is seen as an instructor in the instrumental view, as an explainer in the Platonist view, and as a facilitator in the problem-solving view. According to

Beswick (2005), the three different views are connected to the teaching and learning of mathematics as follows:

- The instrumental view implies that mathematical learning is seen as the passive reception of knowledge and the adoption of different skills. The teaching of mathematics needs to be content-focused, with an emphasis on performance.
- The Platonist view implies that the learning of mathematics means understanding and adopting an existing knowledge structure. The teaching of mathematics needs to be content-focused but also emphasizing active understanding.
- The problem-solving view implies that the learning of mathematics is seen as an autonomous exploration of one's own interests. In the teaching of mathematics, it is the learner that needs to be in focus rather than the content.

Beswick (2005) concludes that the problem-solving view is most consistent with the *constructivist view* of mathematics learning.

Grigutsch et al. (1998) studied mathematics teachers' beliefs in Germany by using a questionnaire consisting of 75 statements concerning the nature of mathematics and the teaching and learning of mathematics. The researchers found four orientations for teachers' beliefs (see Felbrich et al., 2008, p. 764):

- *Formalism-related orientation*: Mathematics is "an exact science that has an axiomatic basis and is developed by deduction".
- *Scheme-related orientation:* Mathematics is "a collection of terms, rules and formulae".
- *Process-related orientation*: Mathematics is "a science which mainly consists of problem-solving processes and discovery of structure and regularities".
- *Application-related orientation*: Mathematics is "a science which is relevant for society and life".

Grigutsch et al. (1998) also found a positive correlation between the formalism-related and scheme-related orientations, and, correspondingly, between the process-related and application-related orientations. According to Grigutsch et al., the formalism and scheme orientations describe the *static* aspect of mathematics, whereas the process and application orientations refer to the *dynamic* nature of mathematics.

The instrumental view in Ernest's classification is similar to the scheme orientation in the classification proposed by Grigutsch et al. In addition, the problem-solving view and the process orientation correspond to each other. Both in the Platonist view and in the formalism orientation, mathematics is regarded as a static system. In the Platonist view, however, the objective nature of mathematical knowledge is crucial, but this is not mentioned in the definition of the formalism-related orientation proposed by Felbrich et al. (2008). Further, Ernest's classification contains no view that corresponds with the application orientation.

Our purpose in this study is to analyze students' beliefs about the nature of mathematical knowledge and about the goals of mathematics teaching and learning by using the four orientations as a framework. Hence, we need to specify what each orientation means with respect to the nature of mathematics and of the teaching and learning of mathematics. In this study we apply the following working definitions for the mathematical orientations:

- The formalism-related orientation means that mathematics is considered as an existing static system of knowledge. The purpose of learning is to learn to know and understand the structure of this system. Mathematical concepts, theorems and notations are thought to be determined beforehand, and they should be acquired in the process of learning. It is important that mathematics is expressed exactly as it is. In consequence, details and exact notations are emphasized.
- The scheme-related orientation means that mathematics is a collection of different rules, formulae and calculation methods. In the learning process the goal is to achieve a proficient use of them. The origin or construction of rules, formulae and methods is not emphasized in this approach.
- The process-related orientation means that mathematics is seen as an active construction process. The crucial goals of learning are the acquiring of skills in reasoning and constructing new things. Creativity also belongs essentially to mathematics. Instead of details, larger ideas and holistic understanding are emphasized.
- The application-related orientation means that mathematics is seen as a method for describing the phenomena of reality and real life. The origin of mathematics is located in the phenomena of reality, and the value of mathematics is dependent on its applicability. In the learning of mathematics it is important to understand the connections between mathematical concepts and the phenomena through which they model and utilize mathematical knowledge in a different context. It is, however, difficult to draw a strict border between mathematics and the world beyond its confines. In consequence, the application of mathematical knowledge and modeling within mathematics is also included, provided that they occur in a different context.

With respect to understanding as a learning goal, we set the following conditions: *Understanding of a given knowledge* refers to the formalism-related orientation, but *understanding of person's own behavior* refers to the process-related orientation. In addition, all metacognitive skills, such as reflection of one's own thinking, belong to the process-orientation.

The different orientations are not necessarily contradictory, but a person's mathematical worldview may consist of beliefs connected to various different orientations.

RESEARCH METHODS

Sample

The sample for this study consisted of Finnish first-year university students who were motivated to be educated as mathematics teachers. All of the students, who had either already been accepted for the teacher education studies program or who had a clear plan to apply for these studies were invited at the start of the autumn term 2012 to participate in the present study. A total of 18 mathematics student teachers enrolled as participants.

Data collection

This qualitative study was based on an open-ended questionnaire and semi-structured interviews (Wengraf, 2001). First, the students completed a survey in which they were asked to list three goals that they considered centrally involved in mathematics teaching. The question was the following[2]:

Please mention three different goals for mathematics teaching that you consider essential. If necessary, you may divide up your responses depending on students' age, class level, skills etc.

A total of 16 students completed the questionnaire, while all 18 students were interviewed. In the interviews the students were requested to discuss this issue further: hence, their answers to the questionnaire formed a starting point for the discussion. The following questions were presented:

What are the goals of mathematics teaching?
What is essential in the learning of mathematics?

With respect to Theme 1, the interviewer asked the students to start a preliminary discussion about the nature of mathematics and mathematical knowledge. The following questions were used:

What is most essential in mathematics (as a discipline)?
What kind of discipline is mathematics?
Please describe the nature of mathematical knowledge.

In order to make these questions easier and attainable for the students, the interviewer might, for example, ask the students to compare mathematics to other domains such as the natural sciences and to think about the differences.

The nature of the interviews was semi-structured so that the main questions were planned beforehand, but the interviewer was allowed to ask additional questions, depending on the interviewee's answers. The questions concerning mathematics teaching and learning (Theme 2) came before questions about the nature of mathematics (Theme 1). Theme 1 was assumed to be difficult for the students, and hence, for this reason, it was not dealt with in the questionnaire. For similar reasons, in the interviews Theme 2 was discussed before Theme 1[3]. The survey was conducted during the first lesson of the mathematics course, and the interviews were conducted during the first month of the semester. The interviews were later recorded and transcribed.

Analysis

The analysis was based on qualitative content analysis (Mayring, 2000). In the analysis, all of the different features that referred either to the nature of mathematics and mathematical knowledge (Theme 1) or to the goals of mathematics teaching and learning (Theme 2) were identified. In the case of Theme 1, these features were looked for in the interview transcripts, but for Theme 2 the written responses in the survey and also the interview transcripts were taken into account. The features that were detected were first grouped into categories and then the categories were connected with the four orientations.

In the following account, students are referred to by using code-names such as Student A, Student B, etc. Each individual code-name – for example, Student A – refers everywhere in this article to the same student.

RESULTS

Students' beliefs about the nature of mathematics (Theme 1)

Students' descriptions of the nature of mathematics as a science and the nature of mathematical knowledge in the interviews were analyzed by applying mathematical orientations as a framework. Each student's response was analyzed as an entity without selecting out any part of it.

References to the different orientations are listed in Table 1. In the following, the students' answers are depicted in more detail.

Table 1. Orientations to which students referred to when they described the nature of mathematics.

Orientations	Number of responses (*n*=18)
Formalism	10
Formalism and scheme	1
Formalism and process	1
Scheme	1
Process	1
Application	2
No clear reference to orientations	2

Descriptions referring to the formalism-orientation

In total, 10 students described mathematics in a way that could be connected solely with the formalism-related orientation. In the following, a review is presented of how the students described the nature of mathematics.

Students C and N thought that *proving exactly* is characteristic of mathematics.

"It is exact: there needs to be complete proof for why an issue is what it is." (Student C)

Interviewer: What kind of a discipline is mathematics? What is characteristic of it?

Student N: Proving.

Interviewer: Just that?

Student N: It seems to be very important at university. Everything needs to be proven. And precise: you have to be exact. This seems to be yet another key word! Everything needs to be defined exactly, rather than just approximately.

Also Student L was probably thinking about proving exactly:

"Yes, everything requires proving, to demonstrate why something is just how it is and why this follows from that." (Student L)

Exactness and exact proving can be connected with the formalism-related orientation on the basis of its definition. The formalism-related orientation emphasizes that it is important to verify that all the details of mathematical reasoning are in accordance of the existing system.

Student R emphasized that mathematical truth is *changeless*:

Interviewer: What kind of knowledge exists in mathematics?

Student R: For one thing, it is absolutely true, if something is concluded mathematically. It is true now and it will be true in a thousand years' time. It is said that science develops, but in mathematics the things which have been true earlier are still true now.

In addition, Student G thought that development in mathematics was slower than in other disciplines:

Student G: I see mathematics as a discipline that's similar to all the others, but mathematics does not progress in the same way all the time, or develop like physics or chemistry, for example. But, I don't know what else to say.

Interviewer: How so? What is the difference in the development?

Student G: Fewer new things are discovered.

Interviewer: OK.

Student G: The development is less continuous, in the sense so that something new should appear on the foundations of the old knowledge. According to the way I understand it, the process is bit slower than it is in the other disciplines.

The stability of mathematical knowledge also belongs to the formalism-related orientation on the basis of its definition.

Student O described mathematics as *an axiomatic system*. He also thought that mathematics was *detached from practice.*

"It *(mathematics)* is a bit like philosophy, but even less practical. [….] It is such an axiomatic system." (Student O)

In addition, Student R regarded mathematics as separated from practice:

"It is quite different than anything else. How can I put it? The difference is that it does not serve any practical purposes. It does not attempt to study any natural conditions or other issues in the world, but is a completely separated thing." (Student R)

Mathematics as an axiomatic system is a basic idea in the formalism-related orientation. The independence from practice also belongs to the formalism-related orientation.

Students N and C emphasized that *mathematics has its own instruments* for communication and that acquiring them is necessary.

"There are quite a few different kinds of symbols, signs and so on, so that not everything is expressed in a plain language. If you are going to study mathematics, you need to know certain basic things. It is not possible to study mathematics simply by reading, because there are quite a few kinds of concepts that are peculiar to it. There is much that…" (Student N)

Student C: …It has a single language that can be understood by everyone.

[…]

Interviewer: What kind of knowledge is mathematical knowledge?

Student C: Is it something to do with understanding the language? In other words, you have to understand the notations and formulas and so on?

Student C seemed to understand the question about mathematical knowledge as referring to the acquisition of mathematics or mathematical knowledge.

Students N and C seemed to understand mathematics as a language system that has its own concepts, notations, and rules. Both responses include the idea that there exists a system outside the learner, and in the learning process the components of this system need to be acquired. Hence, it is reasonable to connect these responses to the formalism-related orientation.

In addition, the following features were mentioned once: definitions, following of given rules, complexity, conventionality and extensiveness. All of these refer, at least in some sense, to the formalism-related orientation.

Other descriptions

Student H, who refers to the *formalism-related and scheme-related orientations*, considers mathematical methods and logical connections important.

Interviewer: If we consider mathematics as a discipline, what do you think, what is most central or most essential in it? Or what is characteristic of mathematics as a discipline?

Student H: All the methods of mathematics, that is those that come first to mind…

Interviewer: Do you mean calculation methods?

Student H: Yes, those that can be calculated by a particular method. Or formulas. […]

Student H: All the knowledge in mathematics, it is so logical that if you hear a new piece of knowledge, you can… where it comes from… There are no separate things.

Interviewer: Everything is connected?

Student H: Yes, they are in some way logically connected so that it is not possible to find something totally separated in mathematics, something that would be totally different, so that everything would work in a totally different way.

Student H first mentions calculation methods and formulas, but then she also refers to logical connections and mathematics as an entity.

Student I (*formalism-related and process-related orientations*) emphasized that the nature of mathematics is dynamic.

Student I: Mathematics is quite dynamic. I think… even though at school it's always said that there's only one correct answer, but that isn't the case… Mathematics is like a strange lump that is always changing… […]

Student I: You build up a knowledge base so that, for example, you start from addition, after then you can move on to training multiplication. In that way you construct a tower or building that grows and acquires side-branches… But everything is connected and affected by the rest…

Student I initially emphasizes first the dynamic process nature of mathematics, but, on the other hand, he understood that mathematics is a connected construction.

Student Q (*scheme-related orientation*) sees formulas as the essential elements of mathematics. In addition, her attitude toward mathematics is quite negative.

Student Q: With regard to mathematics, I can't help thinking of it as only a very disgusting subject that inspires no one. […]

Interviewer: What kind of knowledge is mathematical knowledge?

Student Q: It's a schematic knowledge…

Interviewer: What do you mean?

Student Q: I don't really know. I feel that mathematics, in some way, is schematic nowadays… Everything is produced on the basis of formulas, and every solution is achieved by locating it within a formula.

Student Q mentions the word "schematic" twice. By this, however, she probably means simply the central role played by formulas.

For student J, (*process-related orientation)* personal reasoning and the invention of new ideas seems to be essential in mathematics.

Student J: It is very large… New branches are being found all the time, and then new things are discovered by combining this and that together. It is very large and confusing… But you can yourself invent a lot of new things, setting up new links between different things.

Interviewer: Is there something that is very essential?

Student J: Yeah, just the chance to use your own reasoning!

Student A (*application-related orientation)* emphasizes that mathematics offers a tool for describing phenomena in for example, physics or chemistry. In doing so, he emphasizes the instrumental nature of mathematics.

Student A: I think mathematics offers a language for physics and chemistry, too. It is connected to them. Without mathematics it would be difficult to say anything about physics. What else?

Interviewer: You mean that mathematics offers a language for other disciplines?

Student A: I mean that, through mathematics, it is possible to talk about other disciplines.

Student M (application-related orientation) considers mathematics to be the basis for other disciplines.

Interviewer: If we speak generally about mathematics, if we think about mathematics as a discipline, what kind of is it, as far as you can see? Or what is most essential in it?

Student M: Well, I think that it is the basis for everything, for example, for physics and chemistry. And mathematics is very important, calculations and so on.

Interviewer: All right. What kind of knowledge is mathematical knowledge? Can you suggest some of the qualities, or characteristics of mathematical knowledge.

Student M: Mathematical knowledge, what might that be? I suppose, at least everything connected with problem solving. But I don't know what else to say..

At the end of this extract there is also a weak reference to the process-related orientation. It is not,

Table 2. Students' views concerning the goals of mathematics teaching and learning. The views expressed concerning the goals were gathered from both the survey (third column) and the subsequent interviews (fourth column).

Goal	Orientation	Number of mentions in the written responses (*n*=16)	Number of mentions in the interviews (*n*=18)	Number of mentions in total (*n*=18)
Understanding the content of mathematics	F	4	3	**6**
Understanding reasons	F	4	0	**4**
Mastery of calculation skills	S	10	10	**13**
Development of thinking and problem solving skills	P	8	10	**14**
Learning of skills needed in everyday life	A	3	11	**12**
Learning of skills needed in other domains or in professional life	A	1	8	**9**
Other responses referring to application	A	8	3	**9**
Enhance students' interest in mathematics	-	3	1	**3**
Enhance students' mathematical self-confidence	-	1	0	**1**
Increase in general knowledge	-	0	2	**2**
Other goals		1	1	**1**

however, reliable, because it is not clear what student M means by "everything connected with problem solving".

Some students describe mathematics in a way that cannot be connected with any of the four orientations. two students state that numbers are essential in mathematics. For example, Student P mentions numbers and number language:

"It is like English, but, or, it is like a new language." (Student P)

Student D mentions the combining of numbers and symbols.

Interviewer: What do you think: what is most essential in mathematics?
Student D: Numbers.
Interviewer: Numbers, really?
Student D: Yes, numbers, and all those fine…with "a" (refers to symbol "a") and suchlike… combining them and things like that…
Interviewer: Combining numbers and symbols?
Student D: Yes.

In addition, *logic* and *unambiguousness* are both mentioned by two students, but it would be difficult to connect these with any of the orientations.

The extracts presented in this section reveal that a majority of the students referred mainly to formal features of mathematics, but it should be noted that many other aspects were also mentioned.

Students' beliefs concerning the goals of mathematics teaching and learning (Theme 2)

Both in their responses to the questionnaire and in the interviews conducted with them, the students express a variety of goals related to the teaching and learning of mathematics. The goals mentioned are summarized in Table 2. Students' answers concerning each goal are described in more detail below and several direct quotations are also included. A majority of the goals are connected with one or more of the four mathematics orientations. Many of these connections are obvious, while some of them are discussed in greater detail.

In Table 2 the numbers in the second column show how many of the students mentioned the goal in the survey. The numbers in the third column also show how many students mentioned the goal *for the first time* only in the interview. In the interviews the responses gathered in the survey have been dealt with in greater detail, but these discussions are not taken into account in this particular Table. The numbers in the fourth column show how many students mentioned the goal either in the survey or in the interviews.

Several examples of the students' descriptions concerning the nature of mathematics are presented in the following analysis. Most of the extracts have been taken from the interviews, although those originating from the written responses of the survey are clearly indicated.

Goals referring to an understanding of content or reasons

Four students in the written responses and three students in the interviews (six students in total) mentioned *understanding of the content* of mathematics. In the interviews Students C and G described this in the following way:

"The thing is that you should understand it, mathematics." (Student C)

Student G: The most important thing is to understand the content.

Interviewer: Understand the content? OK, yes.

Student G: Because then you won't simply repeat it but you'll really understand what's going on.

These responses reflect the kind of thinking according to which mathematics simply exists and is ready-made, but also that it needs to be understood. Hence, these can be connected to the formalism-related orientation.

In the interviews four students spoke about *understanding the reasons*. For example, Students F and A described this in the following way:

Interviewer: What do you think, what is most essential in learning mathematics? Or rather, what is most essential, most important?

Student F: The most important thing is that you understand that, if a process is undertaken in a certain way, you understand why it's being done like that.

Interviewer: What do you think are the goals of mathematics teaching? Why is it studied and taught in general?

Student A: Well, you learn logical reasoning and also to understand why things are as they are.

It is evident that, in the descriptions concerning both understanding the content and understanding reasons, students considered mathematics to be an existing body or entity that needs to be understood. Both of these are based on understanding the structure of mathematics. No references to the invention of one's own ideas are included in these responses. Hence, if the broadened definition of the formalism-related orientation is applied, it would be natural to include these goals in the orientation.

Goals referring to mastery of calculation skills

Either in the survey or in the interviews more than two thirds of the students mentioned the learning of *calculation techniques* as a goal of mathematics teaching and learning. In the interviews, the students talked about calculation skills, basic skills, training of routine, etc.:

"... that they will be automated so that you don't need to ponder on how they go" (Student I)

"Through doing calculations you will learn and get a routine."(Student R)

"...internalization of mathematical solving methods" (Student G)

Three students (Students M, D and I) emphasized that training in procedural skills was the most important goal in the learning of mathematics:

Interviewer: What is most important in mathematics?

Student D: Training. I mean that one does these exercises. Well, I mean there are quite common calculations. The ones that are similar are repeated so many times that... You need to see the answer in front of your eyes before you write it down.

Acquiring a routine through training seemed to be emphasized in these responses.

Goals referring to the development of thinking and problem-solving skills

The development of *thinking and problem-solving skills* was mentioned by a total of 14 students. They talked either about general thinking skills or, in particular, about mathematical or logical thinking skills:

"I think that for those who don't continue (studying mathematics), it also develops those (thinking skills)."(Student E)

"I think the most important (goal) is that it fosters logical thinking and suchlike, or problem solving in general, not only in mathematics but in general, too."(Student N)

With respect to problem solving, the existence of alternative solution methods, personal insight, and the ability to perceive the students mentioned such aspects as the following:

"There isn't simply a single correct answer or only one correct solution method..." (Student I)

"A personal insight..."--- "I mean, a deep understanding."(Student J)

"If you can perceive certain things, then you can see things that others do not necessarily take into account."(Student R)

In these responses mathematics was connected to personal development. The dynamic process-nature of mathematics also became apparent.

Goals referring to the application of mathematics

12 students mentioned that it is important to learn skills needed in everyday life. Many of them referred to the skills involved in applying very basic mathematics:

Interviewer: What do you think, what is most essential or most important in learning mathematics?

Student P: Most important... Oh, it must be to calculate practical things, like when you buy things in a market, how much what you buy costs. You need to be able to calculate such things, at least approximately.

Generally, the skills needed in everyday life were connected to the learning of mathematics at primary or lower secondary level.

"If we think about the compulsory level, [the goal is] that you should be able to survive in everyday life, such as shopping, etc., so that it can be taken for granted that you won't need to struggle with a calculator whether... you can afford to buy these two products and whether you can pay for them with a ten euro note. And, of course, at a higher level you need to be able to explore all kinds of things, such as... the prices of insurance, etc." (Student K)

Nine students emphasized that it was important to learn skills *needed in other domains* or *skills needed in professional life.*

"[One goal is] that you are able to study other subjects such as physics and chemistry. This can't be done if you don't have mathematical... Mathematics is necessary. And mathematics is also needed in different occupations, of course, everywhere: civil engineering, etc. Certainly, they need mathematics." (Student H)

This student mentioned physics, chemistry, and civil engineering as examples of areas where mathematics was needed. Student F also mentioned physics, while medical science was mentioned by Student L. Otherwise, students spoke about the importance of mathematics at a more general level:

"It is important in companies that there are people who master [mathematics] very well." (Student N)

Nine students referred to the applications of mathematics without specification. It is not possible to see whether these responses refer to the application of mathematics within or beyond its own field. Many of these references were brief: for example, when the goals of mathematics teaching and learning were listed in the written responses:

"Calculation routine, understanding, application, the skills of logical reasoning" (Student C, written response)

Three students (Students M, B, and G) mentioned that one goal of mathematics teaching should be to promote understanding of the importance of mathematics:

"Applying mathematics is very important for me, because it helps me to see where it is possible to use the knowledge that I have." (Student B)

Other goals

Three students mentioned *enhancing interest in mathematics* as an important goal of mathematics teaching and learning. This goal refers to affective factors. Student J thought that the enhancement of interest was important especially at the lower secondary level. Student L responded that pupils' enjoyment and experience of success were important.

Student E referred to pupils' mathematical self-confidence:

"For some students mathematics is like a scary bugbear... Even so, it should be possible to learn it." (Student E)

Students O and R referred to general knowledge when they spoke about the goals of mathematics teaching and learning:

"And then, of course, there are things that are hardly needed in practice. They simply get consigned to general knowledge, like everything else." (Student R)

These responses cannot be connected to any particular orientation.

Total numbers of mentioned orientations

Table 3 shows how many students mentioned each orientation as well as how many orientations the students mentioned in total. All of the references to the orientations that appeared either in the written responses or in the interviews were counted.

Table 3 shows that the students' responses concerning the goals of mathematics teaching and learning were quite varied with respect to the orientations. Each orientation was mentioned by more than half of the students, and most of the students mentioned goals related to three or four orientations.

Connection between the beliefs about Theme 1 and Theme 2

Subjects' views about the goals of mathematics teaching and learning (Theme 2) were distributed quite evenly between all four orientations. In fact, the formalism-related orientation was strongest in terms of views concerning the nature of mathematics and mathematical knowledge (Theme 1). Students' response-profiles with respect to the four orientations are presented in Table 4. These profiles reveal which orientations students referred to in their responses concerning each theme.

Table 3. Total numbers of orientations mentioned in students' responses regarding the goals of mathematics teaching and learning

Orientation	**Number of students (*n*=18)**
Formalism-related	11
Scheme-related	13
Process-related	14
Application-related	16
Number of mentioned orientations	
One orientation	1
Two orientations	4
Three orientations	7
Four orientations	6

Table 4. Students' response-profiles with respect to the orientations.

	Theme 1	**Theme 2**
Student A	A	FSPA
Student B	F	FSPA
Student C	F	FSPA
Student D	–	SPA
Student E	F	FSPA
Student F	F	FA
Student G	F	FSPA
Student H	FS	SPA
Student I	FP	FSA
Student J	P	FPA
Student K	F	FA
Student L	F	FA
Student M	A	SPA
Student N	F	SPA
Student O	F	SP
Student P	–	SPA
Student Q	S	P
Student R	F	FSPA

(F= Formalism-related orientation, S = Scheme-related orientation, P = Process-related orientation, A = Application-related orientation)

No clear connection could be detected between the beliefs related to the two themes. Of the ten students who mentioned only formalism-related views vis-à-vis Theme 1, five mentioned views connected with all four orientations, three mentioned views that could be identified with the formalism- and application-related orientations, while two did not refer to the formalism-related orientation at all in the case of Theme 2. Three students mentioned formalism-related views in connection with Theme 1 but not with Theme 2. This suggests that even though several students' beliefs about the nature of mathematics were quite strongly directed toward the formalism-related orientation, their beliefs about the goals of mathematics teaching and learning were more diverse.

CONCLUSIONS

Our study has consisted of an analysis of students' beliefs about the nature of mathematics and also of their beliefs about the goals of mathematics teaching and learning by using as a framework the mathematics orientations published by Felbrich et al. (2008), although in a slightly modified form. The most essential modification made to the original framework was the broadening of the formalism-related orientation so that all the views referring to mathematics as an existing system of knowledge could be included within it. The majority of beliefs presented by the students could be classified by applying this framework, and it worked well in our analysis of the students' beliefs.

Students' beliefs about the nature of mathematics were directed towards the formalism-related orientation in several cases. They considered mathematics to be a ready-made, static system. This result is in line with the results found by Nisbet and Warren (2000). They applied Ernest's framework in their study concerning primary school teachers' beliefs about mathematics and its teaching, and found that a problem-solving perspective was missing from among the subjects. The question about the nature of mathematics was also difficult for many of our students, and they probably had not explicitly thought about the matter earlier. On the one hand, the relatively narrow answers to the questions may indicate that the nature of mathematics and its different aspects are not often discussed in the context of Finnish school-level mathematics teaching. On the other hand, we believe that if the different aspects of the nature of mathematics are asked about by using direct, explicit questions, the students will simply attempt to recall some of the ideas that they have come across in the introductory sections of their math textbooks. Thus, it can be argued that the explicit question is not necessarily the best possible method of studying this kind of implicit knowledge. In fact, the difficulties seem to be similar to other studies concerning the nature of science, to the extent that some teacher students even suffer from a lack of the necessary terminology (Nivalainen, Asikainen & Hirvonen, 2002).

The questions related to the goals of mathematics teaching and learning were more familiar and concrete for the students, and therefore the answers were more varied. Obviously, the students felt more comfortable reflecting on their own concrete learning activities and the models of teaching that they have experienced by themselves rather than on the nature of mathematics per se. Based on our results, however, the important goals were all connected with knowing and understanding the structure of existing mathematical knowledge, the training of calculation skills, the development of reasoning and problem-solving skills, and the application of mathematics. These goals are well in line with the general national goals of upper secondary mathematics teaching (NBE, 2003). This may indicate that upper secondary mathematics teaching in Finland has been successful in conveying the main goals of its teaching. In addition, the results reveal that students' beliefs about the nature of mathematics are not so constricted as our results for the first theme indicate. We would suggest, therefore, that students' beliefs about the nature of mathematics should be studied by using concrete contexts such as their own learning experiences rather than by using explicit questions about the nature of mathematics.

From the viewpoint of teaching mathematics, the key question is how much the different aspects of the orientations should be emphasized in the teaching. Traditionally, until the early 1990s the learning of calculation skills was considered to be a central goal of mathematics learning in Finnish schools (Hassinen, 2006). Several students also referred to this aspect in this study by emphasizing the training, acquiring a routine and automatizing calculation skills, activities that are all clearly related to the scheme-related orientation. However, one of the major concerns among mathematics educators in recent decades has been that the teaching of mathematics is too much focused on these aspects, and that a deep conceptual understanding and the development of students' own reasoning skills is being ignored (e.g. Schoenfeld, 1992).

The results highlight the importance of taking student teachers' beliefs into account in teacher education. It is quite probable that all mathematics teacher education programs affect students' beliefs in one way or another, even if teacher educators do not explicitly recognize their influence. The question is indeed a difficult one, especially because the current literature offers few guidelines. Most scholars, however, agree that it is very important to take into account the whole learning environment, including the learner's own activities and the social interaction between learners in the construction of knowledge structures (Crawford et al. 1998). This means that the aims of learning could be planned according to the principles of the process-related orientation, in which problem-solving is emphasized. Even though the students in this study considered the development of thinking and problem-solving skills to be an important goal of learning, they did not specify what this might mean in practice. Hence, careful consideration of this issue in teacher education and further studies devoted to this topic would be important.

In the present study we have used *a theme-centered approach* and collected different views about two essential themes. By using this method, general trends concerning these themes as they are held by the students interviewed are revealed. This method does not, however, reveal the whole image of mathematics or all mathematics-related beliefs that may impact on the success of students' study of mathematics or the process of maturing for mathematics teachers. In particular, this approach does not reveal which aspects are the most dominant in students' thinking at a personal level. *A student-centered approach* based on a deep-analysis of each student's thinking would be needed for that.

Endnotes

[1] Cited in Felbrich, Müller & Blömeke (2008).

[2] The questionnaire and interviews were conducted in Finnish, but the sample questions have been translated into English.

[3] Both in the survey and in the interviews, themes other than those identified as Theme 1 and Theme 2 were dealt with. These have not, however, been dealt with in the present paper.

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