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Meanings of mathematics teaching forged through reflection in a lesson study

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Abstract
This article discusses the meanings of mathematics teaching forged in the reflections promoted in a lesson study. Guided by the question "What meanings of teaching in mathematics are constructed from reflections promoted in a lesson study?" and based on Isabel Alarcão's (2018) concept of teaching, we examined four editions of a lesson study, two with teachers of the early years of elementary school, one with mathematics teachers from the final years of elementary school, and one with high school mathematics teachers. Each lesson study cycle was organized in twelve two-hour meetings and involved an average of eight teachers per cycle. The participants are teachers from the state public education system in Rio Grande do Sul, Brazil. The empirical material consists of the researcher's field notes, the participants' logbooks, transcripts of the sessions, and interviews conducted at the end of the cycles. Qualitative and interpretative analysis showed that the reflections promoted in the lesson study revealed that mathematics teaching involves various meanings including educational action, social commitment, and pedagogical transformation.

Keywords: mathematics teaching, lesson study, reflection, meanings of teaching

INTRODUCTION

Teaching, the act of educating (Roldão, 2017), as a social construction forged individually and collectively in the constitution of the profession, characterizes the complex activity carried out by teachers in different educational spaces. The teaching profession is affirmed or denied through teaching praxis[1] and the relations of teachers with their interlocutors (Alarcão, 2018). It is *in* action and *through* action that the teaching profession develops (Lessard, 2009) and establishes itself, characterizing teaching as a relational phenomenon (Carvalho, 1999).

Understood as such, teaching presupposes knowledge, values, intentions, and practices related to the specificity of this activity and concretizes the educational and social mission of teachers and schools. Thus, teacher education is a process by which these aspects of teaching can be developed.

Teacher education must promote reflection on teaching to prepare educators to understand and intervene in the face of changes and uncertainties in contemporary education (Imbernón, 2011). Lesson study emerged through the confrontation of challenges in school education, as an approach to the professional development of teachers that focuses on teaching practices (Neves & Fiorentini, 2021; Ponte et al., 2016; Richit, 2020; Richit & Tomkelski, 2022; Stigler & Hiebert, 1999). It originated at the turn of the 19th to the 20th centuries in response to educational changes implemented in Japan (Isoda, 2007).

Lesson study is a process of professional development of teachers supported by two principles: collaboration and *reflection*. According to Richit (2022), lesson study was established in Japan as an approach to professional development within a process of implementation of educational changes, expanding and embracing the educational system.

[1] Praxis refers to the collective elaboration, in a group, of the practices experienced in everyday life. [...] It presupposes a collective: An articulated collective, never en masse or agglutinated (Imbert, 2003, p. 74).

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Contribution to the literature

- The study explores the meanings of mathematics teaching formed in the reflection promoted in a lesson study, based on the analysis of four editions of lesson study, conducted with teachers of mathematics in basic education (elementary school and high school), revealing that their engagement in this process allows them to broaden the understanding of the dimensions of the action of mathematics teaching in the classroom.
- The findings showcase the potential of lesson study and demonstrate attributes of this approach and to clarify the guiding elements that characterize its dynamization in a Brazilian reality. They also reveal the eminently social nature of lesson study in relation to the promotion of mathematics learning in Brazil.
- The findings suggest that the reflections developed in lesson study can contribute to the personal and professional growth of the participants, especially regarding how the meanings attributed to teaching are expressed in their action in classrooms and their potential to facilitate the education of students.

In Western countries lesson study gained visibility through its dissemination in the United States in the late 1990s, with the publication of research results in the English language (Stigler & Hiebert, 1999). In recent decades lesson study has spread through the United States, Canada, and Europe as one of the main cooperative strategies for the development of teachers (Pérez Gómez & Soto Gómez, 2011, Richit et al., 2022b).

In Brazil lesson study appeared at least ten years ago, especially in research by Yurico Baldin. There is now research focused on lesson study throughout Brazil. A search on the Biblioteca Digital Brasileira de Teses e Dissertações (BDTD) [the Brazilian Digital Library of Theses and Dissertations], for the descriptor "lesson study" and the filter "matemática", we identified twenty studies, including three doctoral dissertations (Bezerra, 2017; Borelli, 2019; Utimura, 2019) and seventeen master's theses (Araújo, 2018; Batista, 2017; Carrijo Neto, 2013; Coelho, 2014; Felix, 2010; Freire, 2018; Franzen, 2022; Gaigher, 2017; Mello, 2018; Moura, 2018; Müller, 2021; Neves, 2017; Rodrigues, 2021; Tapparello, 2021; Tomasi, 2020; Utimura, 2015; Vieira, 2021). Moreover, there is a growing number of articles focused on Brazilian experiences with lesson study. A search on the Scielo (Scientific Electronic Library Online), with the descriptor "lesson study e matemática", found fifteen papers.

The results of the experiences analyzed in these studies (theses, dissertations, and articles) have revealed important contributions made by lesson study to the professional development of teachers and to the realization of changes in mathematics education. Moreover, they have indicated specific aspects about the dynamization of this approach in the Brazilian context, especially regarding reflection.

In lesson study, reflection characterizes a reflexive attitude towards teaching through which teachers, in collaboration with colleagues, critically and deeply examine their practice, seeking to understand it in relation to the context in which it is implemented and the phenomena that interfere with it. Based on this understanding, teachers establish lines of action and ways of acting that reveal the meanings of teaching collectively forged in that context (Richit et al., 2022a).

We conducted an investigation to analyze the meanings of mathematics teaching established by teachers who participated in lesson study. Based on Alarcão (2001, 2014), and understanding that meanings are collectively constituted phenomena, we analyzed the dynamization of four editions of lesson study–conducted with basic education teachers in the public school system of northern Rio Grande do Sul state, Brazil. The research was supported by Alarcão's (2018) concept of teaching, in which reflection is a central concept, to conceptualize the principle of reflection in lesson study. Alarcão's (2001, 2014) discussions about didactics help understand teaching based on social, pedagogical, and ethical elements that influence school education and contribute to efforts to improve education.

This research is relevant to the discussions in the realm of mathematics education because it is focused on school education of mathematics, as the primary object of study (Gascón, 1998).

The research can contribute to research about lesson study in Brazil by revealing attributes of this approach and clarifying guiding elements that characterize its dynamization in Brazil. It can also help reveal the eminently social nature of lesson study in the promotion of mathematics education.

REFLECTION AND PROFESSIONAL DEVELOPMENT

Professional development of teachers is a continuous and dynamic phenomenon by which teachers personally and professionally develop, transcending from an individual to a collective level (Richit, 2021). This phenomenon underpins and guides teachers' actions, which include the set of intentionally planned actions conducted in the interactions between teachers and students and knowledge (Alarcão, 2001).

Alarcão (2018) proposes a concept of teaching in which reflection is a central concept. In this concept,

school teaching sparks changes in the practice of teachers as a way of enhancing the autonomy and critical capacity of students.

Corroborating this understanding, Roldão (2007) explains that the implementation of teaching and learning processes guides teaching, around which the expertise of teachers develops and changes. In this way, teaching is anchored by and carries attributes of professional identity, which is historically and socially constituted within the profession, and incorporates or rejects elements depending on the contexts and times. For Alarcão (2014, p. 27), professional identity

is a highly relational, intra and intersubjective concept. It is constructed through relationships, in the union of efforts. It is constructed along a culture's lines of force, emerges as an ideal, materializes in a project (of professional life, of school life, of the life of the profession).

In this sense, the professional identity of teachers, which is intrinsic to teaching, reveals very particular aspects of this profession, among them the reflexive dimension of the action of teachers, which consists "in asking what is happening or what will happen, what can we do, what should we do [...]" (Perrenoud, 2002, p. 30). Serrazina (1999), in turn, emphasizes that reflection gives teachers the opportunity to "rethink their way of teaching mathematics" (p. 143).

Alarcão (2014), supported by Goodson (2000), refers to reflexive action as an attitude of constant inquiry and interaction. She emphasizes that critical and in-depth reflection implies a contextualized approach in which the particularities of the work context are carefully considered, while also being fundamentally questioned. Reflection, in this way, promotes understanding of particular actions, positioning them within the realm of specific institutions and schools, at a particular time, and in particular social, political, and cultural contexts (Goodson, 2000).

When considering the roles assumed by people in school environments, Alarcão (2001) affirms that school is constituted by spaces created and recreated in the context in which teachers, students and school officials interact and socialize. She adds that the relationship between people, and between them and the profession and the school, constitute the bases for teaching and the quest to improve education (Alarcão, 2001).

Selingardi and Menezes (2017, p. 273) add that faced with the challenges of everyday practice, a reflective teacher "performs a reflective movement that consists in turning to the study of a theory in search of a solution. She returns to her practice, now altered in light of the reflection on that theory." Thus, "more than just a process through which teachers look at experiences from their past practice, reviewing episodes, emotions and events, the reflection is also projected to the practice to be developed" (Vieira, 2021, p. 61).

For Roldão (2017), reflection is a device that gives teachers the opportunity to attribute meaning to the teaching experience; and Schön (1983) conceives of reflection as a means for teacher development because it generates knowledge based *on* and *for* the action of teaching.

The individual and collective dimension of professional development supports Alarcão's (2001) reflective school perspective, conceived as the place, the space, and the context for realizing teaching in the distinct senses forged by educators.

Thus, the professional development of teachers can be understood as a permanent attitude of inquiry, the formulation of questions and search for solutions (Marcelo, 2009) about teaching, supported by collective reflection.

However, a reflective attitude, according to Alarcão (2001), cannot be limited to the teachers; on the contrary, it must be nourished by the atmosphere at school. In this light, lesson study is positioned as a context in which teachers assume a reflective posture, reviewing, questioning, and modifying their practices and how they conduct and conceive teaching.

LESSON STUDY

Lesson study, an approach to professional teacher development officially instituted in Japan since the 1960s, is considered primarily responsible for the improvement of teaching in that country (Isoda, 2007; Richit & Tomkelski, 2020; Yoshida, 1999).

Understanding teaching to be a complex and constantly changing activity, Japan established a system that promoted incremental and gradual improvements to education over time. This system, which is based on lesson study, presupposes "clear learning objectives for students, a shared curriculum, the support of administrators and the hard work of teachers who strive to promote gradual improvements in their practice" (Stigler & Hiebert, 1999, p. 109).

According to the NCTM[2] (2000), to improve mathematics teaching, teachers must be able and willing to analyze what they and their students do, seeking to understand how these actions influence student learning. Teaching mathematics requires continuous effort on the teacher's part in order to improve it and help student learning. Therefore, educators must have the opportunity to reflect, together with their colleagues, about their activities and the distinct influences intrinsic to them (NCTM, 2000).

For Lewis (2002), lesson study is a device with which teachers advance in theory and practice through in-

[2] National Council of Teachers of Mathematics

depth, detailed study of the classroom, experimenting and developing best practices.

Lesson study is organized into four distinct moments: *identification of a study question* and definition of objectives for the development of a lesson (research lesson); lesson *planning*, in which small groups of teachers work collaboratively in several sessions to prepare the lesson for a specific group of students, defined at the outset of the process, to address a curriculum topic chosen by the group; *conducting* the class, which is voluntarily taught by one of the teachers participating in the process and observed by her colleagues; and *post-class debriefing*, in which the group meets to discuss and reflect on the students' actions in the research lesson, considering aspects recorded by the observers during the class (Ponte et al., 2016; Richit, 2020). Thus, lesson study is work that develops collaboratively, in which small groups of teachers (Lewis, 2009; Stigler & Hiebert, 1999) study or examine teaching practice (Fernandez & Yoshida, 2004). This work allows educators to reflect on their daily practice; plan and develop new practices; reflect on these and compare them to the established practices in school culture.

The reflection promoted in lesson study favors teachers with a critical and reflective attitude of periodic self-re-examination, of their practice, the knowledge and values which support this practice, and the goals of teaching (Richit et al., 2022a).

Examining the dynamics of collaboration and reflection in lesson study, Quaresma and Ponte (2019) emphasize that the involvement of participants in a process in which they shared goals, facilitated the development of three levels of reflection. The first involves a shallow reflection, based on a description of events, in which teachers tend to associate students' difficulties with external factors. The intermediate level goes beyond description, teachers identify problems and propose solutions, showing that they understand the students' work. Finally, at the most in-depth level of reflection, the educators base their proposals and their analyses on the students' work. Thus, the study showed that at the beginning of the process teachers tended to attribute the students' difficulties to their immaturity or to nature of the math itself. At the end, they began to identify the students' learning problems, and to suggest and establish ways to help them overcome difficulties, transposing the suggestions presented to the group into changes to be carried out in their practice (Quaresma & Ponte, 2019).

Rincón and Fiorentini (2017) conducted an investigation into the educational process and learning by future educators in a Pedagogical Practices in Mathematics course. The course, which was constituted within the context of the study, sought to understand, and problematize school practices for math teaching and learning.

In it, Rincón and Fiorentini (2017) examined the dynamization of a lesson study, from which they sought to answer several research questions, including: what reflections emerge from teaching-learning practices during the participation of future educators in a lesson study? Among the aspects demonstrated in the reflections, the future teachers reaffirmed the importance of planning lessons and anticipating all the possible responses that may emerge during a class, so that the future teachers learn various things from the reflection on this process.

Therefore, reflection in lesson study has the potential to support the development of professional teaching in various ways. According to Murata (2011), lesson study provides educators time and opportunity to reflect on practice and student learning. He adds that the knowledge acquired *from* and *for* reflective practice must be shared with the broader educational and teaching communities (Murata, 2011).

Reflection, in this sense, is an intentional, conscious, and dynamic act related to professional experience, and to the knowledge accumulated over years of professional activity. Lewis (2000) emphasizes this aspect, highlighting that lesson study is a context in which teachers can demonstrate and broaden their skills by identifying their own deficiencies and by requesting and accepting the criticisms of their colleagues. She adds that these criticisms promote reflection because they are not focused on a single individual and their actions, but on teaching as an act conducted by a professional collective.

Iksan and Rahim (2017) investigate how teachers reflect and the level of their reflections by watching videos of teachers teaching. The results of the study indicate that reflection is a central dimension of teaching, because by reflecting on teaching, teachers are able to evaluate their effectiveness in the classroom, which can strengthen the practice of the profession and improve the assessment of students.

Posthuma (2012) analyzed a lesson study (in an adapted form) realized with five mathematics teachers in a rural school in South Africa. The objective of the research was to investigate the reflective practice among teachers while they collaboratively planned mathematics lessons and reflected on teaching the lessons. Lesson study provided a safe space for teachers to reflect on their teaching and they reported an increase in self-knowledge and that they found new ways to teach mathematics to students. The author understands that lesson study enhanced the "reflection-in-practice and reflection-on-practice that enables the teacher to become an agent of change to improve social conditions of practice" (Posthuma, 2012, p. 7). This issue is especially applicable in South African society where

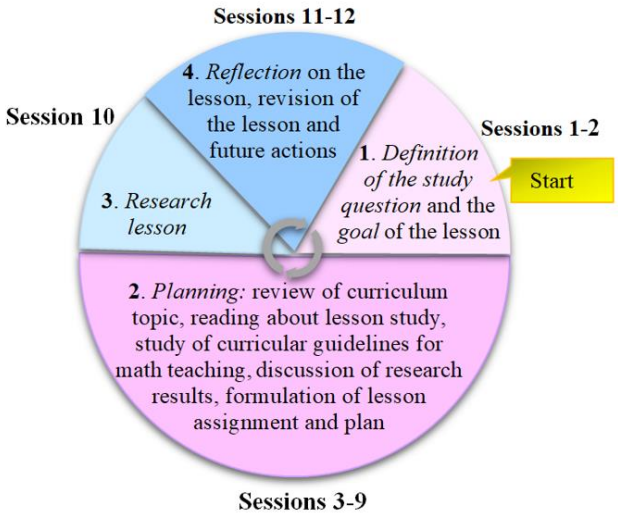

Figure 1. Lesson study structure (Richit, 2021)

teachers' histories need to be considered when planning and implementing professional development programs.

Moreover, the reflection in lesson study, which is exercised individually and collectively, is not restricted to the teacher alone. It is a process that extends to colleagues, instructors, the training procedure, students, the school, and education (Alarcão, 2001), concretizing the shared reflection (Richit et al., 2021a).

From this perspective, lesson study is an important path for the professional development of mathematics teachers, since it creates a context in which they work in collaboration and, above all, have the opportunity to observe, analyze and discuss teaching and students' thinking (Stigler & Hiebert, 1999).

In summary, given the specificities of lesson study, various aspects of professional practices in mathematics are placed at the center of the reflection processes, from which it becomes possible to overcome beliefs, revise conceptions and modify classroom practices. Therefore, lesson study promotes opportunities for teachers to reflect and improve math teaching practice, before, during and after the lesson (as happens in the research lesson) (**Figure 1**).

METHODS

Research Design and Participants

A qualitative and exploratory study (Bogdan & Biklen, 1994) was conducted to analyze the meanings of math teaching forged from reflections promoted by lesson study.

Recognizing the complex (Stigler & Hiebert, 1999) cultural, social, and reflective nature of teaching (Alarcão, 2001, 2014), this study examined the dynamization of four editions of lesson study, two with teachers in early grades, one with mathematics teachers in the final grade of intermediary school and one with mathematics teachers from high school–all of whom are identified with fictitious names. Each cycle was organized into twelve encounters of approximately two hours each and involved an average of eight teachers per cycle.

The participants of the four editions, approximately thirty teachers, are educators who teach in public elementary and high schools in the state public school system in northern Rio Grande do Sul, Brazil. They had a range of 5 to 25 years of teaching experience.

In each lesson study cycle the participants were invited to participate in a lesson study through an invitation sent by e-mail to the schools in a public school district. The e-mail was addressed to mathematics teachers who were asked to respond to the message to indicate their interest in participating.

One characteristic of the lesson study editions that were conducted, is the definition of a theme (a thematic context) on which the research lesson focuses. In each cycle, the research lesson was planned and conducted using a theme close to the students' reality, and from which they were given the opportunity to explore specific curricular topics, defined by the teams of participating teachers (Richit, 2021) (**Table 1**).

Each lesson study cycle was conducted by a team of teacher educators from the Universidade Federal da Fronteira Sul–UFFS, who made an effort to welcome the participants through dialogue, listening and encouragement. By establishing a welcoming context, the participating professors raised the level of engagement in the process. The table below presents the activities conducted in each step of the lesson study in the different editions conducted (**Table 2**).

Data Collection and Analysis

The study's empirical material is comprised of the researcher's field notes, the participants' logbooks, the transcriptions of the sessions and the interviews conducted at the end of each lesson study cycle. We interviewed 20 teachers, who had agreed to participate in this research phase. Approximately five teachers from each cycle of the lesson study were interviewed.

Table 1. Editions and participants

Lesson study (year)	Level of education/topic	Theme/task context
2017	High school/ Function	Maximization of profit
2018	Initial years/metric unit of measurement	School contest
2019	Final years/area and perimeter	Mosaic art and geometry
2021	Initial years/division	School meals

Table 2. Dynamic and activities in lesson study

Meeting	Activities
1st	Welcoming participants Professional experiences in mathematics teaching The reflections began with the following question: What knowledge is essential for the professional practice of mathematics teachers? Brief historical and theoretical presentation of lesson study
2nd	Presentation on lesson study: It's definition, origin, structure, development dynamic, dissemination around the world, possibilities Survey of students' frequent difficulties in mathematics Definition of the topic to be addressed in the lesson study cycle Formulation of objectives for a class as a way to solve the difficulties identified
3th	Discussion and reflection on lesson study by reading articles on this topic Review of the stages of the lesson study Review of students' difficulties in the mathematics topic chosen Review of the objectives for the research lesson
4th	Development of a sequence of exploratory tasks previously prepared by the teacher educators Discussion about the limits and possibilities of the proposed tasks Notes on aspects to be improved in the tasks Start of planning of the exploratory task for the research lesson
5th	Discussion of national curriculum guidelines for teaching mathematics in elementary school Discussion about characteristics and needs of a class of students in which the research lesson would be realized Planning of the exploratory task for the research lesson Definition of the necessary resources for the research lesson
6th	Discussion about the structure of the school and materials available for conducting the research lesson Continuing the preparation of the research lesson Preparation of materials for the research lesson
7th	Discussion on the distinctions between exercises, problems, and exploratory tasks Continuing the preparation of the research lesson Presentation of task developed for research lesson and the suitability of the aspects mentioned by participants Planning of the observation of the students during the research lesson.
8th	Task solving by participating teachers to identify aspects that are lacking Discussion and modification of the aspects needed Preparation of materials for the research lesson Preparation of an observation plan
9th	Finalization of the exploratory task for the research lesson Final organization of the research lesson planning Review and finalization of the observation plan for the research lesson Final preparations for the research lesson
10th	Realization of the research lesson in two classes of two hours each Students' assessment of the experience
11th	Reflection session on students' actions during the problem-solving task Positive and negative aspects of the research lesson Modifications needed in the plan for the research lesson Synthesis of what the participants learned about student learning
12th	Evaluation of the lesson study Review of the research lesson Planning future actions Conclusion of the cycle of the lesson study

Ethics

The investigation complied with ethical research criteria and was approved by the Research Ethics Committee of UFFS (Process nº. 4.764.981), and priority was given to protecting the identity and integrity of the participants.

The qualitative and interpretive analysis (Erickson, 1986), based on content analysis (Bardin, 2003), demonstrated that mathematics teaching, based on the reflections promoted in lesson study, takes on different meanings: educational action, social commitment, and pedagogical transformation.

The categories emerged in the analysis process, which underwent the following steps:

1. careful reading of the empirical material to identify reference units that could reveal how the development of collaboration in lesson study takes place,

2. reference units were systematized and pre-organized into thematic blocks that could answer the research question, and
3. the reference units were again analyzed and grouped by approximations, constituting the categories of analysis: emergence of collaboration, development of collaboration, and valuing collaboration.

Limitations

The limitations of this study are related to the fact that the participants of the lesson study editions were not familiar with the approach. For this reason, reflections appeared more timidly at the beginning of the process and became more developed in the second half of the cycle.

FINDINGS

By reflecting on classroom practices in the mathematics teaching, especially by comparing previous teaching experiences and the research lesson, the participants identified various meanings for teaching.

Educational Action

Teaching as an educational act, involving intentionally planned actions, undertaken, and analyzed in relation to their main purpose (student learning), was expressed by participants in the four lesson study editions.

> **Sofia**: Perceiving the students' difficulties is very important for the mathematics teacher since it is a way of trying to help them in their math learning. Errors can be the basis of the lesson and through the class the teacher seeks to overcome this difficulty (2017).

> **Filipa**: [The students] must understand our intention [for the class]. And this was what I asked them, since I wanted to get to the topic of "money" from whole numbers to numbers with a comma, decimals. I asked them: where do we find numbers? They began to say the number of their shoe size, their clothes, prices, the weight of goods. [It's incredible], they have an imagination! [Thus] we arrived at the topic of "money", which was subject I wanted to work on with them (2019).

Sofia and Filipa showed an understanding that associates the teacher's actions to the dynamic and reflective educational process conceived and implemented to encourage student learning. Sofia values errors as a starting point for this process, while Filipa begins with the students' everyday experiences.

Nick, a participant in a lesson study in 2017, emphasized that a math teacher must identify and understand the students' questions to be able to help them. Janaina, a participant in the 2018 edition, added that errors provide the teacher an opportunity to understand how students think when solving math problems and thus assist their learning. Ivy adds a new facet by emphasizing teaching as a collective educational act since it can involve other school agents.

> **Ivy**: One thing that struck me was that we all entered the classroom, and everyone got involved in something. When the students did the "treasure hunt" [activity] everyone participated, even the secretarial staff and the school administration (2018).

The teaching realized in the research lesson characterizes the educational act as being purposefully thought out and planned to promote learning, whose execution involves other educational agents beyond the teacher.

The teachers found that the nature of lesson study, especially with regard to reflection, led them to understand teaching as a student-centered process.

> **Jade**: [Lesson study] contributed to the professional development of each math teacher [...] especially by stimulating reflection about the pedagogical practice [...] that we usually do and the one we discovered here, which is more concerned with the student and her understanding (2017).

> **Erika**: [I] could understand the importance of lesson planning and, mainly, the observations and the evaluation of the application of the lesson. This evaluation was new, because we compared the goal that was formulated at the beginning to where we ended up, how much we helped them in their learning (2018).

By experimenting with a classroom practice, which was carefully planned and implemented with the goal of helping students with their math learning, enabled the participants to understand teaching from the perspective of what the educator is seeking to promote with their practice. In other words, teaching as the development of a learning path for students, guided by specific goals, forged in response to the concrete context and conditions in which the teaching is carried out.

Moreover, the participants had the opportunity to compare two math teaching perspectives, one focused on the content, as are the practices culturally associated with school mathematics teaching, while the other, the research lesson, emphasized student learning. This helped them to understand that the meaning of teaching differs from other professional practices, especially in terms of its goals and the interferences that emerge from the context.

> **Vick**: [The possibility to] explore mathematics using their day-to-day life, which is where I start, from address, house number, age, birthday, the number of letters in their name or the name of their classmates, [is essential] for math. So, valuing this math in their world [encourages] their understanding, their learning (2021).

> **Marina**: [We conducted] a reflective exercise about our daily classroom procedures, using the students' ideas as a basis for their knowledge building and to help them reflect on their learning, considering the importance of the interaction [between them] to learning. Therefore, the organization of classes should offer moments for small group discussions and moments for large group discussions (2017).

The planning of a different practice, the research lesson, gave participants the opportunity to experiment with math teaching, using different strategies and perspectives from those used in schools. Additionally, the reflection conducted at all stages of lesson study, especially on the objectives and purposes that guide daily math teaching, helped reveal teaching's most intrinsic meaning: teaching as an educational act committed to students' learning and development.

Social Commitment

An understanding of teaching as a social commitment of school and education, which is implemented daily by work of teachers in classrooms, emerged in the reflections about math learning and its consequences in the education of students.

> **Maggie**: The mathematics teaching goes far beyond working with content. The BNCC [Base Nacional Comum Curricular–the National Common Curriculum Base] states that we have to "develop or discuss projects that address urgent social issues, based on solidarity and sustainable and democratic ethical principles, valuing the diversity of opinions and individuals and social groups, without prejudice of any kind" (2019).

In light of this discussion, each teacher related a classroom experience to the group, through which they approached, or could approach, mathematics in a broader perspective, considering socially relevant themes and situations.

> **Adelle**: [I'm developing a project with the 6th grade that we call Geometric Garden, to work on perimeter and area]. Later we will work with the amount of material used, with measurements, with time, observing the time of germination and growth of the plants. This project, which is conducted with the science teacher, is a project [in which we reuse] empty milk cartons to make the plant beds. In addition to mathematics, we are trying to work, with 'environmental education' (2019).

Math teaching, in Adelle's experience, has the potential to transcend the boundaries of the discipline, promoting educational processes focused on socially relevant themes and issues, such as environmental education, in accordance with the national curriculum guidelines (BNCC) (BRASIL, 2017).

Thus, the social dimension of math teaching, expressed in the official curriculum, permeated the choice of thematic context for the development of the curricular topic in each lesson study edition. In the 2021 edition, for example, at the outset of the planning of the research lesson, the possibility to address broader issues permeated the group's reflection, so that they defined 'school nutrition' as the topic for tackling 'the division of natural numbers.

> **Layla**: [With] this class we addressed other topics, such as wasted food in meals served at school and, also, the municipality's school food program, contributing to the civic education of these children (2021).

> **Lilly**: True. It's a totally different math class, because we will explore 'division' based on a situation in their everyday lives and one that will make them think about nutrition, about healthy nutrition and even the lack of food in other realities (2021).

Thus, the choice of a thematic context for the research lesson, which was the basis for the exercise, established the social dimension of the math instruction. In the group mentioned, the theme proved to be suitable for addressing the topic of division, using the school's snack and lunch food distribution process, discussing, among other things, wasted food.

From a different perspective, in the 2018 edition, which dealt with 'a meter as a unit of measurement' using an activity called 'treasure hunt', math instruction was adopted as a commitment shared by the entire team.

> **Janaina**: When we planned the class, the math activity and how we would conduct the class, we considered everything that involved the students in the class–the school space, their conditions, and difficulties, who could help in the research on "measurements", who could help in the treasure hunt. It wasn't simple. Each part of the lesson was a responsibility, a commitment, because we wanted them to understand the meaning of a meter, emphasizing their knowledge and that of their family (2018).

However, the instruction carried out by the educator, in this case a math teacher, wasn't limited to the classroom. The process transcends the classroom space.

In addition, reflection upon the research lesson class, conducted with this group, emphasized an understanding of teaching that goes beyond a purely mathematical approach.

> **Ranya**: We do a lot of this every day, but we often forget that our work in the classroom will influence the student. If he is valued, he is more interested and learns more. So, we build this every day, together with the school, making our students grow (2018).

To promote student learning, teaching needs to be a device for achieving greater school and educational goals. Every day we fulfill a commitment to students' learning and development throughout the students' school trajectory, including in a social sense.

When reflecting on guidelines for math teaching, recommended in Brazil's national curriculum guidelines, educational strategies and dimensions of the students' educational path emerged that reveal new meanings for teachers' work. Additionally, the reflection promoted in lesson study led them to view their classroom work as a means of promoting a broader education. They were also able to glimpse the social purpose of their work, which is achieved throughout the students' schooling.

In summary, teaching is a social process, since it involves educational officials in differing roles in school communities, and because it is a way to promote more general school and educational objectives, promoting the development of students. In the same way, it is a process through which to pursue a broader spectrum of student education, encompassing social, cultural, and ethical issues, which may influence their present and future actions.

Pedagogical Transformation

Teaching as a process of pedagogical transformation emerged from reflection during the preparation and realization of the research lesson. First, class planning was characterized by a search for balance between curricular guidelines and the concrete conditions of the group of students with which the class would be conducted.

> **Roger**: One aspect that I emphasize, which I feel should be the starting point of a teacher's work, is to observe the student's difficulties and seek to consider them in their lesson planning and activities for the students (2017).

> **Ellie**: This practice [the research lesson] made us think of ways of putting into practice much of what the official curriculum, the BNCC, the state guidelines say. But it is not possible to apply, to do, everything, it depends on the reality, on the students' conditions (2019).

> **Chloe**: Because in mathematics the student has to comprehend, understand the content. It's useless just copying and doing exercises from the book. So [when I prepare an] activity [using a textbook], I select what I think is important to work on, if it's on their level, if it's cool. [Later] I explain, question them, because I think it's hard to follow [what is written] in the book (2021).

This characterizes teaching that is a process of transformation, pedagogical in nature, in which students' difficulties and context were incorporated as objectives defined by the teacher and the available materials, which were transformed into specific strategies and activities (tasks) to promote the learning of particular curricular topics. Intrinsic to this process, the teachers emphasized the importance of considering the students' previous knowledge and daily experiences.

> **Léa**: I try to work with issues within their reality, what they have. In the case of the monetary system, they will work on simple problems, things closer, more familiar to their day-to-day lives. [...] there is a small market near the school, so students in the fourth or fifth grade will note down the prices to later work out problem situations (2021).

The participants understand that the teaching of math, based on students' experiences and interests, can stimulate their learning, because it establishes a relationship between prior knowledge and knowledge to be developed. From this relationship, the needs of the students, school conditions and teaching objectives are transformed into educational processes.

They emphasized that the research lesson achieved the transformation *of* the established intentions for the activity *into* a student learning process. This transformation begins by recognizing that the students' reality guides the limits of the class's thematic context, the elaboration of the task and materials for the students, as well as the definition of classroom strategies.

For the participants, the realization of the research lesson and reflection upon student actions in its dynamics, characterized an act of pedagogical change. This change involved the nature of the students' learning experience, adopting a process of building paths and task-solving strategies, which enabled the students to make math discoveries.

> **Adelle**: I think [that this way of teaching math] in which they develop, and we question them [...], later they themselves, with a little knowledge, either make their own conclusion, or they

> question other things and making discoveries helps a lot in their learning (2019).

Adelle emphasizes how students are involved in learning math in the investigation class, revealing a transformational shift in terms of the nature of the learning process. Thus, learning math is conducted in the interaction of students with a task, focused on a curricular topic and fed by the questioning intervention of the teacher, leading them to make discoveries and justify them.

The participants emphasize the transformation that occurs during the dynamization of the class, explaining how interactions between teachers and students make possible established objectives for educational action.

> **Mateus**: Our attitude was different. We weren't there to say if what they were doing was right or wrong. Of course, we policed ourselves [...]. Even if [initially the students said something wrong], somehow, we would talk with them and let them arrive at this conclusion (2017).

> **Filipa**: As Judy said at the first meeting, "They [can] calculate [it], but can't explain to us what area is." So, our effort from [the research lesson] was in this sense, to help the students understand what area is and not just calculate area (2019).

Reflection upon the dynamics of the research lesson, upon its different stages and its characteristics, revealed the process of transformation of events that emerge in the classroom, by means that seek to promote student learning and comprehension. Alice emphasized the process of transforming previously defined goals into resources and materials to work on specific topics.

> **Alice**: [Lesson study] helped us reflect upon our practice, starting from what we want from the class and how we are going to reach that goal. We had to think about the resources, the activities to meet that goal and later evaluate if they were good or not. It was like transforming a goal into a class, into resources for the class (2019).

Reflection on math teaching conducted in the lesson study editions, revealed aspects intrinsic to the activities proposed in the research lesson.

> **Vick**: This way requires much more than thinking of our teaching in terms of lesson organization and the child himself, because it is not a schematized thing that will have a ready answer. [On the contrary], the activity for this lesson will have new possibilities, several possibilities for resolving it. [...]. So, it will encourage their learning because, from the moment that the child steps forward and tests a strategy that they [proposed] and that worked, they'll never forget it (2021).

> **Marie**: [This way of teaching] math avoids the technical idea and, therefore, develops the student's sense of security, because when he finds out for himself, he becomes more secure, [he feels] secure and [improves his] self-esteem. He believes more in himself. I think that maybe this is where math [becomes] a little more human (2019).

The aspects of the research lesson emphasized by the participants reveal the complex process of preparing and developing the learning path for math students. That is, math teaching was characterized as a phenomenon of transformation, operating in a pedagogical dimension of the educational process, and enabled by conditions, strategies and resources outlined by the teacher, using knowledge intrinsic to curricular programs, to promote student learning, encompassing both educational and social dimensions.

DISCUSSION

Educational Action

Reflections promoted in lesson study characterized teacher's actions in mathematics teaching that were dedicated to its primary goal, which is student learning (Lewis, 2000; NCTM, 2000; Roldão, 2014), encompassing different aspects. By promoting a practice that begins with student difficulties, while at the same time facing these difficulties in order to overcome them (Stigler & Hiebert, 1999; Vieira, 2021), the participants of the lesson study editions experienced and understood mathematics as an educational act (Lewis, 2000; Murata, 2011; Richit, 2021). The experience allowed the participants to view teaching from the standpoint of what the teacher tries to promote with their daily practices, understanding teaching objectives as dimensions intrinsic to broader purposes, such as those of schooling and education.

By treating teaching as an educational act, revealed in movements of individual and collective reflection, as realized in all stages of lesson study (Hummes et al., 2020; Murata, 2011; Ponte et al., 2016; Rincón & Fiorentini, 2017; Richit & Tomkelski, 2020), allowed the participants to more accurately gauge the scope of this activity. In addition, experiences in lesson study led them to understand how teacher action can enhance or support student development.

Teaching conducted in the research lesson characterized the purposefully considered and planned educational action (Alarcão, 2001, 2014), whose realization involved a process centered on students and their development. The teachers recognized that the nature of lesson study, especially in terms of reflection (Iksan & Rahim, 2017; Lewis, 2002; Richit et al., 2022a;

Vieira, 2021), led them to view teaching as the development of a student's learning path, guided by specific goals, forged in regard to the context and conditions in which it is carried out.

In addition, through collective reflection (Alarcão, 2018; Quaresma & Ponte, 2019; Richit et al., 2021b), the participants in lesson study revealed attributes of mathematics teaching such as the nature of the approach, the role of error, the importance of considering the context and experiences of students, as well as the complementarity of these aspects and the goals of teaching.

Thus, lesson study encouraged the professional development of the participants based on activities that allowed them to try new practices (Lewis, 2002; Neves & Fiorentini, 2021; Ponte et al., 2016; Richit & Tomkelski, 2020) and reflect on how teaching is performed, revealing its different meanings.

Social Commitment

The teachers' reflections on teaching mathematics, and its consequences for education, revealed aspects about the process's objectives. First, the participants emphasized that teaching presupposes an expanded understanding of the teacher's mission, which is carried out daily and focused on the future (Selingardi & Menezes, 2017), and how it influences students' actions.

The analysis demonstrated that teaching mathematics is a social process, since it involves educational agents in different roles within school communities (Alarcão, 2001, 2018). It is also social because it is a way to promote broader school and educational goals (Alarcão, 2014; Goodson, 2000; NCTM, 2000), promoting student development in ways that transcend the limits of just mathematics. Thus, it is a process through which broader student education is sought, comprising social, cultural, and ethical issues that can influence their current and future actions.

Reflection in lesson study allowed participants to view math teaching as a commitment shared by school teams (Murata, 2011; Posthuma, 2012; Richit & Tomkelski, 2022), placing it within specific social, political, and cultural moments and contexts (Goodson, 2000).

Reflection promoted in the lesson study editions encouraged the participants to adopt a critical attitude of intermittent re-evaluation of themselves, their practice, the knowledge, and values that sustain this practice, as well as the purpose of teaching (Richit et al., 2022a).

Therefore, teaching as a social commitment emerged from reflection on the action of teachers as a phenomenon that transcends the space and time of schools, involving school agents with different roles (Murata, 2011). Understood in this way, teaching is a phenomenon that expands, transforms, and looks toward the future (Selingardi & Menezes, 2017; Vieira, 2021), that realizes the social mission of the teacher, the school and education.

Pedagogical Transformation

Lesson study provided time and opportunities for teachers to reflect on math education and student learning (Murata, 2011; NCTM, 2000; Ponte et al., 2016; Richit, 2022), in a movement to transform curricular guidelines and teaching goals into actions and resources. The planning of the research lesson, which focused on specific thematic topics and contexts, allowed establishing a balance between the curricular guidelines for mathematics teaching and the concrete cultural, social, structural, and educational conditions in which the classes were held.

Based upon the planning and execution of the research lesson, the participating teachers examined the students' learning difficulties, and suggested and established ways to help the students overcome these difficulties and put them into practice (Quaresma & Ponte, 2019), leading to a transformation focused on a practice to be executed, and that is committed to the development of students.

Teaching as a pedagogical transformation was materialized through a relationship established between previous knowledge and knowledge yet to be developed. On this base, teaching objectives were transformed into actions and resources carefully prepared for specific groups of students, in specific contexts.

The teaching implemented in lesson study characterized a phenomenon of transformation, operated in a pedagogical dimension of the educational process. This phenomenon is enabled by the social, structural, and cultural conditions, as well as strategies and resources outlined by the teacher, using knowledge intrinsic to curricular programs to promote student learning, encompassing educational and social dimensions.

Finally, reflection upon teaching was found to be a continuous and dynamic phenomenon (Alarcão, 2001), through which the participants in the lesson study editions developed both personally and professionally (Marcelo, 2009), transcending from an individual to a collective level (Richit, 2021).

CONCLUSIONS

The analysis focused on the meanings of mathematics teaching forged in lesson study, considering that these meanings are collectively constituted phenomena that reveal three aspects: educational action, social commitment, and pedagogical transformation.

The meanings revealed were understood as structural elements of teaching and therefore define the professional identity of teachers and can be influenced

by elements internal and external to the profession, such as curricular guidelines and the concrete conditions under which teaching is conducted. We thus understand that the forged meanings are complementary to each other, while they also stimulate the practice of reflection; triggering changes in ideas, dispositions, propositions, and actions, established individually and collectively.

Teachers from lesson study editions highlighted the contributions reflection has made to their personal and professional growth in relation to how meanings attributed to teaching develop in their classroom action and potential to assist student education. The meanings of mathematics teaching revealed in lesson study constitute a sparsely explored field in math education. Therefore, many questions are unanswered about the nature of reflection promoted in lesson studies, or even how reflection contributes to the re-signification of teaching itself, and its different meanings.

Limitations of This Research

A possible first limitation of this research concerns the categories of analysis generated by a specific theoretical framework (Alarcão 2001, 2014); It is understood that the generation of categories from other theoretical frameworks could reach different results in relation to the meanings of teaching attributed by teachers, according to the study by Breda et al. (2021b) and collaborators. A second possible limitation could be related to the fact that the teachers participating in the LS did not obtain another theoretical tool in their training that would help them guide and structure reflection based on specific criteria (Breda et al., 2021a); perhaps, using reflection development tools in combination could have generated different results from those found. Finally, the third limitation of this research is related to the categories of analysis and their relationship with the professional context of the participants. Because the teachers are in the same school district, we understand that the meanings of mathematics education forged through reflection in lesson study are influenced by the social, cultural, and educational aspects shared in this school district. Therefore, an analysis involving teachers from other realities may lead to different results.

Similarly, we understand that an analysis supported by a different theoretical framework than this could reveal new meanings of teaching mathematics and help reveal the intrinsic dimensions of this process.

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