OPEN ACCESS
EURASIA Journal of Mathematics Science and Technology Education
ISSN: 1305-8223 (online) 1305-8215 (print)
2017 13(7):3539-3552
DOI10.12973/eurasia.2017.00743a

Methodological Potential of Computer Experiment in Teaching Mathematics at University

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Received 12 May 2016 ▪ Revised 29 January 2017 ▪ Accepted 23 March 2017

ABSTRACT
The study is relevant due to the opportunity of increasing efficiency of teaching mathematics at university through integration of students of computer experiment conducted with the use of IT in this process. The problem of there search is defined by a contradiction between great potential opportunities of mathematics experiment for motivating and developing modern scientific world outlook of students and beliefs about general methodology of cognition and between the lack of methodology for its implementation for teaching mathematics at university. The aim of the research is to explore methodological potential of computer experiment for improving the quality of teaching mathematics at university and to develop an appropriate methodology. The article describes student activities that allow using computer experiment for formation of individual research skills and skills of working with modern tools for solving theoretical and practical problems and presents methodology of studying certain sections of mathematical analysis including setting up problems that require experimental research of mathematical objects and their properties. The article may be used for content design of mathematics courses for raising motivation, preliminary studies of abstract notions, such as limit of a sequence and continuity, and organization of in-class, individual and research work of students aimed at exploring ways of scientific cognition and corresponding information technologies.
Keywords: computer experiment, teaching mathematics at university, use of information technologies in teaching, activity approach to teaching.

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State of the literature

- Works on theory and methodology of teaching mathematics do not properly describe the issues of integrating experimental activity in teaching mathematics at university, although some research for school education has been conducted.
- Most research on modernization of teaching mathematics at university see the use of ICT tools and computer experiment as an instrument to increase visualization and motivation and it does not fulfill its methodological potential in improving cognitive activity of students.

Contribution of this paper to the literature

- This research describes opportunities to use computer experiment as methodological instrument contributing to increasing efficiency of teaching mathematics to students at university.
- This paper is the first to emphasize the experimental line that accompanies the study of theoretical facts and gives examples of software products which make computer experiment in classroom, individual and research activity possible.
- This paper suggests the methodology of using computer experiment in teaching mathematics to students at universities, and it is directed at positive motivation and development of modern scientific world outlook, ideas of general methodology of cognition, and modern mathematical software products.

INTRODUCTION

Nowadays technologies develop so fast that the whole education system faces the problem of teaching not only a certain amount of knowledge but introducing students to activities that help to gain knowledge and practical skills. This concept is called lifelong learning. The problem of data deterioration is serious for the teaching students because their professional life will be strongly connected with the use and development of new information technologies. The solution of this problem may lie in adding research elements to the education process at university, and it is confirmed by corresponding competencies in federal state educational standards for all degree programs. This approach is already used in European and American education systems (Karpov, 2011). Moreover, network technologies play a significant role in the formation of modern education (Hiltz & Turoff, 2002; Golubev & Testov, 2015).

The fundamental core of education in IT sphere is mathematical knowledge that allow to build, research and use various mathematical models and use corresponding algorithms. However, teaching mathematics at universities is still traditional. Innovative approaches are poorly used because of the gap in views on education between the older generation and students (Cuban & Kirkpatrick & Peck, 2001; Yushau, 2006; Guo, Dobson & Petrina, 2008), although mathematics has great potential for efficient organization of scientific studies of students (Kalinin & Sokolova, 2013). Computer experiment helps to overcome this gap by integrating in itself new knowledge in mathematics and the active use of IT. The accumulation of results obtained from computer

experiments in mathematics has led to the identification of specifics of methodology of the experimental approach which is now called "experimental mathematics" (Shabanova et al., 2016).

The urgency of the research approach to teaching contributed to the inclusion of experimental mathematics in school education (Marco, 2013) and its usage for the organization of project activity of students (Altshuler, 2005; Lipatnikova & Kosikov, 2013). Systems of dynamic mathematics such as GeoGebra are used not only for the visualization but also for experimental research in school geometry (Sgibnev, 2007; Shabanova et al., 2013). In order to raise interest among students of 7-9 grade to mathematics and improve their creativity, the Northern (Arctic) Federal University named after M.V. Lomonosov annually holds a tournament in experimental mathematics (Website of the tournament in experimental mathematics, 2017). Experimental mathematics and corresponding activities for higher education are not researched enough.

Research Aims

The aim of the research is to show the methodological potential of computer experiment and give recommendations on how to use it for teaching mathematics to students at higher education institutions. Basic tasks of the paper are to study the experience of using the experimental approach in teaching mathematics at universities, find out the activities where the use of computer experiment is reasonable and contributes to the enhancement of teaching efficiency, select models of computer experiment that correspond to general aims and curriculum of students learning mathematics, and give recommendations for its use.

LITERATURE REVIEW

Analysis of Russian academic literature

The role of experiment in mathematical studies was acknowledged by famous mathematicians such as Arnold (2006), Polya (1957), and Poincare (1983). Gnedenko (1991) devoted attention to the development of mathematical methods of research as an independent scientific direction. Experiment is an integral part of any scientific study (Nalimov, 1971) that is why when experiments on the studied issue are preceding theoretical studies it forms scientific style of thinking, and this is one of the main educational tasks of modern course in mathematics at higher education institutions. Positselskaya (2012) considers mathematics experiment held with the help of GeoGebra as an instrument of supporting evidence in studying mathematics.

Analysis of psychological and pedagogical, mathematical and methodological literature and professional experience of teachers shows that modern education environment cannot exist without information technologies. For example, specialized software programs are actively used for teaching mathematics (Safonov, 2009). The examples of effective use of computer mathematics systems for teaching pupils and university students can be found in the works of Klekovkin and Ivanyuk (2009), Sergeyeva (2010), Shirikova and Shabanova (2013). However, said studies concern teaching geometry and barely mention other sections of university course in mathematics.

Analysis of International studies

Foreign scientists studied dependence of successful scientific and learning activity on motivation, interest and academic engagement (Singh, Granville & Dika, 2002) and concluded that the strongest effects were given by academic time spent on homework. A series of research on the influence of web-based tools and computer simulations used in classroom work and homework prove their efficiency (Jacobson, 2006; Campbell et al., 2010; Wooten & Eggers, 2013; Leong & Alexander, 2014; Wang et al., 2014). Moreover, this positive effect is stable and lasts for over 10 years. Due to the importance of Internet technologies some researchers call present days "Online Learning Era" (Hsieh, 2016). The model of interactive educational environment that considers different learning styles and can be implemented in teaching computer science and engineering courses was introduced by Hamada & Hassan (2017). However, teachers are not ready enough to use all methodological opportunities of modern technologies (Dogru, 2017).

Cloud technologies are a promising direction to develop interactive educational environment. Scientists actively study the efficiency of cloud technologies and cloud computing in education. Kavitha (2013) classifies cloud services that may be used in education process. Yadav (2014) points out such benefits of cloud computing for institutions and students as personalized learning, reduced costs, accessibility, no extra infrastructure, go green, user-friendly.

Sociological research on the behavior of mathematically promising students carried out by Budak & Kaygin (2015) showed that such students are more active in classroom activities and communicated more frequently with their teacher compared to their normal ability peers. Therefore, the realization of individual approach to education and development of research skills of students require corresponding activities that would allow fulfilling their intellectual potential.

Fundamental courses in mathematics are the base for applied disciplines which use mathematical modeling for solving problems in various knowledge areas. The curriculum of these courses and their efficiency was studied by Brake & Geiger (2011), Kaiser, Schwarz & Buchholtz (2011), Zeytun, Cetinkaya & Erbas (2017). Modeling courses stimulate students' interest in mathematics, develop creativity, communication skills and solving practical problems (Carlson, Larsen & Lesh, 2013). Working with a model requires some experience in experimental research, and it can be gained during first years at university.

The work (E1-Menoufy, 1991) studies the experiment in teaching statistics to students, but generally the experimental approach in teaching mathematical disciplines at universities is left without proper attention.

The analysis of literature allows us to conclude that studies on theory and methodology of teaching mathematics at university describe only some aspects of the problem. It should be noted that the abovementioned studies do not properly develop methodological concepts that systematically use experimental approach, all the more that provide methodology or techniques of conducting computer experiment with information technologies in individual and research work of students.

MATERIALS AND METHODS

Theoretical and empirical methods

- Theoretical methods – analysis of psychological, pedagogical, methodological and technical literature and methodological studies on the issues of teaching mathematics at university; analysis of the literature on the issues of using ICT in education; analysis of software products for conducting computer experiment, method of mental experiments, forecasting, systematization and generalization of facts and concepts, modeling;
- empirical methods – participant observation, summative and formative pedagogical assessment, questionnaire, testing, analysis of the results of experimental studies.

Research base

Evaluation, generalization, and implementation of results of the research are done through:

- Competent teaching of the course "Mathematical analysis" to the students of degree program 01.03.02 Applied mathematics and informatics (1-2 courses) based on the studies of Sokolova (2012); since 2008 the course has been taught at Vyatka State University of Humanities and since 2016 at Vyatka State University;
- Reports and speeches at scientific conferences and workshops of different levels, including international, publications in collections of scientific articles and scientific and methodological periodicals.

Stages of research

The research consisted of three stages.

The first stage included summative assessment: the state of the problem of the use of computer experiment in theory and practice of teaching mathematics at different levels of education was studied. For this purpose, psychological, pedagogical, mathematical, scientific and methodological literature on the issue was analyzed, the study and comparative analysis of experience in teaching mathematics at universities in Russia and other countries in order to identify possible organizational forms of including experimental parts in educational process were carried out.

The second stage was devoted to the development of methodological approaches to the design of a university course in mathematics including computer experiment organized with the use of information technologies. Moreover, existing experience in using ICT in mathematical education was analyzed and most useful tools for the organization and carrying out of computing experiments in individual and research work of students were selected.

The third stage is teaching and further improvement of the course in mathematical analysis that includes the experiment part. During this stage, pedagogical experiment was carried out, and its results proved the efficiency of the developed methodology for improving the quality of

learning the theory of mathematical analysis. Implementation of the course is accompanied by constant monitoring of the results of the education and it leads to further improvement of the proposed approach. The results of the research are discussed through publications in journals and reports at conferences and workshops of different levels.

RESULTS

Computer modeling shows the fundamental meaning of mathematical models in scientific description of reality. The key moment in the work with a computer model is the study of its characteristics, i.e. computing experiment. It should be noted that unlike the experimental work with, for example, physical or chemical equipment that allows watching a process or a phenomenon, computing experiment allows studying unobservable processes such as to check inequalities, track the values of a function and so on.

On the one hand, computer experiment helps to solve theoretical problems. On the other hand, its results may lead to formulation of a new hypothesis which would need analytical proofs (Pankratova & Sokolova, 2013). Also, computer experiment may reveal a counterexample for the proposed hypothesis that will allow specifying conditions for analytical analysis (Sokolova, 2010). Situations when one needs to find refutation of some statement but it is hard to do analytically are not rare in mathematics. Computer-aided exhaustive search of admissible values of the studied variables often helps to find the required counterexample quickly. Therefore, it can be said that computer experiment is one of the modern methods of mathematical studies.

It should be noted that under the circumstances of implementation of the activity approach in higher education, computer experiment contributes to gaining theoretical knowledge as a result of individual work of a student, his personal experience (Johnson & Lesh, 2003). Corresponding emotional experience makes the education process significant to a person and therefore ensures a high degree of its efficiency. Moreover, empirical way of cognition not only reflects historical development of the studied information but also helps to discover new facts that do not have a theoretical justification yet. This information may become the basis for individual scientific studies of students.

Since students are more interested in forms of work involving ICT (Hsu, 2011), computer models can be created with a wide range of software products from spreadsheets to powerful systems of computer mathematics (for example Maxima, MathCad, MatLab, GeoGebra and others), cloud technologies (http://cloud.sagemath.com) and programs in high-level languages created by students. The choice of means to create a model depends on a specific specialization of students and should correspond to the content of educational program.

The results of a correctly organized and carried out computing experiment do not depend on chosen means of implementation of a computer model and it proves the universality of this approach.

Computer experiment in teaching mathematics is reasonable because of the following advantages:

- It ensures the learning of modern technologies of scientific cognition by students.
- Model visualization allows to better understand rather abstract concepts.
- All stages of research such as problem formulation, choice of methods of its solution, planning and carrying out the experiment, analytical review and assessment of its results, and drawing conclusions (English & Sriraman, 2010; Lesh & Doerr, 2003) are presented.
- Education process integrates with research activities.

Depending on complexity of a task, computer experiment can be conducted in one of the following forms:

1) Tasks on computing values during classes. Such experiments precede new definitions.
2) Series of experiments with the use of software tools during the individual work of students. This type of experimental work is aimed at extension and generalization of knowledge, formation of practical skills for working with specialized mathematical software programs. Students can write their own programs for performing calculations if their degree program allows it.
3) Experimental study of the properties of a mathematical object as a part of research work of students. This task is more time-consuming than individual work but a student may work out their own scientific results such as a hypothesis or an analytically proved statement.

At the stage of interpreting the results of computing experiment it is important to note that despite the apparent high accuracy of calculations there is a rounding-off error because of the limited word size of a computer. That is why a researcher should understand that it is important to find an analytical proof or a refutation of hypotheses.

Let us take a closer look at the methodology of using computing and computer experiments in the study of the course "Introduction to mathematical analysis". Students study this course in the first semester of their first year at a university that is why it is hard for them to understand such abstract concepts as limit of a sequence or continuity of a function. Preliminary experimental work with a studied concept helps to form initial views, motivate students and form the basis for understanding mathematical definitions.

Topic 1. Number sets

The concept of ε-neighborhood of a point is a key concept in this topic that is used in the course of mathematical analysis for formulating definitions and proofs of statements.

Formulation of experimental task №1: Find such values of ε where the points fall within $U_{\varepsilon}(\sqrt{3})$: 1; 2; 1,5; 1,35; 1,732. Present these points on a number axis.

Formulation of experimental task №2: Is it true that there are infinitely many real numbers between any two different real numbers?

These problems don't require special tools and students can solve them in class.

Topic 2. Limit of a numerical sequence

Formulation of experimental task: Fill the following table for n∈[1, 100] and draw a conclusion on patterns of values in the columns (**Table 1**):

Table 1. Values of elements of numerical sequences

n	$\frac{1}{n}$	$\frac{4n^2+5n+7}{3n^2}$	$\frac{2n+1}{3n^2+5n+1}$	$\left(1+\frac{1}{n}\right)^n$	$\sqrt[n]{n}$
1					
2					
…					

This task is designed for the individual work of students before they learn the definition of a limit of a numerical sequence. The more examples of sequences will be considered, the more the tendency to a single numerical value will be presented. Spreadsheets or a system of computer mathematics, such as MathCad, can be used as IT-tools.

Topic 3. Functions and their limits

Formulation of experimental task: Plot graphs of functions defined parametrically. Find out the effect of changing the value of the parameter a on the graph. What values of the parameter a are invalid? Can these functional dependencies be presented in explicit form $y = f(x)$?

a) $\begin{cases} x = a\cos t, \\ y = a\sin t \end{cases}$ b) $\begin{cases} x = a(t - \sin t), \\ y = a(1 - \cos t) \end{cases}$ c) $\begin{cases} x = a\cos^3 t, \\ y = a\sin^3 t \end{cases}$

Parametrically defined functions occur frequently in modeling of physical or technological processes when a parameter t corresponds to a timepoint that is why it is important to help students understand functional dependencies of this kind. It is recommended to complete this task with the use of graphics packages that can plot graphs of different types of equations or with GeoGebra. Experiment with function graphs can be done as a part of homework or individual work.

Topic 4. Continuity of functions

The condition of continuity of a function is used in the studies of differential and integral calculus, so it is important to give students solid knowledge on theory and practical skills to determine continuity of a function from its graph.

Formulation of experimental tasks (Berman, 2001):

1. The function is defined as follows:

$$y=\begin{cases}0, x<0;\\ x, 0\le x<1;\\ -x^2+4x-2, 1\le x<3;\\ 4-x, x\ge 3.\end{cases}$$

Is this function continuous? Graph the function.

2. Let
$$f(x)=\begin{cases}-2\sin 0, x\le -\dfrac{\pi}{2};\\ A\sin x+B, -\dfrac{\pi}{2}\le x<\dfrac{\pi}{2};\\ \cos x, x\ge \dfrac{\pi}{2}.\end{cases}$$

Find numbers *A* and *B* so that the function *f(x)* is continuous. Graph the function.

3. Study continuity and graph functions: a) $y = x - [x]$; b) $y=\dfrac{1}{x-|x|}$; c) $y=(-1)^{[x]}$.

Students can also use graphics packages or GeoGebra.

In the course of "Introduction to mathematical analysis" research activity of students may be organized on the following educational material: statements complementing arithmetic properties of limits, inequality theory, special types of convexity of functions.

This article has studied task formulations connected with the study of basic concepts included in any mathematics course. Range of tasks to conduct computer experiment can be extended depending on the depths of curriculum content.

The use of computer modeling to solve mathematical problems helps students to acquire skills of research activity, analysis of experimental data and formation of independent critical thinking (Karakelle, 2009). Thus, computer experiment has a great methodological potential for preliminary studies of complex mathematical concepts. Also, since computer experiment is a modern method of scientific cognition it can be used to organize scientific and research activity of students. Experience

of teaching mathematical analysis at universities shows that students are interested in solving practical problems especially using IT.

DISCUSSIONS

Computer experiment can be successfully integrated in interactive educational environment of a university. Implementation of systems of computer mathematics and cloud technologies, such as sagemath, is a promising direction to improve organization of computer experiment.

Computer experiment carried out in class plays a motivating role and increases visualization which is important in studying abstract concepts of mathematical analysis. During individual homework, the experiment is aimed at consolidation of knowledge, deepening and extension of knowledge about the object of study, and acquisition of modern tools. Experimental work with hypotheses opens the way for serious scientific work of students, provides a full research cycle from the formulation of an open problem to the description of observation results and their analytical testing. Implementation of systems of computer mathematics and cloud technologies in research activity of students brings mathematical experiment to a new level while maintaining its semantic content: collection and analysis of empirical data, formation of hypotheses, their initial testing and search of counterexamples.

Alongside with fundamental mathematical courses the proposed methodology can be used in special courses connected with the issues of mathematical modeling.

CONCLUSION AND RECOMMENDATION

The approbation of possible content and organizational forms of computer experiment in teaching mathematics to students of higher education institutions has allowed to develop methodology aimed at creating conditions for effective organization of individual and research work of students, preparing to solve professional problems using mathematical models and modern software tools.

Methodological approaches to the planning of classroom, individual and research work of students have been formed during experimental teaching of mathematical analysis that include systematic experimental activity determining the formation of professional competences that are required in information society.

The article can be useful in practical terms for teachers of higher education institutions in planning of courses in mathematics that include scientific and research work of students. Moreover, presented ideas can be used for teaching students whose future professional activities will be connected with information technologies because computer experiment can serve as an integrative approach that combines mathematical modeling and the use of software tools to solve problems. A promising direction of improving the methodology lies in adding aspects related to technologies of online education.

ACKNOWLEDGEMENT

1. The work is performed according to the Russian Government Program of Competitive Growth of Kazan Federal University.
2. 2015 Guangdong Provincial Quality Engineering Construction Project (Innovative Experimental Areas for Talent Cultivation): Innovative Experimental Areas for Interdisciplinary Talents Cultivation in University Town;
3. 2016 "Internet + Resources" Project Construction Virtual Simulation Experiment Project of South China Normal University--Innovative and Entrepreneurial Simulation Operation of College Students' Enterprise;
4. 2014 Education and Teaching Achievement Prize (Higher Education) of Guangdong Province Education Department--Exploration and Practice of Cultivation Mode of Business-Management Talents Based on Multiplatform Co-sharing

AUTHOR NOTE

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