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Misconception on addition and subtraction of fractions in seventh-grade middle school students

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Abstract

As national and international assessments continue to show students struggle with rational concepts, which are seen as a major stumbling block for future career in STEM fields, we used the certainty of response index (CRI) as a method of analysis to investigate a group of 40 United Arab Emirates (UAE) seventh grade students' understanding of basic fraction concepts. The types of errors and misconceptions displayed by students show that over 60% of all participants assumed that adding fractions procedurally is the same as adding whole numbers with many not being able to distinguish a numerator from a denominator. The results lead us to believe that the manner in which students were taught did not ensure the necessary transitional shift from learning whole numbers to learning rational numbers. Applying the theories of Dienes (1960) and Bruner (1966), we propose a method of teaching addition and subtraction of fractions that will allow students to build their own understanding of the rules using the graphing calculator as the medium for concept formation.

Keywords: addition, subtraction, fraction, certainty response index (CRI), United Arab Emirates (UAE)

INTRODUCTION

Rational-number concepts are among the most complex mathematical ideas children encounter during their presecondary school years (Behr et al., 1983). In fact, the learning of rational number concepts is seen as a serious obstacle in the mathematical development of children (Behr et al., 1992). National assessments in the United States (NAEP, 2017, 2019) continue to show students struggle with rational concepts, sounding the alarm for the use of improved approaches to teaching them.

Rational numbers are among the most important to learn as research shows that knowledge of fractions can predict mathematics achievement later in school (Brown & Quinn, 2007; Jarrah et al., 2020; National Mathematics Advisory Panel, 2008; Siegler et al., 2010). STEM fields highly depend on knowledge of rational and irrational numbers. It will be unlikely for a student who cannot go beyond whole numbers and basic rational numbers to have a career in the sciences. At a time when future competitiveness will depend on how well we prepare our students for the 21st century STEM careers, we need to equip the United Arab Emirates (UAE) students early with the tools that will enable them to develop the capacity for critical analysis, evaluation and synthesis of new and complex ideas (Stoica & Wardat, 2021).

Mathematics as a foundational STEM subject has the potential to help develop such capacity in students. That is why it is introduced at a very early age and is made mandatory in the UAE (Alkhateeb, 2001) for all students at all levels from early to primary and secondary education. However, many students have stereotyped mathematics as difficult considering it not a favorable subject to study (Wolff, 2021). Such attitude toward mathematics makes it challenging for teachers to find ways to motivate their students to learn it (Abdallah & Wardat, 2021). Many teachers take the approach to show its usefulness while making it fun. For Tella (2007), such method is highly likely to motivate students to learn and perform better in the mathematics classroom (Alarabi & Wardat, 2021).

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Contribution to the literature

- A quick and easy test of fractions understanding has been created. The test is extremely reliable.
- Applying the theories of Dienes (1960) and Bruner (1966), we propose a method of teaching addition and subtraction of fractions that will allow students to build their own understanding of the rules using the graphing calculator as the medium for concept formation.
- Also, from a theoretical point of view, this article shows how students did types of errors and misconceptions displayed by students show that over 60% of all participants assumed that adding fractions procedurally is the same as adding whole numbers with many not being able to distinguish a numerator from a denominator.

However, mathematics performance in the classroom, at all levels, has been a source of concern for all education stakeholders for as long as one can remember (Alotaibi et al., 2021). Beginning in the elementary school grades, students' mathematics development is often affected by how poor the transitions from whole number systems to fraction concepts to algebraic processes are implemented (Booth et al., 2014). Since the shift from whole to rational numbers starts with fractions concepts early in third grade in most curricula, competence with rational numbers becomes hence an increasing problem as students' progress through the school spectrum (Durkin & Rittle-Johnson, 2015). By the time they reach 7th grade, the majority of students have accumulated quite a set of misconceptions rending difficult the possibility to do advanced mathematics (Amalia et al., 2018; Hamad et al., 2022; Namkung et al., 2018).

The study of fractions represents therefore an essential element of the mathematics curriculum making students' performance in it a barometer for student success in school. Dealing with fractions, unlike whole numbers, brings about numerical development and increases the students' knowledge in mathematics (Aqel et al., 2021). Its applications can be seen as both procedural and conceptual, whereby conceptual allows the student to classify fractions according to sizes, whereas procedural involves arithmetic skills. These arithmetic skills involve subtraction, addition, division, and multiplication (Saban et al., 2021).

It is important to understand how procedural and conceptual fractions relate (Ozpinar & Arslan, 2021). Without understanding fractions as conceptual, it becomes difficult to understand fractions procedures and at the same time predict arithmetic problems associated with it (Braithwaite & Siegler, 2021). It will also be difficult to come up with required solutions or identify an unreasonable answer related to the arithmetic problem (Powell & Nelson, 2021). Copur-Gencturk (2021) examined the results of a sample of 20 students in grade 7 and concluded that difficulty operating arithmetic problem with fractions was still current, especially with the addition and subtraction of fractions having different denominators. A good example supporting this fact is when students were offered an arithmetic problem such as {1/5+2/6=□}, students approached the problem by adding numerators (1+2) and denominators (5+6), resulting to (3/11) as their answer (Xu et al., 2021). These misconceptions can be seen in almost all schools globally, including the UAE grade 7 students.

This study's primary purpose was to understand the mastery-concept level on addition and subtraction of fractions for grade 7 students in the UAE. Using the certainty of response index (CRI) as a method of analysis (Harel & Weber, 2020), we tried to understand why students reasoned out in a manner that produced the types of errors and misconceptions described above. As national and international assessments (NAEP, 2013, 2017; TIMSS, 2019) continue to show students struggle with fractions, it is important to consider other ways of introducing and teaching rational concepts to students as they progress in the school system. Methods that allow students to build their own understanding of the rules that govern concepts formation as prescribed by Piaget (1971), described by his contemporaries Dienes (1960) and Bruner (1966) and tested by (Behr et al., 1992). To that effect, we also present and describe a method that combines the theories of Dienes (1960) and Bruner (1966) using the graphing calculator as the medium for concept formation. Such description will motivate researchers to conduct more studies to improve students' mastery of mathematics concepts involving rational numbers.

LITERATURE REVIEW

Trivena et al. (2017) examined the concept of mastery of the students in mathematics, particularly in addition and subtraction of fractions at primary school levels. Researchers collected data from 23 students' fifth graders (10-11 years old) using qualitative research methods. In addition to the test, a CRI was used. Interviews were conducted with students and teachers as well. Once the test results are obtained, the results were analyzed by examining the students' answers to each item. The categories of CRI are determined based on the combined interview responses of students and teachers. According to the results, students' understanding of addition and subtraction was clouded by 'misconception.' Trivena et al. (2017) concluded that fifth-grade students have low mastery of addition and subtraction of fractions. For them, both teachers and students do not realize that addition and subtraction of fractions are extremely challenging and can significantly impact students' confidence in mathematics.

In their study about misconception in fraction, Fitri and Prahmana (2019) used a descriptive approach to investigate grade 7 students' error in solving fraction problems. The authors found six different types of misconceptions committed by grade 7 students in solving fraction problems. First, students were rewriting the known components of the problem incorrectly; second, they made on the application of fractional counting operations; the third misconception was that they converted wrongly mixed fractions into ordinary fractions and vice versa; the fourth misconception carried regarded carrying incorrectly changing integers to fractions; the fifth was about how they were less careful when performing fractional additions; finally, they considered that fractions should be sorted alphabetically, not by size.

Similarly, Lestiana et al. (2017) conducted a study to Identify students' errors on fractions. They found that significant research studies have revealed fractions to be an extremely difficult topic for students to learn. Many students have trouble figuring out how to add and compare fractions. Nevertheless, there are some common math mistakes students sometimes make when solving problems. In solving mathematics problems, there are three types of errors: factual, procedural, and computational. Lestiana et al. (2017) sought to find how students made fraction-related mistakes. Third-grade students at SD N Laboratorium Unesa Surabaya were assigned a set of validated problems comparing and adding fractions. From the results, there was a lack of awareness about comparing and adding fractions among some students. The majority of these students used incorrect strategies that were categorized as procedural and conceptual errors.

Ghani and Mistima (2018) conducted a study about the misconception of fraction and found that fractions could have two meanings for students: part of a whole and part of a group. It is an exciting topic which can confuse students especially when adding fractions with different denominators. This analyzed the participants' written responses as well as interviews to determine the mistakes and misconceptions made by year four middle school students when solving addition of fractions problems. Results showed that participants made 10 different types of mistakes leading them to conclude that misconceptions with fraction concepts were the cause of their wrong answers.

Aksoy and Yazlik (2017) studied student errors with fractions and the possible causes of the errors. They asked 105 5th graders and 84 6th graders in middle school to define the mistakes and misunderstandings regarding fractions. They used an intentional maximum diversity sampling method to do a qualitative analysis of the data Participants, who voluntarily accepted to be part of the study were from private and public secondary schools of various levels of achievement. Researchers prepared two tests to determine students' errors and misconceptions about fractions, ten open-ended questions for 5th graders and twelve open-ended questions for 6th graders. The data were analyzed by using the content analysis method. As a result of coding the answer papers, students' solutions were categorized as correct, incorrect, and blank. Furthermore, the wrong category of solving was examined in detail, and any mistakes made by students were recorded and discussed (Aloufi et al., 2021). Students misunderstanding and common mistakes when using fractions were obvious. It was discovered that the lack of use of models to describe fractional operations was the main cause of all mistakes and misunderstandings about fractional operations (Wardat et al., 2021).

METHODS

We used a qualitative–descriptive method to examine students' mastery concept regarding the addition and subtraction of fractions. Participants did not receive any special privileges hence results attained were candid and played a significant role in achieving legit results. Of the 40 students involved in the study, 20 were boys and 20 were girls all picked from grade 7 schools within the UAE. We should note that the selected students also had prior knowledge of fractions.

The study relied wholly on interviews and tests as instruments for data collection. We tried to replicate the model used by Trivena et al. (2017), but this time with seventh grade middle school students. The CRI involved five test items which measured the students' mastery-concepts on additions and subtractions of fractions (Yang & Sianturi, 2019). The test involved one open ended and four multiple choice questions. According to their reasoning and understanding, the answers provided helped them gain insight into their level of confidence and mastery-concept on fractions (Trivena et al., 2017). Applying the CRI method used by Trivena et al. (2017) helped us categorize the level of understanding in solving fractions problems according to: understanding of the concept, understanding concept with no confidence, misconceptions and complete lack of knowledge of the concept (Copur-Gencturk, 2021).

Data Analysis

We used a descriptive research approach to examine seventh-grade students' misconceptions in solving fraction problems (Prediger et al., 2015). The research procedure consisted of three phases: preparation, implementation, and data analysis. In the preparation phase, the researchers collected a set of five fraction

Table 1. Seventh grade students' concepts of fractions mastery using CRI categories

Fractions concepts examined	Indicators of fractions concepts mastery [n (%)]			
	Understands the concept well	Displays some understanding but shows no confidence	Displays misconception	Shows no understanding about the concept
Knowing numerator and denominator concepts	31 (77.5%)	3 (7.5%)	5 (12.5%)	1 (2.5%)
Adding fractions with the same denominators	8 (20%)	2 (5%)	26 (65%)	4 (10%)
Adding fractions with different denominators	12 (30%)	3 (7.5%)	22 (55%)	3 (7.5%)
Subtracting fractions with the same denominators	10 (25%)	3 (7.5%)	25 (62.5%)	2 (5%)
Subtracting fractions with different denominators	13 (32.5%)	5 (12.5%)	20 (50%)	2 (5%)
Average	37%	8%	49%	6%

problems from examination tests. The five concepts tested were about students' knowledge of:

1. numerator and denominator,
2. addition of fractions with the same denominators,
3. addition of fractions with different denominators,
4. subtraction of fractions with the same denominators, and
5. subtraction of fractions with different denominators.

In the implementation phase, the researchers gave instructions to students to answer the questions on the worksheet. Students were encouraged not to erase their trials and errors, or the process used to arrive to an answer. We wanted them rather to simply cross them out. We believed that examining crossed out possible answers and scribbles would provide some illustration about students' thinking and how they intellectualized the fraction procedures. Lastly, in the analysis phase, the researchers examined the students' answers in order to identify and describe the mistakes they made, if any. We then classified the mistakes into one of four categories based on indicators that have been identified in the literature as measuring the mastery concepts of addition and subtraction of fractions. A student may

1. understand the concept well,
2. understand the concept but shows no confidence,
3. displays some misconception(s) about the concept, or
4. may show no understanding at all about the concept (Trivena et al., 2017).

RESULTS

Table 1 shows the numbers and percentages of students who show mastery of addition and subtraction of fraction concepts according to the CRI categories we developed. The misconceptions category takes the most significant part as 49% of students display some form of misconception. Except for the concept of "identifying numerators and denominators" in which more than 80% of students were able to respond correctly, there is a large gap between those "who understand the concept well" (27%) and those who "show misconception" or "no understanding" (66%) in four of the five concepts examined. Surprisingly, the data show that students "showed misconception or no understanding" in "adding fractions with the same denominators" (75%) than in adding fractions with "different denominators" (67.5%). In sum, if we consider that those seventh graders who seem to understand the concept but lack confidence can be brought perhaps easily into the category of those who understand it well (total 45%), we still have a lot of work to do with those showing misconceptions or have no understanding of the concepts of fractions (55%).

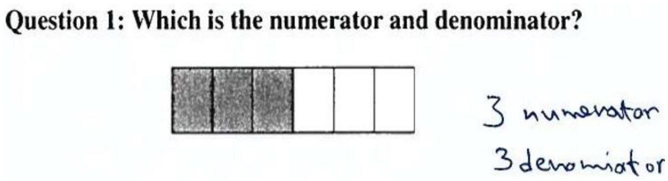

Figure 1. Sample students response for question 1

Analysis by Concepts Studied

Question 1: Knowing numerator and denominator concepts

Presented in the form of a bar graph, question 1 examined students' ability to identify a numerator and denominator of a fraction. More than 77% (n=31) totally understood it and about 8% did but lacked confidence in showing so. Only 15% (n=8) showed misconception or had no understanding of the concept. This result supports conclusions made by other studies that students don't have difficulties recognizing and connecting fraction values when they are represented in rectangular (bars) or circular forms. **Figure 1** shows the question and sample response given by one or more students showing some misconception.

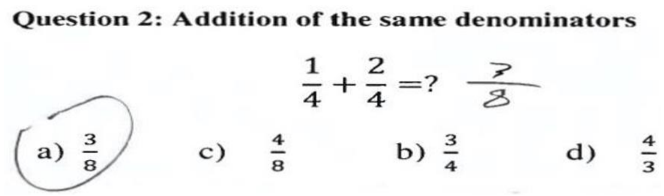

Figure 2. Sample students response for question 2

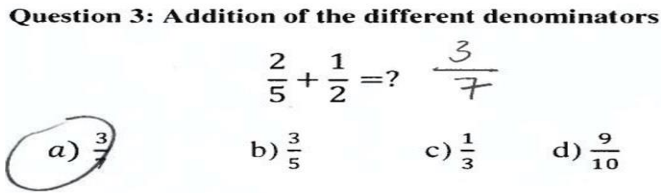

Figure 3. Sample students response for question 3

Some students also 6/3 while others wrote 3/6 without saying which one was the numerator or denominator. Those who wrote 6/3 understood a concept involving "part" and "whole," numerator and denominator but could not distinguish the correspondences. In other terms, the fraction was about a "3" and a "6" but without knowing which one was the part and which one was the whole. Those who wrote 3/6 probably know that there is a "part" and a "whole" but could identify them using the labels "numerator" or "denominator." We believe that with some students, the use of mathematical labels or terms create additional cognitive difficulties to the concept to be learned, not just fractions. Another misconception was shown by students who wrote "3 numerator" and "3 denominator" (**Figure 1**) showing thus no understanding of the concept of "part" and "whole."

Question 2: Adding fractions with the same denominators

Only 20% of students (n=8) successfully responded to the question of adding two fractions with the same denominators. A large majority of students (75%; n=30) showed misconceptions (65%; n=26) or did not understand the problem at all (10%; n=4). Adding the denominators in $\frac{1}{4}+\frac{2}{4}$ to get $\frac{3}{8}$ was the most common response given by students. It seems like the UAE seventh grade students treated fractions as if they were whole numbers. It is a sign that a completely new approach about the introduction and teaching of fractions needs to be considered in schools (**Figure 2**).

Question 3: Addition fractions with different denominators

For the question 2/5+1/2=?, close to 40% of students showed total understanding or understanding without confidence. What is surprising here is that some students who could not respond correctly to addition of fractions with common denominators were able to go through the process of adding fractions with different denominators and do it correctly? This could be interpreted as if the different denominators activated the knowledge of the procedures to be used when facing these problems. Their understanding was procedural not conceptual. They could remember these procedures but not the easy ones when the denominators were the same.

More than 60% of students showed misconceptions (55%) or did not understand it at all (7.5%). Most students with misconceptions gave the reason that numerators should be added, and the same should be done for denominators. To these students, 2/5+1/2=3/7 seemed right, making it a clear indication that they did not understand the procedures of the addition of fractions with different denominators. It is important to understand that students without prior knowledge of distinguishing which fractional value are incapacitated to solve or handle fractional procedures since they cannot predict a fractional arithmetical problem. Understanding fractional value is essential in helping students determine what to expect from specific arithmetic problems and, hence, predict when they are wrong (**Figure 3**).

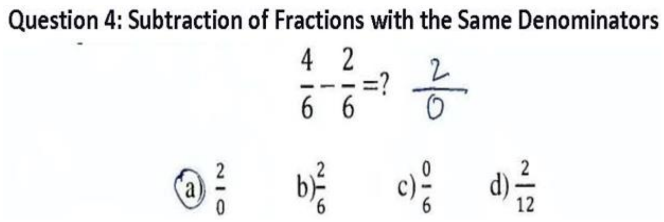

Figure 4. Sample students response for question 4

Question 4: Subtraction of fractions with the same denominators

Subtraction (Q4) and addition (Q2) of fractions with the same denominators follow the same processes. However, responses to question 4 were somehow better than the ones from question 2. Overall, 67.5% of students showed misconceptions or did not understand at for Q4 compared to 75% in Q2. Responses similar to "4/6-2/6=2/0" show deep misunderstanding of fractions concepts (**Figure 4**).

Question 5: Subtraction of fractions with different denominators

It is surprising to see students do better in questions with different denominators (Q3 & Q5) than in questions with same denominators (Q2 & Q4). In Q5, 45% of students showed total understanding (32.5%) or understanding without confidence (12.5%). Only 55% of students showed misconceptions (50%) or no understanding at all (5%). When adding or subtracting fractions with the same denominators, 75% and 67.5% showed misconceptions or no understanding at all, respectively. Students showed more procedural than conceptual understanding of fractions (**Figure 5**).

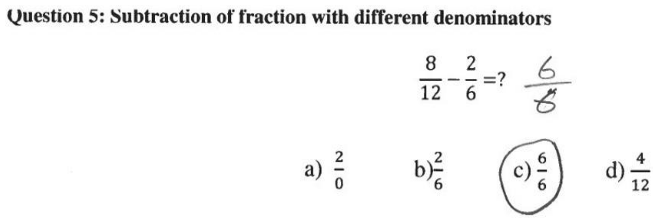

Figure 5. Sample students response for question 5

DISCUSSION

With the question of identifying numerator and denominator concept, students probably learned the concept procedurally which usually lead to poor remembrance of the meaning of each. As Kavramasi's (2003) and Copur-Gencturk's (2021) results show, students at the primary or secondary level have issues distinguishing numerator and denominator. Activating prior knowledge is a key element of teaching and is most effective when done conceptually. Are teachers doing it effectively? It is a question to be asked and investigated.

The questions with the highest frequency of misconception were the addition (Q2) and subtraction (Q4) of fractions with the same denominators. Students assume that adding fractions procedurally is the same as adding whole numbers. They just add across, horizontally, probably assuming, as explained by Trivena et al. (2017) that the numerators and denominators have different place values.

What was surprising being that students performed better on the addition (Q3) and subtraction (Q5) of fractions with different denominators than with Q2 and Q4. Seeing the different denominators probably acted as a trigger on the students' mind to remember the "complicated" procedures shown by their teachers. Teachers may have spent more time practicing these seemingly more difficult problems than practicing with the addition or subtraction of fractions with the same denominators. For teachers, the latter is simpler. They usually just say to the class: "add the numerators" and "keep the denominators the same" but spend a considerable amount of time showing the steps needed to find the common denominators without explaining why. How many teachers spend time getting students to discover a concept and make sense out of it? With respect to addition of fractions, how many teachers would lead their students to discover that the rule is due to the distributive property of addition over multiplication? Or do they perhaps think that it's faster and safer to just show and explain the procedures? Or is it because it will complicate things and lengthen the time needed to teach a lesson to activate prior knowledge the right way when teaching a concept? Most teachers do not realize the power of teaching for conceptual understanding. It takes more time to implement but students will discover the concept and own their learning, not memorize. That is why the difficulties and misconceptions shown by the students in this study are not surprising. These concepts are introduced early in the career of these students and taught multiple times and yet, at the age 12-13 in 7th grade, the majority still show almost no understanding of them. This is a phenomenon that has been recognized by multiple studies worldwide.

And yet, throughout the history of mathematics education as a field, researchers have continued to demonstrate that children understanding of concepts requires much more than the procedural methods used in the classroom. From Dewey's (1938) argument that children need first-hand experiences to learn a concept; Piaget's (1971) elaboration of the stages of development leading him to affirm that children concepts formation takes place through an enacting of reality, not through an imitation of it; Bruner's (1960, 1966) proclamation that knowing is a process, not a product, formulating that learning goes through three stages of representation, enactive, pictorial, and symbolic; and Dienes' (1960) variability principles describing how to achieve what the others have prescribed in the teaching and learning of mathematics concepts; to today's newly developed theories on cognition and brain development (Gabriel et al., 2012), the message is the same: learning requires that children build or construct their own concepts from within rather than having those concepts imposed upon them. We believe that students in this study were mostly taught fractions concepts procedurally rather than conceptually.

For Behr et al. (1983) who have extensively studied rational concepts throughout the years beginning in 1979 when the US National Science Foundation funded the rational number project (Behr et al., 1988), what makes a complete comprehension of rational numbers a formidable learning task emanates from the multiple forms and ways used to represent a rational number. According to Behr et al. (1983, p. 2), "rational numbers can be interpreted in at least these six ways, referred to as sub-constructs: a part-to-whole comparison, a decimal, a ratio, an indicated division (quotient), an operator, and a measure of continuous or discrete, quantities." Kieren (1976 as cited in Behr et al., 1983) "contends that a complete understanding of rational numbers requires not only an understanding of each of these separate sub-constructs but also of how they interrelate."

For children in third to seventh grade classrooms, the goal in most curriculum is not to have them have a complete understanding of rational numbers and the mechanism of how they interrelate across the six sub-constructs, but to guarantee that the shift from whole to rational numbers takes place smoothly through a consolidation of the principles of whole numbers and rational numbers into a single numerical framework (Siegler et al., 2011). It is clear from the results of this study and the misconceptions shown that such shift from whole numbers to rational numbers did not take place with the large majority of the seventh-grade participants. Reaching such a goal, that is, ensuring a smooth transition from the arithmetic of whole numbers to that of rational numbers, will require methods of teaching that are aligned with the approaches prescribed and described by the likes of Dienes (1960), Bruner (1966), and Piaget (1971) as cited above. We believe that these students were not taught using such methods.

Dienes (1971) developed a system of teaching mathematics based upon two principles of learning: The perceptual (embodiment) variability principle and the mathematical variability principle. He believed that when applied to teaching a mathematics concept, the combination of the two principles will help the student gain a better understanding of the concepts to be learned. Bruner (1966) established his theory of stages of representation proposing that children move through three modes or levels of representation as they learn: The enactive (hands-on) level, the iconic (pictorial) level, and the symbolic level. His three modes of representational thought are basically analogous to Piaget's (1971) constructivist proposition that children learn by moving from the concrete to the abstract. A method combining Dienes (1960) and Bruner's (1966) theories of learning for instance could be effective in enabling primary and early secondary children to improve their understanding of fraction concepts and remove the misconceptions related to fractional additions.

CONCLUSIONS & RECOMMENDATIONS

We investigated a group of 40 UAE seventh grade students' understanding of basic fraction concepts using the CRI, as national and international assessments continue to show students struggle with rational concepts, which are seen as a major stumbling block for future careers in STEM fields.

Over 60% of all participants assumed that adding fractions is the same as adding whole numbers procedurally, with many unable to distinguish a numerator from a denominator, according to the sorts of errors and misconceptions revealed by students. The findings suggest that the way students were taught did not allow them to make the essential transition from learning whole numbers to learning rational numbers.

A variety of people have encountered the majority of the student's mastery-concept on addition and subtraction of the same and the most common numbers 'Misconception.' Many students have misconceptions regarding questions 2 and 3 when combining the same denominators and when adding the different denominators. Subtraction with different denominators, and the number 5 with different denominators. On the basis of these, based on the findings, it can be assumed that the pupils are still having trouble understanding the material.

Procedures for fractional arithmetic, particularly addition and subtraction. Students who do not yet grasp that addition and subtraction operations must first balance the numerator are at a disadvantage. The denominator must be removed from the denominator, and the numerator must be added to the numerator.

Every learner has the potential to make mistakes. As a result, in order to notice and understand problems, teachers must acquire a reflective mindset. Educators should seek out new information and knowledge on a regular basis in order to address any misconceptions that may arise. Teachers must identify their students' blunders in order to give them an idea of how to describe and choose the appropriate teaching style to address the issues and ensure that they do not occur again.

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REFERENCES

Abdallah, R., & Wardat, Y. (2021). Teachers' perceptions on the effectiveness of professional development programs in improving the curriculum implementation at Jordanian schools. *Elementary Education Online, 20(5)*, 4438-4449. https://doi.org/10.17051/ilkonline.2021.01.126

Aksoy, N. C., & Yazlik, D. O. (2017). Student errors in fractions and possible causes of these errors. *Journal of Education and Training Studies, 5*(11), 219. https://doi.org/10.11114/jets.v5i11.2679

Alarabi, K., & Wardat, Y. (2021). UAE-based teachers' hindsight judgments on physics education during the COVID-19 pandemic. *Psychology and Education Journal, 58(3)*, 2497-2511.

Alkhateeb, H. M. (2001). Gender differences in mathematics achievement among high school students in the United Arab Emirates, 1991-2000. *School Science and Mathematics, 101*(1), 5-9. https://doi.org/10.1111/j.1949-8594.2001.tb18184.x

Alotaibi, A., Khalil, I., & Wardat, Y. (2021). Teaching practices of the mathematics male and female teachers according to the PISA framework and its relation to their beliefs towards their students. *Elementary Education Online, 20(1)*, 1247-1265. https://doi.org/10.17051/ilkonline.2021.01.126

Aloufi, F., Ibrahim, A. L., Elsayed, A. M. A., Wardat, Y., & Ahmed, A. O. (2021). Virtual mathematics education during COVID-19: An exploratory Study of teaching practices for teachers in simultaneous virtual classes. *International Journal of Learning, Teaching and Educational Research, 20*(12), 85-113. https://doi.org/10.26803/ijlter.20.12.6

Amalia, R., Saiman, S., Sofiyan, S., & Mursalin, M. (2018). Designing computer-based fraction worksheets for junior high school. *Journal of Physics: Conference Series*, *1088*, 012110. https://doi.org/10.1088/1742-6596/1088/1/012110

Aqel, O. A., White, M. T., Khader, M. A., & Sayma, A. I. (2021). Sensitivity of transcritical cycle and turbine design to dopant fraction in CO2-based working fluids. *Applied Thermal Engineering*, *190*, 116796. https://doi.org/10.1016/j.applthermaleng.2021.116796

Behr, M. J., Harel, G., Post, T. R., & Lesh, R. (1992). Rational number, ratio, and proportion. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning: A project of the National Council of Teachers of Mathematics* (pp. 296-333). Macmillan Publishing Co, Inc.

Behr, M. J., Wachsmuth, I., & Post, T. (1988). Rational number learning aids: Transfer from continuous models to discrete models. *Focus on Learning Problems in Mathematics*, *10*(4), 1-18.

Behr, M., Lesh, R., Post, T., & Silver E. (1983). Rational number concepts. In R. Lesh, & M. Landau (Eds.), *Acquisition of mathematics concepts and processes* (pp. 91-125). Academic Press.

Braithwaite, D. W., & Siegler, R. S. (2021). Putting fractions together. *Journal of Educational Psychology*, *113*(3), 556-571. https://doi.org/10.1037/edu0000477

Bruner, J., S. (1960). *The process of education*. Harvard University Press.

Bruner, J., S. (1966). *Toward a theory of instruction*. Harvard University Press.

Copur-Gencturk, Y. (2021). Teachers' conceptual understanding of fraction operations: Results from a national sample of elementary school teachers. *Educational Studies in Mathematics*, *107*, 525-545. https://doi.org/10.1007/s10649-021-10033-4

Dewey, J. (1938). *Experience and education*. Macmillan Company.

Dienes, Z. P. (1960). *Building up mathematics*. Hutchinson Educational.

Dienes, Z. P. (1971). An example of the passage from the concrete to the manipulation of formal systems. *Educational Studies in Mathematics 3* (3/4), 337-352. https://doi.org/10.1007/BF00302302

Fitri, N. L., & Prahmana, R. C. I. (2019). Misconception in fraction for seventh-grade students. *Journal of Physics: Conference Series*, *1188*, 012031. https://doi.org/10.1088/1742-6596/1188/1/012031

Gabriel, F., Coché, F., Szucs, D., Carette, V., Rey, B., & Content, A. (2012). Developing children's understanding of fractions: An intervention study. *Mind, Brain, and Education*, *6*(3), 137-146. https://doi.org/10.1111/j.1751-228X.2012.01149

Ghani, S. N. A., & Mistima, S. (2018). Misconception of fraction among middle grade year four pupils at primary school. *Research on Education and Psychology*, *2*(1), 111-125.

Hamad, S., Tairab, H., Wardat, Y., Rabbani, L., AlArabi, K., Yousif, M., Abu-Al-Aish, A., & Stoica, G. (2022). Understanding science teachers' implementations of integrated STEM: Teacher perceptions and practice. *Sustainability, 14*(6), 3594. https://doi.org/10.3390/su14063594

Harel, G., & Weber, K. (2020). Deductive reasoning in mathematics education. In S. Lerman (Ed.), *Encyclopedia of mathematics education*. Springer, Cham. https://doi.org/10.1007/978-3-030-15789-0_43

Jarrah, A. M., Khasawneh, O. M., & Wardat, Y. (2020). Implementing pragmatism and John Dewey's educational philosophy in Emirati elementary schools: Case of mathematics and science teachers. *International Journal of Education Economics and Development*, *11*(1), 58. https://doi.org/10.1504/ijeed.2020.104287

Lestiana, H. T., Rejeki, S., & Setyawan, F. (2017). Identifying students' errors on fractions. *JRAMathEdu (Journal of Research and Advances in Mathematics Education)*, *1*(2), 131-139. https://doi.org/10.23917/jramathedu.v1i2.3396

Ozpinar, I., & Arslan, S. (2021). Investigation of basic mathematical knowledge of preservice maths teachers: Procedural or conceptual? *International Journal of Mathematical Education in Science and Technology*. https://doi.org/10.1080/0020739x.2020.1867915

Piaget, J. (1971). *The psychology of intelligence*. Routledge and Kegan.

Powell, S. R., & Nelson, G. (2021). University students' misconceptions about rational numbers: Implications for developmental mathematics and instruction of younger students. *Psychology in the Schools*, *58*(2), 307-331. https://doi.org/10.1002/pits.22448

Prediger, S., Gravemeijer, K., & Confrey, J. (2015). Design research with a focus on learning processes: An overview on achievements and challenges. *ZDM*, *47*(6), 877-891. https://doi.org/10.1007/s11858-015-0722-3

Saban, W., Raz, G., Grabner, R. H., Gabay, S., & Kadosh, R. C. (2021). Primitive visual channels have a causal role in cognitive transfer. *Scientific Reports*, *11*, 8759. https://doi.org/10.1038/s41598-021-88271-y

Siegler, R. S., Fazio, L. K., Bailey, D. H., & Zhou, X. (2013). Fractions: The new frontier for theories of numerical development. *Trends in Cognitive*

Sciences, *17*(1), 13-19. https://doi.org/10.1016/j.tics.2012.11.004

Stoica, G., & Wardat, Y. (2021). An inequality can Change everything. *The American Mathematical Monthly, 128*(9), 810. https://doi.org/10.1080/00029890.2021.1949218

Tella, A. (2007). The impact of motivation on student's academic achievement and learning outcomes in mathematics among secondary school students in Nigeria. *Eurasia Journal of Mathematics, Science and Technology Education*, *3*(2), 149-156. https://doi.org/10.12973/ejmste/75390

Trivena, V., Ningsih, A. R., & Jupri, A. (2017). Misconception on addition and subtraction of fraction at primary school students in fifth-grade. *Journal of Physics: Conference Series*, *895*, 012139. https://doi.org/10.1088/1742-6596/895/1/012139

Wardat, Y., Belbase, S., & Tairab, H. (2022). Mathematics teachers' perceptions of Trends in International Mathematics and Science Study (TIMSS)-related practices in Abu Dhabi Emirate schools. *Sustainability, 14*, 5436. https://doi.org/10.3390/su14095436

Wardat, Y., Jarrah, A. M., & Stoica, G. (2021). Understanding the meaning of the equal sign: A case study of middle school students in the United Arab Emirates. *European Journal of Educational Research, 10*(3), 1505-1514. https://doi.org/10.12973/eu-jer.10.3.1505

Wolff, F. (2021). How classmates' gender stereotypes affect students' math self-concepts: A multilevel analysis. *Frontiers in Psychology*, *12*, 599199. https://doi.org/10.3389/fpsyg.2021.599199

Xu, Y., Smeets, R., & Bidarra, R. (2021). Procedural generation of problems for elementary math education. *International Journal of Serious Games*, *8*(2), 49-66. https://doi.org/10.17083/ijsg.v8i2.396

Yang, D., & Sianturi, I. A. J. (2019). Assessing students' conceptual understanding using an online three-tier diagnostic test. *Journal of Computer Assisted Learning*, *35*(5), 678-689. https://doi.org/10.1111/jcal.12368