Numerical Modelling with Spreadsheets as a Means to Promote STEM to High School Students

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The article gives an account of an experiment in which sixty-eight high school students of age 16 – 19 developed spreadsheet applications that simulated fall and projectile motion in the air. The students applied the Euler method to solve the governing differential equations. The aim was to promote STEM to the students and motivate them to study STEM at universities. A questionnaire survey was carried out to find out if the participants found the lessons interesting and if developing and using the spreadsheet models benefited the learning and contributed to the mathematical and technological knowledge of the students.

Keywords: excel, inquiry, scientific models, simulation, visualization

INTRODUCTION

It is a well-known fact that there is a huge demand for STEM experts in the world (PCAST, 2010; European Commission, 2010). The reason is that high school students find school STEM disciplines 'dull' and 'boring'. A study carried out in the UK with more than 1,500 students aged from 14 to 18 showed that 44% believed that STEM subjects, uptake of which is considered vital to the economy, were 'uninteresting'. In addition, more than 53% considered them harder to study than the arts and humanities, and 40% thought them less 'fun' (Garner, 2014). Within five years, there will be 2.4 million STEM job openings in the USA. However, according to a survey of more than a million students, nearly 90% of high school graduates were not interested in a career involving STEM (The New York Times, 2013).

A way to solve the problem is to foster the students' interest in STEM at an earlier age, which could lead them to take careers in STEM fields (Narayan et al., 2013). STEM disciplines should be taught in a more enriching and interesting manner based on an interdisciplinary approach to keep curiosity alive (Kim & Cho, 2015; Yarker & Park, 2012; Ezra & Nahmias, 2015). Real-world learning is critical in STEM subjects therefore more emphasis should be put on inquiry and authenticity.

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There is a wide range of everyday phenomena that students observe and experience (see e.g. Ng, Nguyen, 2006). Fall and projectile motion are among them (Griffith, 2008). They have been subjects of interest to investigators for centuries with Galileo being the first who studied them scientifically (Swerdlow, 2013). They are common topics of physics courses today, starting at secondary but even at primary level (Eckstein, 1997; Davies & McMahon, 2004; Erickson, n. d.). The solutions to fall and projectile motion in a vacuum are well-known (Halliday, Resnick & Walker, 2011). The solutions to low altitude fall, that is up to 500m when the air density may be taken constant, comprises hyperbolic functions (Greiner, 2004) while the solution to projectile motion is given by analytically unsolvable integrals (Hayen, 2003). The solution to high altitude fall is given by analytically unsolvable integrals (Benacka, 2010), while the solution to projectile motion is unknown. The overview implies that fall and projectile motion in the air are inappropriate topics for high school students. Therefore, fall and projectile motion *in a vacuum*, that is, under idealised conditions, are studied at the level (Niaz, 1999). The students are familiar with the fact that the nearest place where the condition holds is the Moon. The question arises among the interested students of what the solution is and what it looks like in the air, that is, in real practice.

State of the literature

- There is a huge demand for STEM experts in the world. The reason is that high school students find school STEM disciplines 'dull' and 'boring'.
- Many papers have been written on promoting STEM to students through realistic tasks and real-life examples.
- Computer modelling is a way to make school science interesting through visualization and animation.

Contribution of this paper to the literature

- Physical systems are given by differential equations. Such problems are typically considered not appropriate to be solved at high school.
- This paper shows that the problems are solvable even at high school if the Euler method of solving differential equations is applied.
- The paper shows three real-life tasks governed by differential equations and solved with high school students in Excel by using the Euler method. The paper gives also the result of a questionnaire on students' opinion of the method and lessons.

A way to study dynamic systems, e.g. fall or projectile motion in the air, at high school level is by applying numeric methods to solve the governing ordinary differential equations (ODE; see Erickson, n.d., p. 11-13). The Euler method is the simplest numeric method (Butcher, 2003). It is easy to comprehend and work with. Spreadsheets allow implementing the method in a transparent way. No programming is required. The feedback is instant. That makes spreadsheet numerical modelling an excellent way to study dynamic systems and integrate computational experiment in learning at high school (Benacka, 2008).

The use of spreadsheets in science education has been researched for decades. Baker and Sugden (2003) gave a detailed account from 1979 to 2003. They show the potential of spreadsheets to enhance learning on a wide range of papers, and call for further topics that can be covered by spreadsheets. Most articles and books on spreadsheets bring ideas and useful recommendations for teaching practice (Healy & Sutherland, 1992; Brosnan, 1994; Keune & Henning, 2003; Neuwirth & Arganbright, 2004; Haspekian, 2005; Benacka & Stubna, 2009; Benacka, 2011; Benacka & Ceretkova, 2013), but few give feedback from students. Sivasubramaniam's research (2000) with student of age 14-15 showed that applying spreadsheets had a positive impact on understanding Cartesian graphs. Neurath and Stephens' (2006) research into teaching algebra with students of age 14-17 showed that 77% enjoyed the course because solving tasks with Excel made algebra more interesting to them. Lingefjärd and Meier (2011) reported on an experiment in which Swedish and German students of age 17-18 investigated the declination of the Sun during one year in the same time of day. The problem ended up in a linear combination of two trigonometric functions and was resolved in Excel by using a

graph with parameters governed by scrollbars. Nisawa and Moriya (2011) gave account of an experiment in which 30 high school students analysed the stopping distance of a car as a function of speed, friction coefficient and thinking time. Almost all students made sense of the relationship between the braking distance and speed, and between the braking distance and friction coefficient. Benacka and Ceretkova (2014) gave the results of an experiment in which 27 high school students developed a numerical model of harvesting the world population of the Sperm whale. All students found the lesson interesting and 89% would have liked to continue with developing a more complicated model, the predator-prey one. Benecka (2013, 2015a) reported on two experiments in which the principles of 3D graphics were explained through making a rotatable and sizable orthographic parallel projection of simple figures in Excel (1) without and (2) with face overlapping taken into account. Altogether 181 participants took part in the first experiment, and 34 of them were high school students of age 16-19. All students found the lesson interesting and 82% would have liked to continue with solving the problem of face overlapping. Thirty high school students aged 17–19 took part in the second experiment. All students found the lesson interesting and 87% would have liked to continue with projecting more complicated figures. Benecka (2015b, 2015c) reported on two experiments in which 38 and 30 high school students developed Excel applications that simulated projectile general motion in a vacuum and found the point of shot and impact of a projectile by using Gaussian elimination. All student in the first group and 28 in the second group found the lesson interesting. Benecka (2015d) gave an account of an experiment in which 33 high school students aged 16–19 acquired the principles of the integral calculus through applying the rectangle method in spreadsheet to calculate the area of a planar figure bounded by the graph of a function, and the volume of the spatial figure created by rotation of the graph. Altogether 91% of the students found the lesson interesting and 61% would have liked to continue with a more difficult task – to calculate the volume of a rugby ball. Crook, Sharma and Wilson's (2015) research of 967 high school students showed that schooling with 1:1 laptops had a positive effect on examination in physics, biology and chemistry. The greater effect in physics corresponded with greater use of simulations and spreadsheets by students and teachers.

The overview suggests that spreadsheet modelling may promote STEM to high school students and motivate, inspire and encourage them to study the disciplines at universities. Doing STEM with spreadsheets involves Creativity, Visualization, Inquiry and Authenticity, which are vital to enjoyable learning. Creating an application that enables one to solve a real problem is a strong motivating factor. Learning this way gives young people the great feeling of achievement (Hromkovič & Steffen, 2011). Being the maker of a meaningful product is a success. Visualization is central to learning, especially in sciences, because students have to learn to navigate within and between the modes of representation (Gilbert, 2005). Uttal and O'Doherty (2008) define visualization as any type of physical representation designed to make an abstract concept visible. Visualization helps because it makes complex information accessible and cognitively tractable. It allows one to perceive, and to think about, relations among items that would be otherwise difficult to comprehend. Inquiry and problem-based approach involve the learner, with appropriate guiding, in the practice and conceptualization of the discipline and in that way promote the construction of knowledge recognized as learning (Hmelo-Silver, Duncan & Chinn, 2007). The 'Aha!' effect is important in inquiry learning. It yields positive affective experience – joy, "a feeling of great pleasure and happiness" (Oxford Dictionaries, n.d.). A solution with an' Aha!' experience is remembered better (Danek et al., 2013). Authenticity concerns the truthfulness. Science education and technology education should be as authentic as possible, and models

can form a bridge between them in the context of *reality*, *theory* and *model* (Gilbert, 2000). A model is as authentic and reliable as the theory upon which the model was developed. Students can improve their spreadsheet models and involve other variables to make the model more authentic (fall in a vacuum; fall in the constant density air; fall in the real air; etc.). Such a spreadsheet model is not a black box in any case.

This article reports on an experiment in which 68 high school students developed spreadsheet numerical models of dynamic physical systems by applying the Euler method to solve the governing ODE or system of ODE. The first group simulated free fall in a vacuum and in the air of constant density and drew consequences relative to the mass, size and shape of the body. The second group modelled high-altitude fall of a record-holder speed skydiver. The goal was to determine whether he would reach supersonic speed if he jumped from 11km altitude, the edge of the troposphere, where airliners cruise. The third group simulated projectile motion in the air of constant density and drew consequences relative to the mass, size and shape of the projectile. The aim of the experiment was to find out if the students found the lessons interesting, and whether developing the spreadsheet models and using numeric modelling contributed to the mathematical and technological knowledge of the students. A questionnaire survey was carried out. The results are discussed.

FREE FALL IN A VACUUM AND IN THE AIR

Free fall in a vacuum is usually taught in the first year at high school. Fall in the air is not taught as the required mathematics is beyond the high school level.

Theory

Let a body start falling in a vacuum at time $t = 0$ s from altitude $y = h$. The only force that acts on it is weight $G = gm$, where g is acceleration due to gravity and m is the mass of the body. Speed v and altitude y are given by the formulas (Halliday, Resnick & Walker, 2011)

$$v = gt, \tag{1}$$

$$y = h - \frac{1}{2} gt^2. \tag{2}$$

The graphs are in Figure 1. The time of impact is given by the formula
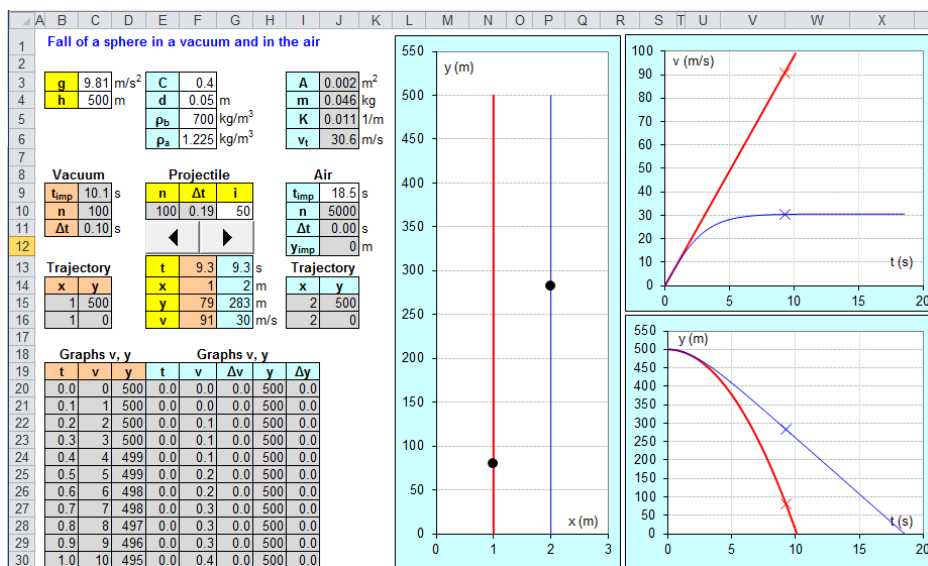

Figure 1. Simulation of free fall in a vacuum (bold) and in the air of constant density

$$t_{\text{imp}} = \sqrt{\frac{2h}{g}} \, . \tag{3}$$

In the air, drag acts, which is given by the formula (Halliday, Resnick & Walker, 2011)

$$F_{\text{D}} = \frac{1}{2} C A \rho_{\text{a}} v^2 \, , \tag{4}$$

where C is the drag coefficient dependent on speed v and the shape of the body, A is the cross-section area of the body perpendicular to the velocity, and ρ_{a} is the air density. If the speed is below 80 % of the sound speed, that is, $v < 270$ m/s, then C is constant (Thornton & Marion, 2003). The air density at sea level is $\rho_{\text{a}} = 1.225$ kg/m³ (US Standard Atmosphere, 1976). It drops with the altitude but if 5% drop is tolerable then it may be taken constant up to 500 m. The equation of motion is

$$\frac{\mathrm{d}v}{\mathrm{d}t} = g - Kv^2 \, , \; v(0) = 0 \, , \; y(0) = h \, , \tag{5}$$

where

$$K = \frac{C A \rho_{\text{a}}}{2m} \, . \tag{6}$$

Replacing the differentials in Equation (5) by differences and rearranging gives

$$\Delta v = \left(g - Kv^2 \right) \Delta t \, , \tag{7}$$

$$v = v + \Delta v \, , \; v(0) = 0 \, , \tag{8}$$

and

$$\Delta y = v \Delta t \, , \tag{9}$$

$$y = y - \Delta y \, , \; y(0) = h \, , \tag{10}$$

where notation $v = v + \Delta v$ means that the "new" value of speed is obtained by adding Δv to the "old" value of speed. Equations (7) – (10) enable one to graph the speed and altitude.

As the weight G is constant and the resistance force F_{D} increases with the speed from nought, the two forces become equal after some time. From then, the body moves on uniformly, that is, at a constant speed, which is the terminal speed v_{t} of the fall. The equation $G = F_{\text{D}}$ yields

$$v_{\text{t}} = \sqrt{g/K} \, . \tag{11}$$

The graph is a horizontal line which is the asymptote to the speed graph. After reaching this speed, the body moves steadily, therefore the altitude becomes a linear function. The graph is an aslant line which is the asymptote to the altitude graph.

Spreadsheet application

The Excel application is in Figure 1. It is available at https://www.dropbox.com/s/bd0pgwtx7dnudev/fall_vacuum_air.xlsx?dl=0.

The body is a homogenous sphere of diameter d and density ρ_{b}. Density $\rho_{\text{b}} = 700$ kg/m³ is for hard wood. It holds for a sphere that $C \approx 0.4$ (Timmerman & van der Weele, 1999). The inputs are in the white cells.

The graphs in a vacuum are made over 101 points (100 steps, cell C10). Time of impact t_{imp} is calculated in cell C9 by Equation (3). The time is calculated from 0 to t_{imp} in range B20:B120 by step $\Delta t = t_{\text{imp}} / 100$ (cells C11). The speed is calculated in range C20:C120 by Equation (1). The altitude is calculated in range D20:D120 by Equation (2). The graphs are of type XY. The bold graphs are for a vacuum. The $y(x)$

graph is made over range B15:C16. The fall is supposed to take place at $x = 1$. The $v(t)$ graph is made over range B20:C120. The $y(t)$ graph is made over ranges B20:B120, D20:D120.

To simulate the fall, time t_{imp} is divided into 100 steps (cell E10). The step is in cell F10 (in Figure1, it holds for fall in the air, see below). The number of elapsed steps is in cell G10. The total time is in cell F13. The corresponding speed and altitude are calculated in cells F15 and F16 by Equation (1) and (2). The ball is a one-point XY graph made over range F14:F15. The number of elapsed time intervals is governed by the spinbutton placed over range E11:G12. Its properties Max, Min, SmallChange and LinkedCell are set to 100, 0, 1 and G10. Clicking the spinbutton increases the number of elapsed time steps by 1 and puts the ball into the position given by the new coordinates *x* and *y*. Clicking the spinbutton and holding down causes the ball to fall along the trajectory according to the laws of physics. The altitude and speed at a given time can be obtained in cells F15 and F16 if the time is written in cell F13. The position on the $v(t)$ and $y(t)$ graphs is marked with "x", which are one-point XY graphs made over cells F13, F16, and F13, F15.

The graphs in the air are made over 5001 points (5000 steps, cell J10). Time of impact t_{imp} (cells J9) is estimated and then found with the *Goal Seek* tool (see below). The time is calculated in range E20:E5020 from 0 to t_{imp} by step $\Delta t = t_{\text{imp}}/5000$ (cells J11). The speed, change in speed, altitude and change in altitude are calculated in range F20:I5020 by Equations (7) – (10). The graphs are of type XY. The $y(x)$ graph is made over range I15:J16. The fall is supposed to take place at $x = 2$. The $v(t)$ graph is made over range E20:F5020. The $y(t)$ graph is made over range E20:E5020, H20:H5020. Cell J12 refers to cell H5020 and it is to find impact time t_{imp} in cell J9 by the *Goal Seek* tool (table *Data*, *What-If Analysis*) in which box S*et cell* is set to J12, box *To value* is set to 0 and box *By changing cell* is set to J9.

To simulate the fall, the impact time (cell J9) is divided into 100 steps (cell E10). The step is in cell F10. The number of elapsed steps is in cell G10. The total time is in cell G13. The corresponding altitude and speed are in cells G15 and G16 returned from range E20:H5020 by function VLOOKUP. The ball is a one-point XY graph. The number of elapsed time intervals is governed by the spinbutton. The altitude and speed at a given time can be obtained in cells G15 and G16 if the time is written in cell G13. As the impact time in a vacuum is smaller than in the air (see in Figure 1), the time in cell F13 has to stop when it reaches the value of the impact time in cell C9.

Lessons

Twenty-nine high school students aged 16-19 in three groups took part in the experiment. Two 90-minute optional applied informatics lessons were taught to each group. The students were familiar with Equations (1) – (4) from the first year physics. The aim was to learn the Euler method, practise Excel, and learn to simulate motion with Excel.

The first lesson started with a discussion on free fall in a vacuum and in the air. Equations (1) – (3) were recalled and discussed. The students found that the formulas were at variance with the fact that the speed in the air becomes constant after some time, e.g., a parachutist with open parachute falls uniformly. However, the students found that the formulas give the upper limits for the speed and distance travelled in the air.

The teacher showed the application in Figure 1 using a data projector. The students downloaded the template (the application in Figure 1 with no formulas and

no chart) and developed the part for a vacuum. The teacher was developing the application along with the students, discussing the steps of creation and projecting his work to give feedback but with a delay so that the clever student could work individually. After accomplishing the model, the students experimented with it. The teacher projected a pdf file with the theoretical background and went through the derivation. He applied the method of questioning (Petty, 2004) and encouraged the students to recall the necessary formulas. Then, he sent the pdf file to the students via emails. That was the end of the first lesson.

The second lesson started with revising the theory of fall in the air. The students opened the application and continued developing. The teacher worked along, discussing the steps with the students. After accomplishing the model, the students experimented with it. They found that if parameter K dropped, the graphs in the air got closer to the graphs in a vacuum and finally merged with them. They found that parameter K could be decreased by decreasing parameter C (more streamline shape) or ρ_a (thinner air), or increasing ρ_b (a heavier sphere with unchanged diameter). The effect of parameter d was a surprise to the students. They presupposed that if the sphere was smaller, then the air resistance would drop and the graphs in the air would get closer to the graphs in a vacuum. However, the model showed the opposite – the graphs moved away. The teacher challenged the students to take the mass into account. Afterwards, the students found that if they halved diameter d, the drag dropped to a quarter but the mass dropped to an eighth and coefficient K doubled.

The students filled in a questionnaire at the end of the lesson. The result is in section Survey.

HIGH-SPEED SKYDIVER'S JUMP

Speed skydiving is the fastest non-motorized sport in the world. The competitor jumps from an altitude of 4000 m and the score is the average vertical speed that s/he achieves through the course of a vertical kilometre starting at 2700 m and ending at 1700 m above the ground. The world record holder is Marco Wiederkehr who reached 557.57 km/h on May 14-17, 2015 (ISSA, n. d.). On October 14, 2012, Felix Baumgartner jumped from an altitude of 39 km and reached the speed of 1358 km/h, or Mach 1.24, to be the first human to break the sound barrier (Red Bull Stratos, n. d.). Airliners fly at the altitude of about 11 km, which is the upper bound of the troposphere, the first layer of the atmosphere (US Standard Atmosphere, 1976). The question is: Would Marco Wiederkehr be supersonic if he jumped from 11 km?

Theory

Let a body start falling at time $t = 0\,\text{s}$ from altitude $y = h$. Two forces act on the body, which are weight $G = gm$ and drag (see Equation (4))

$$F_{\text{D}} = \frac{1}{2}CA\rho v^2 , \qquad (12)$$

where ρ is the air density. Up to 11 km, it is given by the formula (US Standard Atmosphere, 1976)

$$\rho = \rho_0\left(1 + \frac{L_0}{T_0}y\right)^{\alpha} , \quad \alpha = 4.258 , \qquad (13)$$

where $\rho_0 = 1.225$ kg/m³ is the air density at sea level, $L_0 = -0.0065$ K/m is the temperature lapse rate and $T_0 = 288.15\,\text{K}$ is the temperature at sea level.

The equation of motion is

$$\frac{dv}{dt} = g - B\rho v^2, \ v(0) = 0, \ y(0) = h, \tag{14}$$

where

$$B = \frac{CA}{2m}. \tag{15}$$

Replacing the differentials in Equation (14) by differences and rearranging gives

$$\Delta v = \left(g - B\rho v^2\right)\Delta t, \tag{16}$$

$$v = v + \Delta v, \ v(0) = 0, \tag{17}$$

and

$$\Delta y = v\Delta t, \tag{18}$$

$$y = y - \Delta y, \ y(0) = h. \tag{19}$$

Equations (16) – (19) enable one to graph the speed and altitude of the body.

The speed of sound is given by the formula (US Standard Atmosphere, 1976)

$$c = c_0 \sqrt{1 + \frac{L_0}{T_0} y}, \tag{20}$$

where $c_0 = 340.29$ m/s is the speed of sound at sea level. Mach number M is the ratio of the speed and the speed of sound at the altitude. A body is supersonic if $M > 1$.

We suppose that the skydiver's mass is 95 kg (80 kg for the body plus 15 kg for the parachute). To survive a jump from 11 km, we suppose plus 10 kg of mass for special equipment.

Spreadsheet applications

Two applications are necessary to solve the task. They are depicted in Figure 2 and 3. They are available at https://www.dropbox.com/s/xigz6g5hiokrw10/speed_skydiving.xlsx?dl=0.

The aim of the application in Figure 2 is to find the value of B to reach the average speed of 557.57 km/h at 2700 m and 1700 m. The inputs are in the white cells.

The graphs are made over 5001 points (5000 steps, cell F4). Time of impact t_{imp} in cells F3 is estimated and then found by the *Goal Seek* tool (see below). The time is calculated in range B14:B5014 from 0 to t_{imp} by step $\Delta t = t_{imp}/5000$ (cell F5). Parameter B is calculated in cell I4 by Equation (15). The speed, altitude and air
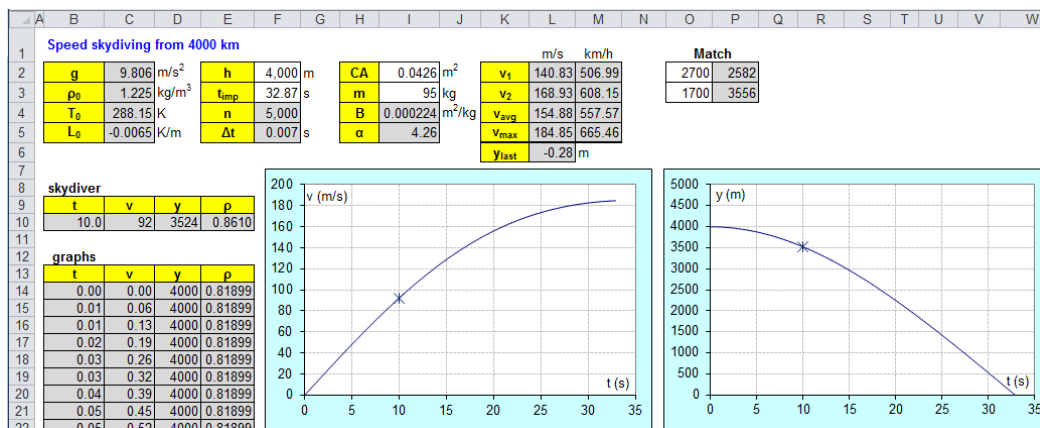

Figure 2. Model of a speed skydiver's fall in the troposphere from 4km

density are calculated in range C14:E5014 by Equations (16) – (19) and (13). The graphs are of type XY.

Cells P2 and P3 contain function MATCH that gives the ordinal number of 2700 and 1700 in range D14:D5014. Cells L2 and L3 contain function INDEX that gives the speed in range C14:C5014 at the ordinal number given by function MATCH. Function AVERAGE in cell L4 gives the average of the speed at 2700m and 1700m. Function MAX in cell L5 gives the maximum speed in range C14:C5014. Cell L6 refers to cell D5014 and it is to find impact time t_{imp} in cell F3 by the *Goal Seek* tool (see below). Formulas in range M2:M5 convert speed from m/s to km/h.

The speed, altitude, air density, sound speed and Mach number at the time written in cell B10 are returned to cells C10:G10 from range C14:E5014 by function VLOOKUP. The position on the $v(t)$ and $y(t)$ graphs is marked with "x", which are one-point XY graphs made over ranges B10:C10 and B10, D10.

The user enters the inputs. The values of t_{imp} and product CA have to be estimated. Note that Equation (3) gives $t_{\mathrm{imp}} = 28.3\,\mathrm{s}$ in a vacuum. The projected area of a human body is approximately $A = 0.6 \times 0.3 = 0.18\,\mathrm{m^2}$, and $C \approx 0.4$ for a sphere, therefore one can take $t_{\mathrm{imp}} = 30\,\mathrm{s}$ and $CA = 0.07\,\mathrm{m^2}$. The correct values can be found by iteration by means of the *Goal Seek* tool. For CA, *Goal Seek* box *Set cell* is M4, *To value* is 557.57 and *By changing cell* is I2. For t_{imp}, *Goal Seek* box *Set cell* is L6, *To value* is 0 and *By changing cell* is F3. The result is $t_{\mathrm{imp}} = 32.87\,\mathrm{s}$ and $CA = 0.0426\,\mathrm{m^2}$.

The application in Figure 3 is an adjusted copy of the application in Figure 2. Range F13:G5014 is added to calculate speed of sound c by Equation (20) and Mach number v/c. Cell L4 contains function MAX that gives the maximum Mach number in range G14:G5014.

The user enters the inputs taking $CA = 0.0426\,\mathrm{m^2}$. The value of t_{imp} has to be estimeted. Equation (3) gives $t_{\mathrm{imp}} = 46.9\,\mathrm{s}$ in a vacuum therefore one can take $t_{\mathrm{imp}} = 60\,\mathrm{s}$. The correct value $t_{\mathrm{imp}} = 59.61\,\mathrm{s}$ can be found by *Goal Seek* setting *Set cell* to L5, *To value* to 0 and *By changing cell* to F3. The result of the investigation is in range L2:L5. It can be seen that the maximum speed is 892 km/h and the maximum Mach number is 0.77. Hence, the answer to the task question is that Marco Wiederkehr would not be supersonic if he jumped from the altitude of 11 km in which airliners cruise. Still, his maximum speed would be imposing anyway – equal to that of an airliner.
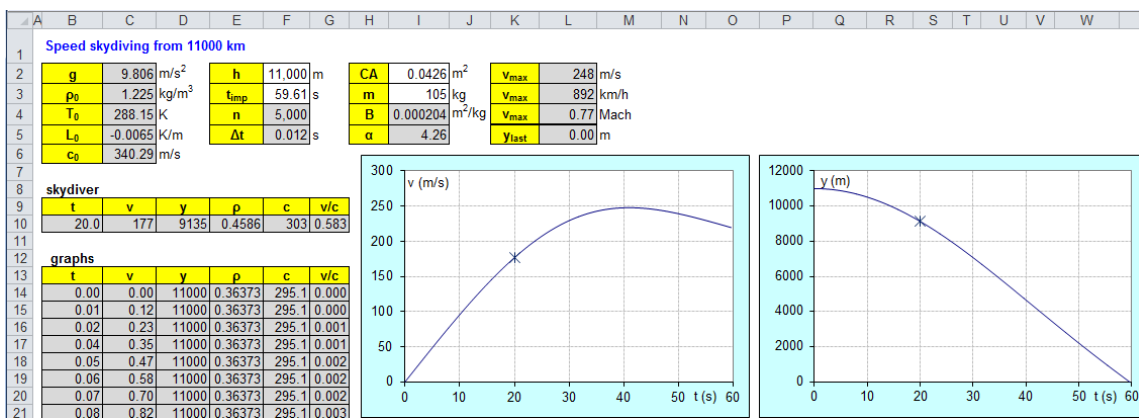

Figure 3. Model of a speed skydiver's fall in the troposphere from 11km

Lesson

Nineteen high school students aged 18-19 took part in the survey in two groups. A 90-minute optional applied informatics lesson was taught to each group. The aim was to learn the Euler method, practise Excel modelling, and learn to analyse problems scientifically. The students were familiar with Equation (12) and formulas $G = gm$, $F = am$, $a = \Delta v / \Delta t$ and $v = \Delta y / \Delta t$ from the first year physics.

The lesson started with a discussion on speed skydiving and Baumgartner's jump. The teacher presented pictures using a data projector and posed the task question. He showed the application in Figure 2 via a data projector. Then, he projected the theoretical background and went through it. He applied the method of questioning and encouraged the students to recall the necessary formulas. Then, he sent the students the pdf file with the theory via emails.

The students downloaded the template (the application in Figure 2 with no formulas and charts and empty cells for t_{imp} and *CA*). They developed the application while the teacher was developing the application along with them and projecting it to give feedback but with a delay, so that the clever student could work individually. Range B10:E10 was left empty due to lack of time. After accomplishing, the students found the correct t_{imp} and *CA*. Then, they downloaded the template of the application in Figure 3 (empty cells for t_{imp} and *CA* and empty range F14:G5014), pasted in the value of *CA* from the previous application and estimated t_{imp}. They added the formulas for the speed of sound and Mach number in range F14:G5014, iterated t_{imp} and found the maximum speed.

The students filled in a questionnaire at the end of the lesson. The result is in section Survey.

PROJECTILE MOTION IN THE AIR

Projectile motion in a vacuum is taught at high school in the author's country in an optional physics subject. Projectile motion in the air is not taught. The problem is governed by a system of two ODE. The numeric solution by the Euler method is easily obtainable by applying formulas of dynamics, vectors and trigonometric functions.

Theory

Two forces act on a projectile moving in the air, which are weight $G = gm$ and drag. The drag is opposite to the velocity and its amplitude is given by Equation (4). The trajectory is a ballistic curve.

Let a projectile be fired at time $t = 0 \, \text{s}$ at speed v_0 and elevation angle α_0 from altitude $y = h$ above the target level. Decomposing drag $\mathbf{F}_D$ and velocity $\mathbf{v}$ to x and y direction gives two similar right-angle triangles (Figure 4; note that $v_x > 0$, $F_{Dx} < 0$; if going up, then $v_y > 0$ and $F_{Dy} < 0$; if going down, then $v_y < 0$ and $F_{Dy} > 0$. It holds that

$$\frac{\left|F_{Dx}\right|}{F_D} = \frac{\left|v_x\right|}{v}, \tag{21}$$

and

$$\frac{\left|F_{Dy}\right|}{F_D} = \frac{\left|v_y\right|}{v}, \tag{22}$$
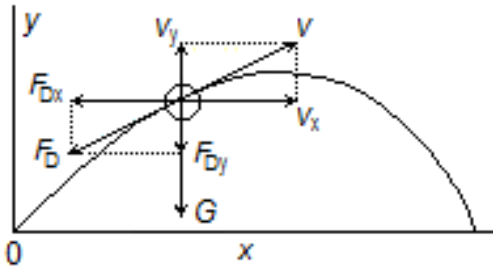
Figure 4. Trajectory, forces and velocity of a projectile moving in the air

where speed v is given by the formula

$$v = \sqrt{v_x^2 + v_y^2}\,. \tag{23}$$

Substitution from Equation (4) and taking the sign of components F_{Dx}, F_{Dy} and v_x, v_y into account yield

$$F_{\mathrm{Dx}} = -\frac{1}{2}CA\rho_a v_x v \tag{24}$$

and

$$F_{\mathrm{Dy}} = -\frac{1}{2}CA\rho_a v_y v\,. \tag{25}$$

In the x direction, force F_{Dx} acts only, so

$$F_x = F_{\mathrm{Dx}}\,. \tag{26}$$

Applying the formula $F_x = m\Delta v_x / \Delta t$ and rearranging give

$$\Delta v_x = -K v_x v \Delta t\,, \tag{27}$$

where K is given by Equation (6). Then,

$$v_x = v_x + \Delta v_x,\; v_x(0) = v_0 \cos\alpha_0\,, \tag{28}$$

$$\Delta x = v_x \Delta t\,, \tag{29}$$

$$x = x + \Delta x,\; x(0) = 0\,. \tag{30}$$

In the y direction, force F_{Dy} acts plus weight G acts downwards, thus

$$F_y = F_{\mathrm{Dy}} - G\,. \tag{31}$$

Applying the formula $F_y = m\Delta v_y / \Delta t$ and rearranging give

$$\Delta v_y = -\left(K v_y v + g\right)\Delta t\,. \tag{32}$$

$$v_y = v_y + \Delta v_y,\; v_y(0) = v_0 \sin\alpha_0\,. \tag{33}$$

$$\Delta y = v_y \Delta t\,, \tag{34}$$

$$y = y + \Delta y,\; y(0) = h\,. \tag{35}$$

Equations (27)–(30) and (32)–(35) enable one to draw the trajectory.

Spreadsheet application

The Excel application is in Figure 5. It is available at https://www.dropbox.com/s/jk5ba0r05u7z497/projectile_air.xlsx?dl=0.

The projectile is a homogenous sphere of diameter d and density ρ_b. Density $\rho_b = 700\,\mathrm{kg/m^3}$ is for hard wood. It holds for a sphere that $C \approx 0.4$. The inputs are in the white cells.
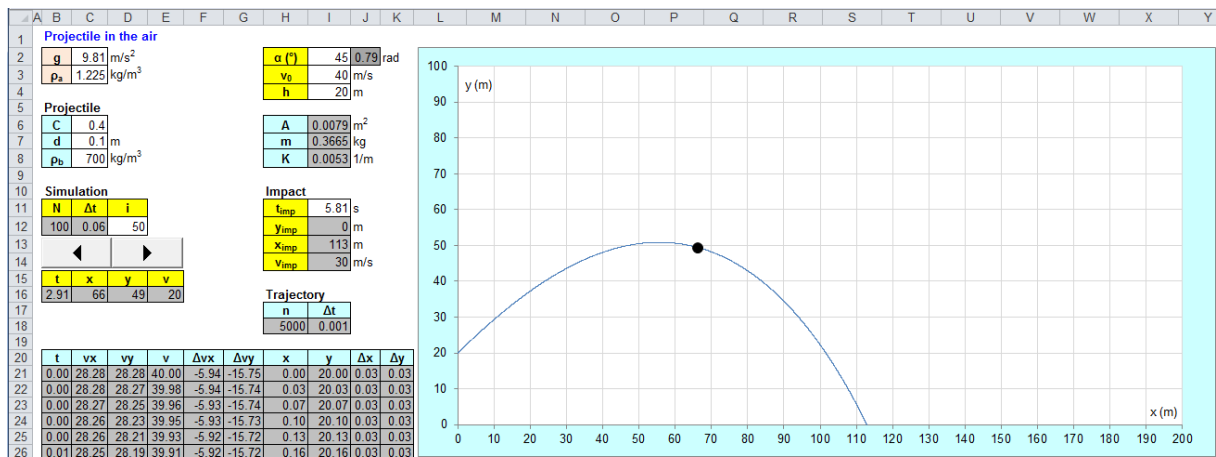
Figure 5. Simulation of projectile motion in the air of constant density

The trajectory is a graph of type XY made over 5001 points (5000 steps, cell H18). Time of impact t_{imp} is estimated and then found by the *Goal Seek* tool. The time is calculated in range B21:B5021 from 0 to t_{imp} by step $\Delta t = t_{imp}/5000$ (cell I18). The speed, speed components and coordinates are calculated in range C21:K5021 by Equation (27)–(30) and (32)–(35). Cell I12 refers to cell I5021 and it is to find impact time t_{imp} in cell I11 by *Goal Seek* with *Set cell* I12, *To value* 0 and *By changing cell* I11. The horizontal range and impact speed are in cells I13 and I14.

To simulate the motion, time t_{imp} is divided into 100 (cell B12) steps $\Delta t = t_{imp}/100$ (cell C12). The number of elapsed steps (cell D12) is governed by the spinbutton with properties Max, Min, SmallChange and LinkedCell set to 100, 0, 1 and D12. The total time is in cell B16. The corresponding coordinate *x*, *y* and speed *v* of the projectile are returned into cells C16, D16 and E16 by function VLOOKUP. The projectile is a one-point graph of type XY. A click on the spinbutton increases the number of elapsed time intervals by 1 and puts the projectile in a new position *x*, *y*. Clicking and holding down the spinbutton causes the projectile to move along the trajectory according to the laws of physics. The coordinates and speed at a given time can be obtained in cells C16, D16 and E16 if the time is written in cell B16.

Lesson

Twenty students aged 16-17 took part in the experiment in two groups. A 90-minute lesson was taught to each group. The aim was to exercise Excel, learn the principles of the Euler method to solve a system of ODE, and analyse projectile motion in the air and compare it to the motion in a vacuum. The lesson was a follow-up to a lesson in which the students simulated and analysed projectile motion in a vacuum with Excel (Benecka, 2015a). Then, all students found the lesson interesting and 84% would have liked to continue with modelling projectile motion in the air. The time span between the lessons was four months in the first group and one week in the second group. The students were familiar with Equation (4) and formulas $G = gm$, $F_x = ma_x$, $a_x = \Delta v_x/\Delta t$ and $v_x = \Delta x/\Delta t$ from the first year physics.

The lesson started with a discussion on projectile motion in a vacuum and in the air. The students found that the Moon is the nearest place where the equations for the motion in a vacuum hold, and that they are the upper limits for the speed and distance travelled in the air. The teacher presented the application in Figure 5 via a data projector and demonstrated that the trajectory is not a parabola but a ballistic curve. Then, he projected Figure 4 and Equation (21)–(35) and went through the derivation. He applied the method of questioning and encouraged the students to

recall the necessary formulas. Then, he sent the pdf with the theoretical background to the students via emails.

The students downloaded the template (the application in Fig. 5 with no chart and formulas) and developed the application. The teacher was developing the application along with the students discussing the steps and projecting his work to give feedback but with a delay so that the clever student could work individually. After accomplishing the model, the students experimented with it. They found that the range is maximum not at the elevation angle of 45°, which holds in a vacuum, but at a smaller one. They found that the range increased (1) if they decreased coefficient *C*, from which they deduced why streamline projectiles were introduced in the 19th century instead of balls; (2) if they increased density ρ_b of the projectile, e.g. a lead ball flied further than an iron one of the same size; (3) if they increased diameter *d* of the projectile, from which they deduced why heavy calibre guns have a bigger range; (4) if they decreased the air density. The effect of parameter *d* surprised the students. They assumed that if the projectile was smaller, then it would fly further as the drag dropped but the model showed the opposite. The teacher encouraged them to take the mass into account. After some questioning the students found that if *d* dropped to a half, then drag dropped to a quarter but the mass dropped to an eighth, therefore coefficient *K* doubled.

The students filled in a questionnaire at the end of the lesson. The result is in section Survey.

SURVEY

All students filled in the following questionnaire:
A)   The lesson was (1 = very; 2 = quite; 3 = little; 4 = not) interesting.
B)   I understood (1 = all; 2 = most; 3 = little; 4 = nothing) of the algorithm
C)   I learned (1 = a lot; 2 = quite a lot; 3 = little; 4 = nothing) new in Excel.
D)   I learned (1 = a lot; 2 = quite a lot; 3 = little; 4 = nothing) new in physics.
E)   I learned (1 = a lot; 2 = quite a lot; 3 = little; 4 = nothing) new in mathematics.
F)   I would like to model Felix Baumgartner's fall: (1 = yes; 2 = no).
G)   I am a boy (1 = yes; 2 = no).
We take answers 1 and 2 in questions A – E and 1 in questions F – G for positive ones.

Free fall in a vacuum and in the air

Twenty-nine students took part in the experiment. The number of answers is in Table 1.

Surprisingly, the girls found the lessons more interesting than the boys despite the fact that they understood and would have liked to model a more difficult task less than the boys. However, the number of girls was too small to make strong conclusions.

High-speed skydiver's jump

Nineteen students took part in the experiment. The number of answers is in Table 2.

The boys found the lesson more interesting and understood more than the girls. The girls had the feeling that they learned new in mathematics, physics and Excel

significantly more than the boys. All boys would have liked to model a more difficult task while it was just a small part of the girls. The number of girls was too small to make strong conclusions.

Projectile motion in the air

Twenty students took part in the experiment. The number of answers is in Table 3.

All students found the lesson interesting. The girls understood more and learned significantly more in mathematics and Excel than the boys while the boys learned more in physics. All boys would have liked to model a more difficult task. The number of girls was too small to make strong conclusions.

Summary

Sixty-eight students took part in the experiments. The total number of answers is in Table 4.

The result of 97% in question A suggests that the lessons were successful. Working with the applications was an interesting, productive and motivating activity according to the students. That follows from the positive answer to question A but also to B and F because if the students had found the lesson interesting but

Table 1. Number of answers to simulating free fall in a vacuum and in the air

	all				all %				boys %				girls %				positive %		
	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	all	boys	girls
A	17	11	1	0	59	38	3	0	61	35	4	0	50	50	0	0	97	96	100
B	9	18	2	0	31	62	7	0	35	65	0	0	17	50	33	0	93	100	67
C	8	14	6	1	28	48	21	3	26	52	17	4	33	33	33	0	76	79	67
D	2	17	8	2	7	59	28	7	9	57	26	9	0	67	33	0	66	66	67
E	8	17	4	0	28	59	14	0	26	61	13	0	33	50	17	0	86	87	83
F	24	5			83	17			87	13			67	33			83	87	67
G	23	6			79	21													

Table 2. Number of answers to modelling high-speed skydiver's fall

	all				all %				boys %				girls %				positive %		
	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	all	boys	girls
A	6	12	1	0	32	63	5	0	43	57	0	0	0	80	20	0	95	100	80
B	11	6	2	0	58	32	11	0	64	29	7	0	40	40	20	0	89	93	80
C	1	10	8	0	5	53	42	0	0	43	57	0	20	80	0	0	58	43	100
D	2	9	7	1	11	47	37	5	7	43	43	7	20	60	20	0	58	50	80
E	0	6	10	3	0	32	53	16	0	21	57	21	0	60	40	0	32	21	60
F	15	4			79	21			100	0			20	80			79	100	20
G	14	5			74	26													

Table 3. Number of answers to simulating projectile motion

	all				all %				boys %				girls %				positive %		
	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	all	boys	girls
A	5	15	0	0	25	75	0	0	25	75	0	0	25	75	0	0	100	100	100
B	9	10	1	0	45	50	5	0	50	44	6	0	25	75	0	0	95	94	100
C	0	18	2	0	0	90	10	0	0	88	13	0	0	100	0	0	90	88	100
D	1	16	3	0	5	80	15	0	6	81	13	0	0	75	25	0	85	88	75
E	3	12	5	0	15	60	25	0	19	50	31	0	0	100	0	0	75	69	100
F	19	1			95	5			100	0			75	25			95	100	75
G	16	4			80	20													

Table 4. The total number of answers to the questionnaires

	all				all %				boys %				girls %				positive %		
	1	**2**	**3**	**4**	**1**	**2**	**3**	**4**	**1**	**2**	**3**	**4**	**1**	**2**	**3**	**4**	**all**	**boys**	**girls**
A	28	38	2	0	41	56	3	0	45	53	2	0	27	67	7	0	97	98	93
B	29	34	5	0	43	50	7	0	47	49	4	0	27	53	20	0	93	96	80
C	9	42	16	1	13	62	24	1	11	60	26	2	20	67	13	0	75	72	87
D	5	42	18	3	7	62	26	4	8	60	26	6	7	67	27	0	69	68	73
E	11	35	19	3	16	51	28	4	17	47	30	6	13	67	20	0	68	64	80
F	58	10			85	15			94	6			53	47			85	94	53
G	53	15			78	22													

had understood little or had not wanted to continue with a more difficult problem, then developing and working with the applications would have been a play only. The result in questions C, D and E suggests that the lessons contributed to the mathematical, physical and technological knowledge of the students.

The boys answered more positively to questions A, B and F. Significant differences, 41%, 16%, 16% and 15%, were in question F, B, E and C. The boys much more than the girls would have liked to continue with modelling Baumgartner's supersonic jump.

CONCLUSIONS

The paper gave an account of an experiment in which spreadsheet applications that simulate fall and projectile motion in the air by using the Euler method of solving differential equations were developed with high school students. Sixty-eight students aged 16 – 19 took part in the experiment. They filled in the same questionnaire aimed to find out if they found the lessons interesting, and if developing the models and using numeric modelling contributed to the mathematical, physical and technological knowledge of the students. Altogether 97% of the students found the lessons interesting and 85% would have liked to continue with modelling a more complicated problem. The result corresponds well with the findings of Benacka and Ceretkova (2014) (100% and 89%), and implies that developing spreadsheet applications that model real dynamic systems by using the Euler method of solving the governing differential equation has the potential to promote STEM to high school students. Numerical modelling with spreadsheets can help make lessons interesting, benefits the learning and contributes to the scientific and technological knowledge of students.

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