MODESTUM

Research Paper

Origami and GeoGebra Activities Contribute to Geometric Thinking in Second Graders

Anat Klemer [1*], Shirley Rapoport [1]

[1] Western Galilee College, Acre, ISRAEL

Abstract

This article describes the beneficial effects of an intervention program, based on the computerized Origametria program and the GeoGebra environment, for acquiring geometric concepts in the second grade. Eighty-eight second-graders, from both Hebrew-speaking and Arabic-speaking schools, were instructed in groups by eighteen Hebrew and Arabic-speaking college students, respectively. The groups met for twenty-three 45-minute sessions throughout the schoolyear, undergoing different activities in Origametria and GeoGebra about the concepts of triangle, quadrilateral, square, rectangle, right angle and reflectional symmetry. The pupils' geometric knowledge was compared pre and post intervention to that of twenty-nine pupils in the control group. Pre-intervention, the experimental group's geometric knowledge was significantly lower than that of the control group and there were differences in knowledge between pupils from the Hebrew-speaking and Arabic-speaking school. Post-intervention, a considerable improvement was noted in the experimental group, to the point that group differences were no longer significant. Differences between Hebrew and Arabic-speakers in knowledge of most concepts were also eliminated. Due to the intervention program, the knowledge gap between low and high achieving pupils in the pre-intervention test was closed. Thus, all pupils were brought to a similar level of geometric knowledge. There were no gender differences pre or post-intervention.

Keywords: Origami, Geogebra, geometric knowledge, dynamic computer activities, Hebrew-speaking, Arabic-speaking

INTRODUCTION

The current study examines a dynamic, hands-on way to teach and learn geometric concepts. Technology and hands-on activities (GeoGebra – a dynamic computer program, and Origametria – an intervention program with computerised animations, which guide the pupil in learning through origami) were used to help children construct basic geometric concepts in the first steps of elementary school. The contribution of origami for learning geometry and the connection between origami and GeoGebra has been researched in the past (Budinski et al., 2018). The current study adds to the existing body of research and explores in addition the effect of the novel Origametria learning environment (Golan, 2011).

The effect of integrating GeoGebra and origami was examined, particularly on understanding the attributes of triangles, quadrilaterals and specifically squares and rectangles, the right angle and reflectional symmetry. The integration of these tools allows the pupil to be active, creative and learn concepts through experimentation and cooperative learning.

Subsequently, the teaching of geometry in elementary school will be reviewed, focusing on challenges of teaching and learning. Particularly, the teaching of reflectional symmetry and the concept of angles, as they appear within the teaching of triangles, quadrilaterals and polygons. Methods of teaching and learning using origametria and GeoGebra.

LITERATURE REVIEW

Geometric Thinking

Geometry is perceived as the most complicated of all mathematical fields learned at school, due to its

Contribution to the literature

- The present study examines the contribution of Origametria - an innovative computerized environment using origami - for learning geometry.
- While the contribution of origami for learning geometry has been researched in the past, the current study explores in addition the effect of the novel Origametria learning environment.

deductive structure. Pupils are often in a lower level of geometric thinking, than required in order to learn geometry as taught at school (National Council of Teachers of Mathematics, 2000). Learning is a process of conceptual development, constructed by the child himself, out of the learning opportunities given to him (Patkin & Kenner, 2010). Piaget (e.g: Piaget, 1952; Piaget & Inhelder, 1969) describes the development of knowledge as a spontaneous process linked with the development of the nervous system and brain systems responsible for cognitive functions, In addition to the process of learning guided by environmental situations. In order to bridge these two processes, Piaget developed the idea of "operation" – in order to know something, it is not sufficient to look at it or copy it, one has to operate on it: change it and examine its new attributes. Operation, Piaget claims, is the essence of knowledge (e.g.: Piaget, 1952; Piaget & Inhelder, 1969). As in Piaget's theory of cognitive development, Van Hiele presented a model of levels to describe the development of geometric thinking. As in Piaget's theory, the child advances from one level to the other, leading eventually to formal geometric thinking. In order to create a suitable learning environment, the teacher should be aware of the levels of geometric thinking. The development of spatial cognition is at the center of Van Hiele's model, which thoroughly describes the whole range of exploration of geometric shapes, from the level of global visual perception to the level of proofs and accuracy (Van Hiele, 1999). Van Hiele's model describes the way children sort geometric shapes. The current research focuses on an intervention program aiming to transfer 2nd grade pupils from the first level, describing the shape's visual appearance, to the second level, describing the geometric shape's attributes. According to Van Hiele (1999), developing pupils' thinking levels is more contingent on teaching than on age or biological maturity. Certain types of learning experiences can nurture or withhold the development of geometric thinking. When teaching strives to promote the pupil's development from one level to the next, it should include sequences of experimental activities, gradually constructing concepts and the instilling the use of appropriate terms. It should also include activities that help pupils bridge between what they have learned and their prior knowledge. Geometry teaching is a complex process, in which the pupil adds layers of interrelated knowledge, based on an understanding of definitions, concepts, shapes, assumptions and generalizations, depending on the levels of perceptual, cognitive and spatial development. Learning geometric concepts should include the use of concrete aids or computer programs, instead of only formal presentation (The ministry of education, 2006). It is important to put an emphasis on pupils acting in an environment rich with concrete models of geometric concepts and objects demonstrating the learned topics. The curriculum requires that learning activities include free construction of polygons, or according to different constraints, cutting and pasting shapes, sequencing in variety of ways, moving two and three-Dimensional models in space and using dynamic geometry programs (The ministry of education, 2006).

The challenge is to integrate dynamic means of illustration, which enable visualization, in a way that pupils will actively create concrete examples and construct shapes, to better understand their attributes and gradually arrive at formal geometric definitions (Van Hiele, 1999).

The following sections will review challenges in teaching and learning geometry, the concepts included in the first and second grade's curriculum, and how dynamic tools such as origami and computer programs can support the teaching and learning of geometry.

Challenges in Teaching and Learning Geometry

The most important skill for learning geometry is visual abilities. Geometry is a field involving the visualization of abstract concepts. A pupil with underdeveloped visualization skills, may encounter difficulty, the main challenge being understanding of concepts and the interaction between them. Therefore, means of illustration should be used to develop these skills. Additionally, pupils encounter difficulties due to misconceptions, a lack of prior knowledge, difficulty with reasoning and mistakes in basic operation (Özerem, 2012). Another difficulty could arise when the teaching of geometry is based on static illustrations of shapes and concepts. Learning based on static illustrations in a textbook or worksheet, make it harder for the pupil to see a variety of exemplars of the shape or concept and arrive at the relevant generalizations (Klemer et al., 2015). Using only prototypes as examples leads to a difficulty in understanding basic geometric concepts, since the learner tends to judge other shapes by comparing it visually to the prototype, instead of using the shape's **attributes**. Fixation on the prototype makes it more difficult to identify more shapes that fall under the definition of the concept (Hershkovitz, 1991). In a

research about children's understanding of the concept of triangle and suitable means of illustration, a preschool child was asked to draw a triangle and then point to its vertices. She then pointed only at the vertex at the top of the triangle. When asked to point at the angles, she pointed only at the angles at the bottom part. This separation demonstrated may affect 1st and 2nd graders' ability to identify parts of geometric shapes. This ability is one of the most important components of the second level in Van Hiele's model (Abd-Al-Khalek & Daher, 2009). In order to refrain from creating such difficulties, it is recommended to teach according to thinking levels, as detailed in the curriculum, influenced by Van Hiele's model (Van Hiele, 1999).

Basic concepts in the 1st through 3rd grade's curriculum include triangles, quadrangles, angles and reflectional symmetry.

There are specific challenges associated with the learning of each geometric concept. For example, in the topic of **triangles**, learners that are not yet familiar with the shape's attributes categorize shapes as triangles only by their general appearance. They tend to recognize shapes correctly as triangles when they are "friendly" (e.g.: Shaib & Tabach, 2014; Tirosh & Tzamir, 2008) – for example, where the lower edge of the triangle is parallel to the lower edge of the page – but experience difficulty recognizing triangles that are "not friendly", that do not strongly resemble a prototypical triangle. Mistakes can also occur when these learners recognize as triangles shapes, which visually resemble the prototype, but are not in fact triangles by their attributes.

When learning about **quadrangles,** difficulties tend to arise when learning how properties are deduced from one quadrangle to another, such as the understanding that all squares are rectangles (Ben ari & Rosenthal, 1995). When learning about quadrangles, the pupils, mostly the young ones, have misconceptions about shapes because they recognize them by their general appearance. For example, they tend to perceive "long" parallelogram or right trapezoids as rectangles. It seems that children's image of a rectangle is a shape with four edges with two long parallel sides and corners that are approximately right (Sarama & Clemens, 2000). They also tend to perceive a trapeze as a triangle whose top is cropped. Only in later stages, the learner develops their mathematical thinking and starts recognizing shapes by their attributes.

For another example, in the topic of **angles**, the learners are introduced to a geometric shape formed by two rays sharing a common endpoint. The rays are infinite lines, which is an important notion to teach, in order to prevent a misconception tying the size of the angle and the length of its rays. One of the main challenges of teaching the concept of angle is the multifaceted nature of it, and hence, different ways of measuring it. For example, an angle as a geometric shape

is measured by measuring its inner section. An angle as a rotation is measured by the extent of rotation and an angle in a polygon is measured by measuring the inner *or* outer section. Teachers tend to focus on the opening of the angle, a concept that may confuse the pupil when looking at a flat angle or a full rotation. The description of the angle as a geometric shape is static – two rays in a plane. It can become more dynamic if we imagine that one of them or both can rotate around an axis at the vertex of the angle, like the hands of a clock (Barabash, 2010).

As to **reflectional symmetry**, Fischbein, in his intuition theory, claims that mathematical reasoning is affected by intuition, which is one's subjective knowledge about a concept. This knowledge seems so obvious, that it does not require proof. The intuition is based on prior knowledge that is not necessarily relevant, and it takes over formal knowledge about the learned concept (e.g.: Fischbein, 1987; Tzamir & Barkai, 2005). Regarding the concept of reflectional symmetry, there is often a discrepancy between the learner's intuitive image of what it is, and the actual attribute of symmetry. For example, learners tend to regard parallelograms as having a line of symmetry, or regard a rectangle's diagonal as a line of symmetry. That observation is due to their knowledge that if the rectangle is bisected along its diagonal, the triangles created are congruent. Yet, reflectional symmetry requires that each point of the original figure and its image are the same distance away from the line of reflection.

Teaching with Computerized Dynamic Tools

Exposing pupils to a variety of means of illustration, both physical and virtual, is of high importance, since it could greatly improve their understanding and thinking skills. Pupils learn better when relying on means of illustration as an aid when constructing verbal definitions. These are quickly forgotten, unless the pupil has reached them by active learning (Özerem, 2012).

Origametria is a new technological tool recently created in Israel (Golan, 2011). It is a computerized environment, which supports the teaching of geometry through origami. Origami is an activity that develops spatial perception and intelligence (Harris et al., 2013) and it is widely integrated into the teaching of geometry in different places and in different levels (Klemer et al., 2015).

In Origametria, in the first stage, the learners are instructed how to fold the origami papers by a visual presentation on the computer screen. The computerized animations of the origami steps are followed by questions for discussion, and in the second stage, the learners discuss about a mathematical attribute they encountered during the process. Hence, the teacher is no longer the sole source of knowledge, but a participant in

the virtual, visual activity, developing a discussion in various thinking levels (Klemer et al., 2015). Origametria supports the first levels in Van Hiele's model, which are crucial for learning geometry later on. The questions for discussion are compatible with Van Hiele's thinking levels, in both visual and analytical aspects, taking into account the learners' level (Golan, 2011).

Experimenting with origami allows active learning, during which, the learners discover new knowledge that will be familiar to them later, when presented with its formal use (Patkin & Kenner, 2010). In Origametria, the activity leads the learners to explore different shapes' attributes, creating a terminology that is initially based on "pupils' language" and guided by the teacher, they gradually reach the formal geometric terms (Abd-Al-Khalek & Daher, 2009).

Using dynamic computer programs allows the pupils to learn geometry in a representation-rich environment, and develop a deep understanding of concepts. Dynamic programs provide an opportunity to change the image of a concept (Chazan & Yerushalmy, 1998), refuting misconceptions that stem from being presented only with prototypes as examples. In a dynamic geometric environment, the visual stimulus is not a single drawing of a shape, but a dynamic representation, which can be moved on the computer screen while keeping its original geometric attributes. Additionally, the computer enables interactions in class and mathematical discourse (Chazan & Yerushalmy, 1998).

The national council of teachers of mathematics in the United States (National Council of Teachers of Mathematics, 2000) stated that technology should be the main tool in teaching math. The Ministry of Education in Israel mentions four significant characteristics of the digital learning environment and emphasizes the potential in such an environment: it may contain a variety of media, allows presentation of visual information, it is interactive, up-to-date and dynamic. The use of technology is not a substitute for operating on the physical environment, yet, technology may address pedagogical needs that are sometimes difficult to address without it (Ministry of Education, 2014).

When teaching geometry, using programs has the potential to promote learning through processes of experimentation in order to construct knowledge that is based on understanding and not on rote learning (Furner & Marinas, 2012).

One of the dynamic programs that allows the learner to experiment and reach a deeper understanding of geometric concepts is GeoGebra. The program is user-friendly, intuitive and does not require advance technical skills. It enables the construction of points, intervals, straight lines and polygons. It encourages pupils to experiment with mathematics and raises the level of thinking, which is crucial in order to construct knowledge (Budinski et al., 2018).

Arbain and Shukor (2015) found that GeoGebra has a positive effect on academic achievements and the perception of geometry. The learners who studied with GeoGebra displayed more enthusiasm, confidence and motivation, and performed better on tasks (Arbain & Shukor, 2015). With GeoGebra, pupils can explore the world of mathematics and develop creative thinking.

A Malaysian study tested the efficiency of GeoGebra in learning the sum of angles in a triangle. The pupils created triangles, after which, they were able to move, turn and change the length of edges and triangle dimensions, and be convinced that the sum of angles will always remain 180 degress (Boo & Leong, 2016).

In summary, in light of the evidence about the benefit of integrating computerized dynamic tools and means of illustration for promoting geometric knowledge, the current study examined the effect of integrating Origametria and GeoGebra on second grade pupils' geometric knowledge.

Along with the effects of dynamic tools on learning geometry, there are other personal factors affecting math achievements. In the following paragraphs, some of them will be reviewed.

Factors Affecting Achievements in Mathematics

Findings from the PISA (program for international student assessment in reading, mathematics and science) test, 2015, show there are differences between Hebrew and Arabic speaking students, in favor of the Hebrew-speaking ones. Higher economic and socio-cultural background was found to have a positive impact on achievements in math (Karlitz & Keshet, 2018).

However, differences between the achievements of boys and girls, amongst Hebrew and Arabic speakers, were not found to be statistically significant. Conversely, other studies have shown differences in achievements between boys and girls and between sectors. In Israel, in Hebrew-speaking schools, male students were found to have higher math achievements than female ones, as is found in western countries, yet in Arabic-speaking schools, girls' achievements were higher, as is found in the Arab world. Female students' language skills are generally higher, but when controlling for language skills and comparing students with similar language skills, it was found that male students have an advantage in math (Rapp, 2014). Furthermore, a study of high school students, which examined differences in geometric thinking levels, according to Van Hiele's model, found that male students achieved higher Van Hiele levels than female students (Kundu & Ghose, 2016).

Studies about the efficiency of mathematical intervention programs for low and high-achieving pupils, do not agree as to their effects on pupils of differing levels. The effect of the intervention is likely dependent on the specific mathematical field it

addresses. (e.g.: Smith et al., 2013; Fuchs et al., 2016) showed that 1st graders who function below the 25th percentile are more affected by intervention programs than higher-achieving pupils. In contrast, Fuchs, Sterba, Fuchs, and Malone (2016) found that there is no difference in the efficiency of a fractions intervention program for low and high achievers.

THE CURRENT STUDY

The main goal of the current study was to show the benefit of introducing an intervention program, based on the computerized Origametria and GeoGebra environments, for acquiring geometric concepts in the second grade, across sectors, genders and regardless of initial geometric knowledge.

Research Questions

1. How does an intervention program of origami and dynamic computer activities affect second graders' geometric thinking?
2. Does the program have a different effect on Hebrew and Arabic speakers?
3. Does the program have a different effect on boys and girls?
4. Does the program have a different effect depending on the pupil's initial level of geometric knowledge?

Methods

Participants

Participants in the experimental group were sampled from two schools in Acre, Israel – one school belonging to the Hebrew speaking education sector and the other to the Arabic speaking sector. In each school, two second grade classes participated in the study, two Hebrew speaking classes and two Arabic speaking classes overall. Nine college students were assigned to each school as part of their college coursework, and worked with 5-6 second grade pupils each. One hundred second grade pupils participated in the experimental group, of whom 12 did not complete either pre or post tests, and were therefore excluded from the final sample. Thus, the final experimental group was composed of 88 pupils: 48 Arabic speaking pupils (54.5%), and 40 Hebrew speaking pupils (45.5%); 42 boys (47.7%) and 46 girls (52.3%). Arabic speaking pupils included 22 boys and 26 girls, and Hebrew speaking pupils included 20 boys and 20 girls.

The control group was composed of a convenience sample of 29 second grade, Hebrew speaking pupils, from various schools. They were taught by six college students, who worked with 4-5 second grade pupils each. Population in these schools was of a higher socioeconomic level compared to the schools of the experimental group.

Measures

A research questionnaire composed of 19 questions was devised, in order to determine the geometric thinking level (according to Van Hiele's model) of 2nd grade pupils. The questionnaire was based on Usiskin's test (Usiskin, 1982), adapted for 2nd graders. Content validity was judged by three relevant academic experts, who were asked to decide which thinking level each question reflects. After which, suggested changes were made and an agreement was reached on all questions, between all independent judges. Examples of test questions and the thinking levels reflected in them can be found in Appendix A.

Procedure

Before applying the intervention program, every student used our research questionnaire to interview each pupil in her group, in order to discover their existing geometric perceptions and misconceptions. Following, an intervention program based on the Origametria program and GeoGebra environment was adapted to the pupils' thinking levels. Examples for activities can be found in Appendix B.

The students met with their group weekly, for twenty-three 45-minute sessions. The students used laptops to present the animations of the origami steps, and guided their group discussion. In six sessions, each pupil experimented with GeoGebra on a separate computer. The geometric concepts learned were: polygons and their attributes, reflectional symmetry, parallel and perpendicular lines and angles.

After the intervention program ended, there was a post-program test: the students repeated the initial interview. The pupils' answers were compared to their initial answers on the pre-program test.

Variables

Level of knowledge of the following geometry concepts: triangle, quadrilateral, square, rectangle, right angle, and reflectional symmetry. Level of knowledge of each concept was rated on a 0-100 scale, based on the pre and post program tests.

Total score for level of knowledge in Geometry: A total score was composed in the pre and post program tests, from mean knowledge scores across the various concepts (Cronbach α = .74).

Data analysis

Data were analyzed with SPSS ver. 25. First, descriptive statistics were calculated for the background data (education sector, gender) and the study variables. Means, standard deviations, and correlations were

Table 1. Means, standard deviations, and correlations for Geometric knowledge in the pre program test (N = 117)

	M (SD)	.2	.3	.4	.5	.6	.7
1.Triangle	66.37 (23.17)	-.13	.33***	.53***	.01	.11	.56***
2.Quadrilateral	48.44 (17.12)		-.12	-.09	.11	.01	.17
3.Square	63.50 (18.24)			.39***	.14	.21*	.57***
4.Rectangle	54.27 (21.80)				.28**	.18	.70***
5.Right angle	41.67 (25.80)					.21*	.58***
6.Reflectional symmetry	42.69 (28.62)						.60***
7.Total knowledge score	52.82 (12.29)						

*$p < .05$, **$p < .01$, ***$p < .001$

Table 2. Means, standard deviations, t values and F values for knowledge in Geometry, by group and time (N = 117)

	Experimental group (n = 88)		Control group (n = 29)		Pre program differences	Post program differences
	Pre	Post	Pre	Post	$t(115)$	$F(1, 114)$
Knowledge for:	M (SD)	M (SD)	M (SD)	M (SD)		(η^2)
Triangle	62.93 (23.57)	87.64 (13.39)	76.80 (18.62)	91.17 (7.31)	3.24**	0.36 (.003)
Quadrilateral	49.16 (17.11)	63.52 (16.49)	46.28 (17.25)	52.62 (16.19)	-0.79	9.07** (.074)
Square	58.81 (15.43)	78.59 (14.21)	77.72 (18.98)	76.00 (19.31)	5.40***	1.88 (.016)
Rectangle	49.92 (20.12)	69.86 (19.93)	67.48 (21.69)	76.55 (23.55)	4.00***	1.59 (.014)
Right angle	36.55 (24.19)	55.11 (25.03)	57.21 (24.63)	60.38 (22.97)	3.97***	0.16 (.001)
Reflectional symmetry	33.70 (24.86)	72.83 (23.74)	69.97 (21.18)	77.00 (23.17)	7.05***	0.03 (.001)
Total knowledge score	48.51 (10.13)	71.26 (12.71)	65.91 (8.45)	72.29 (11.08)	8.34***	0.19(.002)

*$p < .05$, **$p < .01$, ***$p < .001$

Note. Knowledge for 'triangle', at pre program: $df = 60.01$

calculated for the study variables at pre program. Second, initial differences in the study variables were calculated between the experimental and control groups, using *t*-tests. Similarly, initial differences in the study variables were calculated within the experimental group, by education sector, gender, and the total score in Geometry (between regular and weak pupils), using *t*-tests. In light of the significant initial differences that were found, the extent of change between each two groups or sub-groups (time by group, 2x2), was calculated with analyses of variance of the post program test scores, by group, controlling for the pre program test scores, matching each post program test score with its respective pre program test score.

RESULTS

Descriptive Results

The distribution of the variables representing the level of Geometric knowledge in the pre program test (Table 1) shows that their means range between 41.7 and 66.4. Rank ordering them was found significant ($F(5, 580) = 28.82$, $p < .001$, $\eta^2 = .199$), with the scores for knowledge of 'triangle' and 'square' being the highest, and the scores for knowledge of 'right angle' and 'reflectional symmetry' being the lowest ($p < .001$). The total knowledge score, for the six concepts, was about 53 on average. Positive correlations were generally found between knowledge of the various concepts, but only some are significant. Knowledge for 'triangle' was positively related with knowledge for 'square', knowledge for 'rectangle' was positively related with

knowledge for 'triangle', 'square', and 'right angle', and knowledge for 'reflectional symmetry' was positively related with knowledge for 'square', and 'right angle'.

Group Differences

An examination of group differences in Geometric knowledge, at pre program, revealed significant results (Table 2). Knowledge of all concepts, except for 'quadrilateral', was higher at pre program among pupils in the control group (averaging at 57 to 78) than among pupils in the experimental group (averaging at 34 to 63). A significant difference was found for the total knowledge score as well (average of 66 vs. 49, respectively). In light of these initial differences, the extent of change, between the experimental and control groups (time by group, 2x2), was calculated with analyses of variance of the post program test scores, by group, controlling for the pre program test scores, matching each post program test score with its respective pre program test score.

Results show that in all cases where a significant pre program group difference was detected (regarding 'triangle', 'square', 'rectangle', 'right angle', 'reflectional symmetry', and total knowledge score), the post program group difference was not significant. That is, a significant improvement was noted in the experimental group, to the point that the average scores in this group were not different than the average scores of the control group. The initial score for 'quadrilateral' was not different between the two groups, and in the end was significantly higher in the experimental group than in the control group. This result serves as another finding

Table 3. Means, standard deviations, t values and F values for knowledge in Geometry, by sector and time, in the experimental group (N = 88)

	Hebrew speaking (n = 40)		Arabic speaking (n = 48)		Pre program differences	Post program differences
	Pre	Post	Pre	Post	$t(86)$	$F(1, 114)$
Knowledge for:	M (SD)	M (SD)	M (SD)	M (SD)		(η^2)
Triangle	76.05 (20.54)	88.65 (12.83)	52.00 (20.25)	86.79 (13.92)	5.51***	0.46 (.005)
Quadrilateral	45.43 (18.23)	59.18 (16.40)	52.27 (15.64)	67.15 (15.83)	-1.90	5.79* (.064)
Square	63.13 (16.30)	76.25 (14.13)	55.21 (13.82)	80.54 (14.13)	2.47***	3.46 (.039)
Rectangle	55.13 (18.16)	67.68 (21.96)	45.58 (20.82)	71.69 (18.10)	2.27*	0.53 (.006)
Right angle	25.78 (22.03)	42.15 (21.66)	45.52 (22.36)	65.92 (22.54)	-4.15***	15.91*** (.158)
Reflectional symmetry	26.45 (25.58)	70.83 (24.67)	39.75 (22.78)	74.50 (23.07)	-2.58*	0.71 (.008)
Total knowledge score	48.66 (9.39)	67.45 (12.64)	48.39 (10.80)	74.43 (15.83)	0.12	6.98** (.076)

*$p < .05$, **$p < .01$, ***$p < .001$

that shows the improvement that took place in the experimental group.

At post program, means in the experimental group ranged between 55.1 and 87.6. Rank ordering them was found significant ($F(5, 435) = 44.77$, $p < .001$, $\eta^2 = .340$), with the scores for knowledge of 'triangle' and 'square' being the highest, and the scores for knowledge of 'right angle' and 'quadrilateral' being the lowest ($p < .001$).

Further, difference scores were defined in the experimental group, and change was compared between the various concepts, with a repeated measures analysis of variance. The analysis was found significant ($F(5, 435) = 11.33$, $p < .001$, $\eta^2 = .115$), with the difference score for 'reflectional symmetry' ($M = 39.12$, $SD = 35.03$) being higher than all other difference scores (ranging between $M = 14.36$ $SD = 24.07$ for 'quadrilateral', and $M = 24.70$ $SD = 23.95$ for 'triangle').

Differences by Sector

An examination of differences by sector (Hebrew and Arabic speakers) in Geometric knowledge, at pre program, in the experimental group (i.e., between the two schools), yielded significant results (Table 3). The data show a non-consistent pattern of differences. Pre program scores of Hebrew speaking pupils were higher than pre program scores of Arabic speaking pupils concerning knowledge of 'triangle', 'square', and 'rectangle', while the opposite was found concerning 'right angle', and 'reflectional symmetry'. No significant differences were found for 'quadrilateral'. In light of these inconsistent pre program differences, no significant difference was found in the total score of Geometric knowledge by sector.

In light of these initial differences, the extent of change, between the two sectors (time by sector, 2x2), was calculated with analyses of variance of the post program scores, by sector, controlling for the pre program scores, matching each post program score with its respective pre program score.

Results show that no post program difference were found regarding 'triangle', 'square', 'rectangle', and 'reflectional symmetry', so that the initial sector differences significantly decreased. No initial difference was found regarding 'quadrilateral', and the post program difference was in favor of the Arabic speaking pupils. Regarding 'right angle', the initial difference, that was in favor of the Arabic speaking pupils, was retained at post program, even while controlling for the pre program scores. Regarding the total score of Geometric knowledge, where no initial differences were found, scores were higher at post program among Arabic speaking pupils.

It should be noted that there were generally no significant initial differences in Geometric knowledge between the two classes in the Hebrew speaking sector ($p = .051$ to $p = .691$), except with respect to 'reflectional symmetry' ($p = .006$), and there were generally no significant initial differences in Geometric knowledge between the two classes in the Arabic speaking sector ($p = .060$ to $p = .653$), except with respect to 'square' ($p = .020$). The initial difference for 'reflectional symmetry' is a function of a low mean score in one of the two classes in the Hebrew speaking sector. Excluding this class turned the initial sector difference for 'reflectional symmetry' non significant ($t(68) = -0.57$, $p = .572$). Likewise, the initial difference for 'square' is a function of a low mean score in one of the two classes in the Arabic speaking sector. Excluding this class turned the initial sector difference for 'square' non significant ($t(65) = 1.02$, $p = .311$). In both cases, post program differences, without the exceptional class, while controlling for the pre program score, has yielded non significant results, as with the total sample.

That is, it may be concluded that the level of Geometric knowledge has increased to a greater extent in the Arabic speaking sector, regarding the specific concepts and the total scores, except for 'reflectional symmetry'. The extent of improvement in 'reflectional symmetry' is greater in the Hebrew speaking sector, yet this result is a function of the one class that had an initial lower mean. An examination of the change without this class revealed no significant sector difference.

Table 4. Means, standard deviations, t values and F values for knowledge in Geometry, by the total knowledge score and time, in the experimental group (N = 67)

	Weak pupils (*n* = 33)		Regular pupils (*n* = 34)		Pre program differences	Post program differences
	Pre	Post	Pre	Post	$t(65)$	$F(1, 64)$
Knowledge for:	M (SD)	M (SD)	M (SD)	M (SD)		(η^2)
Triangle	47.06 (23.87)	86.24 (14.05)	75.74 (14.77)	88.15 (13.21)	5.89***	0.67 (.010)
Quadrilateral	45.30 (16.54)	65.15 (14.39)	53.41 (16.48)	60.00 (14.26)	2.01*	1.77 (.027)
Square	50.82 (13.85)	75.64 (12.23)	65.88 (14.32)	79.47 (15.37)	4.38***	0.41 (.006)
Rectangle	38.00 (18.05)	72.09 (19.69)	64.38 (14.07)	67.74 (21.39)	6.68***	0.02 (.001)
Right angle	27.03 (23.51)	56.30 (23.45)	46.91 (21.04)	54.09 (24.91)	3.65***	2.24 (.034)
Reflectional symmetry	22.97 (21.22)	68.39 (26.56)	41.15 (24.61)	72.85 (22.07)	3.23**	0.79 (.012)

*p < .05, **p < .01, ***p < .001

Gender Differences

An examination of gender differences in Geometric knowledge, at pre program, in the experimental group, did not yield significant results: $t(86) = -1.15$ $p = .254$ to $t(86) = 0.79$ $p = .433$. Likewise, at post program, there were generally no significant differences: $F(1, 85) = 0.49$, $p = .487$, $\eta^2 = .006$ to $F(1, 85) = 3.50$, $p = .065$, $\eta^2 = .040$.

Differences According to the Total Knowledge Score

In order to compare the extent of change between weak and regular pupils, two sub-groups were defined, ranging at about half of a standard deviation around the mean of the initial total knowledge score: pupils with a total score up to 46 (*n* = 33, 37.5%), and pupils with a total score of 52 and above (*n* = 34, 38.6%). (The rest of the pupils, with scores of 47 to 51, *n* = 21, 23.9%, were excluded from this comparison). This sub-grouping was conducted in order to clearly distinguish weak from regular pupils on the one hand, and to enable group sizes that are large enough for statistical analyses, on the other hand.

Obviously, an examination of pre program sub-group differences has yielded significant results in favor of the regular pupils (Table 4). Pre program mean scores for the regular pupils range between 41 and 75, and pre program mean scores for the weak pupils range between 23 and 50.

In light of the significant initial differences, the extent of change, between the two achievement sub-groups (time by sub-group, 2x2), was calculated with analyses of variance of the post program scores, by sub-group, controlling for the pre program scores, matching each post program score with its respective pre program score. Results show no significant post program differences. That is, an increase was noted in the weak pupils' scores, to the point that the differences between them and the regular pupils turned non significant.

DISCUSSION

The current study examines a dynamic, hands-on way to teach and learn geometric concepts. The effect of integrating Origami and GeoGebra was examined, particularly on understanding the attributes of triangles, quadrilaterals and specifically squares and rectangles, the right angle and reflectional symmetry. The integration of these tools allows the pupil to be active, creative and learn concepts through experimentation and cooperative learning.

The following research questions were examined:

1. How does an intervention program of origami and dynamic computer activities affect second graders' geometric thinking?
2. Does the program have a different effect on Hebrew and Arabic speakers?
3. Does the program have a different effect on boys and girls?
4. Does the program have a different effect depending on the pupil's initial level of geometric knowledge?

In relation to the **first research question**, findings show that before the intervention program, pupils in the experimental group achieved poorly on the test (mean grade: 48.51% accuracy), reflecting a low level of geometric thinking. The control group scored significantly higher on the test (mean grade: 65.91% accuracy). The knowledge gap may be attributed to the fact that participants in the control group belonged to a higher socioeconomic status than participants in the experimental group. Higher economic and socio-cultural background was found to have a positive impact on achievements in math in the PISA test, 2015.

After the initial test, the experimental group underwent an intervention program based on the Origametria program and GeoGebra environment, while the control group learned geometry with textbooks according to the ministry of education's curriculum. In the post-program test, differences in achievements between the experimental and control group were no longer significant (mean grade: 71.26% and 72.29% respectively).

The post-program test results imply that the knowledge gap was closed. We can also infer that most pupils in the experimental group had reached Van Hiele's second thinking level after twenty-three intervention sessions, since that in order to answer the

test questions correctly, one has to recognize and describe the shape by its attributes (see Appendix A).

The experimental group had reached a more advanced thinking level, due to a teaching method that is hands-on and integrates computerized, dynamic tools. The higher thinking level was reached because they were presented by the teacher with a sequence of experimental activities, as Van Hiele advocates. It can be also said that they reached the more advanced thinking level because they *operated* on the shapes, which coincides with Piaget's idea of "operation" – in order to know something, it is not sufficient to look at it or copy it, one has to operate on it: change it and examine its new attributes.

Working with dynamic tools - Origametria and GeoGebra – gave the pupils an opportunity to construct geometric concepts by physically creating shapes and noticing their attributes. They were led to develop new insights, since in the process they could create and explore non-prototypical shapes. Hershkovitz (1991) claimed that using only prototypes as examples leads the learner to judge other shapes by comparing it visually to the prototype. Therefore, exploring non-prototypical shapes taught the pupils to judge shapes by their attributes rather than comparing it to the prototype.

In summary, the dynamic, computerized intervention program has given the experimental group a learning opportunity that allowed experimentation and exploration, and was suitable to their developmental stage. As a result, the knowledge gap between the higher-achieving control group and the lower-achieving experimental group was no longer significant. Pupils of all socioeconomic backgrounds should be given suitable learning opportunities.

Regarding the **second research question**, before the intervention took place, there were significant differences between Hebrew-speaking and Arabic-speaking participants. Specifically, Hebrew-speakers achieved better scores on the pre-program test in questions about triangles, squares and rectangles, while Arabic-speakers had an advantage in questions related to reflectional symmetry. This picture is concurrent with achievements on the PISA test, 2015, where Hebrew-speaking pupils showed higher scores in math.

After the pupils underwent the intervention program, there were no significant differences in knowledge about triangles, squares, rectangles and reflectional symmetry between these sectors. Hence, the program gave pupils from all sectors an equal opportunity to acquire geometric concepts and eliminated differences in knowledge between sectors. Regarding the concept of quadrangles, there were no sector differences pre-program and after the intervention program, Arabic-speakers achieved significantly higher in questions about quadrangles. That is, it may be concluded that the level of Geometric knowledge has increased to a greater extent in the Arabic-speaking sector**.**

Regarding the **third research question**, no significant differences were found between the achievements of boys and girls, both on the pre and post-program tests. This finding coincides with results from the 2015 PISA test, yet there are many evidences of higher math anxiety amongst female students, compared to their male counterparts (e.g.: Ramirez et al., 2013; Vukonic et al., 2013), of male students having higher math achievements and of male high school students demonstrating higher geometric thinking levels (Kundu & Ghose, 2016). Thus, early interventions where girls can actively and equally participate and notice that they are just as capable as the boys, are of crucial importance.

In relation to the **fourth research question**, pupils with achievements below average on the pre-program test (below 46% accuracy) were compared with pupils who scored above average (above 52% accuracy). It was found that after the intervention program, the low achievers' scores improved to a greater extent than the high achievers' scores, to the point that there were no longer any significant differences in geometric knowledge between these two groups. It can be concluded that due to the intervention program, the knowledge gap between low and high achievers was closed and all pupils were brought to a similar level of geometric knowledge. The current findings coincide with other studies, which claimed that low-achievers are more affected by intervention programs (e.g.: Fuchs et al., 2016; Smith et al., 2013). The efficiency of intervention programs in different mathematical fields, for different populations, should be further examined.

CONCLUSIONS

The dynamic, computerized intervention program inspected in the current study enabled pupils to operate on three dimensional, not necessarily prototypical shapes, whereas the teacher ceased to be the sole source of knowledge and became a participant in the virtual, visual exploration, developing a discussion in various Van Hiele thinking levels. This program has given the participating pupils suitable learning opportunities and thus eliminated the initial knowledge gap between pupils of different socioeconomic levels, sectors, genders and levels of achievement. Therefore, it is highly recommended for the use of all pupils and educators.

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APPENDIX A

Test Questions

Examples of test questions, which the pupils answered before and after participating in the intervention program, and the thinking levels they require:

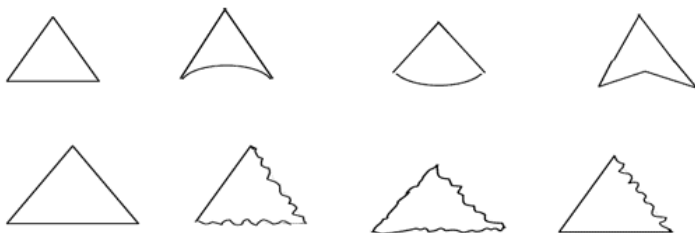"Circle all the squares" (requires the pupil to be in Van Hiele's 1st thinking level).
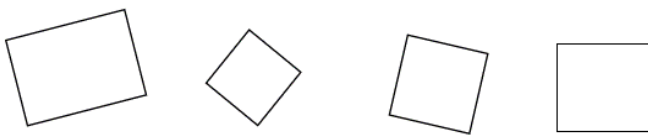
"Circle all the shapes that are created by reflectional symmetry" (requires the pupil to be in Van Hiele's 1st thinking level).
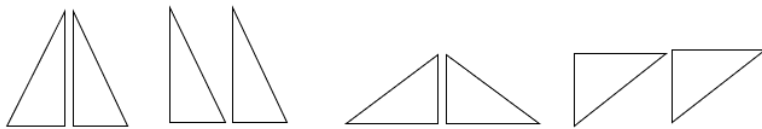

"Circle all the polygons in which there is a right angle" (requires the pupil to be in Van Hiele's 2nd thinking level, since they have to refer to the shape's attributes).

"Circle all the triangles" (requires the pupil to be in Van Hiele's 2nd thinking level, since they have to identify triangles by their attributes).


"A rectangle is a quadrangle which has only right angles. Circle all the rectangles" requires the pupil to be in Van Hiele's 3rd thinking level, since they have to understand that a square is also a rectangle in order to circle all rectangles).
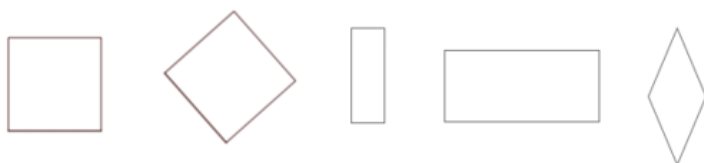
APPENDIX B

An Example of an Origametria Activity

"Fold two edges towards the line of symmetry according to the markings in the animation, to receive a deltoid. Can you count the triangles created by folding the edges?"
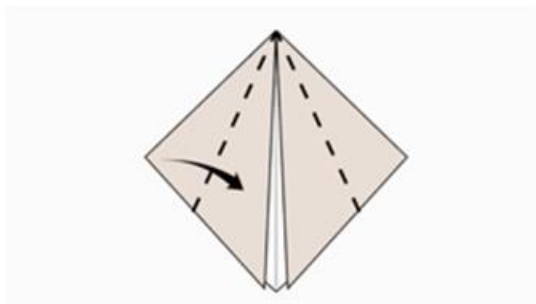
Example of GeoGebra activities:

1. "Here are different kinds of triangles. Move their vertexes and notice the changes in the lengths of the edges.

   Propose an appropriate name for each triangle, based on the lengths of their edges".
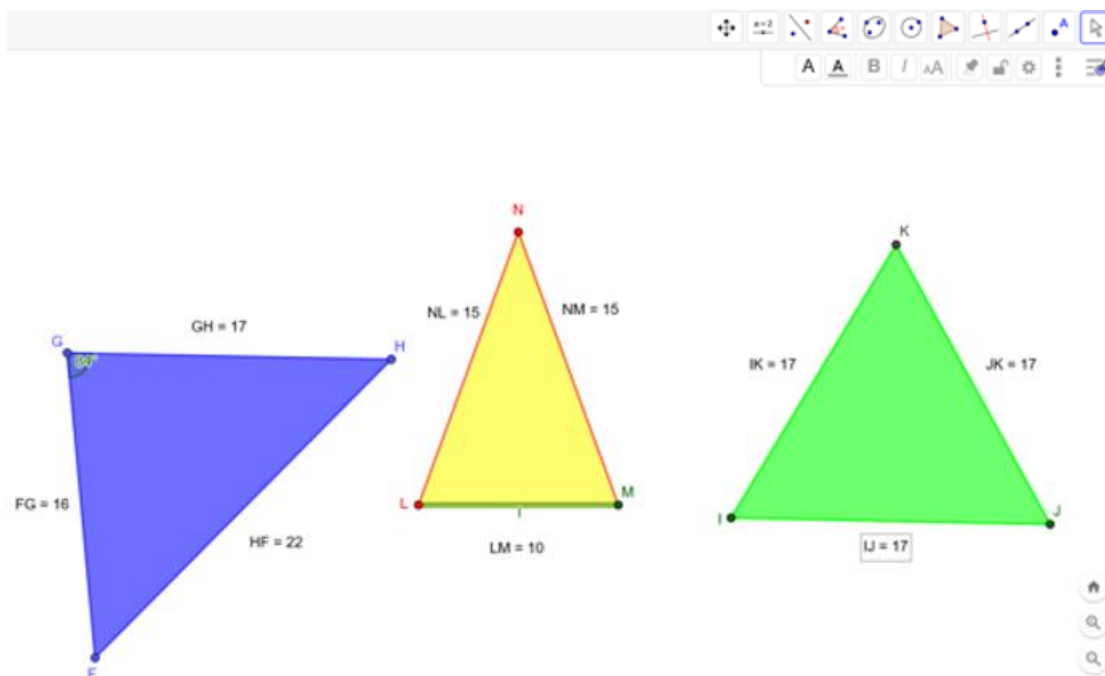
2. Experimenting with the hands of the clock, trying to maintain a right angle between the hands, and also

   reach an understanding that altering the length of the rays does not change the size of the angle.

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