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Pre-Service Mathematics Teachers' Use of Multiple Representations in Technology-Rich Environments

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In this paper, we examine the development of pre-service mathematics teachers' use of multiple representations during teaching in technology-rich environments. The pre-service teachers took part in a preparation program aimed at integration of technology into teaching mathematics. The program was designed on the basis of Technological Pedagogical Content Knowledge (TPCK) framework; and the mathematical content chosen for the program was the concept of derivative. The pre-service teachers' development was scrutinized in terms of their knowledge of representations, of connections established among the representations, and of the aspects of derivative emphasized by these connections. On the basis of our analyses we argue that any attempt to prepare pre-service teachers for effective use of technology in teaching mathematics needs to explicitly focus on the functions of multiple representations in tandem with the mathematical content under consideration. We discuss the educational implications of the study in designing and conducting of the preparation programs related to the successful integration of technology in teaching mathematics.

Keywords: derivative, multiple representations, pre-service teachers, TPCK

INTRODUCTION

The issue of multiple representations (MRs) is an important one in mathematics education and attracted the interest of researchers especially within the last three decades. One reason for the increasing interest is related to the NCTM (National Council of Teachers of Mathematics) Standards (NCTM, 1989) in which use of MRs while teaching mathematics is strongly emphasized:

> *Different representations of problems serve as different lenses through which students interpret the problems and the solutions. If students are to become mathematically powerful, they must be flexible enough to approach situations in a variety of ways and recognize the relationships among different points of view (p.84).*

Here in these lines it is suggested that each individual representation provides students with a point of view through which they can approach to a problem and this

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State of the literature

- Use of multiple representations (MRs) is important as they can potentially create conditions for effective learning and as they lead to deeper levels of understanding of the subject.
- Research on MRs show that unless the links between and among the MRs are stressed, student experience difficulties in connecting the MRs by themselves. However teachers do not explicitly focus on the links in their instructions.
- As the technology has the potential to make these links explicit, we, in this study, focus on pre-service teachers' utilization of MRs in technology rich environments after they took a course designed for the integration of technology into teaching.

Contribution of this paper to the literature

- The study contributed to the extant literature in at least four ways. First of all there was rather limited research on how pre-service teachers make use of technology in addressing MRs and on the ways in which their competence for that matter can be developed. This study contributes to our understanding of these issues.
- Secondly, our participants showed important developments in the use of MRs via technology. This development comes about through a course designed on the basis of Technological Pedagogical Content Knowledge framework (TPCK). The results suggest that TPCK is a useful design tool for that matter.
- Thirdly, integration programs need to be designed in ways that allow participants to eliminate obstacles stemming from the lack of technological pedagogical and technological content knowledge with reference to MRs.
- Finally, in order for future teachers to make an effective use of MRs in their teaching, they themselves need to experience and explore the potentials of technology as a learning resource rather than a computational device.

in turn allows them to become more competent in handling mathematical problems. The research on MRs indicates two important benefits in their use: 1) MRs cater for wider range of students with different learning styles and hence promote conditions for effective learning (Mallet, 2007) and 2) use of MRs leads students into deeper understanding of the subject as each representation emphasizes different aspect of the same concept (Berthold et al., 2009).

The issue of MRs attracted more attention from the Council with the spread of digital technologies in teaching and learning environments. In 2001, for example, the Council's yearbook focused on the roles of representation in school mathematics (Cuoco, 2001). The yearbook attaches considerable importance to the use of digital technologies in making representations available to the students. As NCTM suggests, digital technologies provide visual models or representations that many students are unable to generate through their independent efforts. Zbiek et al. (2007) note that technology can potentially underline the important qualities of individual representations, making it easier for the students to interconnect them and hence achieve a robust understanding.

In an extensive literature review, Ainsworth (1999) examines the representations that educational technologies offer. On the basis of this examination, the author develops a functional taxonomy of MRs. The taxonomy differentiates three main functions that MRs serve in learning situations: to complement, constrain and construct. The three main functions are further divided into several sub-classes (see Figure 1). Ainsworth argues that one single representation could involve more than one function.

The first function Ainsworth cites is that MRs can be used for complementary roles; that is, different representations involve distinct yet complementary information or may support different processes. Combination of MRs with complementary roles is expected to create an environment where learners can benefit from the aggregate of their advantages. Consider, for example, the absolute value function compactly expressed in algebraic form as $y=|x|-5$. This representation affords one to find the value of y for any given value of x, regardless of how large the x is. However, this representation does not show the variation as explicitly as the equivalent graph which also unveils trends and interaction between the values of x and y. Hence these two representations support different processes and carries different yet complementary information from which learners can benefit in understanding the notion of, for instance, absolute value functions.

The second function that Ainsworth points out is that MRs can be employed to constrain interpretations: representations can confine inferences, allowing one to constrain potential (mis)understandings stemming from the use of another one. This can be done either by employing a known representation to construe a less familiar one or by making use of inherent properties of one representation to limit the inferences drawn from a second one. As an example, consider the absolute value functions once again. Students may over-generalize the meaning of absolute value and have a misconception that these functions must take only positive values (as it involves absolute value; see Ozmantar (2005) for more on this) and hence have misinterpretations as to the

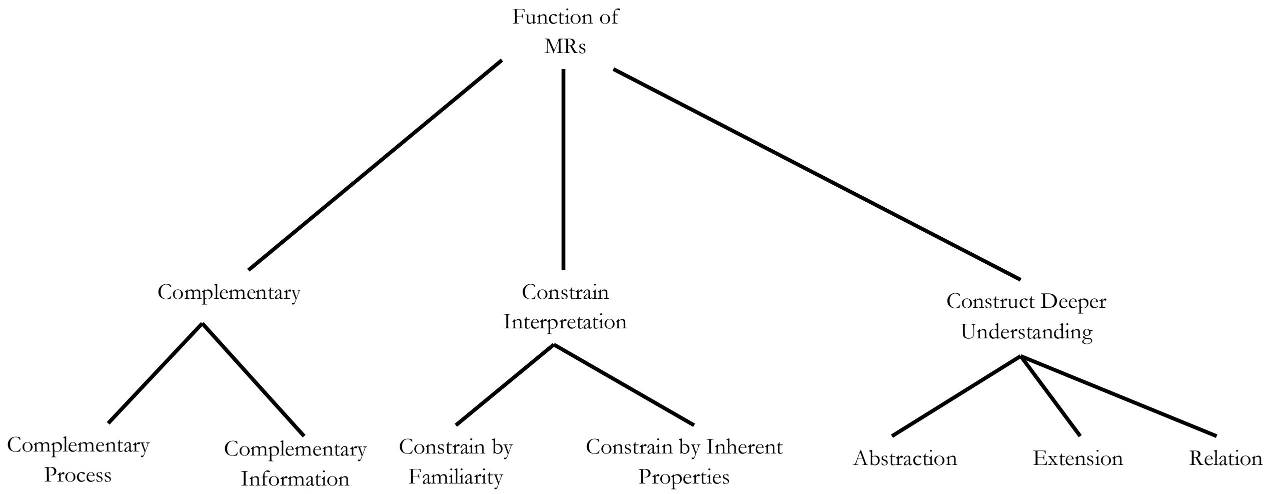

Figure 1. A functional taxonomy of multiple representations (Ainsworth, 1999, p.134).

graphs of such functions. Graphs of absolute value functions such as $f(x)=|x|-1$ can be used to constrain the students' conceptions of the graphical representations of absolute value functions. Hence when the MRs are used for constraining, the purpose is not necessarily provide new information but "to support a learner's reasoning about a less familiar one. It is the learner's familiarity with the constraining representation, or its ease of interpretation, that is essential to its function" (Ainsworth, 1999, p.139).

The third function is that MRs could be used to construct deeper understanding of the concept under consideration. Ainsworth (1999, p.141) cites Kaput (1989) that "the cognitive linking of representations creates a whole that is more than the sum of its parts. ... It enables us to 'see' complex ideas in a new way and apply them more effectively." Ainsworth claims that construction of deeper understanding occurs through abstraction, generalization (or extension) and relations. With regard to abstraction, exposure of MRs is hoped to lead learner to construct references across the representations. This knowledge is then assumed to allow the learner to find out the underlying structure of the concept under investigation. Generalization refers to a learner's extension of his/her knowledge without fundamentally changing the nature of that knowledge. For example, one may know how to interpret increasing or decreasing functions on the basis of their algebraic representations. He/she may later extend this knowledge to the interpretations of such representations as the increasing (or decreasing) graphs or tables of values. Finally, construction of deeper understanding can also occur through teaching the relations among different representations. The pedagogical concern here is not so much with teaching each representation but rather with teaching to translate between two or more representations which are introduced simultaneously.

When the research studies are scrutinized carefully, it is realized that teachers stand out as important factors that make a difference in successful use of MRs in technology rich-environments. Hence teachers' knowledge about the representations, how they use MRs for teaching, and how they make use of technology in addressing the MRs are all important issues to be considered while teaching with MRs through technology. Despite its importance, there does not appear much research on how teachers or pre-service teachers use MRs for teaching in technology-rich environments. We found two studies focusing on this issue (Juersvich et al., 2009; Alagic & Palenz, 2006). In their study, Juersivich et al. (2009) investigated how pre-service teachers utilized the provided technology to generate MRs. They found that pre-service teachers realized the potential of technology to provide MRs that support pupils' sense making in ways that could not be possible under typical conditions. Alagic and Palenz (2006) emphasize that teachers need pedagogical and technological support when integrating technology into teaching and provided technology-based representations in real-life contexts for mathematics teachers as part of a professional development program. They found that teachers learnt how to make connections between MRs using technology. However these studies provide insufficient details as to the way in which pre-service mathematics teachers (PSMTs) employ MRs and of how their competence to effectively use MRs in technology-rich environments can be developed.

With this gap in the literature in mind, our purpose in the paper is to examine the development of PSMTs with regard to the use of MRs during teaching in technology-rich environments. To this end, in the rest of this paper, we first briefly detail the context of our research that aimed to develop a program for PSMTs to integrate technology into teaching. Then we focus on methodology and present data analyses and our findings. The paper ends with a discussion of the issues regarding the effective use of MRs through technology and the educational implications of our findings.

THE RESEARCH AND METHODOLOGY

In this part of the paper, we briefly sketch out the research project that gave rise to this study. To do this, we first attend to the course designed for the pre-service teachers on the basis of "Technological Pedagogical Content Knowledge" (TPCK) framework and second provide the content of the course with regard to MRs.

COURSE DESIGN WITH TPCK FRAMEWORK

This study is part of a research project which aims to develop pre-service mathematics teachers' TPCK (Mishra & Koehler, 2006). For this aim, a course was designed by using TPCK framework which has been recently used to investigate the characteristics of knowledge required by teachers for successful technology integration. TPCK framework was originated from the notion of "Pedagogical Content Knowledge (PCK)" offered by Shulman (1986, 1987). Shulman (1987) drew attention to the significance of "subject matter for teaching" and considered PCK as an important domain of teachers' knowledge. In Shulman's view PCK is an amalgam of content knowledge and pedagogical knowledge. In Shulman's (1987, p.8) view, "pedagogical content knowledge is the category most likely to distinguish the understanding of the content specialist from that of the pedagogue."

The notion of PCK is extensively studied in many domains and guided the efforts to understand the teaching approaches of both in-service and pre-service teachers (e.g., Uşak, 2009; Abd-El-Khalick, 2006).As the importance and potential of technology in teaching and learning is realized, Pierson (2001) has included the

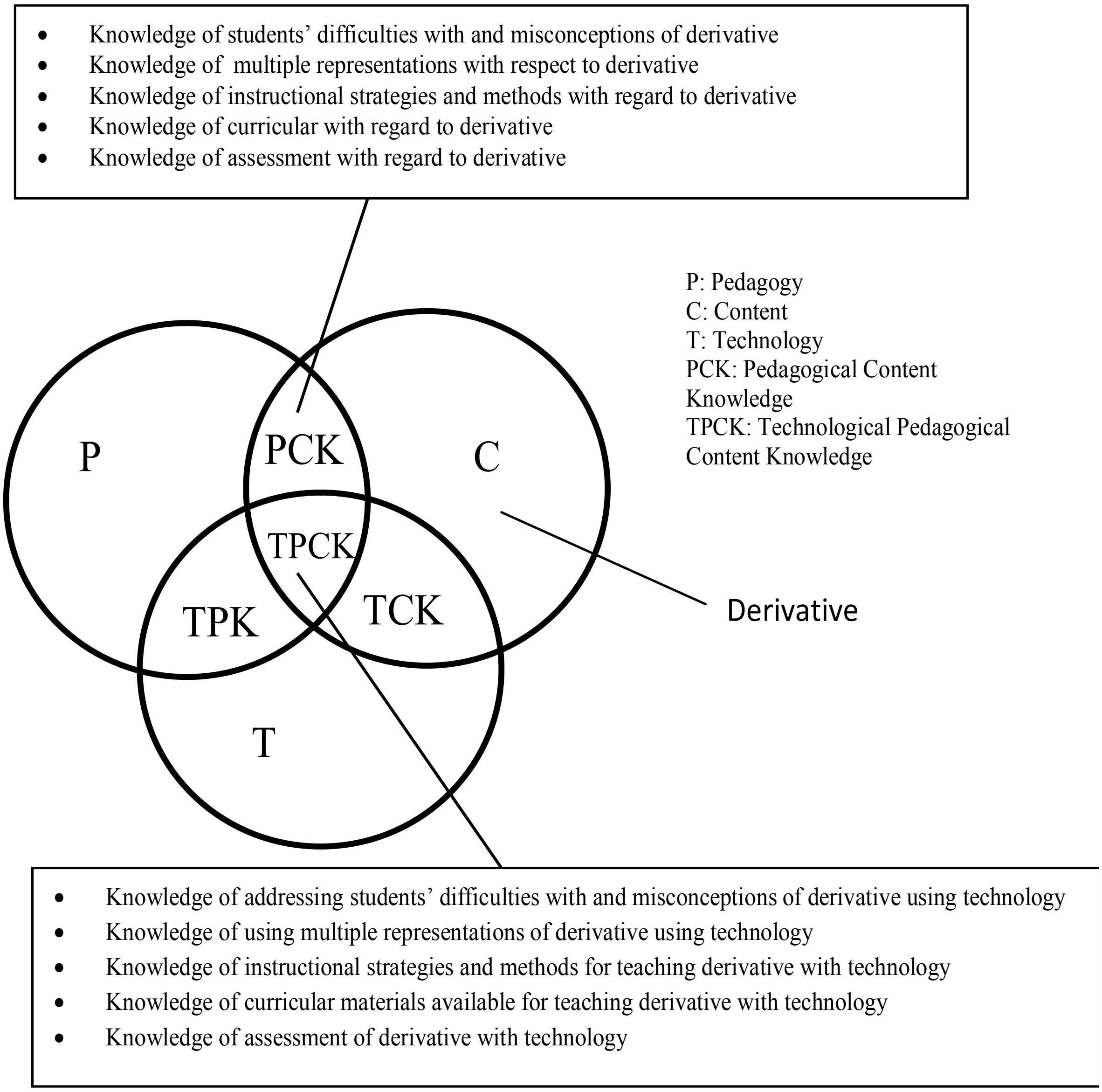

Figure 2. The TPCK framework with different knowledge categories and the components.

technology component into the idea of PCK and considered TPCK as a blend of three categories of knowledge: content, pedagogy and technology. Mishra and Koehler (2006) depict TPCK as an intersection of these three types of knowledge (see Figure 2). The authors also classify where the pairs of different types of knowledge intersects: pedagogical content knowledge (PCK), technological content knowledge (TCK), and technological pedagogical knowledge (TPK). TCK is concerned with the inter-animation between the technology and content; that is, for example, the knowledge of MRs of a concept that a software is able to offer. In this regard, Mishra and Koehler (2006) write "teachers need to know not just the subject matter they teach but also the manner in which the subject matter can be changed by the application of technology" (p. 1028). TPK is "the knowledge of pedagogical strategies and the ability to apply those strategies for use of technologies" (ibid., p. 1028) e.g. knowledge of how to make use of a specific software in establishing the links among the different representations.

We employed TPCK framework to design the course for the PSMTs to successfully integrate the technology in teaching. However, this framework, though provided us with a lens for the design, was lacking in sufficient details with regard to the components that each intersection of the pairs of knowledge categories has. To overcome this problem, we examined PCK components suggested by the relevant literature and used them to determine the components of TPCK. In our examination, we found the components of PCK suggested by Grossman (1989, 1990) and Magnusson et al. (1999) rather useful; these components were:

i.knowledge of instructional strategies and methods for teaching a particular concept
ii.knowledge of representations of a particular concept
iii.knowledge of student misconceptions of the concept
iv.knowledge of purposes for teaching the concept
v.knowledge of curriculum materials available for teaching the concept.

We adapted these components to the TPCK framework for the design of the course. Our endeavor eventually led us to generate the following components of TPCK for the design of our courses:

- *Knowledge of addressing students' difficulties and misconceptions for a particular concept using technology;*
- *Knowledge of using MRs with technology;*
- *Knowledge of instructional strategies and methods for teaching a particular concept using technology;*
- *Knowledge of curricular materials available for teaching a particular concept with technology;*
- *Knowledge of assessment of a particular concept with technology*

We aimed to use TPCK framework with its five components to develop contents for two courses (which we name as Methods for Teaching Mathematics II and Technology-Aided Mathematics Teaching in this paper) as part of a project for PSMTs in Turkey. The course contents were developed by three of the authors collaboratively and they were run by the second author. The aims of these courses were, broadly speaking, to get PSMTs equipped with the skills of teaching mathematics with the aid of technology at secondary level.

We used the five components of PCK to develop contents in five parts. First we amended PCK's five components to generate five corresponding components that we interpret as components of general pedagogical knowledge (PK). The reason we do this was to develop a generic approach and get PSMTs equipped with an overall perspective for any concept in mathematics. Second we brought the content aspect

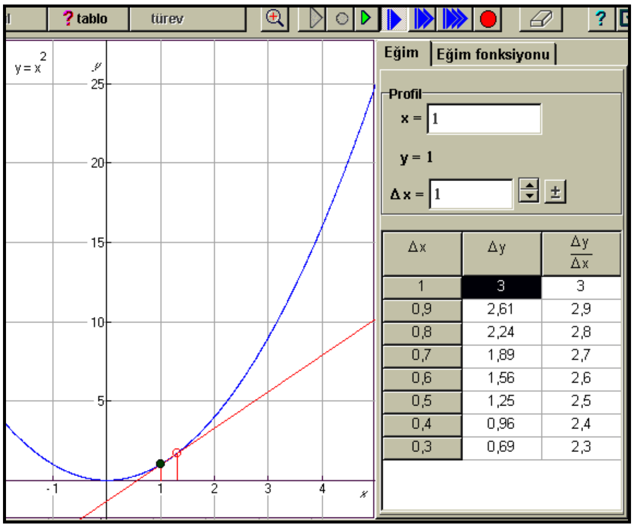

Figure 3. Derivative at a point in Graphic Calculus

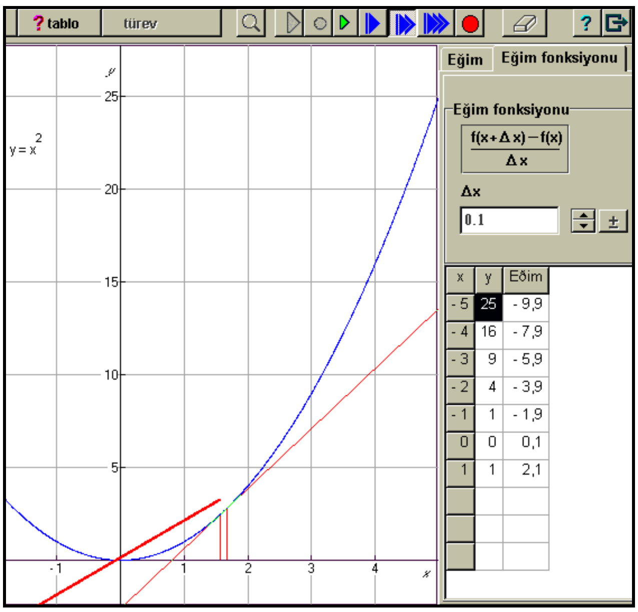

Figure 4. Slope function of derivative

into play and aimed to exemplify how these components can be applied with a particular mathematical concept and we used the concept of derivative for that purpose. Thirdly, we introduced the software (a Turkish version of Graphic Calculus) and planned hands-on activities to explore technological content of the software in general (TK) and technological content of the software for derivative in particular (TCK). Fourth, we amended TPCK's five components to generate five corresponding components that we interpret as components of general technological pedagogical knowledge (TPK). Finally, we brought content aspect into play in the context of technology and aimed to exemplify how these components can be applied with teaching derivative concept using technology.

As our focus in this paper is on MRs, we now turn our attention to "multiple representation component" and first present its content with regard to PCK and TPCK. Before presenting the course content, it will be explicative to mention about how the course content was presented to the PSMTs. During the courses, the instructor made use of PSMTs prior knowledge and asked them questions for discussions using PowerPoint software. Pre-service teachers worked in groups on the discussion points and shared their ideas with the whole class. When technology came into plan, the PSMTs used computers in pairs and used the software in a computer lab. PSMTs were informed about the TPCK framework and objectives of the course which were specified by the course designers for each component of PK, PCK, TCK, TPK and TPCK.

Content for PK with regard to multiple representations

During the course, PSMTs were asked to share their existing knowledge of MRs and provided with the knowledge of algebraic, numerical and graphical representations of mathematical concepts, the relationships among them, and how to take them into account in teaching. Function and limit concepts are used to exemplify the MRs. Limitations and affordances of each representation were also discussed in the contexts of functions and limit.

Content for PCK of derivative with regard to multiple representations

PSMTs were presented with an example of a function and were asked to produce algebraic, numerical and graphical representations of derivative at a given point. They then discussed connections between three aspects of derivative (instantaneous rate of change, the slope of the tangent line to a curve at a particular point and the limit of the difference quotient; see Bingolbali (2008) for more details) and how algebraic, numerical and graphical representations could be linked to relate these aspects of derivative during teaching.

Content for TCK with regard to multiple representations

Technological content introduced to pre-service teachers is a Turkish version of Graphic Calculus software (Blokland, Giessen & Tall, 2006). The software and an activity book in Turkish (Akkoç, 2006) were given to each pre-service teacher. Graphic Calculus software provides graphical and numerical representations of derivative at a point which are dynamically linked as can be seen in Figure 3. As the software calculates the values of rates of change for smaller values of Δx, the secant lines approach to the tangent to the point.

The software also presents slope function by dynamically assigning x values of a function to the slopes of the tangents to the graph of this function at given values of x.

PSMTs were given a worksheet which requires them to evaluate the numerical values of rate of change in the table as seen in Figure 3 and 4. They were also asked to discuss in groups on how the software drew the slope function and the difference between slope function and derivative function.

Content for TPK with regard to multiple representations

Having discovered the technological content of the software with regard to derivative concept, PSMTs discussed TPK with regard to MRs. They were asked to discuss the affordances and limitations of the software

Table 1. The average rate of change of $f(x)=x^2+1$ in the neighborhood of $x=3$

x	2.5	2.6	2.7	2.8	2.9	3.1	3.2	3.3	3.4	3.5
Δx	-0.5	-0.4	-0.3	-0.2	-0.1	0.1	0.2	0.3	0.4	0.5
Δx	-2.75	-2.24	-1.71	-1.16	-0.59	0.61	1.24	1.89	2.56	3.25
$\frac{\Delta y}{\Delta x}=\frac{f(x)-f(3)}{x-3}$	5.5	5.6	5.7	5.8	5.9	6.1	6.2	6.3	6.4	6.5

in terms of MRs. They were asked to draw $y=\sin x$, $y=\sin 2x$, $y=\sin(x/2)$, $y=2\sin x$ and $y=(\sin x)/2$ using the software and discuss how technology provides opportunities to make links between representations. The notion of periods of trigonometric functions was introduced to PSMTs and a discussion started on how it differs from paper and pencil techniques to introduce periods of trigonometric functions. TPK content ends with a discussion on the affordances and limitations of the software with regard to the use of MRs with the availability of technological tools.

Content for TPCK with regard to multiple representations

Following the content for TPK with regard to MRs, TPCK of MRs of derivative at a point was introduced. Two main activities as shown in Figure 3 and Figure 4 were re-examined and the following questions were discussed:

- *In each activity, what kinds of representations of derivative at a point are available and what kinds of opportunities are there to make links between representations?*
- *What kinds of opportunities are there to relate three aspects of derivative (derivative-rate of change, derivative-slope and derivative-limit) using the software?*

In addition to these discussion points, PSMTs' attention was drawn to the limitations of the software in terms of rounded values of rate of change in the table produced by the software (For smaller Δx values, the software rounds up the values of rate of change; e.g. 2.000001 is rounded up to 2). PSMTs were asked to discuss how this limitation could be avoided or potentially used to promote student learning.

After PCK and TPCK contents were given as described above, micro-teaching videos of derivative lessons which were performed by pre-service teachers a year ago were watched and discussed in terms of how MRs were used.

The participants

A cohort of 40 PSMTs participated in the research. The PSMTs initially took a three-and-half years mathematics program at the university and then enrolled the secondary mathematics teacher preparation program. The graduates of the program will teach mathematics at secondary level and have a strong mathematical background. The preparation program for the PSMTs involves such courses as Teaching Methods, Educational Psychology and Assessment. The data for this study collected during the courses "Methods for Teaching Mathematics II" and "Technology-Aided Mathematics Teaching." All forty PSMTs enrolled these courses which were designed on the basis of TPCK framework as explained hitherto. The content of PCK with five components was delivered to the PSMTs during workshops. Then, ten PSMTs did microteachings before their peers who took observation notes on the microteachings. Following this, the TPCK content was delivered to the PSMTs with five components each of which was introduced in separate workshops. For the TPCK workshops the Graphic Calculus software was also demonstrated to the PSMTs who were allowed to work independently for the purpose of exploration. Ten PSMTs did micro teachings once again but this time they employed Graphic Calculus software to introduce the concept of derivative.

DATA COLLECTION

During the research, several data collection tools were employed, including: diagnostic test on derivative, lesson plans, detailed teaching notes used during microteachings, video records of micro-teachings, interviews and questionnaires.

Diagnostic test on derivative: Despite the fact that PSMTs spent three-and-half years in pure mathematics courses, it was important to see their conceptual understandings of the notion of derivative. Hence, a test with several items, which aimed to find out PSMTs' understandings and concept images (Tall and Vinner, 1981) of derivative, was applied to all participants.

Lesson plans: The participants were asked to prepare three lesson plans: one before the course started, one after the PCK workshops and the last was after the TCPK workshops. All three lesson plans were on the same topic: introduction of derivative. The lesson plan format had sections on objectives,

Table 2. PSMTs' responses for the meaning of MRs.

	Number of PSMTs (N=40) (Before the course starts)	**Number of PSMTs (N=40)** (After the TPCK workshops)
Graphical representation	2	36
Tabular (numerical) representation	1	38
Algebraic representation	1	37
Different representations of a concept	7	17
Use of different symbols for the expression of a concept	8	0
Unanswered	15	0
Others	6	9

prerequisite knowledge, materials used, classroom organization, outline of teacher and student activities, and assessment during and after the lesson. To prepare the plans, the PSMTs were allowed to make use of any textbook they wish and also required to examine the curriculum scripts.

Detailed teaching notes: Those who were to do microteachings were asked to prepare detailed teaching notes which were later retained by the research team. These notes included PSMTs' suggestions and personal reminders of particular issues to attend to during their teachings.

Video records of micro teachings: Ten PSMTs were asked to do microteachings before their peers in two occasions. The first microteachings were performed just after the completion of the PCK workshops and the second one took place following the TPCK workshops. After the microteaching sessions, the performing teacher candidates made reflections on their approaches and performances with regard to the components of PCK (or TPCK) and also had a chance to hear the reactions and/or observations of their peers.

Interviews: The research team conducted semi-structured interviews with those doing the microteachings. The interviews took place before and after each microteaching sessions. The PSMTs were asked a series of questions regarding their preparations and the lesson plans. After the microteachings, they were interviewed about their performance. Both of the interviews were shaped on the basis of the components of PCK (or TPCK).

Questionnaires: The open-ended questionnaires were also used to find out the PSMTs' initial understandings of the components of PCK in a general manner (e.g., student difficulties, MRs, instructional strategies, assessment and curriculum). The questionnaire was applied twice, at the start and end of the course.

DATA ANALYSIS AND FINDINGS

Analyses of the data will be presented in two sections. In the first one, we present our quantitative analyses on the basis of mainly lesson plans with regard to the use of MRs. In the second one, we present a case study in which microteaching of a pre-service teacher is examined to give the reader a better appreciation of the nature of PSMTs' development. Before going into data analyses, however, we feel it useful to briefly mention about the MRs of derivative which we focused during the course and employed in our analyses.

Multiple representations of derivative

The concept of derivative at a point can be defined in three main ways: (i) the slope of the tangent line to a curve at a particular point, (ii) the limit of the difference quotient and (iii) the instantaneous rate of change (Bingolbali, 2008). Of these three, the limit of the difference quotient aspect lies at heart of the derivative interpretation and helps to understand the other two aspects. It is through the limit of the difference quotient aspect that we can actually make sense of the slopes of secant lines approaching to the slope of tangent line. Similarly, it is through the limit of the difference quotient aspect that we can make sense of average rate of changes approaching to the instantaneous rate of change at a particular point. A conceptual understanding of the derivative and its teaching for such an understanding, therefore, require an understanding of how all these aspects are related and presented in a connected manner in its teaching.

Among many other things, the successful use of such MRs as algebraic, graphical and tabular (numerical) is a way to make links between and among the aspects/interpretations of derivative. The uses of these different representations are considered to pave the way for the conceptual understanding of the derivative in its teaching (Amoah & Laridon, 2004). The uses of graphical and numerical representations are particularly emphasized alongside the common use of algebraic representation (ibid.). Below the function of $f(x)=x^2+1$ and its derivative at a particular point are presented to illustrate how these representations can be used to make sense of the derivative concept (Akkoç, 2008).

The derivative of the function of $f(x)=x^2+1$ is equal to $f\ '(x)=2x$. This derived function can be obtained through the fundamental rule of differentiation or through the limit of difference quotient. After finding out the derived function, the derivative (slope, instantaneous rate of change) at a particular point can be calculated. The derivative of the given function, for instance, at $x=3$ is equal to 6. Calculating the derivative in this manner is an example of finding it out algebraically.

The derivative of the function of $f(x)=x^2+1$at $x=3$ can also be represented numerically as shown in Table 1. From both sides of $x=3$, the average rates of changes approach to 6 as the width of the intervals goes to zero. Note that the value 6 is the derivative of the function at the point $x=3$.

In addition to its algebraic and numerical representations, the derivative of the function of $f(x)=x^2+1$ at $x=3$ can be represented graphically as well. It can graphically be seen that the slopes of secant lines from both sides of $x=3$ approach to the value 6, which is the slope of tangent line to the curve at $x=3$.

What have been presented so far suggests that the three different aspects of derivative can be better appreciated through the use of three MRs.

QUANTITATIVE ANALYSES OF THE DATA

In this section, we provide the results of our analyses of the questionnaires and lesson plans. The first thing that we desired to find out was PSMTs' understanding of MRs. The data for the PSMTs' understanding of MRs came from general pedagogy questionnaire in which PSMTs were asked what the multiple representation of a mathematical concept is. The PSMTs' responses to this item generated following categories as presented in Table 2.

As can be seen in Table 2, PSMTs gave examples of MRs such as "graphical", "tabular (numerical)" and "algebraic" to explain what MRs meant as the first three categories of responses. However, very few PSMTs gave these examples before the course. The number of PSMTs who gave these examples increases considerably after they took TPCK course. One interesting result is concerned with PSMTs' misunderstandings about MRs. Before the course, eight PSMTs considered "symbols" used in mathematics as MRs. On the other hand, none of them had this misunderstanding after TPCK course. Overall, these results indicate that PSMTs' understanding of what MRs meant had dramatically improved.

After the analyses of the PSMTs' responses to the questionnaire item, we turn our attention to their lesson plans and analyze initial two plans: one before the course starts and one after the PCK workshops. Analyses of the lesson plans before the course suggest that 12 PSMTs used only one representation of derivative and the rest used at least two representations. This figure changes after the PCK workshops in that 38 of the PSMTs used two or more representations while introducing derivative. It was interesting to observe that although many PSMTs did not show a solid understanding of the issue of MRs in the questionnaire item, 28 of them were able to use more than one representation in their lesson plans. One reason for this, we believe, is related to the fact that PSMTs examined the curriculum scripts and textbooks, which guided their preparation of lesson plans.

We then decided to examine lesson plans to figure out if the PSMTs make connections between the MRs

Table 3. Frequency analysis of the links established among different representations

	First lesson plans		Second lesson plans	
Categories	**N**	**%**	**N**	**%**
MRs are not linked	34	85.0	9	22.5
One pair linked	3	7.5	5	12.5
Two pairs linked	1	2.5	8	20.0
Three pairs linked	0	0	2	5.0
All three MRs are interconnected	0	0	12	30.0
No response	2	5.0	4	10.0
Total	40	100.0	40	100.0

Table 4. Frequency analysis of the aspects of derivative addressed in the lesson plans

	First lesson plans		Second lesson plans	
Categories	N	%	N	%
None	28	70.0	6	15.0
Only one aspect	8	20.0	6	15.0
Two aspects	1	2.5	6	15.0
All three aspects	0	0	2	5.0
The three aspects are interconnected	0	0	19	47.5
Unanswered	3	7.5	1	2.5
Total	40	100.0	40	100.0

Table 5. The frequency analysis of the use of technology for the MRs of derivative

Categories	N	%
None	1	2.5
Only one pair	5	12.5
Only two pairs	1	2.5
All three pairs	2	5.0
Three representations are interconnected	27	67.5
Unanswered	4	10.0
Total	40	100.0

used in the lesson plans. Considering that we focus on three common representations of derivative (algebraic, numeric or tabular, and graphical), in our analyses we focused on the categories of possible connections among the MRs of derivative as follows.

- *MRs of derivative (Graphical (G), Numerical (N) and Algebraic (A)) are not linked.*
- *Only one pair of representations are linked (any one of G-N, G-A, or N-A)*
- *Only two pairs of representations are linked (any two of G-N, G-A, and N-A)*
- *Three pairs of representations are linked (pairs of G-N, G-A, and N-A are all present)*
- *All three representations are interconnected to one another (the pairs of G-N, G-A, and N-A as well as G-N-A are present)*

On the basis of these categories, we analyzed the first and second lesson plans and our analyses are presented in Table 3. PSMTs' initial lesson plans prepared before the PCK workshops reveal that 85% of the participants did not link the MRs of derivative used in the lesson plans. However, this figure drops to 22.5% after the PCK workshops. It is also remarkable that among the first plans, there was not a single one that attempted to establish links among all three representations; yet after the PCK workshops there were 12 (30%) plans which explicitly linked all three MRs of derivative. A holistic evaluation of the figures clearly shows that PSMTs gained an awareness of the necessity of establishing links among the MRs of derivative and made an effort to reflect this awareness into their lesson plans.

As mentioned before, there are close relationships between the use of MRs and the different aspects of derivative (the slope of the tangent line to a curve at a particular point, the limit of the difference quotient and the instantaneous rate of change). A true understanding of this concept requires a holistic view of these aspects and a grasp of the relevance of one to the others. Hence we assume that an effective teaching needs to make use of MRs in relating the aspects of derivative to one another. With this assumption in mind, we also analyzed the lesson plans to find out if the PSMTs used MRs to demonstrate the connections between the different aspects of derivative. To this end, we created categories for the aspects of derivative attended to in the lesson plans and carried out our analyses accrodingly. These categories were as follows.

- *None of the aspects of the derivative is introduced.*
- *Only one aspect is addressed: any one of Derivative-Rate of change, Derivative-Limit or Derivative-Slope is focused.*
- *Two aspects are addressed: any two of Derivative-Rate of change, Derivative-Limit and Derivative-Slope relationships are emphasized.*
- *Three aspects are addressed: all the three are emphasized but are not interrelated.*
- *The three aspects are both addressed and interconnected: the three aspects of derivative are related to one another (Derivative-Rate of change-Limit-Slope).*

Our analyses based on these categories show (see Table 4) that while in the initial lesson plans, 70% of the PSMTs did not mention about any aspects of derivative, this figure drops to 15% after the PCK workshops. There is a dramatic increase in the number of those who addressed the three aspects and related them to each other through MRs of derivative: almost half of the second lesson plans (47.5%) did so; compare this with the first plans in which there was none. Generally speaking, in the first lesson plans the aspects of derivative was ignored while in the second plans, except for the 7 PSMTs, at least one aspect was taken into consideration. These figures clearly show that the PSMTs has certainly gained an awareness as to the importance of different aspects of derivative and also gained insights into these aspects. They hence made an effort to prepare the second plans accordingly and reflected their understandings into their approaches.

So far, PSMTs' lesson plans have been analyzed in terms of MRs and different aspects of derivative. We now turn our attention to the use of technology in connecting the MRs and the different aspects of derivative. For this purpose, we analyzed the third lesson plans which were produced after the TPCK workshops. Our initial analyses suggested that 87.5% of the participants used at least two representations of derivative with the help of technology. We further analyzed the lesson plans with a greater detail to see if technology was used to link the MRs of derivative. For this purpose we created the following categories.

- *None: technology was used to link none of the MRs with one other.*
- *Only one pair: any one of the G-N, G-A, or N-A connection was planned with technology.*
- *Only two pairs: any two of G-N, G-A, or N-A connection was planned with technology.*
- *Three pairs: All three pairs of G-N, G-A, and N-A are linked with technology.*
- *Three representations, G-N-A, are interconnected to one another with technology*

The analyses of the third lesson plans along with these categories yield the results as presented in Table 5. As can be seen , a great majority of the PSMTs (67.5%) made use of technology for the purpose of interconnection of MRs. This figure is rather important for PSMTs not only employed the MRs of derivative and made explicit links between and among the MRs but they also integrated technology into their teaching plans and drew on it to establish the links. The importance of this figure becomes even more evident when we consider their first plans where a large number of PSMTs (85%, see Table 3) did not link the representations, let alone doing the links with the help of technology. Hence this analysis indicates PSMTs' developing competence and awareness of using technology.

In order to gain further insights into the development of PSMTs' utilization of technology, we also carried out analyses of the third lesson plans as to how they handled the three aspects of derivative in the lesson plans. With this regard, we sought to set two main issues: 1) if they included any aspect of derivative (slope, limit of the difference quotient and the instantaneous rate of change) in the lesson plans and 2) whether they employed the technology when addressing any of the aspects. Our analyses are presented below as the frequency of the presence of any one of the aspects of derivative and of those who planned to use technology.

As seen in the Table 6 , 75% of the PSMTs aim to establish interconnections among the three aspects of derivative and they planned to do so with the help of technology. This is remarkable in the sense that many of the PSMTs, at the start of the course, were not aware of these aspects; yet, after the TPCK workshops they surely developed insights not only into these aspects and the relations among them but also into the benefits and affordances of technology while making the relations among them explicit. Hence the analyses of PSMTs' third lesson plans provide evidence as to their development for the integration of technology in making connections among the three aspects with the help of MRs of derivative.

Observing the PSMTs' development is important. Yet equally important is the nature of this development. In order to provide the reader with an opportunity to see the nature of PSMTs' developing competences with regard to use of MRs of derivative in technology-rich environments we now present micro-teaching of a pre-service teacher.

QUALITATIVE ANALYSES OF A PSMT'S MICROTEACHING

In this part of the paper, we present the analyses of a PSMT's microteaching. The aim here is to demonstrate the use of MRs with regard to three aspects of derivative through technology. The pre-service teacher (called Arzu) is a female teacher candidate and did microteaching after the PCK workshops. She re-prepared the lesson plan that was produced after the TPCK workshops. In this microteaching she employed technology to introduce the concept of derivative. The analyses of her teaching will be presented in four sections. First we briefly describe her approach to introducing derivative; second detail how she employed

Table 6. The frequency of those addressing the aspects of derivative and using technology for this

Categories	Yes N(%)	No N(%)	Unanswered N(%)	Total N(%)
Do PSMT address the link between the rate of change and slope?	32 (%80)	4 (%10)	4 (%10)	40 (%100)
Do PSMT make use of technology in addressing the link between the rate of change and slope?	32 (%80)	4 (%10)	4 (%10)	40 (%100)
Do PSMT address the link between the limiting process and slope?	34 (%85)	2 (%5)	4 (%10)	40 (%100)
Do PSMT make use of technology in addressing the link between the limiting process and slope?	34 (%85)	2 (%5)	4 (%10)	40 (%100)
Do PSMT address the link between the rate of change and limiting process?	32 (%80)	4 (%10)	4 (%10)	40 (%100)
Do PSMT make use of technology in addressing the link between the rate of change and limiting process?	32 (%80)	4 (%10)	4 (%10)	40 (%100)
Do PSMT address the links among the rate of change, limiting process and slope?	30 (%75)	6 (%15)	4 (%10)	40 (%100)
Do PSMT make use of technology in addressing the links among the rate of change, limiting process and slope?	30 (%75)	6 (%15)	4 (%10)	40 (%100)

Table 7. The tabular representation that Arzu produced

$[t_1, t_2]$	[3, 5]	[4, 5]	[4.5, 5]	[4.9, 5]	[5, 5.1]
V_{average}	11	12	12.5	12.9	13.1

the MRs. Third we consider the aspects of derivative on which she focused and finally we present the way in which MRs are used to interconnect the aspects of derivative.

An overview of Arzu's approach to introduction of derivative

Arzu started her teaching with a problem which involved the calculation of average velocity by using the given distance equation which was the function of time. The distance function was $X(t)=t^2+3t$ and average velocity was calculated in the intervals of [1,5] and [2,5]. In her solution, she emphasized that average velocity can be calculated via $\Delta x/\Delta t$. Later she pointed out that if an interval is given, then it is possible to find the average velocity as there can be changes both in distance and time. She asked her peers about the possibility of finding the velocity at a particular point, when $t=5$. To

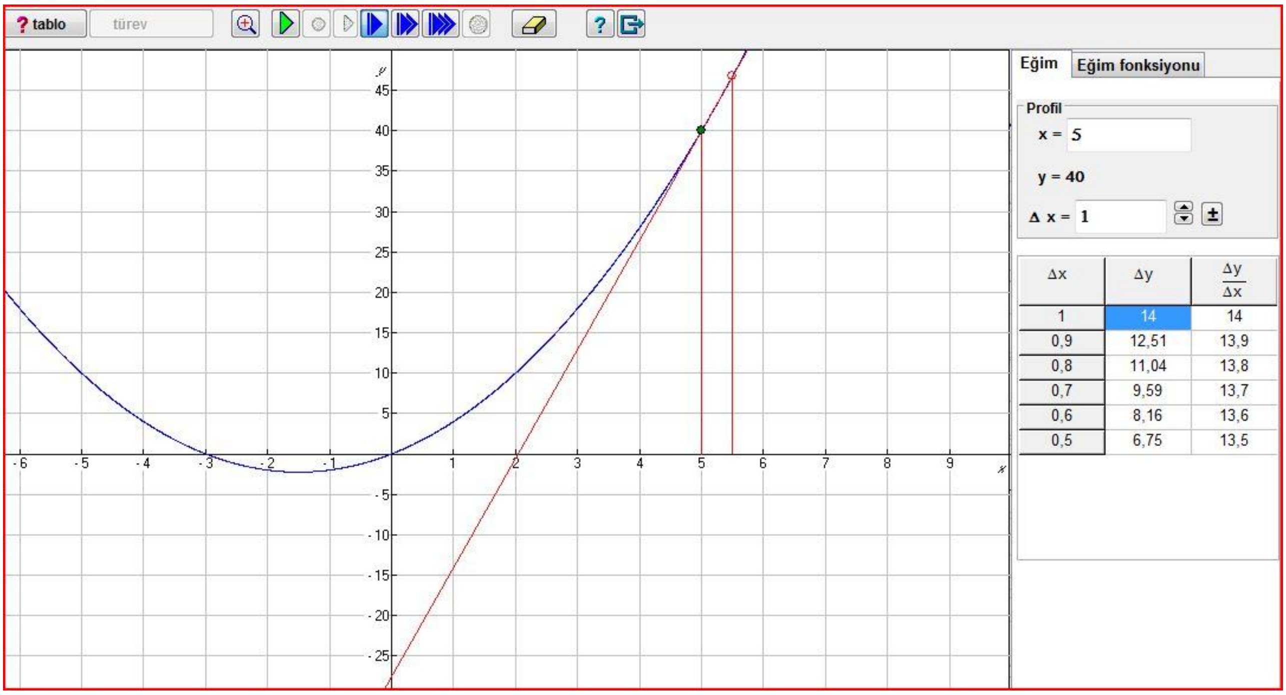

Figure 5. Graphic Calculus window for the function x^2+3x with graphical and numerical representations

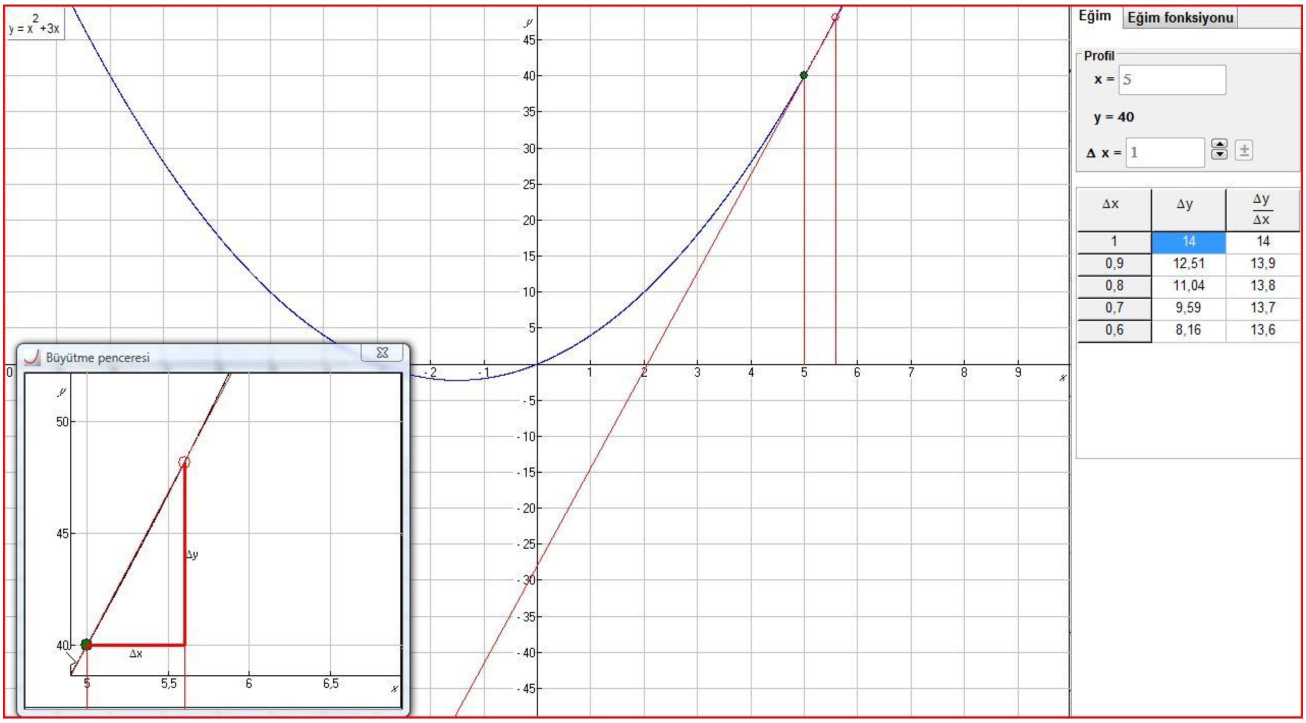

Figure 6. Rate of change and the slope of the secant lines

find out this, she suggested producing a table of values by evaluating average velocity over various intervals in the neighborhood of 5. She produced table 7.

She, based on the graph, commented that as the intervals were getting smaller in the neighborhood of 5, average velocity seemed to be 13; she also noted that with their current knowledge, they could not tell the exact result. She then reminded the concept of limit and noted that if the changes in times in the neighborhood of 5 approach to zero (Δt→0), then it might be possible to find the exact result. She wrote the algebraic representation of limit as $\lim_{\Delta t \to 0} \dfrac{x(t+\Delta t)-x(t)}{t+\Delta t - t}$ and calculated this for t=5 (which yielded the result of 13 for the velocity at t=5).

Following this Arzu got the Graphic Calculus software started. She entered the function and obtained the graph and tabular values for the rate of change (see Figure 5). She then explained how the values for $\Delta y/\Delta x$ were produced and had some of her peers calculate certain values to make sure that the outcomes in the tabular representation were understood.

Arzu later focused on the graphical representation and pointed out that the rate of change (i.e. $\Delta y/\Delta x$) was related to the slope of the secant lines and showed this with the "Zoom-in" tool of the software (see Figure 6). Then she introduced the formal limit definition of derivative and wrote it down on the board.

Having given the formal limit definition, Arzu turned to the software and this time used it to obtain the slope function of x^2+3x. The software dynamically demonstrates the values of slope function in relation to the values of rates of change and represents them graphically (see Figure 7). Arzu explained how the slope function was obtained and related this to the values of rate of changes. Then she articulated that the derivative of the function of x^2+3x was represented by the slope function. She concluded her teaching by explaining the procedure to algebraically obtain the derivative of a polynomial function.

Arzu's use of MRs of derivative

When Arzu' teaching is examined with regard to MRs, it can be said that she employed tabular (or numeric), graphical and algebraic representations during her teaching. Arzu started her teaching by calculating the average velocity in two intervals. Later she raised the issue of finding the velocity of moving object at a particular point in time. As this was the introduction of derivative, she did not use the rules of differentiation to answer this. Hence what she did was to create a table of values for the average velocity in different intervals at the neighborhood of t=5. The tabular representation of the values (see Table 7) certainly helpful in predicting the velocity of the object at t=5. As Ainsworth (1999, p.135) suggests tables tend to "support quicker and more accurate readoff and highlight patterns and regularities across cases or sets of values." In this sense, the function of using table representation here can be considered as constraining and complementing. The values of average velocity in different intervals were computed separately and assembling these values in the table does not convey new information on the part of learner. However, such an assembly and reorganization of the values constrain the learner by giving them a focus and hence directing their attention to the target value of velocity at a particular point in time.

Arzu also used representations for the complementary purposes. For example, having reached to the graph of x^2+3x through the software, she concentrated on both tabular and graphical representations (see Figure 5), each of which was used to explain one another. As seen in Figure 6, table of values as rates of change are complemented by the graphical representation which in turn was used to explain the production of the table of values. To achieve this, Arzu even used the Zoom-in button (see Figure 6) to direct the attention to, and indeed to elucidate, the link between the tabular and graphical representations. Surely each representation carries unique as well as shared information with regard to the meaning of derivative. For instance, graphical representation contains the information that derivative is the limit of the secant lines while the tabular representation relates the slope of the tangent to the rate of change. Hence the advantage of using these two representations was greater than the sum of the parts.

Finally Arzu employed representations to construct deeper understanding of derivative. This we believe is rather valuable and in practice it occurs rarely. Yet Arzu was quite successful in her attempt. She pronounced the derivative while interconnecting the graphical, tabular and algebraic representations (see Figure 6). Having considered the slopes of the secant lines and related them to the tabular representations, she turned her attention to the slope function which Graphic Calculus software produced. The graph of slope function is obtained by matching the values of x with that of y generated via $\dfrac{f(x+\Delta x)-f(x)}{\Delta x}$ (see Figure 7) The tabular representation of the slope function along with the graphical and algebraic one is present on the same window, which gives the teacher a chance to merge them all into a single picture and hence connect them under the concept of derivative. This was the way that Arzu first pronounced the concept of derivative by interrelating the MRs to construct a new understanding, a new concept, derivative.

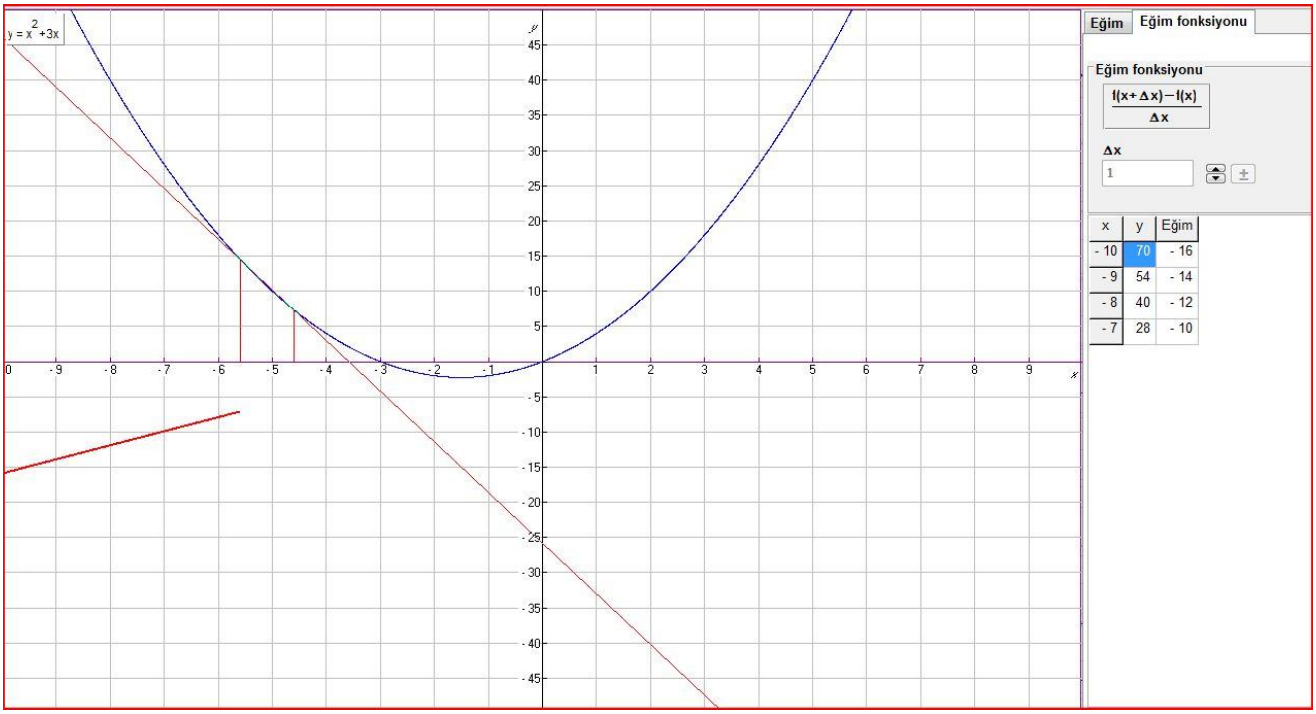

Figure 7. Graphic Calculus shows dynamically occurrence of the slope function

Three aspects of derivative in Arzu's teaching

Arzu incorporated all three aspects of derivative in her teaching. She started her teaching with a problem of average velocity of a moving object. To handle the problem, she made use of rates of change and then problematized the possibility of determining the instantaneous rate of change. To determine the velocity of the object at a particular point in time, Arzu brought the concept of limit into attention. It was through the limiting process that she calculated the instantaneous velocity. Later she used technology to introduce the slopes of secant lines approaching to the slope of tangent line to the graph at the point of 5. She then related the slope of secant lines to the rate of change aspect of derivative. Finally she considered the slope function to emphasize that the derivative of a function is obtained by matching values of x with the instantaneous rate of change at that particular points of x. In doing so, she was not only emphasizing three aspects but also trying to clarify the interrelationships among these aspects of derivative.

The MRs and three aspects of derivative

The first step that Arzu took towards the introduction of derivative was the concept of rate of change, which she brought into consideration through the problem of a moving object. To solve the problem, she employed the algebraic representation which, unlike the other representations, allows the symbolic manipulation and easy and fast calculation with regard to this problem. She then mentioned about the instantaneous rate of change and set off to find out the velocity of a moving object at a particular point in time. To this end, she employed algebraic representation for the calculations but preferred tabular representation to get students sensing the value of velocity. Arzu used tabular representation for the purposes of both constraining and complementing. Tabular representation played constraining role in that it highlighted the regularities across the set of matching values (see Table 7). Tabular representation, unlike the algebraic one, has the potential to evoke the concept of limit and in this sense it also played a complementary role. Following this, Arzu drew on the limit concept and calculated the instantaneous velocity.

She later brought the technology into her teaching. She sketched the graph of x^2+3x with Graphic Calculus software. On the same window, one could see graphical, tabular and algebraic representations. The first thing that Arzu did was to explain and exemplify the relationships between and among these representations. She explained how the tabular values were obtained from the algebraic representations. She also focused on the relationship between the tabular values and the secant lines on the graph and explained how one representation was related to the other. Arzu noted that each value of Δx on the table was used to create a secant line on the graph and when the Δx approached to zero then the secant lines approached to the tangent line at the point under consideration. Based on this, she introduced the limit (formal) definition of derivative and turned the Graphic Calculus to explain this with the help of slope function. In her explanation, she related the tabular representation to the limit of secant lines.

As this brief analysis suggests, Arzu used MRs of derivative for the purpose of constructing: the meaning of derivative by combining the three aspects together into a single picture.

DISCUSSION AND EDUCATIONAL IMPLICATIONS

It is clear from the data that PSMTs taking part in our study has shown great developments in their knowledge of MRs, in their utilization of MRs for teaching, in devising ways to connect the MRs for teaching and in making use of technology for the MRs. First of all, the PSMTs, before the course started, did not have much knowledge about the MRs in mathematics. This is evident in their responses to questionnaire items as 75% of the PSMTs (Table 2) were not able to explain the meaning of MRs. Of course this does not mean that the PSMTs did not have any idea of the MRs. Surely they used different representations in their first lesson plans prepared before the course but their knowledge of MRs was dispersed nor did they appreciate the importance of MRs for teaching and learning mathematics. The way that they used the MRs was lacking in the sense of purpose; that is, they did not relate the representations to one another in the context of derivative. We believe that the PSMTs' use of MRs in the first lesson plans was largely shaped by the textbooks and the curriculum scripts that they were advised to examine for their lesson plans. As we reported elsewhere, the PSMTs tend to follow the approach adopted in the source that they examined (see Ozmantar et al., 2009). Hence their use of MRs in the first lesson plans was as if the MRs employed to introduce derivative were "poured into" their plans without having much thought as to the purpose that those MRs of derivative served. However after the completion of the course the PSMTs were able to explain the meaning of MRs; further to this, they incorporated MRs into their lesson plans purposefully.

The PSMTs' initial deficiency was reflected in their utilization of the MRs for teaching derivative as evidenced in their first lesson plans. It is true that they employed MRs of derivative in the first lesson plans; however, the plans were not structured in such a way that links the representations with the aspects of derivative. However, a change in their approach to utilize MRs is evident in their second lesson plans where the PSMTs' efforts to connect the MRs were all too apparent.

The PSMTs show progresses in the use of technology with regard to the MRs. Arzu's case is exemplary for that matter. In her teaching, she was competent in the utilization of MRs for complementing, constraining and constructing purposes. She was also able to combine different representations to emphasize different aspects of derivative. Arzu's integration of technology for bringing the representations together in relating the aspects of derivative to one another was rather successful. However, Arzu was not alone; as the quantitative analyses show 75% of the PSMTs made use of technology in connecting more than one pairs of the MRs (Table 6). Further to this, 75% of them related the aspects of derivative to one another via MRs with the help of technology.

The question of interest at this point is: why is the cited development of the PSMTs so important? The development of the PSMTs in our study with regard to the use of MRs in technology-rich environment is important for at least four reasons. First of all, one important justification for the use of MRs is that MRs of mathematical concepts provide unique potentials to construct deeper understandings (e.g., Moreno and Mayer, 1999). Further to this, as Berthold et al. (2009, p.346) express, "by combining different representations with different properties, the learners are not limited by the strengths and weaknesses of one particular representation." The literature also provide evidence that the links among the MRs are often not established during instruction by the teachers (Mallet, 2007). It seems that the work of linking the MRs is largely left to the learners. The initial lesson plans that our participants produced provide corroboratory evidence in that 85% of the PSMTs did not relate the MRs with regard to derivative. However, there are many studies showing that the expected outcomes with the use of MRs often do not come about (de Jong et al., 1998) due to the fact that learners find it difficult to relate the MRs to one another and they, more often than not, focus on one type of representation or fail to connect them (Berthold, 2009; Goldenberg, 1988). In fact most students do not spontaneously make an effort to establish connections among the MRs (Yerushalmy, 1991). These findings clearly suggest that unless the links among them is an explicit focus of instruction then the assumed benefits stemming from the use of MRs do not come about. Our participant PSMTs seem to have grasped the importance of linking the representations and hence 67.5% of them linked two or more representations (Table 3) in the context of derivative.

Secondly, the development of the PSMTs informs us about the certain features of successful programs designed for the effective use of MRs with the aid of technology. Three such features that our design implies are the content, method of delivery of the content and hands-on activities (see also Hew and Brush, 2006). First of all, the content of course designed for the PSMTs involved the issue of MRs and particularly focused on the examples of MRs in different topics of mathematics (e.g., limit, function and trigonometry), functions of MRs, strengths and weaknesses of individual representations, representational power of the particular technology chosen for the study (i.e. Graphic Calculus), the importance of connecting the representations and the affordances of the technology for that matter.

We followed certain order in our method to deliver the content in the framework of TPCK. We first considered the definition of MRs, the importance of employing MRs for teaching and examples of MRs from different topics (Pedagogical Knowledge). Then we focused on the ways in which MRs can be employed, on the functions and the importance of interconnecting the particular representations in the context of derivative (Pedagogical Content Knowledge). Following this, we brought the technology dimension into play and discussed the MRs that the particular software (Graphic Calculus) offers (Technological Content Knowledge). Later, the concern was with the questions of how the technology helps in making connections among the MRs, how these connections can be used to achieve a robust understanding of the concepts and what the technology has to offer for teaching with MRs (Technological Pedagogical Knowledge). Finally we concentrated on the particular topic under consideration (derivative) and had the PSMTs devise ways via technology as to how to combine MRs of derivative, how to relate the MRs to three aspects of derivative, what technology has to offer in making explicit the relationships between the MRs and three aspects of derivative, how to employ the technology in making these relations comprehensible to the learners, in what order the aspects should be introduced via MRs (Technological Pedagogical Content Knowledge). Hence the TPCK framework was effectively our design tool for the courses as well as provided us with a framework to shape our method to deliver the course. Considering the development of our participants, we believe that this method of delivering the content was effective.

In our delivery of the content, we paid particular attention to get PSMTs involved in hands-on activities. Our primary aim was to achieve active participation of them into activities as well as to give them a "space" where they can explore the ideas and tools on their own ways. Hence we guide them with discussion questions but largely left the responsibility to the individuals who most of the times worked in groups. We believe that giving the PSMTs a chance to explore the alternatives and individually active involvement into activities are important for them to get acquainted and develop insights and competencies in the integration process of technology. Based on this brief consideration, we argue, the successful integration programs need to have at least these three as features in the design and conduct of the courses.

Thirdly, the research cited the teacher's lack of knowledge and skills in using technology as obstacles to the integration of technology into teaching (Pelgrum, 2001; Hakkarainen et al., 2001). Even though teachers have the necessary skills to use technology, if they are lacking in the knowledge of how to deliver the content by means of technology, this again creates barriers (Hew and Brush, 2006). Such deficiencies in teacher knowledge shape their beliefs as to the usefulness of technology as well. For example, Ertmer et al. (1999) investigated one elementary school in the USA and found that teacher beliefs about the role of technology in the curriculum formed their objectives for the use of technology. Those considering technology just "a way to keep kids busy" (Hew and Brush, 2006) did not appreciate the relevance of technology to the content knowledge. Students were allowed to use computers as a reward of finishing the assigned tasks. The teachers reported that the content knowledge and skills were more important to them. As this study clearly suggests many teachers are not able to see the relation between the use of technology and content dimension of their subject. Further to this they do not believe that technology aid their students for a robust comprehension of the subject matter at hand. For instance, a study carried out in Australia on the perception of computers at secondary level reveal that teachers do not see computers as leading to better understanding or fostering the learning (Newhouse, 2001).

When considered in the context of our study with regard to the TPCK framework, these findings point to the importance of teacher's technological knowledge, technological content knowledge as well as technological pedagogical knowledge (see also Arnold et al., 2009). Our pre-service teachers' development certainly parallels to those areas of knowledge. The PSMTs in our study joined workshops on Graphic Calculus software and performed hands-on activities as described above. Furthermore, they did activities as to the MRs of several mathematical topics, in particular on derivative, via the software. They also participated in the workshops where they developed ideas as to how to make use of technology for connecting and relating the MRs, which in and of itself created the content dimension and gave the PSMTs a sense of purpose in their efforts. All these activities and workshops contribute a great deal to the development of PSMTs with regard to use of MRs. In so doing, we believe, PSMTs overcome some important obstacles to the technology integration. The evidence for this come from the developing competences of our participants both in their lesson plans and the way in which aspects of derivative are interrelated through MRs with the help of technology. This competence was clearly evident in microteachings such as that of Arzu.

Fourthly, there is a tendency among the pre-service teachers that technology functions mainly as a computational device rather than a learning resource (Juersivich et al., 2009). Given that, at least in the case of our research, most PSMTs are "foreigners" to the culture of teaching mathematics with the aid of technology, there is a need for teacher educators to get PSMTs experiencing the contribution of technology to their teaching. MRs provide a venue for the PSMTs to engage in activities where the technology acts as a learning resource and not only paves the way for new possibilities of teaching but also serves to deepen the student understanding of the mathematical concepts. It is important for our PSMTs to experience these features of technology with regard to the use of MRs. The importance does not merely stem from their developing competences in technological knowledge; but also from the fact that our participants' subject matter knowledge has also improved and they themselves experienced the enhancement in their own learning. For example, in our interview on her microteaching, we asked Arzu if she made any changes in the structure of her lesson plan with the involvement of technology. She responded as follows:

[In this process] I got confident, knowledgeable… I mean in this process I corrected my own misconceptions… I knew things [about derivative] but they were shaky… things haven't settled down… [before] ideas were dispersed because I myself couldn't connect them [the representations and three aspects of derivative]… But I learnt the topic [derivative] more… things settled down now.

As Arzu's response clearly suggests, during the TPCK workshops where she did hands-on activities on MRs and three aspects of derivative via technology, she corrected her own misconceptions and she herself "learnt" the derivative. Such remarks were common among the PSMTs that they learnt the relationships among three aspects and among the MRs of derivative during their participation of the workshops. This experience in and of itself was valuable as it convinced many of the usefulness of technology and the necessity of incorporation of it into teaching for the better learning outcomes (Juersivich et al. (2009) also report similar observations).

Having observed PSMTs development, examined their lesson plans and followed their microteachings, we are very much convinced of the importance of MRs for a successful teaching – whatever the teaching medium is (regardless of whether it be chalk-and-board or a technologically rich environment). We believe that in any attempt to design courses for technology integration, the issue of MRs need to be part of the program. Teachers or pre-service teachers need to have a chance to see explicitly the potential of MRs in achieving a robust and deep understanding of the subject at hand. They also need to see the contribution that technology has to offer for that matter.

At this point, we wish to note here that the contribution of technology is often attributed to its power of allowing the visualization. However, our analyses point out that contribution of technology, at least in the case of Graphic Calculus, with regard to MRs goes well beyond its visual power. We do not deny the important effect of visualization that technology offers. Equally important is, we think, the dynamic relations among the MRs that technology is able to put forward. A change in one representation immediately affects the others and all the changes can be seen at the same time on the same window. This dynamic linking makes most of technological tools powerful resources for learning. It is through this feature that the links among the MRs and the aspects of the topic become accessible to the learners. It is one of the features that allows teachers to make the links explicit focus of instruction and to combine MRs with individual unique properties for the purposes of complementing, constraining and constructing deeper understanding.

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