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Primary Teachers' knowledge for teaching ratio and proportion in Mathematics: The case of Indonesia

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In this study, we aimed at examining Indonesian In-Service primary Teachers' Mathematics Content Knowledge (MCK) and Mathematics Pedagogical Content Knowledge (MPCK) for teaching ratio and proportion. The instruments were administered to 271 in-service primary teachers with various education background. There were three underlined factors on MCK and MPCK instrument classified by item analysis and factor analysis. Three teachers' categories (Good, Middle, and Low) were established by cluster analysis methodology. The most teachers' challenge on MCK was figural representation. For MPCK, Indonesian teachers need more opportunity to learn the factor of Knowing students conceptual understanding. The state of teachers' MCK and MPCK and related components are worthy to suggest the design of Indonesian In-service Teachers Professional Development Program.

Keywords: MCK, MPCK, ratio and proportion, teachers' professional development program, Indonesia

INTRODUCTION

Students achievement from The Third International Mathematics and Science Study (TIMMS) could be considered as benchmark for mathematics education reform, school and teaching effectiveness. Reflecting data on students' performance in International assessment such as Third International Mathematics and Science Study (TIMSS) and Program for International Student Assessment (PISA), Indonesian students' performance were categorized as low. To be more specific, for instance in PISA survey 2009, almost all Indonesian students only reached level 3, whereas only 0.1% of Indonesian students reaching level 5 and 6 (Kemdikbud, 2013; Stacey, 2011). Besides, the latest PISA result in 2012 informed that Indonesia ranks 64 out of 65 countries with level attainment relatively low (OECD, 2013). These might be influenced by the material used for teaching mathematics are not the same with material that are evaluated using International Standard (Zulkardi, 2013).

In the case of ratio and proportion which addressed as complex topic and difficult for teachers to teach and for students to learn (Behr et al., 1992; Lamon, 2007) in three years TIMMS evaluation data (1999; 2003; 2011), there were some TIMMS items which Indonesian students performed similar with international average. For instance, the item of finding ratio of shaded to unshaded part and finding the ratio of rectangle width and its perimeter. However, about 70% of problem on ratio and proportion in TIMMS could not be solved appropriately by students and resulted them in the

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State of the literature

- Research suggest that the extent and richness of teachers' Mathematics Content Knowledge and Mathematics Pedagogical Content Knowledge that influence the effective teaching.
- To be more specific on ratio and proportion as content, recent findings on pre-service teachers showed a lack of knowledge of multiplicative thinking, in particular, where multiplication and division were required within the items.
- National Teachers Professional Development (TPD) Program that tied witth teachers' certification does not specifically concern on some factors of MCK and MPCK resulted from this study. Based on these, it could be suggested that it needs to reform TPD program that view teachers as teaching expertise in relation to MCK and MPCK factors suggested.

Contribution of this paper to the literature

- This study investigated In-Service Teachers Mathematics Content Knowledge and Mathematics Pedagogical Content Knowledge on ratio and proportion with developed instruments by item and factor analysis methodology.
- The phenomenological analysis of Indonesian teachers' knowledge on ratio and proportion might informed other periphery countries such as Southeast Asia countries regarding its similar situation. Teachers' understanding were grouped by cluster analysis on factor analysis result and informed the varies of teachers performance.
- The state of in-service teachers' MCK and MPCK as the result of this study could inform Teacher Professional Development designer about suggesting conceptual framework for the in-service Teacher Professional Development program.

category of lower than international average. Most students faced difficulties on the proportion problem might be due to the complexity of situation and the lack of understanding of multiplicative relation on proportion. One example problem that students experienced challenges was "Alice can run 4 laps around a track in the same time that Carol can run 3 laps. When Carol has run 12 laps, how many laps has Alice run?". It could be inferred from this phenomena that students needed more opportunity to understand multiplicative thinking on proportional situation problem. There was astrong relation between students achievement and teachers' knowledge as described by Schmidt et al. (2011). Literatures suggested that the effective teaching in mathematics depends on the extent and richness of teachers' knowledge (Fenstermacher, 1986; Shulman, 1986; Fawns & Nance, 1993). Shulman (1986,1987) first suggested three domains of knowledge that required for teaching such as Subject matter content knowledge, pedagogical content knowledge and curricular knowledge. The way in which the knowledge contributed to mathematics teaching has gradually developed and resulted in its subdivision into two major areas namely Subject Matter Knowledge and Pedagogical Content Knowledge (Pothen, 2011). Specifically in Mathematics teaching, those two knowledge domains could be regarded as Mathematics Content Knowledge (MCK) and Mathematics Pedagogical Content Knowledge (MPCK). MCK includes fundamental mathematical definitions, concepts, algorithms and procedures. Moreover, according to Kwong et al. (2007), MPCK includes complex interactions between knowledge of generic pedagogy, a strong understanding of the discipline of mathematics and a sound grasp of the principles of mathematics specific pedagogy. Numerous studies have researched on the assessment of teachers' MCK and MPCK such as COACTIV (Krauss et.al, 2008); TEDS-M (Blömeke & Delaney, 2012) and Mathematics Teaching in the 21st Century (MT21) (Schmidt et al., 2011). Investigation of different facets of knowledge did not specifically relate to the knowledge on ratio and proportion even though these concepts are fundamental to mathematics and important in many other fields of knowledge (Chaim, Keret & Ilany, 2012).

In this study, we focused at investigating Indonesian In-Service primary Teachers' Mathematics Content Knowledge (MCK) and Mathematics Pedagogical Content Knowledge (MPCK) performance for teaching ratio and proportion. The phenomenological analysis of Indonesian teachers' knowledge on ratio and proportion might informedperiphery regions (Nebres, 2008) such as Southeast Asia countries and others regarding its similar situation. Indonesian primary teachers' knowledge were described with respect to three assigned teachers understanding categories (Good, Middle, and Low) derived from cluster analysis. It regarded factor component of MCK and MPCK resulted from Exploratory Factor Analysis (EFA). The state of teachers' MCK and MPCK is important to be considered for designer of in-service Teacher professional development (TPD). The result would also informed the Teacher Professional Development (TPD) designer regarded the suitable entry for primary in-service teachers.

Conceptual framework of teachers' knowledge in this study

The conceptualization of MCK and MPCK in this study were adapted from the COACTIV study. There

were four level of MCK described by Krauss et. al (2013) in COACTIV such as (1) Everyday mathematics knowledge required by the average adult; (2) A reasonable command of school-level mathematical knowledge; (3)A profound understanding of the content of the secondary school mathematics curriculum; (4) University-level knowledge of mathematics. For this study, the fourth level is not included as it is beyond the scope of the study. In terms of MPCK, synthesizing the stream of the description of MPCK by Kwong et.al (2007), there were four parts of MPCK used in this study such as Knowledge about teaching the concept (include giving feedback); Knowledge about students' understanding of the concept; Knowledge about level of task and Knowledge about the appropriate teaching approach for students understanding. This is in line with PCK framework in another discipline like Technology Education, Rohaan et al. (2011) distinguished three factors on PCK such as knowledge of pupils' concept, pre and misconception related to technology; knowledge the nature and purpose of technology education; Knowledge of pedagogical approaches and teaching strategies.

Ratio and Proportion

The concept of ratio inferred a multiplicative relationship between two values that is calculated by dividing (or multiplying) one quantity by another. It is related to other concept such as fraction and proportion. Students learn the concept of ratio since primary level though the term is not introduced to them explicitly. A ratio is a comparison between two quantities (Livy & Vale, 2011) can be represented by a fraction and subsequently law of fractions can be applied to ratios. Regarding this, the introduction to ratio could be done after students familiar with fraction and its operation. Ratio is the quantification of multiplicative relationship that is calculated by dividing (or multiplying) one quantity by another (Chaim et.al, 2012). There are three common ratio comparisons: ratio, part-part (for example, one part of cements and three parts of sand or 1:3); proportion, part-whole (for example one of the five cordial or 1/5); and scaling, whole-whole (comparing wholes to wholes, where 1cm on the map equals 1.000.000 cm on the ground) (Suggate, Davis & Goulding, 2006). Furthermore, proportion is a relationship between four numbers or quantities in which the ratio of the first pair equals the ratio of the second pair written as a:b = c:d (Borowski & Borwein, 1989). The proportional problem involves the situation in which the mathematical relationship are multiplicative one (opposed to additive) in nature and allow the formation of two equal ratio between them (Chaim et al., 2012). Regarding ratio and proportion content, Simon and Blume (1994) found that prospective elementary teachers demonstrate additive strategies for ratio and proportion problem when multiplicative strategies are appropriate. Furthermore, Livy and Herbert (2013) described that second year pre-service Teachers demonstrated a lack of knowledge of multiplicative thinking, in particular, where multiplication and division were required within the items. These were related to the teachers' proportional reasoning which played important role and had important practical function.

The MCK and MPCK Instruments

To document teachers' understanding of ratio and proportion, we developed paper pencil test instrument that contain both MCK and MPCK Categories. Some MCK problems that posed were routine school mathematics problem and some were non-routine. The MCK item instrument on ratio and proportion elaborated three important variables such as context situations, task type and number structure which were also regarded in Alatorre and Figueras (2004). The context situation regarded in the MCK instruments were stretchers and shrinkers (i.e. enlargement figure) and well-chunked measure problems (i.e. fuel used litre/hour). Different context situations in proportional problem influence the different students reasoning strategy such as the recipe context in which students could use unitary method and another context such as geometrical enlargement required multiple method. In terms of task type, there were classification of tasks by Tourniaire and Pulos (1985) such as missing value problem and ratio comparison problem. Different reasoning strategies would be differ with respect of task type. In addition, the third variable that we considered was number structure which consisted of integer multiple and non-integer multiple number structures.

Regarding MPCK instruments, it includes the knowledge how teachers make the content (ratio and proportion) understandable for students; The suitable teaching method for students that emphasized the topic proportional reasoning; The cognitive demand of mathematics task for students; and the understanding of students error and misconception were also considered. Both MCK and MPCK item problem were created in three different forms such as Multiple Choice (MC), Complex Multiple Choice (CMC) that teachers were asked to answer with more than one choices and Open Problem (OP). The MCK and MPCK instrument were validated by Mathematics Educator expert and pilot tested among 20 pre-service/in-service teachers. Statistic of the pilot study showed that the reliability score for MCK and MPCK were 0.719 and 0.731 respectively which was acceptably consistent. Since the teachers assessed were Indonesian primary teachers and the instrument initially developed in English, the

language translation validation done and took Beaton et al. (2000) guidelines as consideration. Three stages process were applied such as (1) Adaptation the english version instrument into Bahasa Indonesia by two Indonesian with different education background; (2) Two translation result synthesized into one common translation; and (3) Consistency checking by back translation was done by a person who considered English as a source language. The English translation result was reviewed by the instrument developer and the result showed that there were some synonymous terms found without changing meaning of the original instrument. At last, Bahasa Indonesia items with little revision was used. Table 1 and 2 are the overview of MCK and MPCK item instrument.

The maximum score of each MCK and MPCK items is 1. The coding for items were done more than once to keep the consistency.

METHODOLOGY

The participants in this study were in-service primary teachers of grades one to six. We delivered the instruments to 271 in-service primary teachers from 4 districts in East Java, Indonesia. Teachers participants were asked to complete their identity questionnaire regarding their education backgroud, the year of teaching experience and the grade that they taught. Some participants are novice teachers as one with less than 3 years teaching experience (Borko et al., 1992; Drake, 2000) and some others are expert teachers who already have teaching experience more than 3 years in primary level. Teachers were invited to join MCK test voluntary and were given a small present for appreciation.

Assessment of Instrument

The Item analysis was done before the factor analysis method applied. The goal of item analysis is to investigate the item performance considered individually either in relation to some external criterion or in relation to the remaining items on the test (Thomson & Levitov, 1985). Two hierarchical item analysis method for each item were applied such as (1) the classification of teachers (27% top and 27% bottom) regarding the total score. Afterwards, the independent t-test for the two different groups and compare the mean from two samples. (2) Apply correlation analysis of each item to total score and the item was deleted if the correlation

Table 1. MCK items description

Code	Problem Overview	Item Format
MCK1	Rate missing value problem about the use of fuel with non-integer number structure	MC
MCK2	Innate ratio in the congruency of two geometrical figure	MC
MCK3	Reasoning of congruency of two geometrical figure	OP
MCK4	Non-integer scaling and number structure (Mr.Short&Mr.Tall problem)	OP
MCK5	Proportional& Non-proportional situations	CMC
MCK6	Ratio relation of two different objects (speed context)	OP
MCK7	Meaning of proportional relation in situation	OP
MCK8	Meaning of equivalence sign in proportional relation	OP
MCK9	The conditional statement of two proportional statements within speed context	OP
MCK10	Drawing the enlargement figure with integer number structure scaling	OP
MCK11	Missing value problem with non-integer scale factor enlargement	OP
MCK12	Ratio relation in Cartesian coordinate system	MC

Table 2. MPCK items description

Code	Problem Overview	Item Format
MPCK1	Develop proportional problem that fit to primary level	OP
MPCK2	Identify students' responses on contextual proportional problem	OP
MPCK3	Analyze and interpret students' misconception for solving proportional problem	OP
MPCK4	Encourage/guides students to aware of their misconception	OP
MPCK5	Choose appropriate teaching method for students understanding and reasoning ability.	MC
MPCK6	Analyzing of teaching unitary method for ratio and proportion problem	OP
MPCK7	Provide appropriate feedback for students misconception	OP
MPCK8	Evaluate students' mathematics solution on providing different number arrangement of proportional problem.	MC
MPCK9	Analyze the different students strategy in solving proportional problem	OP
MPCK10	Identify tasks difficulty level for students based on their cognitive demand	CMC
MPCK11	Analyze the more demanding task compares to other	OP

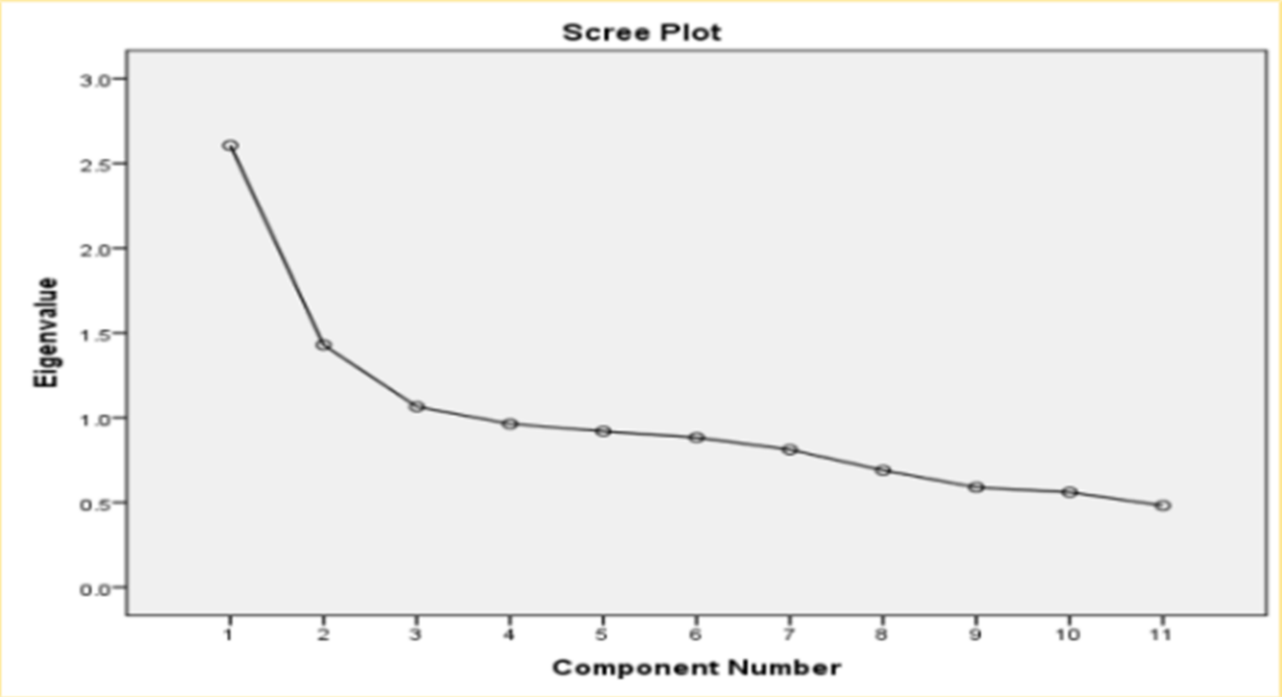


Figure 1. MCK loading factor scree plot

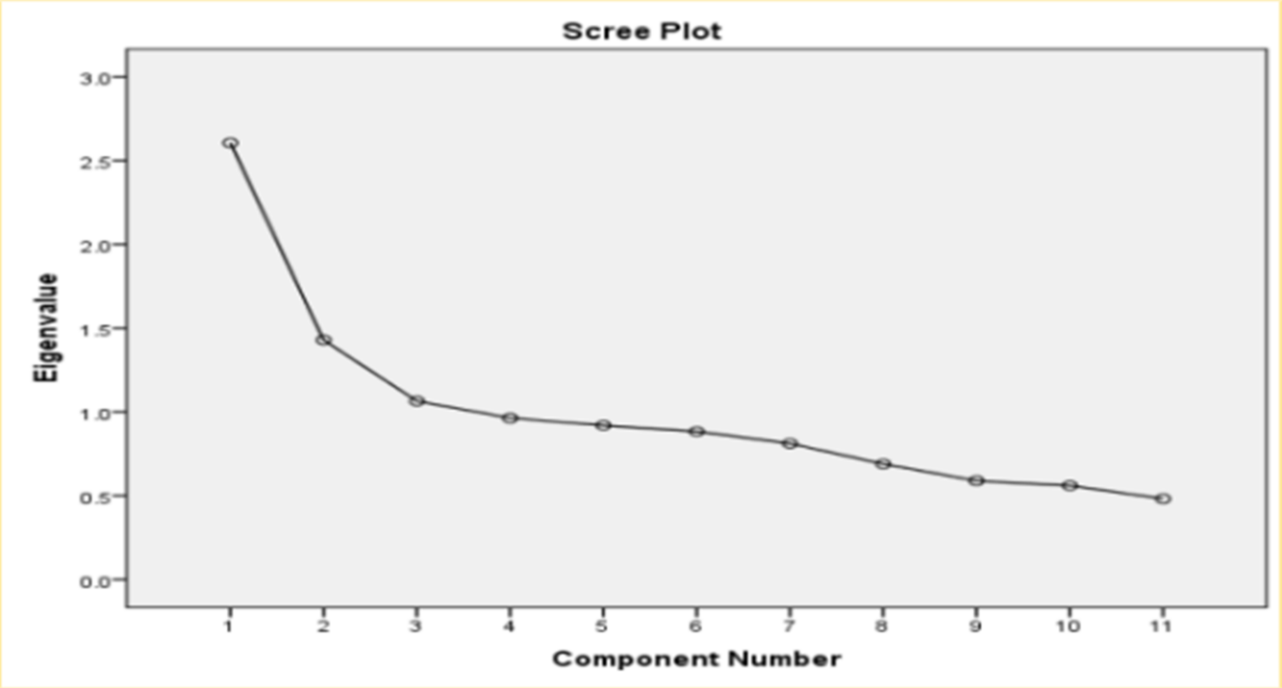


Figure 2. MPCK loading factor scree plot

less than 0.3 (Ebel & Frisbie, 1986). (3) Execute factor analysis and determine the factor loadings for each variable. In order to understand the patterns in the data, each variable in single factor loading should be at high loading (>0.3). The item analysis result should fulfill at least two criterias above and if it could not satisfied, the items would be deleted.

For MCK instrument, we deleted MCK 2 since its score correlation to the total score and the factor loading value of this item using SPSS were less than 0.3. As for MCK, MPCK 5 was deleted because of those two conditions. Among the 271 samples, The cronbach Alpha of MCK was 0.651 and MPCK was 0.641 which were considered as acceptable internal consistent reliability coefficient (Hair et al., 1998) and indicated a reliable measure of both categories. Afterwards, the exploratory factors done in order to explore the dimensionality of the framework of MCK and MPCK

Table 3. MCK Structure Matrix

Description	Factor		
	1	2	3
Proportional& Non-proportional situations	0.551		
Ratio relation of two different objects(speed context)	0.586		
Meaning of proportional relation in situation	0.770		
Meaning of equivalent sign in proportional relation	0.649		
the conditional statement of two proportional statements	0.690		
Rate missing value problem about the use of fuel with non-integer number structure		0.759	
Non-integer scaling and number structure (the problem of Mr.Short& Mr.Tall)		0.492	
Missing value problem with non-integer scale factor enlargement		0.749	
Reasoningthe congruency of two geometrical figures			0.570
Drawing the enlarged figure with integer number structure scale			0.572
Ratio relation in Cartesian coordinate system			0.672

Table 4. MPCK Structure Matrix

Description	Factor		
	1	2	3
Develop proportional question that fit to primary level	0.487		
Identify students responses on proportional problem	0.359		
Analyze students' misconception on proportional problem	0.776		
Encourage students to aware their misconception	0.793		
Identify task difficulty level		-0.901	
Analyze the underline reason of difficulties of problem		-0.854	
Analyze the teaching of unitary method			0.522
Provide appropriate feedback for students misconception			0.621
Evaluate different given students mathematics solution			0.726
Analyze different given students solution			0.490

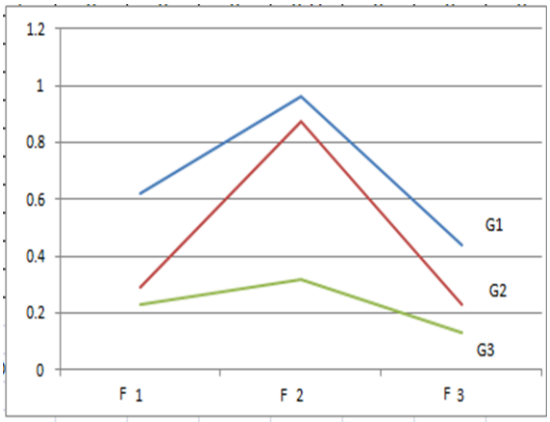
items by exploratory principal component factor analysis with Oblimin and Kaiser Normalization's rotation method. The communalities of items that showed its variances with the factor were from 0.329 to 0.612 and 0.273 to 0.793 for MCK and MPCK respectively. These could be considered as high communality. The three underlying factors for each MCK and MPCK instruments were identified with the eigenvalue greater than 1 (see fig 1.scree plot and the rotated structure matrix). Furthermore, the value of Kaiser Meyer-Olkin Measure Sampling Adequacy (KMO) of MCK and MPCK were 0.717 and 0.654. This could be interpreted as appropriate number for the factor analysis process as described by Coakes and Steed (1997).

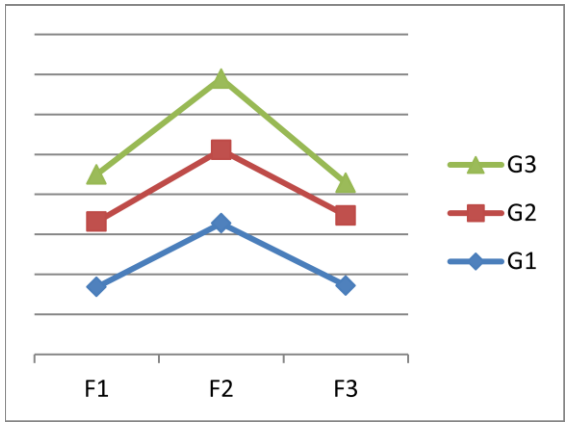
The rotated structure matrix of MCK and MPCK(Loadings of greater than 0.3 were outputted. PCA with Oblimin, Kaiser Normalization)

As described in the table of rotated structure matrix of MCK items (see Table 3), the 11 MCK items were categorized into three factors that interpreted as the meaning of proportional and non-proportional situations (F1), number structures in situation (F2) and figural representation (F3) (Ekawati et.al, 2014). The main underlying base for MCK items is proportional reasoning which Chaim, Keret & Ilany (2012) considered it as the main indicator during the stage of formal development. F1 pertained to understanding proportional/ non-proportional situation problems and the meaning of ratio relations in proportional patterns within situations. F2 pertained to the different numerical structure problems within different situations and could be viewed as items for measuring the ability to find quantitative mathematical solutions for proportional problems. F3 considers ratio relations in geometrical figures and in representations. In regard of MPCK items (see Table 4), three factors were also loaded such as Knowing students conceptual understanding (F4), Ratio and proportion task level feature (F5) and Teaching problem solving strategy of

Table 5. Teachers' MCK and MPCK Result

MCK	MPCK
Good category (G) = 72 teachers Middle category (M) = 162 teachers Low category (L) = 37 teachers	Good category (G) = 127 teachers Middle category (M) = 64 teachers Low category (L) = 80 teachers





The average score of MPCK

	Good	Middle	Low
Factor 1	0,62	0,29	0,23
Factor 2	0,96296	0,874486	0,3153
Factor 3	0,4375	0,228395	0,1307

The average score of MPCK

	Good	Middle	Low
Factor1	0,336811	0,326953	0,235625
Factor2	0,655512	0,367188	0,35625
Factor3	0,344134	0,350469	0,162906

ratio and proportion (F6). F4 regarded students' misconception and thinking on ratio and proportion based on their level. F5 considered the identification of task level difficulty and the underline reason of the hierarchy level that influenced by such factor for example the number structure in ratio and proportion. Finally, F6 included teachers' understanding of initial unit strategies of teaching ratio and proportion concept, solving mathematics problem and their pedagogical problem.

RESULTS

Indonesian Primary Teachers' Performance (MCK and MPCK)

In this study, we drawed attention to discrepant pattern of Indonesian primary teachers' performance on MCK and MPCK on ratio and proportion by cluster analysis on the factor score result. We assigned teachers to three different categories such as Good (G), Middle (M), and Low (L). The outcomes of number of teachers in the cluster classification for MCK and MPCK results and the mean of each categories were described in Table 5.

Indonesian in-service primary teachers found difficulties in the factor of figural representations (F3) and they performed best on factor of number structure in situations (F2) in which teachers were able to represent product of proportional reasoning. Compare to MCK result, the teachers' performance on MPCK on each categories were inconsistent. With respect to the three factors, overall the teachers' performance and within three assigned categories of teachers perform best on the factor of Ratio and proportion task level feature (F5). Furthermore, regarding the most difficulty factor, Good and Middle teachers had challenges on Knowing students' conceptual understanding (F4) and for low performance teachers, they found difficulties on Teaching problem solving strategy (F6). To sum up, Figure 3 and 4 were chart of level of difficulties on MCK and MPCK.

Regarding MCK, Figural representation factor was the most difficult MCK factor in each teacher's categories. Only 6.64% teachers could answer one of the F3 items, which asked to draw the enlargement figure, correctly. Another item related to figural representation that integrated intuitive ratio in similar figure representation, most teachers were not able to describe the underline reason of congruency of two rhombus figures related to ratio of comparing corresponding diagonal lines. For the problem included in F1, 53.9 % of teachers were able to match at least four out of six contextual situations to their corresponding types. In identifying the ratio relation of distance/time, 71.6 % of teachers were able to answer the items correctly. The lower percentages of correct answers appeared in the problems of explaining the meaning of equivalence sign on proportions with ratio relations $\frac{s1}{t1} = \frac{s2}{t2}$ and $\frac{s1}{s2} = \frac{t1}{t2}$ were 35.4 % and 9.59%, respectively. In addition, in distinguishing the

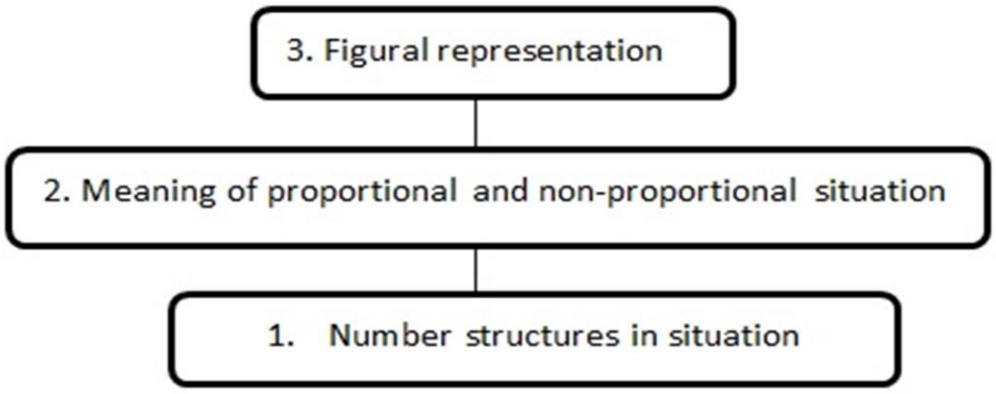


Figure 3. Hierarchical level of MCK of all teachers

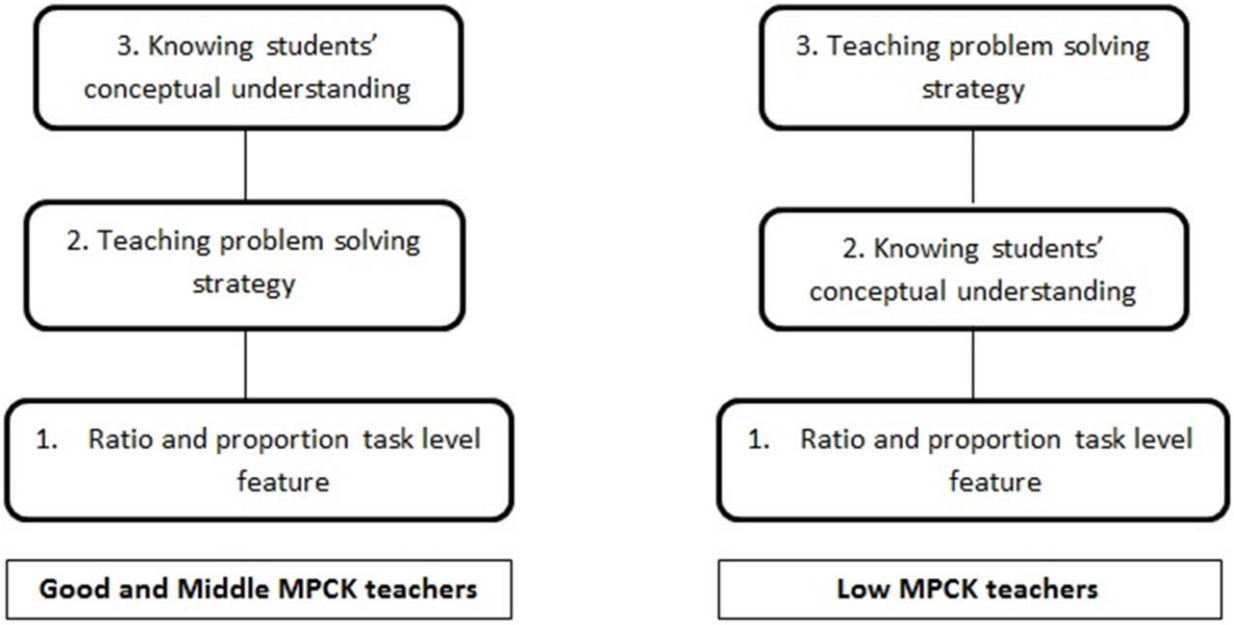


Figure 4. Hierarchical level of MPCK of teachers' assigned categories

proportional and non-proportional situations, it found that some teachers misunderstood the given situations and tried to describe it with proportional algebraic pattern and regard it as proportional situation. It showed that the proportional or non-proportional situations was an essential component that teachers should aware of.

Regarding teachers' MPCK, there were two groups of teachers faced different challenges. Most teachers (Good and Middle teachers' categories) needed more sensitivity on students in the factor of knowing students' conceptual understanding. Regarding the primary teachers' challenges on MPCK items, it showed the inconsistent on low performance teachers compare to Good and Middle ones. Low MPCK performance teachers needed more understanding on teaching problem solving strategy factor. It consisted of several units such as teaching unitary method, giving feedback, evaluating and analysing students solution became the most challenging items for low performance teachers. These findings were due to teachers might only believe on single teaching method such as deliver cross multiplication strategy. They did not consider other strategy such as unitary method that could help students develop their proportional reasoning. One teacher responded, "if teachers used the unit strategy, it will not explore detail calculation. It is better to use cross multiplication due to its logical thinking". Those teachers assumed that introducing 'unit strategy' would need more time to develop students understanding of ratio and proportion compared to efficient cross multiplication method. These situations could influence teachers' difficulty in giving feedback and analysing students' different strategy. Furthermore, most teachers also tend to present single solution strategy such as in comparing ratio; they consider one kind of number

arrangement in ratio that could be interpreted as within ratio strategy. It required to put the same unit in the same position, either in numerator or denominator.

For teachers with Good and Middle categories, they needed more sensitivity of students understanding such as misconception and different students strategies in solving proportional problems. Take an exemplary problem for the factor of knowing students conceptual understanding, there were only eight out of 271 teachers' participants who able to analyse the additive strategy that students used in Mr. Tall and Mr. Short problem. From this phenomenon, it could be reflected that teachers did not pay attention to their students' relative difficulties on ratio and proportion. However, all primary teachers' participants performed best in the factor of ratio and proportion task level. Items in this factor asked teachers to identify the level task for sixth graders and explained the underlined reason of the more difficulties tasks. There were 51% teachers could state the two most difficult task properly and 29.5% could partially decide one task as the most difficult. There were only about 35.42% of teachers could provide underline reason of deciding two most difficult tasks.

The combination of Indonesian primary teachers' MCK and MPCK performance on ratio and proportion were elaborated in Table 6.

The distribution of 271 teachers' participants showed that some categories consist of small number of teacher such as Low MCK Middle MPCK (LM teachers) and Good MCK Low MPCK (GL teachers) categories. From the nine cells of combination MCK and MPCK categories, there was no tendency of to which category that highlights Indonesian teachers mostly performed. However, there were still high percentages of teachers need to have opportunity to learn to be in Good MCK and Good MPCK.

DISCUSSION

The result of the quantitative analysis with Exploratory Factor Analysis (EFA) of 271 teachers' responses on paper and pencil test formulated three MCK and MPCK factors on ratio and proportion. Furthermore, cluster analysis method was applied to the factor score from EFA and resulted three different categories of MCK and MPCK (Good, Middle and Low). Regarding MCK, Indonesian primary in-service teachers performed best in the factor of number structures in situation and faced challenge in the figural representation factor. One component that could influence this was the textbook that teachers used for teaching. The national mathematics textbook that is used by teachers does not fit to the factors as framework for this study analysis. For example, there is no example and problem exercise that contain figural representation factor. Although figural representation is important for instructional feature that influenced students' mathematical thinking and reasoning. Ball (1993), Goldin(1987), Leinhardt (1993) , NCTM (1991) argued that representations are both an inherent part of mathematics and an instructional aid in making sense of mathematics. Figural representation could be used as tool for teachers to guide students to explain the concept and giving feedback to students' errors and misconception.

In terms of MPCK, teachers with low MPCK category experience challenges on Teaching problem solving strategy factor. This might also be influenced by the textbook content on ratio and proportion that highlighted formula or rules that can be memorized by students when they solve proportional problem as shown in Figure 5.

Besides, knowing students' conceptual understanding factor also another factor that difficult for teachers in middle and low categories. A teacher shared her idea in interview session that they might not able to provide appropriate feedback for students' errors due to the lack of guidance from teachers' guide book and also need more opportunity to learn it in in-service teacher education program.

Implication for Teacher Education Program in Indonesia

In Indonesia, there was an in-service teachers' professional development program that tied with the National Teaching certification program called PLPG (Professional Education and Training for Teachers). It stipulates Indonesian teachers to hold a national teaching certificate as their teaching license. Teachers must have academic qualifications, competencies (pedagogical, social and professional), national certification for teaching, good physical and spiritual health, and the desired ability to achieve national education goal. In PLPG, there were several aims to increase teachers' ability in teaching. For instance, teachers would be able to design teaching activity that leads students' active learning. To be more specific, it related to media used for teaching, developing students' skills based on their level of understanding and developing assessment for students' active learning. There is no specific concern on knowing students' conceptual understanding such as related to students' errors. Considering the primary teachers challenges phenomena of MCK and MPCK on ratio and proportion, generally teachers' participants need meaningful suggestion program to increase knowledge understanding. Saxe et al. (2001) described that the program with four areas need teachers understanding of mathematics they teach, children's mathematics, children's achievement motivation in mathematics and opportunity for teachers to work with other

Table 6. Primary Teachers' assigned Categories

		MPCK		
		Good (G)	Middle (M)	Low (L)
MCK	Good (G)	43 (15,87%)	21 (7,75%)	6 (2,21%)
	Middle (M)	73 (26,94%)	40 (14,76%)	52 (19,19%)
	Low (L)	13 (4,8%)	3 (1,10%)	20 (7,38%)

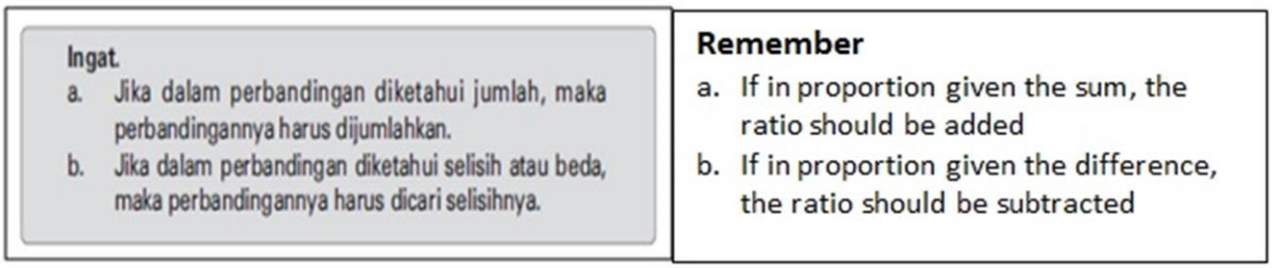


Figure 5. Rules in textbook and its translation

professionals showed greater gains on conceptual scale compare to other.

Regarded the current In-Service TPD in Asian such as Thailand, Inprasitha (2013) mentioned that teacher education is in crisis. 5-year teacher education program in 2004 there had feature of the program that emphasized on the highest number of credits for majoring courses (84-170 credits) and one-year internship. However, after a decade, this new type of teacher education program could not provide a successful result and the missing link in that 5-year teacher education program is still question. On the other hand, reflection from a Novice Mathematics Teacher Educator-Researcher (MTE-R) in designing In-Service Mathematics Teachers' Professional Development who did reflection of the emergent problem that he perceived in educating practice that teachers not wishing to or not being able to design teaching activities (i.e. conjecturing activities) (Chen et.al, in press). Based on that, MTE-R intended to focus mainly on those conjecturing activities for students' conceptual learning. It regards the facilitating students' conceptual understanding as the greatest concern. Therefore, the entry for In-Service Teacher Professional Development based on students' learning approach was on students' misconceptions. To be more specific, MTE-R invited to give critical comment on the task that their peers developed. The outcome of those TPD led teachers to be able to acquisition of new knowledge. Based on these phenomena, it could be suggested that the balance and combination of MCK and MPCK. The combination regards the hierarchical level of MCK and MPCK factors on paper and pencil test. The conceptualization described in the framework represented Figure 6.

Teachers might started with level 1 conceptual framework that consider activity that intertwine the number structures in situation (MCK) and ratio and proportion task level features (MPCK). It could be done in authentic situation which teachers familiar and expertise on these. For instance, teachers were given opportunity to explore a number of activities that contain task that include different number structures and explored the level of difficulties. (Several authentic activities also available in Chaim et.al, 2012)

Opportunity to learn is not only essential for in-service teachers but also in any discussion of teachers preparation program in which it had already the structure of the course. The focus on the teachers' preparation program is usually in the study of mathematics, mathematics pedagogy and general pedagogy (Schmidt et al., 2008). However, the issues within mathematics pedagogy that build relation between theoretical and practical aspects of pedagogy become the essential concern (Blomeke, 2002; Grossman, 2005; Gundem & Hopmann, 1998). The coverage of practical aspects of instruction in mathematics, instructional planning and management courses could include some aspects such as understanding standards and choosing textbooks; instructional instructions around mathematics; assessing, diagnosing, analysing and understanding how students learn mathematics etc. In addition, future teachers need to be given opportunity to have some form of educational practice in real classroom (Hsieh et al., 2011). Those components for pre-service teachers' education program that build the relation of theoretical and practical of mathematics pedagogy should be more highlighted for the reform needed.

Conclusion

The description of Indonesian Primary Teachers Mathematics Content Knowledge and Mathematics

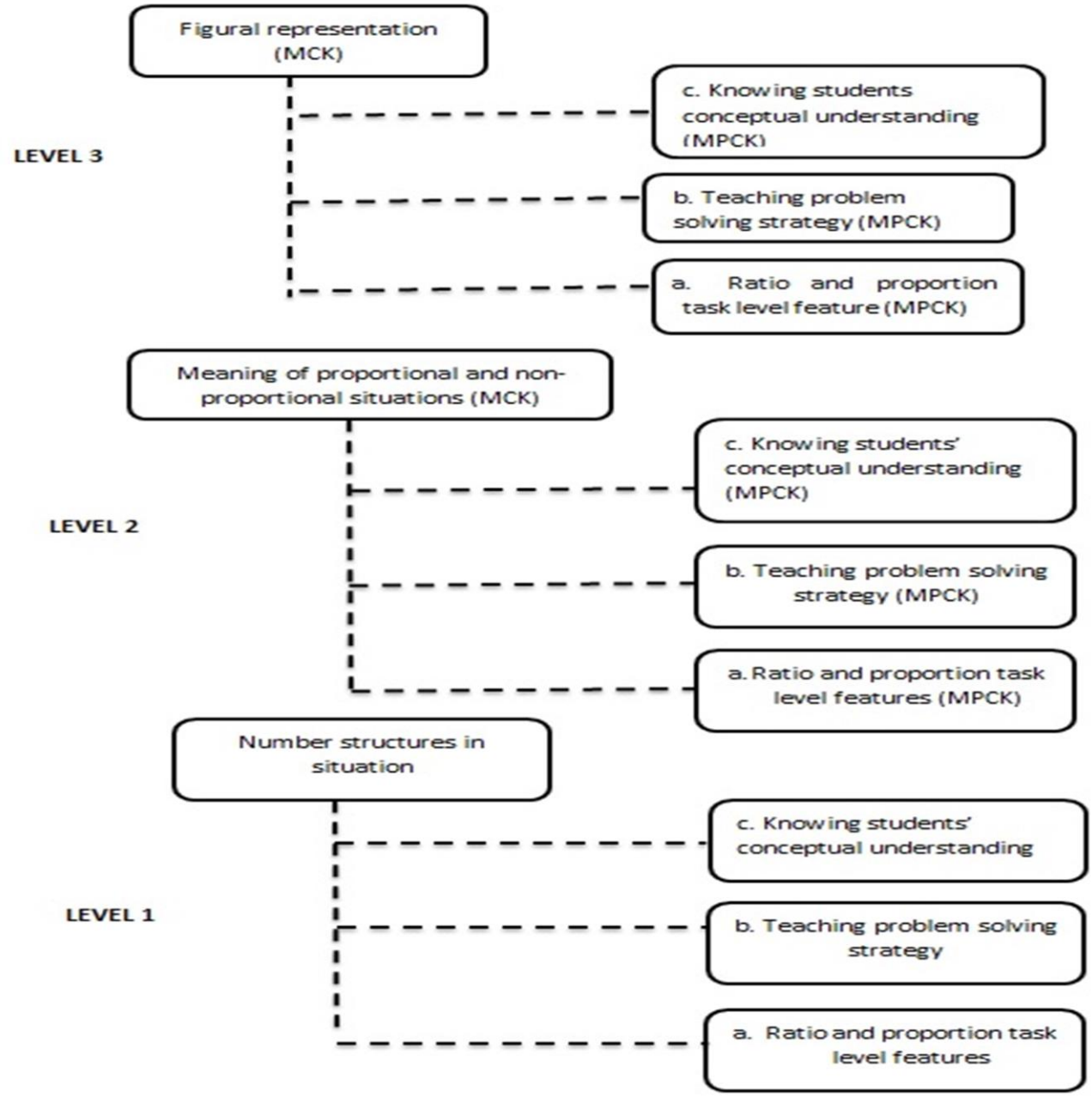


Figure 6. Exploratory conceptual frameworks for TPD from medium to high level knowledge

Pedagogical Content Knowledge on ratio and proportion were the main result of this study. For both MCK and MPCK, teachers perform best on factor of number structure in situations and ratio and proportion task level features respectively. Furthermore, teachers faced the most challenges on MCK factor of figural representation. The most challenges regarding MPCK were experienced by two groups of teachers (Good and Low MPCK) on knowing students conceptual understanding. However, Low MPCK group had most challenge in Teaching problem solving strategy factor. These similar exploration studies could be applied to other countries and it is possible that Indonesian phenomena described appear in other periphery countries such as some countries in Southeast Asia and others. The Teacher Professional Development need to consider view of teachers as teaching practice experts with regards MPCK in relation to MCK within hierarchical level. In addition, the investigation of effectiveness of the suggesting program could be regarded as the future study based on the knowledge phenomena given.

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◈⊙◈

Appendix A. The MCK instrument on ratio and proportion

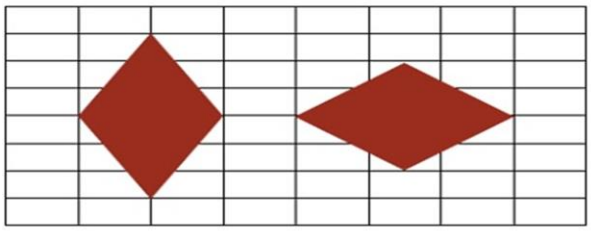
Code: MCK1	Outcome: MCK	Max point: 1

(a) A machine uses 2.4 litres of fuel for every 30 hours of operation. How many litres of fuel will the machine use in 100 hours if it continues to use fuel at the same rate?

A. 7.2 B. 8.0 C. 8.4 D. 9.6

Code: MCK2	Outcome: MCK	Max point: 1

a. There are two rhombuses figure on the grid paper with **size 3 cm x 2 cm** below.



Are A and B congruent?

a.Yes, A and B are congruent b. No, A and B are not congruent

Code: MCK3	Outcome: MCK	Max point: 1

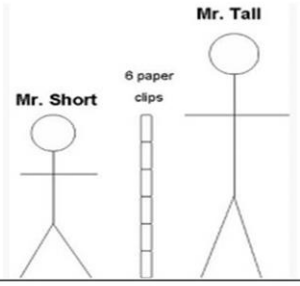
Explain the reason of your chosen answer above!(in MCK2)

Reason :

Code: MCK4	Outcome: MCK	Max point: 1

a. You can see the height of Mr. Short measured with paper clips is 6 paper clips. Mr. Short has a friend Mr. Tall. When we measure their heights with matchsticks:

Mr. Short's height is four matchsticks
Mr. Tall's height is six matchsticks
How many paperclips are needed to measure Mr. Tall's height? Explain your answer!



Code: MCK5	Outcome: MCK	Max point: 1

Match the situational problems on the left side with corresponding situations on the right side by drawing lines!

A. Robot Lia and Robot Matt did running at the same speed. Lia started before Matt. When Robot Lia had run 4 minutes, Robot Matt had run 2 minutes. How long had Robot Matt run when Robot Lia had run 12 minutes?

B. 4 tents can house about 12 campers. How many tents are needed for 30 campers?

C. When Dina is 1 year old, Budi is 2 year old. What will be Budi's age when Dina's age is 10 years old?

D. The recipe of onion soup for 6 people such as 5 onion, 4 gingers and 8 chicken brotch. How many gingers do we need to make soup for 4 people?

E. A video store charges $25 per month for unlimited rentals. Sally rented 5 videos last month. This month she rented 6 videos. How much Sally paid for 6 videos she rented ?

F. Heny bought 3 candies for Rp. 2400,-. How much should Heny pay if she want to buy 4 more for her brother?

Non-Proportional situation

Proportional situation

Code: MCK6	Outcome: MCK	Max point: 1

Given s_1 =distance 1, s_2= distance 2, t_1=time 1, t_2= time 2

a. What does it mean by $\frac{s_1}{s_2}$?

Code: MCK7	Outcome: MCK	Max point: 1

Given statement **p** → $\frac{m_1}{n_1} = \frac{m_2}{n_2}$, explain the meaning of sign "=" in the statement p!

Code: MCK8	Outcome: MCK	Max point: 1

Given statement **q** → $\frac{m_1}{m_2} = \frac{n_1}{n_2}$, explain the meaning of sign '=' in the statement q!

Code: MCK9	Outcome: MCK	Max point: 1

Explain why if the statement p correct, then the statement q is also correct!

Code: MCK10	Outcome: MCK	Max point: 1

Picture B is the enlargement of picture A. Draw the missing vertical line in picture B so that it keeps the same.

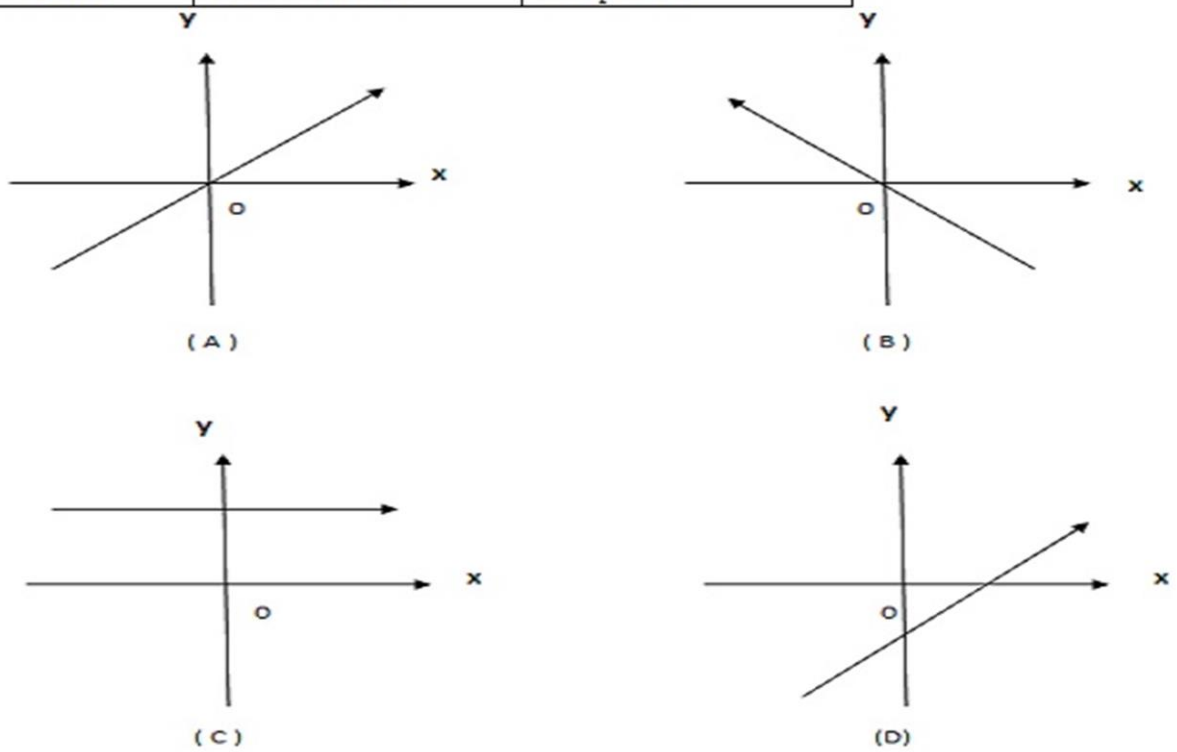
Code: MCK 11	Outcome: MCK	Max point: 1

Below is the figure of rectangle with size 5 cm length and 3 cm width.

a. Dina wants to enlarge the above rectangle so that the new base becomes 12 cm long. What will be the new width? _____

Explain how you attain the answer?

Code: MCK12	Outcome: MCK	Max point: 1



Which of the graphic representation above would present the ratio relationship of x and y?

a. A b. B c. C d. D

Appendix B. MPCK Instrument of ratio and proportion

A machine uses 2.4 litres of fuel for every 30 hours of operation. How many litres of fuel will the machine use in 100 hours if it continues to use fuel at the same rate?

A. 7.2 B. 8.0 C. 8.4 D. 9.6

Code: MPCK1	Outcome: MPCK	Domain: Number	Sub-domain: Generate question	Max Points: 1

(a) Create a different problem of the same type as the problem in (a) (same processes/operations) that is EASIER for <primary> students to solve

Code: MPCK2	Outcome: MPCK	Sub-domain: Predicting typical students responses	Max point: 1

The following problem was given to 6th grade students"Some children are making two glasses of sweetened water. Henny uses 3 sugar cubes and 7 glasses of water.Ria uses 5 sugar cubes and 9 glasses of water.Which one is sweeter?

Some teachers have identified five different students' solutions and reasoning methods to come to the conclusion for that problem. According to you, List all the students' solution and reasoning method ! (Note: Based on you, you may write as many as possible)

1.

2.

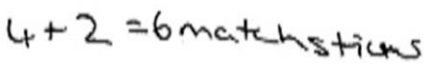
3.

4.

5.

Code: MPCK3	Outcome: MPCK	Sub-domain: -Enacting- Analyzing students strategy and misconception - Encourage students to solve math problem	Max point:1

Mr. Dodi (as teacher) gave the problem in a (problem of Mr. Short and Mr. Tall) aboveto students. One of his students, Ina answered and shared strategy as follow



Analyze the strategy that Ina show and Interpret all Ina's misconception based her solution?(Hint: It may more than 1)

Code: MPCK4	Outcome: MPCK	Sub-domain: -Enacting- Analyzing students strategy and misconception - Encourage students to solve math problem	Max point:1

Design a question for Ina so that she awares of her misconception!

Code: MPCK5	Outcome: MPCK	Sub-domain: Planning appropriate methods for representing mathematics ideas	Max point: 1

There is a problem in a textbook as follow: "Indah and Gana want to paint wall together. They want to use each exactly the same color. Indah use 3 cans of yellow paint and 6 cans of red paints. If Gana use 5 cans of yellow paint, how much red paint does Gana needs?" If there are three teaching strategies in solving the problem above described as below

Indah use 3 cans of yellow paint and 6 cans of red paint.

Therefore if she use 1 can of yellow paint, so there will be 2 cans of red paint.

If Gana want to make the same color and use 5 yellow paint, so that the red paint will be 5 cansx 2 = 10 cans of red paint.

(A)

Indah use 3 cans of yellow paint and 6 cans of red paint.

To find the number of red cans with 5 yellow cans of Gana

(x5) (x1/3)
3 yellow ---> 15 yellow --- > 5 yellow (Gana)

(x5) (x1/3)
6 red --- > 30 red -------- > Gana red cans will be 10

(B)

Indah use 3 cans of yellow paint and 6 cans of red paint. If Gana has 5 yellow paints...

Indah	3 yellow	6 red
Gana	5 yellow	x ? red

$\frac{3}{6} = \frac{5}{x}$

(Cross multiplication) $3x = 30$

$x = 30/3 = 10$

(C)

Among A, B and C teaching strategies, which strategy wil youl choose for students' ***better understanding and reasoning*** ?

a. (A) b. (B) c. (C)

Code: MPCK6	Outcome: MPCK	Sub-domain: - Planning appropriate method for representing mathematics ideas - Support students to be able to solve ratio and proportion	Max point: 2

In teaching ratio and proportion, Mr. Indra shared an example of problem to students to lead them to understand it as follow: 'A company can produces in average 21 bird cages in 3 hours. How many bird cages that can be produced in 5 hours?' He prefer to begin theteachingby using concept 'every one' (finding the one unit of 'hour').

Give two reasons she could have for preferring to do this rather than simply teaching the children use cross multiplication strategy?

Reason 1: Reason 2:

Code: MPCK7	Outcome: MPCK	Sub-domain: Provide appropriate feedback	Max point: 1

a. One of the students solve the problem and concentrate on the difference 12 and 5. The difference is 7 so the new width is 7 more than 3, the answer is 10. How do you give feedback on the this students' solution strategy?

Code: MPCK8	Outcome: MPCK	Sub-domain: Evaluate students mathematics solution	Max point: 1

Students are asked to solved the problem below as homework:

Luis mixed 5 ounces of orange juice concentrate with 7 ounces of water to make orange juice. Martin mixed 3 ounces of the same orange juice concentrate with 5 ounces of water. Who made the drink with the stronger orange flavor?

Student Amin, Brandon and Charlie shared different strategies to come to the result. Each student compares two ratios by stating different number arrangements

a. What do you think about the different ratio arrangement above? may check more than one box

A. Amin's ratio number arrangement is correct ☐
B. Brandon's ratio number arrangement is correct ☐
C. Charlie's ratio number arrangement is correct ☐

Code: MPCK9	Outcome: MCK	Sub-domain: Knowing the properties of ratio and proportion	Max point: 1

b. Analyze Amin's and Charlie's differences number arrangement, write your analysis below!

Code: MPCK10	Outcome: MPCK	Sub-domain: Planning	Max point: 1

A <grade 6> teacher asks her students to solve the following four story problems, in any way they like, including using materials if they wish

a. The teacher notices that two of the problem are more difficult for her students than the other two.

Identify the TWO problems which are likely to be more DIFFICULT to solve for <Grade 6> students

Problem _____ and Problem _____

Problem 1: To make chicken soup for 8 people need some ingredient such as 2 glass of water, 6 onion and 4 chicken cubes. How many chicken brotch cubes needed for 4 people soup?

Problem 2: To make chicken soup for 8 people need some ingredient such as 2 glass of water, 6 onion and 4 chicken cubes. How many chicken brotch cubes needed for 6 people soup?

Problem 3: What is the size of the missing vertical line enlargement below !

Problem 4: What is the size of the missing vertical line enlargement below !

Code: MPCK 11	Outcome: MPCK	Sub-domain: Planning	Max point: 1

b.Explain why those two tasks yo choose above are more difficult than others for your sixth grade students?